USING CAS TECHNOLOGY IN A COURSE DESIGNED FOR PRESERVICE TEACHERS

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Abstract: The Common Core articulates the role of reasoning and proof throughout the grade spectrum. This paper serves to illustrate how the TI-89 CAS enhances the conceptual understanding of preservice teachers when accompanied by rich problems to explore.

The focus of this course and paper will be to explore and discover key concepts in elementary number theory with the interface of CAS technology. The target population consists of preservice secondary teachers. The structure of such a course is to be hands-on with much participation and interaction between students and the faculty member. The solution of rich problems via the exploration of patterns and the formation of conjectures based upon the analysis of such patterns will be emphasized. For example, the CAS calculator can explore engaging problems involving number and operations including primes, factors, multiples and recursive sequences. Number theory is a rich source for many problems that are easily posed, but in many cases remain open and the resolutions have escaped even the greatest minds in mathematical lore including Euler. In addition, CAS technology opens an avenue for exploration that is not feasible by mere paper and pencil. Time consuming algebra can be accomplished by this technology (after the student has mastered the important algebra basics) freeing both teacher and student to explore activities more deeply while furnishing proofs of theorems involving algebra. Since many students who are aspiring to become future teachers particularly at the secondary school level require a history of mathematics course as well as an elementary number theory course, CAS can place both concepts and history into perspective and serve as a neat ancillary to traditional teaching and foster creative and independent thinking. The inclusion of simple programs designed for the calculator and MATHEMATICA as well as referencing the World Wide Web to sites such as MathWorld – A Wolfram Resource, enables participants to obtain a deeper understanding of problems that have fascinated mathematicians throughout the ages and might serve as the impetus to achieve new results. In addition, one can display neat problems involving algebra, geometry and patterns that explore connections in the spirit of engagement and discovery. Such problems include connections between Pythagoras and Fibonacci as well as results intersecting the branches of number theory, geometry, algebra and technology.

Our first activity involves Goldbach's Conjecture which asserts that every even integer >2 is seemingly expressible as the sum of two primes while every even integer greater than 4 is the sum of two odd primes. Goldbach in a letter to the preeminent mathematician of his day Leonhard Euler in 1742 presented this conjecture, but neither he nor Euler were unable to either prove it or arrive at a first counterexample. While a large number of mathematicians are in strong agreement that this conjecture is indeed true, no one has been able to resolve the problem although partial solutions are known. During the history of this problem, one mathematician was able to prove that every even integer >2 can be written as the sum of no more than 300,000 primes. On the other hand, 2 is a far cry from 300,000. Another version of Goldbach's conjecture asserts that every odd integer >5 can be expressed as the sum of three primes while every odd integer >7 can be expressed as the sum of three odd primes. Moreover, every integer >17 can be written as the sum of three distinct odd primes. In this activity, we use the TI-89 to express the even integer 120 as the sum of two primes in all possible ways. Using the TI-89, we define four functions (the third and fourth functions are identical) in the Y= EDITOR as follows in **FIGURE 1** where the keystrokes in FUNCTION MODE are GREEN DIAMOND W (Y=). In **FIGURE 2,** set up a table using the keystrokes GREEN DIAMOND T (TBLSET). We start the table with the initial odd prime 3 and proceed in increments of 2. One sees the resulting table in **FIGURES 3-6** where the keystrokes GREEN DIAMOND Y (TABLE) will access a Table. We seek the outputs that are true in the columns headed by $y1\, and\ y3(y4)$ in a given row. These will generate the proper representation of the even integer as the sum of two odd primes.

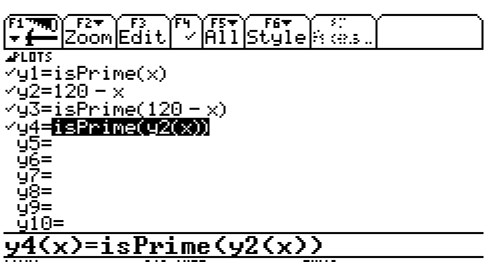


FIGURE 1: The Function Inputs

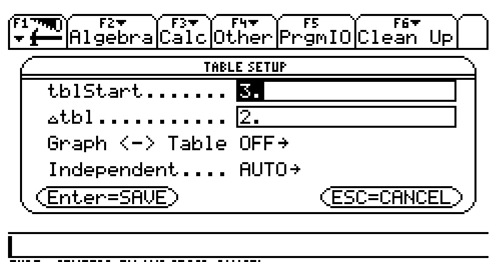


FIGURE 2: The Table Setup

x	y1	y2	y3	y4
3.	true	117.	false	false
5.	true	115.	false	false
7.	true	113.	true	true
9.	false	111.	false	false
11.	true	109.	true	true
13.	true	107.	true	true
15.	false	105.	false	false
17.	true	103.	true	true

x=3.

FIGURE 3: The Table

x	y1	y2	y3	y4
19.	true	101.	true	true
21.	false	99.	false	false
23.	true	97.	true	true
25.	false	95.	false	false
27.	false	93.	false	false
29.	true	91.	false	false
31.	true	89.	true	true
33.	false	87.	false	false

x=19.

FIGURE 4: The Table

x	y1	y2	y3	y4
35.	false	85.	false	false
37.	true	83.	true	true
39.	false	81.	false	false
41.	true	79.	true	true
43.	true	77.	false	false
45.	false	75.	false	false
47.	true	73.	true	true
49.	false	71.	true	true

x=35.

FIGURE 5: The Table

x	y1	y2	y3	y4
51.	false	69.	false	false
53.	true	67.	true	true
55.	false	65.	false	false
57.	false	63.	false	false
59.	true	61.	true	true
61.	true	59.	true	true
63.	false	57.	false	false
65.	false	55.	false	false

x=51.

FIGURE 6: The Table

Since $\frac{120}{2}=60,$ we do not need to consider any $x>60.$ Otherwise one will obtain duplicates of previous outcomes only in the reverse order. Thus 120 is represented as the sum of two odd primes in the following dozen ways:

$$120=7+113=11+109=13+107=17+103=19+101=23+97=$$
$$31+89=37+83=41+79=47+73=53+67=59+61.$$

When holding the 2nd and down arrow cursors simultaneously, we can jump down to the next eight rows in the Table. Moreover, one can check that 120 is the first positive even integer that can be expressed as the sum of two primes in twelve different ways.

Our next activity focuses on the idea of twin primes which are a pair of odd primes that differ by two (such as 3 and 5). Whether there are finitely many or infinitely many pairs of twin primes remains unresolved to this day. This is in contrast to the set of all primes which is well known to comprise an infinite set known since the time of Euclid. While the mathematical community appears to be overwhelmingly convinced that the number of twin prime pairs is indeed infinite, a proof still awaits. We determine all twin prime pairs in the range from 1-300.

Since twin primes are odd primes that differ by two, our functions are depicted in **FIGURE 7,** the TABLE SETUP in **FIGURE 8** and the TABLE in **FIGURES 9-27.** We seek the outputs that are true in the columns headed by $y1\, and\ y3(y4)$ in a given row. These will generate the associated twin prime pairs.

