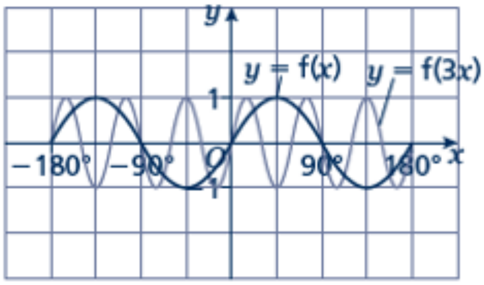
Transforming trigonometric graphs

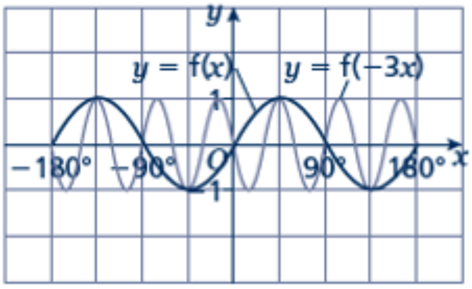
A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

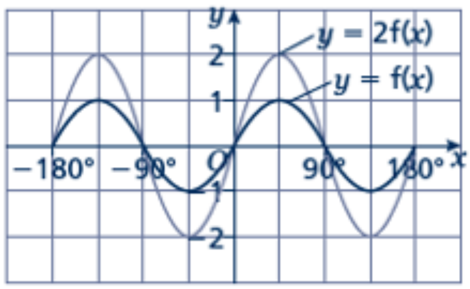
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis.



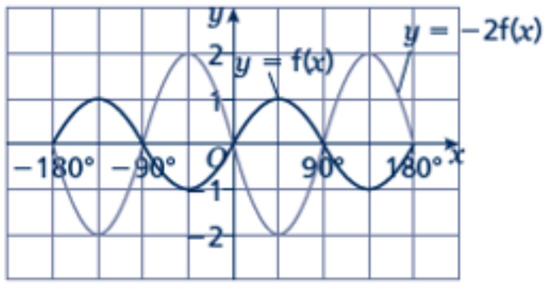
- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis and then a reflection in the y-axis.



- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis.



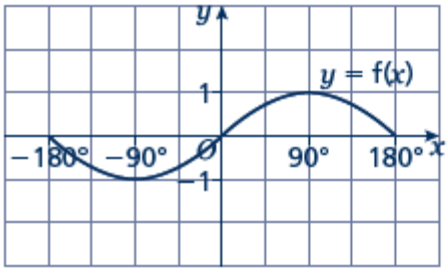
- The transformation $y = -af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis and then a reflection in the x-axis.

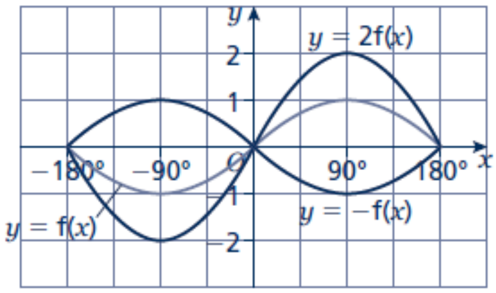


Examples

Example 1 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = 2f(x)$ and $y = -f(x)$.



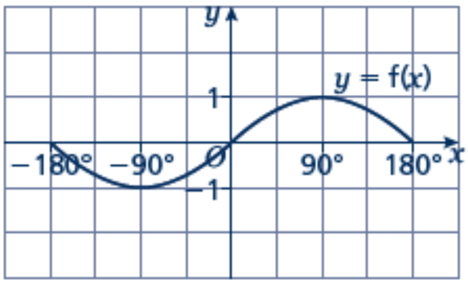


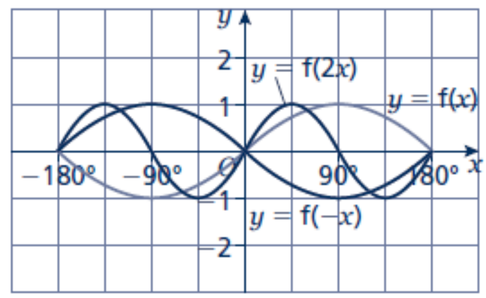
The function $y = 2f(x)$ is a vertical stretch of $y = f(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -f(x)$ is a reflection of $y = f(x)$ in the x-axis.

