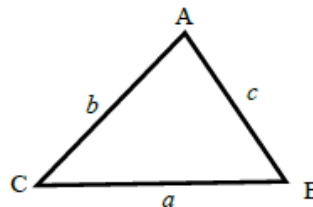# The cosine rule to find missing sides

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
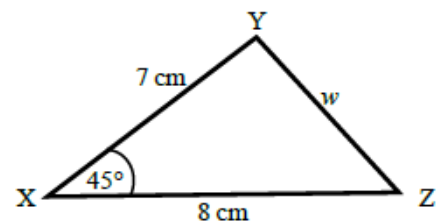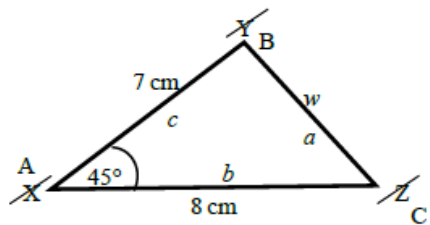
Key points

- a is the side opposite angle A.
  b is the side opposite angle B.
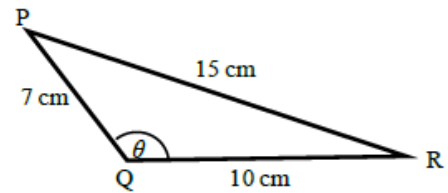 c is the side opposite angle C.

- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.

- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.

- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.
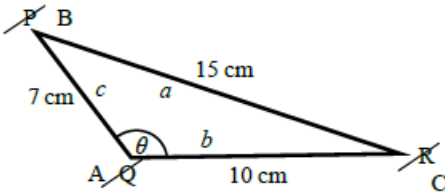
Example 1   Work out the length of side w.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc \cos A$

$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$

$w^2 = 33.804\ 040\ 51...$
$w = \sqrt{33.80404051}$
$w = 5.81$ cm

1   Always start by labelling the angles and sides.

2   Write the cosine rule to find the side.

3   Substitute the values a, b and A into the formula.
4   Use a calculator to find w^2 and then w.
5   Round your final answer to 3 significant figures and write the units in your answer.

Example 2   Work out the size of angle θ.
Give your answer correct to 1 decimal place.
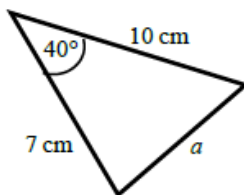

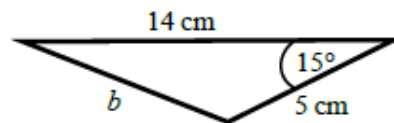|	
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \dfrac{-76}{140}$ $\theta = 122.878\,349...$ $\theta = 122.9°$	**1** Always start by labelling the angles and sides. **2** Write the cosine rule to find the angle. **3** Substitute the values a, b and c into the formula. **4** Use $\cos^{-1}$ to find the angle. **5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$. **6** Round your answer to 1 decimal place and write the units in your answer.

Practice questions

1   Work out the length of the unknown side in each triangle.
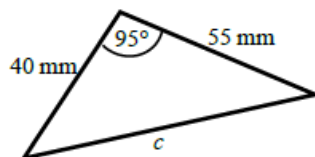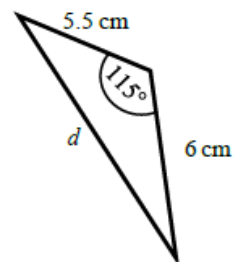Give your answers correct to 3 significant figures.

a

b

c

d

Answers

6   **a**   6.46 cm   **b**   9.26 cm   **c**   70.8 mm   **d**   9.70 cm