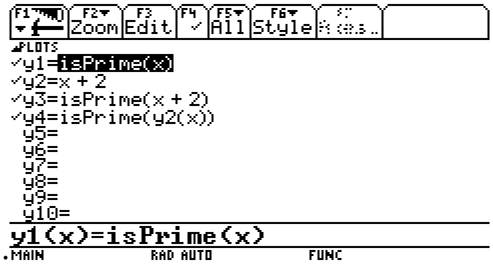


FIGURE 7: The Function Inputs

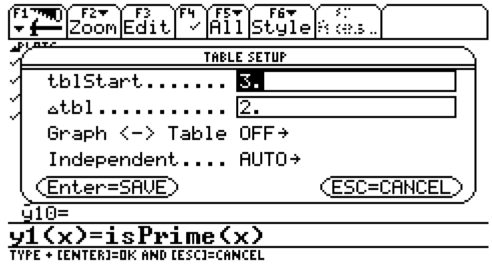
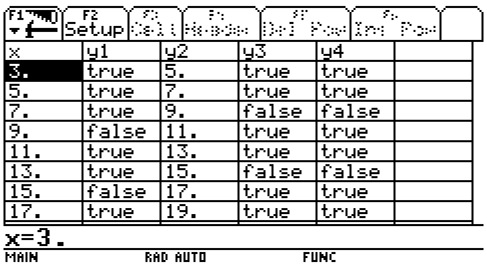


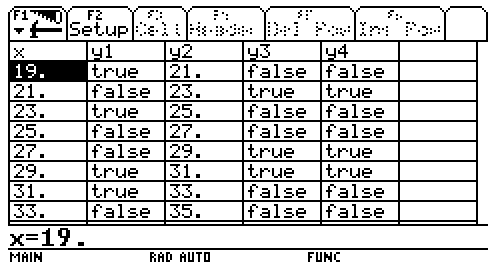
FIGURE 8: The Table Setup



x	y1	y2	y3	y4
3.	true	5.	true	true
5.	true	7.	true	true
7.	true	9.	false	false
9.	false	11.	true	true
11.	true	13.	true	true
13.	true	15.	false	false
15.	false	17.	true	true
17.	true	19.	true	true

x=3.
MAIN RAD AUTO FUNC

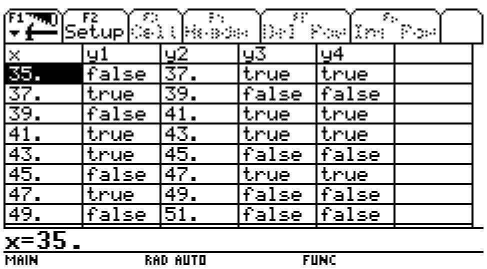
FIGURE 9: The Table



x	y1	y2	y3	y4
19.	true	21.	false	false
21.	false	23.	true	true
23.	true	25.	false	false
25.	false	27.	false	false
27.	false	29.	true	true
29.	true	31.	true	true
31.	true	33.	false	false
33.	false	35.	false	false

x=19.
MAIN RAD AUTO FUNC

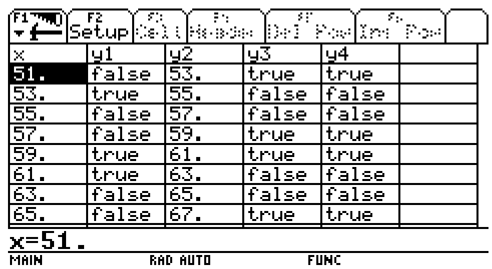
FIGURE 10: The Table



x	y1	y2	y3	y4
35.	false	37.	true	true
37.	true	39.	false	false
39.	false	41.	true	true
41.	true	43.	true	true
43.	true	45.	false	false
45.	false	47.	true	true
47.	true	49.	false	false
49.	false	51.	false	false

x=35.
MAIN RAD AUTO FUNC

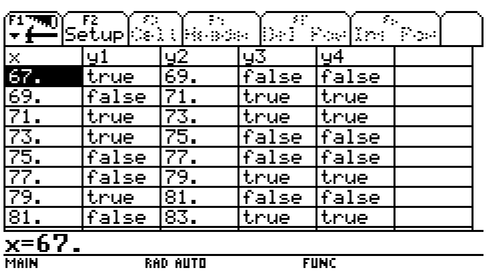
FIGURE 11: The Table



x	y1	y2	y3	y4
51.	false	53.	true	true
53.	true	55.	false	false
55.	false	57.	false	false
57.	false	59.	true	true
59.	true	61.	true	true
61.	true	63.	false	false
63.	false	65.	false	false
65.	false	67.	true	true

x=51.
MAIN RAD AUTO FUNC

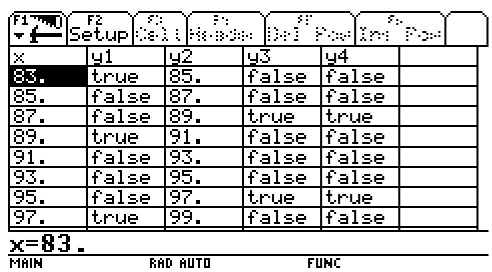
FIGURE 12: The Table



x	y1	y2	y3	y4
67.	true	69.	false	false
69.	false	71.	true	true
71.	true	73.	true	true
73.	true	75.	false	false
75.	false	77.	false	false
77.	false	79.	true	true
79.	true	81.	false	false
81.	false	83.	true	true

x=67.
MAIN RAD AUTO FUNC

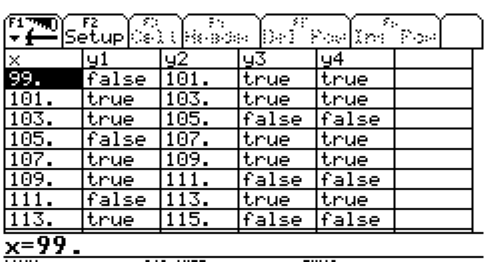
FIGURE 13: The Table



x	y1	y2	y3	y4
83.	true	85.	false	false
85.	false	87.	false	false
87.	false	89.	true	true
89.	true	91.	false	false
91.	false	93.	false	false
93.	false	95.	false	false
95.	false	97.	true	true
97.	true	99.	false	false

x=83.
MAIN RAD AUTO FUNC

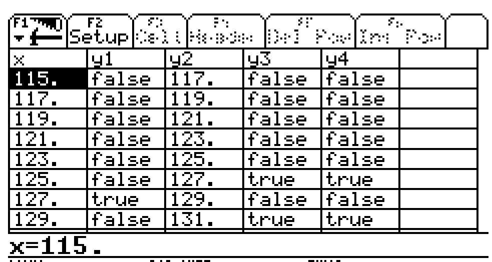
FIGURE 14: The Table



x	y1	y2	y3	y4
99.	false	101.	true	true
101.	true	103.	true	true
103.	true	105.	false	false
105.	false	107.	true	true
107.	true	109.	true	true
109.	true	111.	false	false
111.	false	113.	true	true
113.	true	115.	false	false

x=99.
MAIN RAD AUTO FUNC

FIGURE 15: The Table



x	y1	y2	y3	y4
115.	false	117.	false	false
117.	false	119.	false	false
119.	false	121.	false	false
121.	false	123.	false	false
123.	false	125.	false	false
125.	false	127.	true	true
127.	true	129.	false	false
129.	false	131.	true	true

x=115.
MAIN RAD AUTO FUNC

FIGURE 16: The Table

x	y1	y2	y3	y4
131.	true	133.	false	false
133.	false	135.	false	false
135.	false	137.	true	true
137.	true	139.	true	true
139.	true	141.	false	false
141.	false	143.	false	false
143.	false	145.	false	false
145.	false	147.	false	false

x=131.

MAIN RAD AUTO FUNC

FIGURE 17: The Table

x	y1	y2	y3	y4
147.	false	149.	true	true
149.	true	151.	true	true
151.	true	153.	false	false
153.	false	155.	false	false
155.	false	157.	true	true
157.	true	159.	false	false
159.	false	161.	false	false
161.	false	163.	true	true

x=147.

MAIN RAD AUTO FUNC

FIGURE 18: The Table

x	y1	y2	y3	y4
163.	true	165.	false	false
165.	false	167.	true	true
167.	true	169.	false	false
169.	false	171.	false	false
171.	false	173.	true	true
173.	true	175.	false	false
175.	false	177.	false	false
177.	false	179.	true	true

x=163.

MAIN RAD AUTO FUNC

FIGURE 19: The Table

x	y1	y2	y3	y4
179.	true	181.	true	true
181.	true	183.	false	false
183.	false	185.	false	false
185.	false	187.	false	false
187.	false	189.	false	false
189.	false	191.	true	true
191.	true	193.	true	true
193.	true	195.	false	false

x=179.

MAIN RAD AUTO FUNC

FIGURE 20: The Table

x	y1	y2	y3	y4
179.	true	181.	true	true
181.	true	183.	false	false
183.	false	185.	false	false
185.	false	187.	false	false
187.	false	189.	false	false
189.	false	191.	true	true
191.	true	193.	true	true
193.	true	195.	false	false

x=179.