Example 2 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = f(2x)$ and $y = f(-x)$.





The function $y = f(2x)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.

The function $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.

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Practice questions

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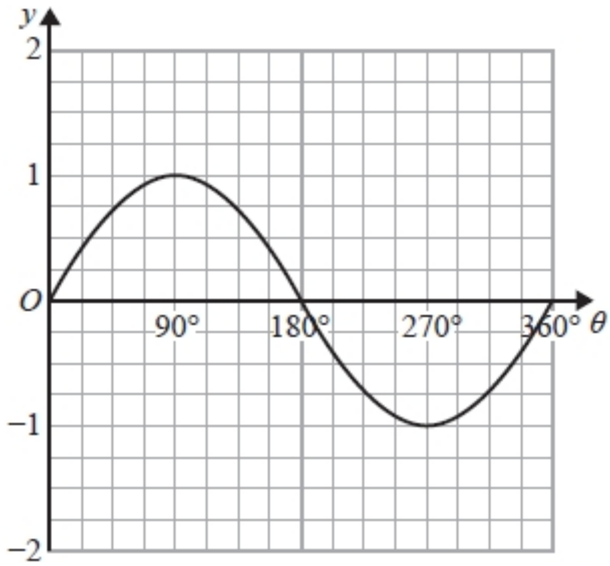


Figure 3

Figure 3 shows a plot of the curve with equation $y = \sin\theta$, $0 \leq \theta \leq 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leq \theta \leq 360°$$

A copy of Figure 3, called Diagram 1, is shown here.

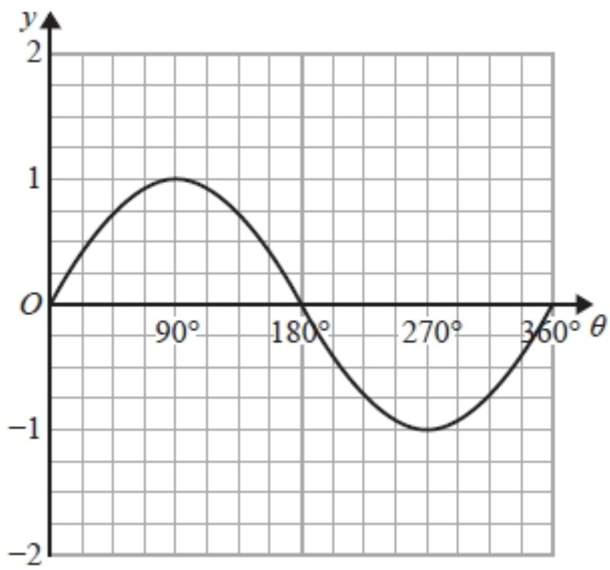


Diagram 1

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta$, $0 \leq \theta \leq 360°$

(ii) $y = \tan\theta, \quad 0 \le \theta \le 360°$

(c) Hence find the number of solutions of the equation

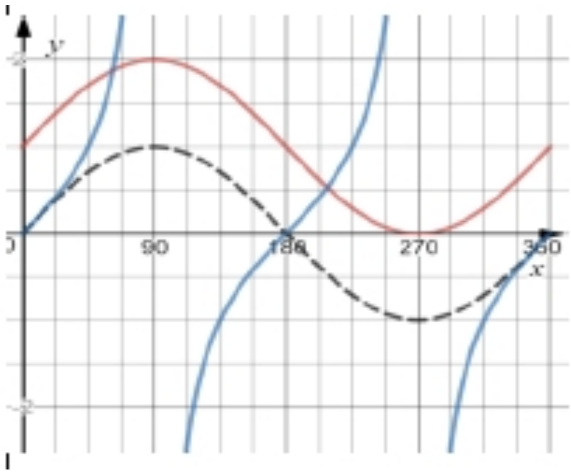
(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \le \theta \le 2160°$

(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \le \theta \le 1980°$

Answers

1 (a) $(270°, -4)$

(b)



(c) (i) $6 \times 2 = 12$

(ii) 11