MAIN RAD AUTO FUNC

FIGURE 21: The Table

x	y1	y2	y3	y4
211.	true	213.	false	false
213.	false	215.	false	false
215.	false	217.	false	false
217.	false	219.	false	false
219.	false	221.	false	false
221.	false	223.	true	true
223.	true	225.	false	false
225.	false	227.	true	true

x=211.

MAIN RAD AUTO FUNC

FIGURE 22: The Table

x	y1	y2	y3	y4
227.	true	229.	true	true
229.	true	231.	false	false
231.	false	233.	true	true
233.	true	235.	false	false
235.	false	237.	false	false
237.	false	239.	true	true
239.	true	241.	true	true
241.	true	243.	false	false

x=227.

MAIN RAD AUTO FUNC

FIGURE 23: The Table

x	y1	y2	y3	y4
243.	false	245.	false	false
245.	false	247.	false	false
247.	false	249.	false	false
249.	false	251.	true	true
251.	true	253.	false	false
253.	false	255.	false	false
255.	false	257.	true	true
257.	true	259.	false	false

x=243.

MAIN RAD AUTO FUNC

FIGURE 24: The Table

x	y1	y2	y3	y4
259.	false	261.	false	false
261.	false	263.	true	true
263.	true	265.	false	false
265.	false	267.	false	false
267.	false	269.	true	true
269.	true	271.	true	true
271.	true	273.	false	false
273.	false	275.	false	false

x=259.

MAIN RAD AUTO FUNC

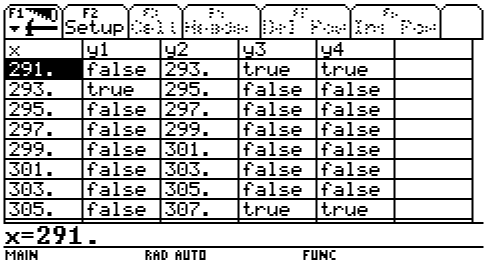
FIGURE 25: The Table

x	y1	y2	y3	y4
275.	false	277.	true	true
277.	true	279.	false	false
279.	false	281.	true	true
281.	true	283.	true	true
283.	true	285.	false	false
285.	false	287.	false	false
287.	false	289.	false	false
289.	false	291.	false	false

x=275.

MAIN RAD AUTO FUNC

FIGURE 26: The Table



x	y1	y2	y3	y4
291.	false	293.	true	true
293.	true	295.	false	false
295.	false	297.	false	false
297.	false	299.	false	false
299.	false	301.	false	false
301.	false	303.	false	false
303.	false	305.	false	false
305.	false	307.	true	true

x=291.

FIGURE 27: The Table

For the twin prime pairs ≤ 300:

$(3,5),(5,7),(11,13),(17,19),(29,31),(41,43),(59,61),(71,73),$

$(101,103),(107,109),(137,139),(149,151),(179,181),(191,193),(197,199),$

$(227,229),(239,241),(269,271),(281,283)$

One hence have a total of nineteen twin prime pairs.

Our third activity focuses on prime decades which are also known as prime quadruples. A prime decade is a set of ten integers from $n0-n9$ (where n is a positive integer of any length) such that $n1, n3, n7$ *and* $n9$ are all primes. (Of course all primes with the exception of 2 and 5 terminate in one of the digits 1, 3, 7, and 9). Thus in the sequence of ten integers $n0,\ n1,\ n2,\ n3,\ n4,\ n5,\ n6,\ n7,\ n8\ n7$ *and* $n9$, one has a pair of twin primes twice. For example, 11, 13, 17, and 19 constitutes a prime decade while the next one is 101, 103, 107, and 109. The question that remains unresolved is whether there are infinitely many prime decades. Note that if there indeed were, then the twin prime problem would be solved in the affirmative in the sense that there would be infinitely many twin prime pairs.

We secure all prime decades <1200. To find the next prime decades after the prime decade $\{11,13,17,19\}$, we employ the TI-89 in FUNCTION MODE with the following seven functions in **FIGURE 28** with the TABLE SETUP in **FIGURE 29** (it is shown below that the minimum distance between any two prime decades is 30) and TABLE in **FIGURES 30-39: (**In order to see the full table, use the right arrow cursor to view columns headed by $y6$ *and* $y7$.**)**

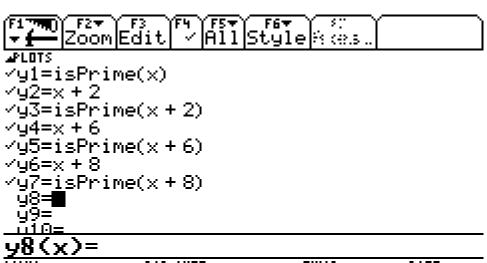


FIGURE 28: The Function Inputs

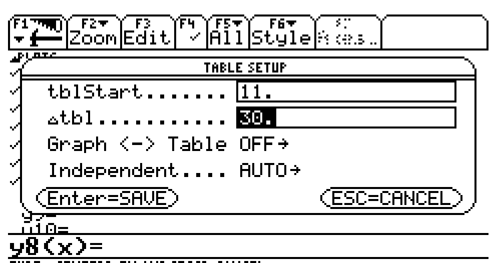


FIGURE 29: The Table Setup

x	y1	y2	y3	y4	y5
11.	true	13.	true	17.	true
41.	true	43.	true	47.	true
71.	true	73.	true	77.	false
101.	true	103.	true	107.	true
131.	true	133.	false	137.	true
161.	false	163.	true	167.	true
191.	true	193.	true	197.	true
221.	false	223.	true	227.	true

x=11.

FIGURE 30: The Table

x	y3	y4	y5	y6	y7
11.	true	17.	true	19.	true
41.	true	47.	true	49.	false
71.	true	77.	false	79.	true
101.	true	107.	true	109.	true
131.	false	137.	true	139.	true
161.	true	167.	true	169.	false
191.	true	197.	true	199.	true
221.	true	227.	true	229.	true

y7(x)=true

FIGURE 31: The Table

x	y1	y2	y3	y4	y5
251.	true	253.	false	257.	true
281.	true	283.	true	287.	false
311.	true	313.	true	317.	true
341.	false	343.	false	347.	true
371.	false	373.	true	377.	false
401.	true	403.	false	407.	false
431.	true	433.	true	437.	false
461.	true	463.	true	467.	true

x=251.

FIGURE 32: The Table

x	y1	y2	y3	y4	y5
251.	true	253.	false	257.	true
281.	true	283.	true	287.	false
311.	true	313.	true	317.	true
341.	false	343.	false	347.	true
371.	false	373.	true	377.	false
401.	true	403.	false	407.	false
431.	true	433.	true	437.	false
461.	true	463.	true	467.	true

y1(x)=true

FIGURE 33: The Table

x	y1	y2	y3	y4	y5
491.	true	493.	false	497.	false
521.	true	523.	true	527.	false
551.	false	553.	false	557.	true
581.	false	583.	false	587.	true
611.	false	613.	true	617.	true
641.	true	643.	true	647.	true
671.	false	673.	true	677.	true
701.	true	703.	false	707.	false

x=491.

FIGURE 34: The Table

x	y3	y4	y5	y6	y7
491.	false	497.	false	499.	true
521.	true	527.	false	529.	false
551.	false	557.	true	559.	false
581.	false	587.	true	589.	false
611.	true	617.	true	619.	true
641.	true	647.	true	649.	false
671.	true	677.	true	679.	false
701.	false	707.	false	709.	true

y7(x)=true

FIGURE 35: The Table

x	y1	y2	y3	y4	y5
731.	false	733.	true	737.	false
761.	true	763.	false	767.	false
791.	false	793.	false	797.	true
821.	true	823.	true	827.	true
851.	false	853.	true	857.	true
881.	true	883.	true	887.	true
911.	true	913.	false	917.	false
941.	true	943.	false	947.	true

x=731.

FIGURE 36: The Table

x	y3	y4	y5	y6	y7
731.	true	737.	false	739.	true
761.	false	767.	false	769.	true
791.	false	797.	true	799.	false
821.	true	827.	true	829.	true
851.	true	857.	true	859.	true
881.	true	887.	true	889.	false
911.	false	917.	false	919.	true
941.	false	947.	true	949.	false

y7(x)=true

FIGURE 37: The Table

x	y1	y2	y3	y4	y5
971.	true	973.	false	977.	true
1001.	false	1003.	false	1007.	false
1031.	true	1033.	true	1037.	false
1061.	true	1063.	true	1067.	false
1091.	true	1093.	true	1097.	true
1121.	false	1123.	true	1127.	false
1151.	true	1153.	true	1157.	false
1181.	true	1183.	false	1187.	true

x=971.

FIGURE 38: The Table

x	y3	y4	y5	y6	y7
971.	false	977.	true	979.	false
1001.	false	1007.	false	1009.	true
1031.	true	1037.	false	1039.	true
1061.	true	1067.	false	1069.	true
1091.	true	1097.	true	1099.	false
1121.	true	1127.	false	1129.	true
1151.	true	1157.	false	1159.	false
1181.	false	1187.	true	1189.	false

y7(x)=false

FIGURE 39: The Table

Hence one has four prime decades <1200:
$\{11,13,17,19\}$, $\{101,103,107,109\}$, $\{191,193,197,199\}$ *and* $\{821.823,827,829\}$.

ANALYSIS OF THE TABLE SETUP IN FIGURE 29:

If $\{p, p+2, p+6, p+8\}$ constitutes a prime decade, then clearly each of the integers in the set below is divisible by 3 and hence is not prime:
$\{p+1, p+4, p+7, p+10, p+13, p+16, p+19, p+22, p+25, p+28\}$ (In any set of three consecutive integers, one of them is always divisible by 3.) Hence if the set $\{p, p+2, p+6, p+8\}$ comprises a prime decade, then each of the following sets cannot constitute a prime decade:

1. $\{p+10, p+12, p+16, p+18\}$
2. $\{p+20, p+22, p+26, p+28\}$

Hence there must be a minimal distance of 30 between prime decades. Using the divisibility of an integer by 7 will show that no pair of prime decades can have a distance of 60 between them. The details are left to the interested reader. Examining **FIGURES 30-39,** the four prime decades <1200 are $\{11,13,17,19\}$, $\{101,103,107,109\}$, $\{191,193,197,199\}$ *and* $\{821,823,827,829\}$.

Our fourth activity involves the Collatz problem. In 1937, the German mathematician Lothar Collatz, (1910-1990) considered the following problem accessible to most fifth grade arithmetic students: Consider any positive integer. If it is even, divide by two. If it is odd, triple and add one. Repeat the above process on each new number obtained. Eventually after finitely many steps, this iterative sequence will converge to one. While this problem has been successfully tested on all integers $\leq 19\cdot 2^{58} \approx 5.48\cdot 10^{18}$, it has not been proven in general. A variation on this conjecture generally requiring fewer iterations is to take any odd integer, triple its value, add one and divide the resulting sum by two. The problem takes on various names including Ulam's Conjecture and The Syracuse Problem as indicators of the people and places that have studied the problem in depth. More computer time has been allocated to this problem than any other in the annals of mathematics. In our activity, we determine the number of iterations needed to reach 1 for the following positive integers 18, 128, and 25 respectively. See **FIGURE 40** for the Collatz Program**, FIGURES 41-43** for 18, **FIGURE 44** for 128 and **FIGURES 45-48** for 25.

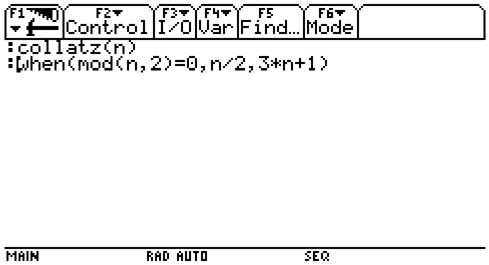


FIGURE 40: The Collatz Program

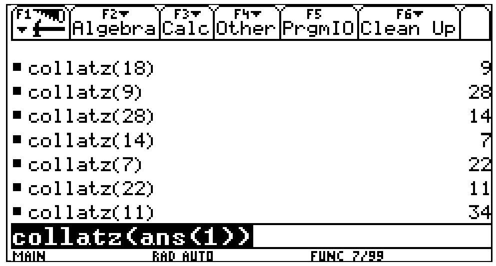


FIGURE 41: The Table

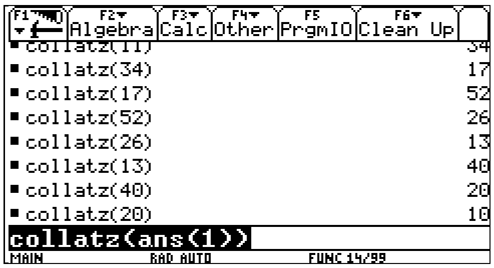


FIGURE 42: The Table

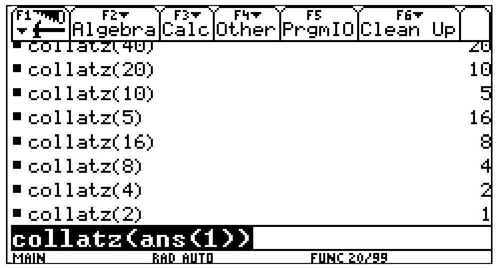


FIGURE 43: The Table

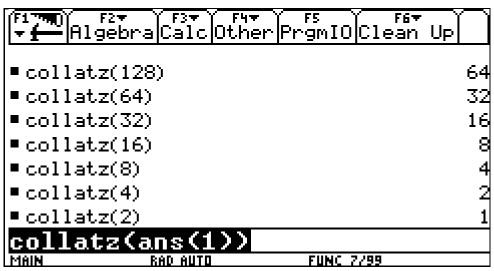


FIGURE 44: The Table

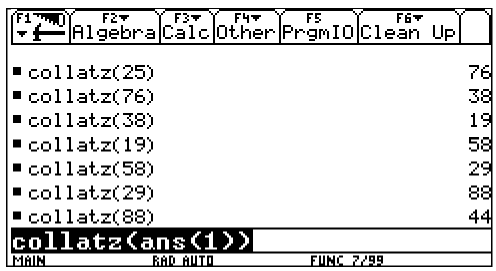


FIGURE 45: The Table

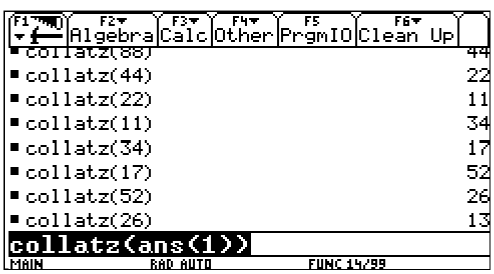


FIGURE 46: The Table

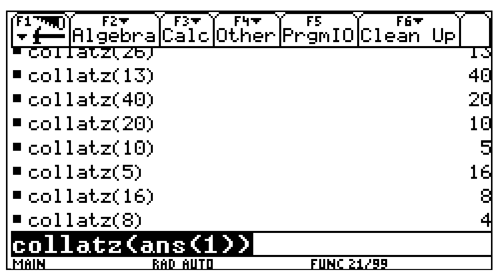


FIGURE 47: The Table

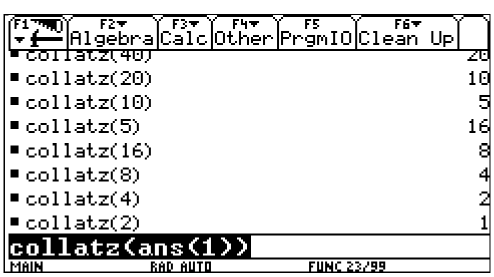


FIGURE 48: The Table

Observe that if we do not count the seed value, it takes, in turn, 19, 7, and 23 steps respectively, for the integers 18, 128, and 25 to reach 1. Note that if one has the time and patience, they could try the Collatz problem with the integer 27. 27 is the first integer which requires more than one hundred steps to reach 1 (111 not counting the seed value).

Alternatively, with another example, let us employ the TT-89 to determine the number of iterations required for the sequence $\{65, 66, 67\}$ to reach 1. Our inputs and outputs are provided in **FIGURES 49-57** where we are in SEQUENCE MODE:

FIGURE 49: SEQUENCE MODE

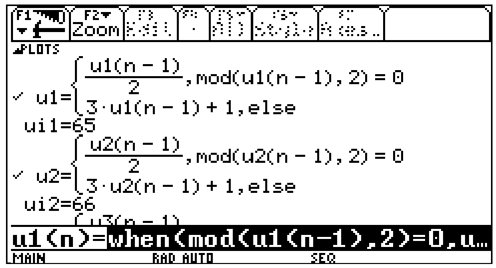


FIGURE 50: Sequence Inputs

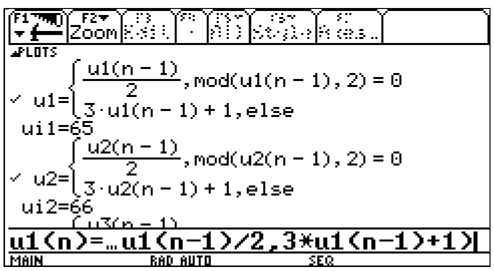


FIGURE 51: Sequence Inputs

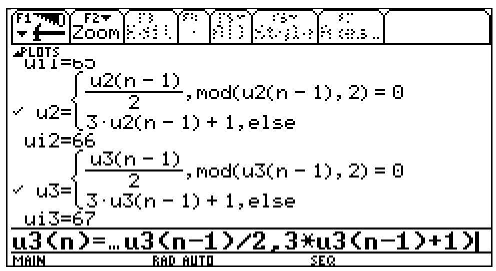


FIGURE 52: Sequence Inputs

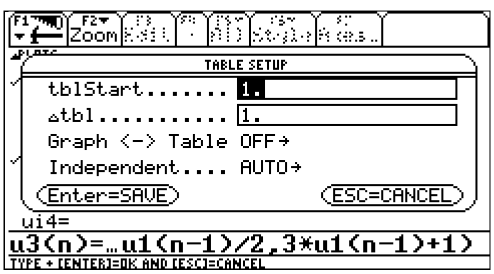
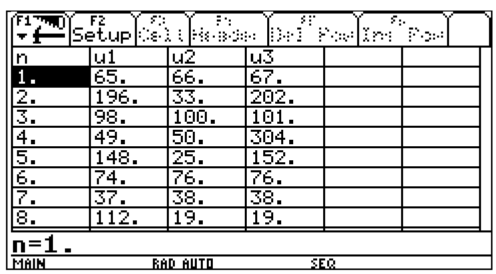


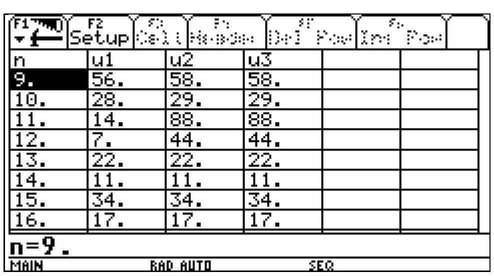
FIGURE 53: The Table Setup



n	u1	u2	u3
1.	65.	66.	67.
2.	196.	33.	202.
3.	98.	100.	101.
4.	49.	50.	304.
5.	148.	25.	152.
6.	74.	76.	76.
7.	37.	38.	38.
8.	112.	19.	19.

n=1.

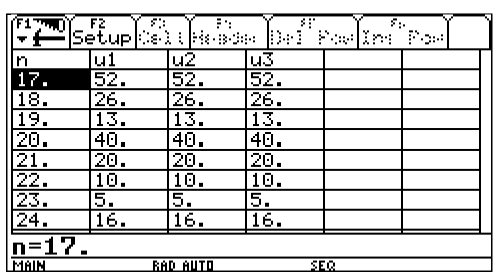
FIGURE 54: The Table



n	u1	u2	u3
9.	56.	58.	58.
10.	28.	29.	29.
11.	14.	88.	88.
12.	7.	44.	44.
13.	22.	22.	22.
14.	11.	11.	11.
15.	34.	34.	34.
16.	17.	17.	17.

n=9.

FIGURE 55: The Table



n	u1	u2	u3
17.	52.	52.	52.
18.	26.	26.	26.
19.	13.	13.	13.
20.	40.	40.	40.
21.	20.	20.	20.
22.	10.	10.	10.
23.	5.	5.	5.
24.	16.	16.	16.

n=17.

FIGURE 56: The Table

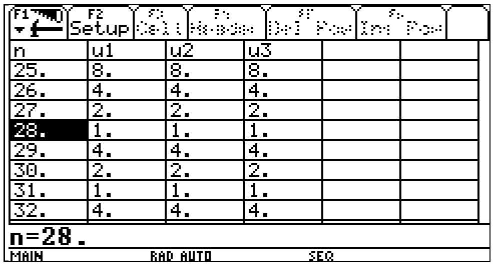


FIGURE 57: The Table

It thus requires 27 steps (not including the initial integers) for the Collatz 3-tuple to reach 1. We are asserting here that 65, 66, and 67 are three consecutive integers for which the Collatz sequence has the same length.

Our fifth activity searches for Home Primes. The Home Prime Conjecture was introduced to me by Dr. Neil A.J. Sloane at an MAA lecture in Philadelphia, PA in 2002 and refers to the concatenation of composite integers subsequent to factoring them. The process is quite simple. Take a composite integer, factor it and concatenate the factors into a new integer. Repeat the process. Eventually after finitely many iterations, you should obtain a prime, called the Home Prime of the original integer. This conjecture is stalled after 100 steps for the integer 49. It is unknown whether every integer has a Home Prime. We employ the Home Prime Conjecture for the following composite integers: (a). 8, (b). 25, (c). 75, (d). 78

(a). To secure the Home Prime of 8, we resort to **FIGURES 58-60:**

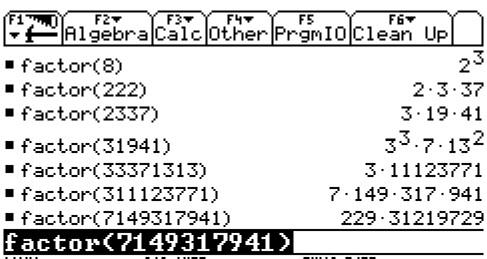


FIGURE 58: Home Prime Iterations

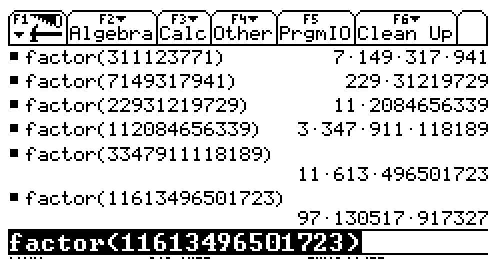


FIGURE 59: Home Prime Iterations

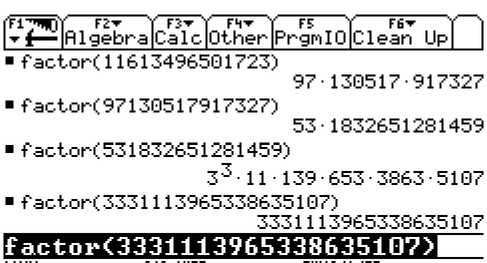


FIGURE 60: Home Prime Iterations

Hence 3331113965338635107 is the Home Prime of 8 secured in 13 steps.

(b). To obtain the Home Prime of 25, proceed as in **FIGURE 61:**

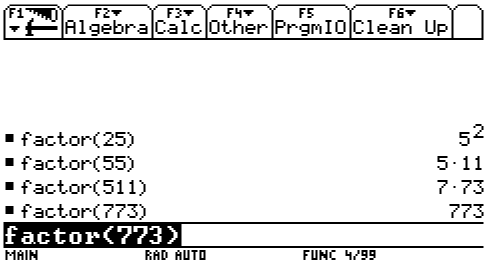


FIGURE 61: Home Prime Iterations

Thus 773 is the Home Prime of 25 obtained in 3 steps.

(c). To obtain the Home Prime of 75, we appeal to **FIGURE 62:**

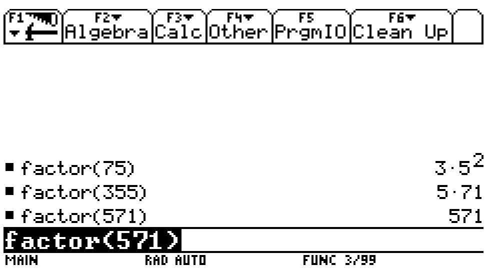


FIGURE 62: Home Prime Iterations

Hence 571 is the Home Prime of 75 achieved in 2 steps.

(d). To obtain the Home Prime of 78, we proceed as in **FIGURES 63-64:**

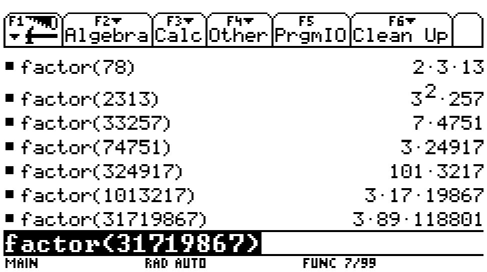


FIGURE 63: Home Prime Iterations

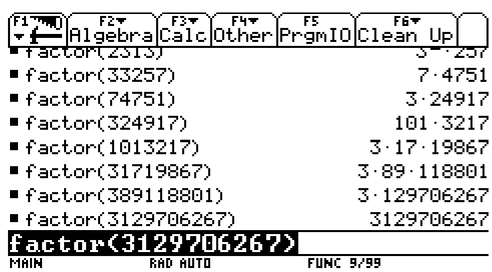


FIGURE 64: Home Prime Iterations

Thus 3129706267 is the Home Prime of 78 obtained in 8 steps.

80 is the integer <100 that requires 31 steps to reach its Home Prime 313169138727147145210044974146858220729781791489. **FIGURES 65-68** give a start to the sequence of iterations, but after more than thirty minutes, the calculator is stalled while attempting to factor the composite integer 49742531769473451811578 67 in the 19th step. Hence MATHEMATICA is employed to complete the task commencing with the seventeenth iteration.

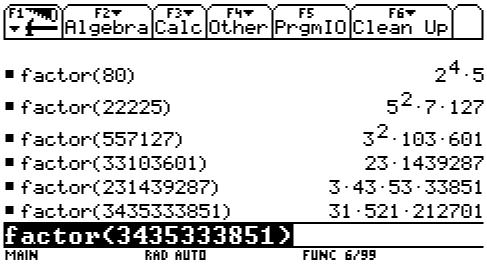


FIGURE 65: Home Prime Iterations

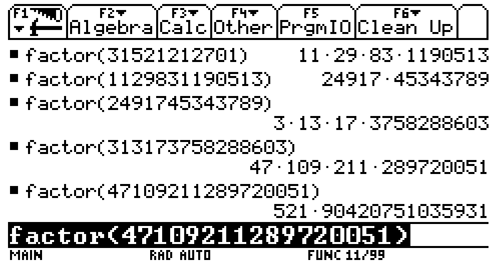


FIGURE 66: Home Prime Iterations

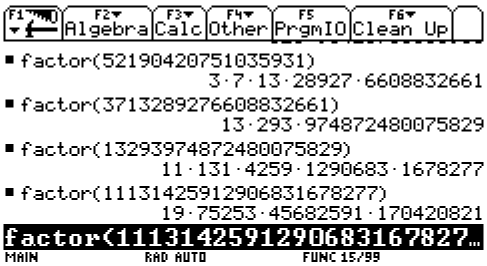


FIGURE 67: Home Prime Iterations

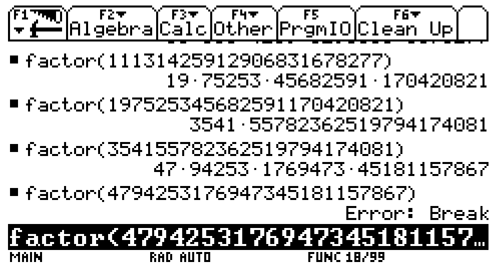


FIGURE 68: Home Prime Iterations

The following table completes the repeated concatenation and factoring required to secure the Home Prime of 80 using MATHEMATICA:

Step:	Integer and its Prime Factorization:
17	$4794253176947345181157867 = 13 \cdot 35801984243 \cdot 10300789571213$
18	$133580198424310300789571213 = 24144697 \cdot 1012307071 \cdot 5465225099$
19	$2414469710123070715465225099 = 7 \cdot 344924244303295816495032157$
20	$7344924244303295816495032157 = 25084266359 \cdot 292810008440530123$
21	$25084266359292810008440530123 = 3 \cdot 103 \cdot 187547 \cdot 449917889 \cdot 962054203309$
22	$3103187547449917889962054203309 = 3 \cdot 17 \cdot 1031 \cdot 59017279006673853482475689$
23	$317103159017279006673853482475689 =$ $3 \cdot 4091 \cdot 84942079 \cdot 1022090777 \cdot 297603119071$
24	$34091849420791022090777297603119071 =$ $3 \cdot 7 \cdot 10457 \cdot 12329 \cdot 16693 \cdot 50392193 \cdot 14969202179383$
25	$371045712329166935039219314969202179383 =$ $3 \cdot 30259 \cdot 71055159937 \cdot 57524912931153279285967$

26	$330259710551599375752491293115327928596 7 =$ $3 \cdot 89 \cdot 13961402129 \cdot 8859624151256362891884 63869$
27	$38913961402129885962415125636289188463869 =$ $3 \cdot 293 \cdot 41233 \cdot 110384367575484 71 \cdot 97266672953292277$
28	$3293412331103843675754847197266672953292277 =$ $3 \cdot 7 \cdot 11 \cdot 1853767605161 \cdot 76909296498934871307602 22347$
29	$37111853767605161769092964989348713076022 2347 =$ $2887 \cdot 128548159915501079906799324521471122535581$
30	$2887128548159915501079906799324521471122535581 =$ $31 \cdot 3169 \cdot 1387271471 \cdot 452100449741 \cdot 468582207297817 91489$
31	$3131691387271471452100449741468582207297817 91489 \ (\text{Prime})$

In our next activity we consider the calendar for the month of MARCH 2017:

MARCH 2017

S	M	T	W	R	F	S
			1	**2**	**3**	**4**
5	**6**	**7**	**8**	**9**	**10**	**11**
12	**13**	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**	**25**
26	**27**	**29**	**30**	**30**	**31**	

a. We select several 3×3 groups of numbers and find the sum of these numbers and determine how the obtained sums related to the middle number.

b. We then prove that the sum of any 9 integers in any 3×3 set of numbers selected from a monthly calendar will always be equal to 9 times the middle number using algebra and technology to furnish a convincing argument.

We first consider the group highlighted in blue above. The integers are 6, 7, 8, 13, 14, 15, 20, 21 and 22 with the middle number being 14. Adding these nine numbers, we obtain $6+7+8+13+14+15+20+21+22=126=9\cdot14.$

Let us next select a second 3×3 group of numbers highlighted in green above. These numbers are 9, 10, 11, 16, 17, 18, 23, 24 and 25 with 17 serving as the middle number. We note that $9+10+11+16+17+18+23+24+25=153=9\cdot17.$

We finally select a third 3×3 group of numbers highlighted in red below. These numbers are 10, 11, 12, 17, 18, 19, 24, 25 and 26 where 18 is the middle number. Note that $10+11+12+17+18+19+24+25+26=162=9\cdot18.$

MARCH 2017

S	M	T	W	R	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

b. We next prove that the sum of any 9 digits in any 3×3 set of numbers selected from a monthly calendar will always be equal to 9 times the middle number using both algebra and technology to furnish a convincing argument.

Based on our analysis of three specific cases, it seems plausible to conjecture that the sum of any nine elements in the 3×3 group is always nine times the middle number. To show that this is always true, one can employ algebraic reasoning. We let $x=$ the first number in the array. The next numbers are thus $x+1,\ x+2,\ x+7,\ x+8,\ x+9,\ x+14,\ x+15\ \textit{and}\ x+16.$ Next note that $x+x+1+x+2+x+7+x+8+x+9+x+14+x+15+x+16=9x+72=9\cdot(x+8).$ Observe that $x+8$ is the median (middle number) in the array completing our proof.

Technology can play a role as well. Let us use a graphing calculator (TI-89) to furnish the specific cases as well as a formal proof. See **FIGURES 69-72:**

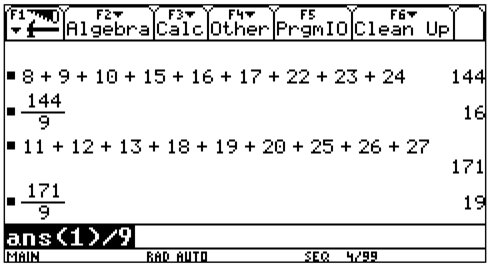


FIGURE 69: Example Calculations

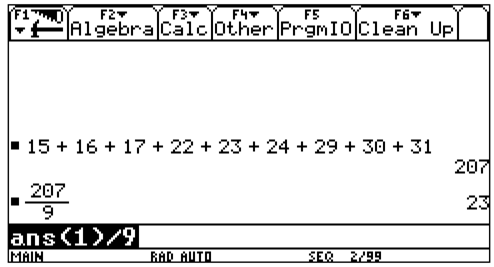


FIGURE 70: Example Calculations

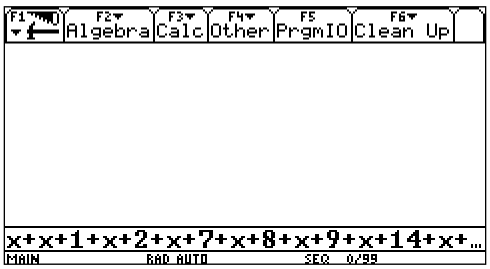


FIGURE 71: The Deductive Proof

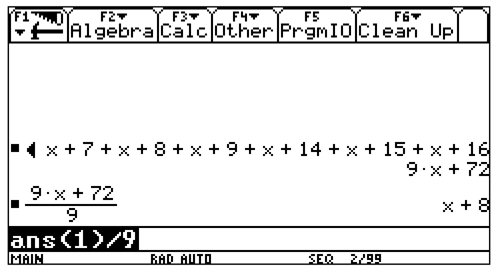


FIGURE 72: The Deductive Proof

Our next activity focuses on the distinction between inductive reasoning which is the reasoning often utilized in the sciences and deductive reasoning which is the mathematician's proof tool. The use of number puzzles is a neat vehicle to engage students and teachers in this distinction. An example follows courtesy of the popular text *Mathematics for Elementary Teachers* by Albert Bennett and Ted Nelson where we utilize both inductive reasoning (five cases) and then deductive reasoning to solve the following number puzzle employing the given directives:

a. Pick any number.
b. Add 221 to the given selected number.
c. Multiply the sum by 2652.
d. Subtract 1326 from your product.
e. Divide your difference by 663.
f. Subtract 870 from your quotient.
g. Divide your difference by 4.
h. Subtract the original number from your quotient.

In inductive reasoning, we reason to a general conclusion via the observations of specific cases. The conclusions obtained via inductive reasoning are only probable but not absolutely certain. In contrast, deductive reasoning is the method of reasoning to a specific conclusion through the use of general observations. The conclusions obtained through the use of deductive reasoning are certain. In the following number puzzle, we employ five specific numbers 5, 23, 12, 10, and 85 for the inductive case and then employ algebra to furnish a deductive proof. The puzzle and the solutions are provided below:

Pick any Number.	5	23	12	10	85	n
Add 221 to the given selected number.	226	244	233	231	306	$n+221$
Multiply the sum by 2652.	599352	647088	617916	612612	811512	$2652\cdot n+586092$
Subtract 1326 from your product.	598026	645762	616590	611286	810186	$2652\cdot n + 584766$
Divide your difference by 663.	902	974	930	922	1222	$4\cdot n +882$
Subtract 870 from your quotient.	32	104	60	52	352	$4\cdot n +12$
Divide your difference by 4.	8	26	15	13	88	n+3
Subtract the original number from your quotient.	3	3	3	3	3	3

The answer we obtain is always 3. We next deploy the calculator to show the inductive cases in **FIGURES 73-82** and the deductive case in **FIGURES 83-84:**

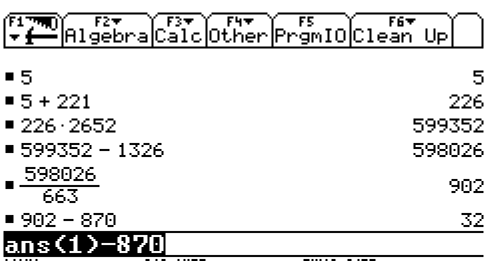


FIGURE 73: Example Calculations

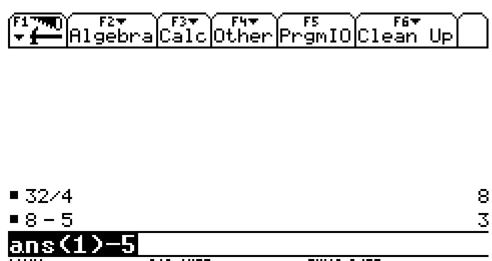


FIGURE 74: Example Calculations

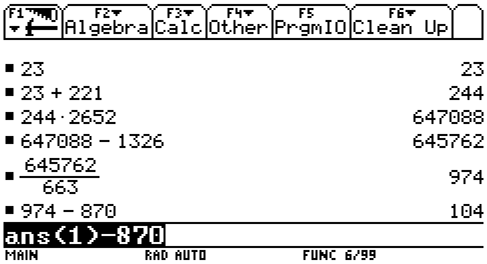


FIGURE 75: Example Calculations

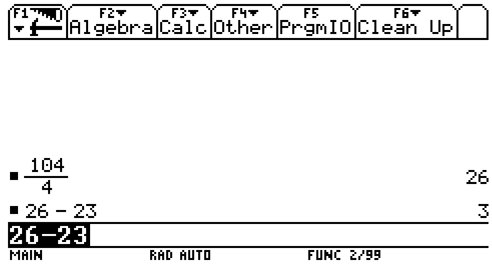


FIGURE 76: Example Calculations

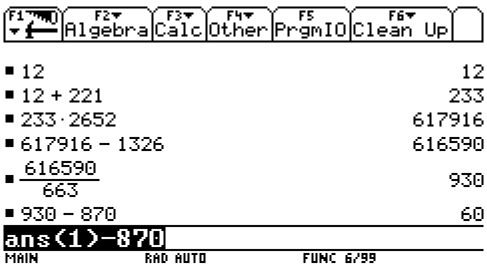


FIGURE 77: Example Calculations

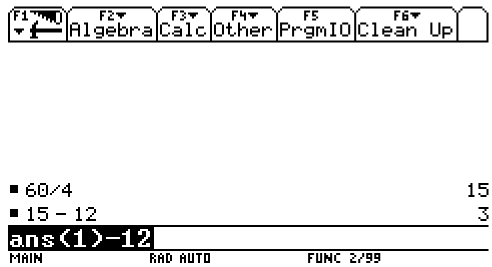


FIGURE 78: Example Calculations

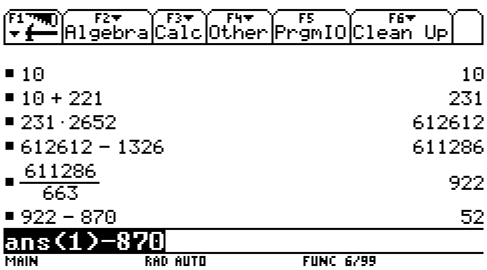


FIGURE 79: Example Calculations

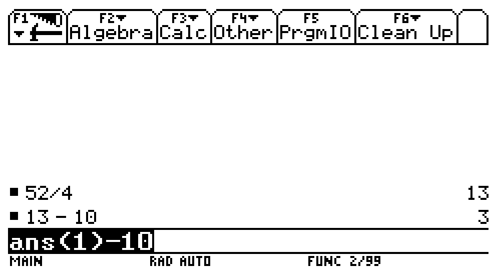


FIGURE 80: Example Calculations

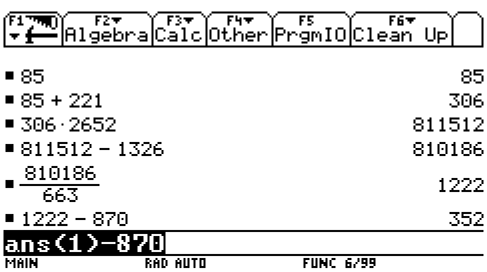


FIGURE 81: Example Calculations

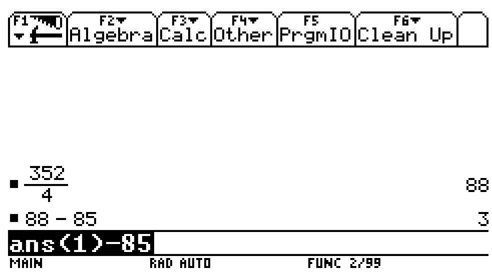


FIGURE 82: Example Calculations

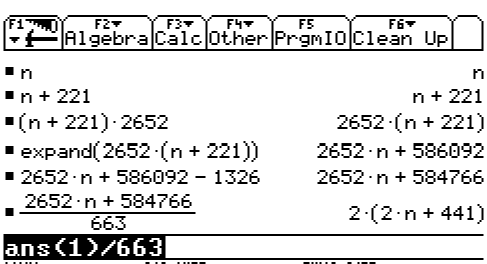


FIGURE 83: The Deductive Proof

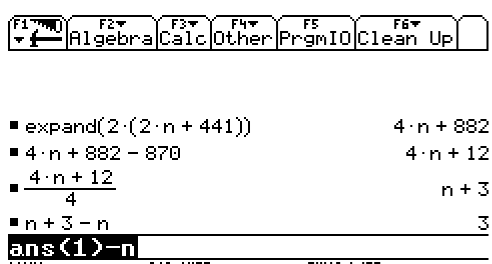


FIGURE 84: The Deductive Proof

Our final activity involves everyone's favorite sequence, the Fibonacci sequence. Recall that the Fibonacci sequence satisfies the following recursion relation:

$F_1 = F_2 = 1 \; and \; F_n = F_{n-2} + F_{n-1}; \; n \geq 3.$

In this activity, we next take any four consecutive Fibonacci numbers. Form the product of the first and fourth terms of the sequence. Next take twice the product of the second and terms. Finally take the sum of the squares of the second and third terms. Observe the relationship to the Pythagorean Theorem in plane geometry. We gather some empirical evidence via the following three examples:

Consider, in turn, the following three sets each entailing four consecutive Fibonacci numbers: $\{3,5,8,13\}$, $\{8,13,21,34\}$ *and* $\{13,21,34,55\}$ respectively. Observe the truth of the following with the TI-89 in **FIGURE 85, FIGURE 86 and FIGURE 87** respectively:

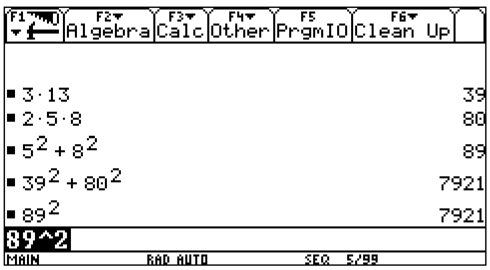


FIGURE 85: Example Calculations

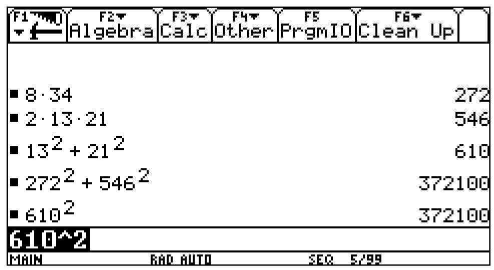


FIGURE 86: Example Calculations

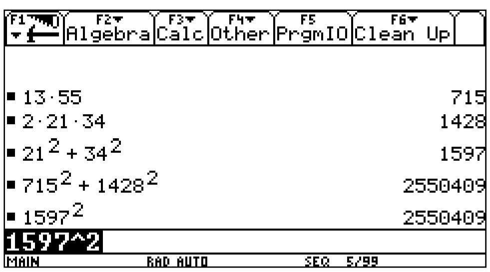


FIGURE 87: Example Calculations

We observe that the Pythagorean Triples (39, 80, 89), (272, 546, 610) and (715, 1428, 1597) are respectively formed. The first and third of these Pythagorean triples are primitive, but the second one is not primitive; (for 2 is a common factor among each of the components). The associated primitive Pythagorean Triple is (136, 273, 305). Note that the hypotenuses of each of the right triangles formed are Fibonacci numbers. (89, 610, 1597).

Based on the observations in the three examples, one suspects that a Pythagorean triple is always formed and this is indeed the case. We justify our conjecture with the aid of the TI-89:

Suppose $\{x, y, x+y, x+2\cdot y\}$ represents any four consecutive terms of the Fibonacci (or Fibonacci-like) sequence. We view our inputs and outputs in **FIGURE 89** using the expand command (See **FIGURE 88)** from the Algebra menu on the HOME SCREEN:

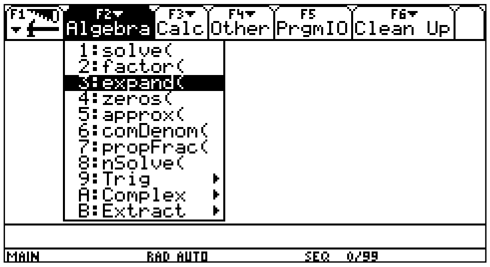


FIGURE 88: The Expand Command

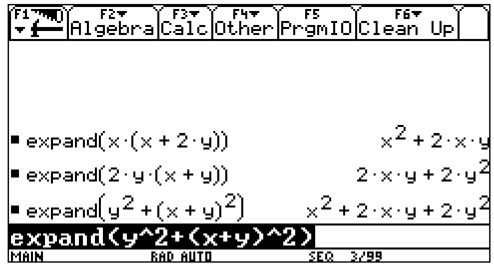


FIGURE 89: The Deductive Proof

To show that $\left(x^2+2\cdot x\cdot y, 2\cdot x\cdot y+2\cdot y^2, x^2+2\cdot x\cdot y+2\cdot y^2\right)$ forms a Pythagorean Triple, see **FIGURES 90-92** for our inputs and outputs:

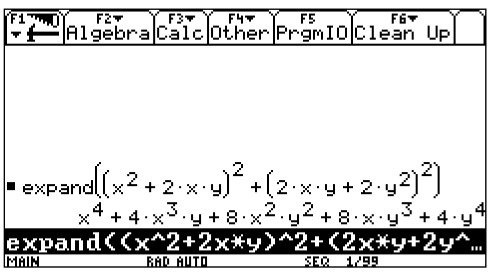


FIGURE 90: The Deductive Proof

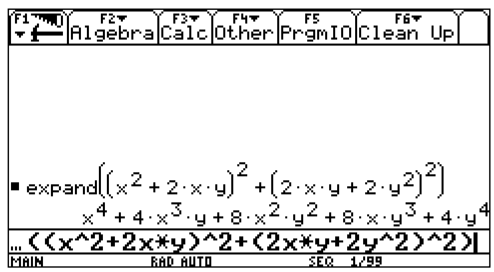


FIGURE 91: The Deductive Proof

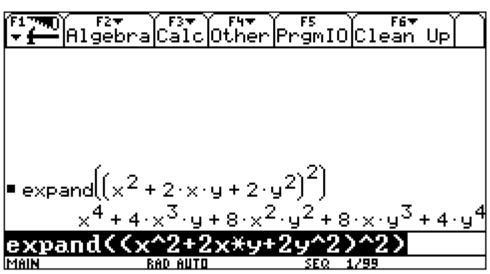


FIGURE 92: The Deductive Proof

Conclusion: This paper served to furnish palatable and engaging activities with the integration of technology to explore mathematical connections among disciplines that are amenable to teachers in the secondary curriculum. It is in this spirit that students can discover mathematical insights and ensure that mathematics is meaningful and both conceptual and contextual. Technology when utilized judiciously enables both teachers and students to explore a world of possibilities.