SIMULATIONS: FROM PLAYING CARDS TO STATCRUNCH™ AND BEYOND

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Abstract

In this paper, I will explain the format of the different simulations that can be used to introduce abstract concepts in an introductory statistics course. The different formats will be discussed regarding the time needed to implement, the necessary technology available to the students, and the outcomes and limitations that may need to be considered when executing the activity. Furthermore, an example of each type of format will be included in this paper.

1. Introduction

Incorporating active learning into an introductory statistics course has been one of the leading course redesign practices that I have used in the past decade or so. Every semester, I am looking for ways to incorporate activities that help students gain a deeper understanding of statistical concepts. Students often struggle with abstract concepts such as sampling distributions, central limit theorems, confidence intervals, and hypothesis tests in an introductory statistics course. Using simulations has proven to be a successful tool to introduce these concepts so students gain a deeper understanding and a stronger foundation in these statistical ideas.

In 2005, the American Statistical Association published the Guidelines of Assessment and Instruction in Statistical Education (GAISE) [1]. In 2016, a revised GAISE report [2] was published. In both reports, the recommendations remained the same. Of the six recommendations made in the reports, five support the use of simulations. These recommendations are (1) teach statistical thinking, (2) focus on conceptual understanding, (3) integrate real data with a context and a purpose, (4) foster active learning, and (5) use technology to explore concepts and analyze data.

Furthermore, the 2016 GAISE [2] highlights the many changes that have occurred since its first report was completed. Some of the changes identified include (1) more students are studying statistics, (2) there is an increase in the available data, (3) there is better technology, and (4) alternate learning environments are used today more than in the past.

These recommendations and changes have been the catalyst for a transformation within my own teaching practice. Statistics is now a general education course required of all students at Flagler College. So, students taking the course are not necessarily strong mathematics students and they are not necessarily in a major that would require statistics otherwise. Thus, the course has been redesigned to be taught as an applied course that focuses on statistical thinking and interpretations. Traditional lectures in a traditional classroom are a thing of the past in my courses. My class meets in a computer lab for 75-minutes two times a week. Students work in groups on computers throughout the class discovering statistical concepts. Although every class period is different, typically, half of the time involves a full group lesson and half involves activities designed to develop conceptual understanding of statistics. In my courses, I use StatCrunch™ because it is cloud-based, easy to use, integrated into MyStatLab and has both computational ability and built-in applets. Many of the applets in StatCrunch™ can be customized with original data sets. This helps students to truly understand the simulation.

Overall, starting with an activity that uses a simulation has been very effective in getting the students actively involved in the course material. This active introduction to the topics gives students the foundation to understand the statistical principles underlying the tests commonly used in statistical research.

In this paper, I have divided these simulations into groups based on the format of the activity. An outline and at least one example of each of the different formats are provided, so the reader will be able to adapt these activities affectively in his/her own course and design his/her own activities using a similar format.

2. Walk – Jog – Run Simulations

Walk – Jog – Run simulations all follow a similar general format; first students conduct a hands-on physical simulation, second data is gathered from all students in the classroom to look at the overall picture, and third a computer simulation is conducted to see what happens in the long run. This follows the notion that you must first walk, then jog, and finally run with the idea to truly understand and embrace the concept. Once the students have experienced the activity, the groundwork has been set to introduce the statistical principles. Furthermore, after the activity has been completed, when students are working through exercises using the theoretical results, it is always helpful to refer to the activity and build connections. As an instructor, it is delightful to see the "ah-ha" moment the students have when the association between the simulation and the theory are realized.

The Walk – Jog – Run activities require considerable time to complete. Typically, 30 – 45 minutes is needed for these activities. These activities involve student use of a computer or some electronic device to take surveys and then run the simulations. Throughout a semester, I use approximately four or more of these types of activities. I use the Walk – Jog – Run method to introduce the central limit theorem for proportions and for means, a two-proportion test, and randomization tests. There are certainly other concepts that could be introduced using this technique, but with time constrains during the semester, it is best to choose your top favorite.

The following two sections give examples of Walk – Jog – Run simulations. There are many more that I have used in class, but these should give the reader an overall understanding of how to design these activities.

2.1 Is Duct Tape as Effective as Cryotherapy in Testing Wards – Two Proportion Tests

This activity in adapted from the Student Activity Workbook for the Sullivan Statistics Series. [3]

Exercise: A study was conducted by researchers to determine if application of duct tape is more effective than cryotherapy (liquid nitrogen applied to the wart for 10 seconds every 2 to 3 weeks) in the treatment of common warts.

The researchers randomly divided 51 patients into two groups. The 26 patients in Group 1 had their warts treated by applying duct tape to the wart for 6.5 days and then removing the tape for 12 hours, at which point the cycle was repeated, for a maximum of 2 months. The 25 patients in Group 2 had their warts treated by cryotherapy for a maximum of six treatments.

Once the treatments were complete, it was determined that 22 patients in Group 1 (duct tape) and 15 patients in Group 2 (cryotherapy) had complete resolution of their warts.

(*Source:* Dean R. Focht III, Carole Spicer, Mary P. Fairchok. "The Efficacy of Duct Tape vs. Cryotherapy in the Treatment of Verruca Vulgaris (the Common Wart)," *Archives of Pediatrics and Adolescent Medicine*, 156(10), 2002.)

Conversation: In the sample, 85% of the duct tape patients had complete resolution of their warts while only 60% of the cryotherapy patients did. We need to determine whether the observed difference of 25% is due to random error (and there really is no difference in the treatments), or if the differences are significant (so that one treatment is superior to the other). To answer this question, we will randomly assign a treatment to each of the outcomes.

Walk: Each student will take a standard deck of cards and let the 26 red cards represent the patients who had complete resolution of their warts using duct tape and let 25 of the black cards represent the patients who had complete resolution of their warts using cryotherapy. They will shuffle the cards and then deal 37 cards. These 37 cards represent the 37 individuals who had complete resolution of their warts. They will count the number of red and black card dealt and then calculate the percent of the red cards and black cards dealt. Lastly, they will calculate the percent difference red cards and black cards dealt.

Jog: Each student will then take the survey "Warts" available on the StatCrunch^{TM} group page to collect the student results (percent red, percent black, and percent difference) of the card simulation. As a class, a histogram is created of the percent difference gathered in the survey. The mean, median, and shape of the distribution is discussed and then the probability of getting the sample difference of 25% or more is determined. Based on the class simulation and the probability, it is then determined if there is evidence that duct tape is superior to cryotherapy.

Run: What we really want to know is if we conducted this experiment many, many times, how likely is it to obtain a sample difference in proportions of as extreme, or more extreme than 0.25 (25%) that was observed if there is no difference in the treatments. Each student will open a StatCrunch™ spreadsheet. They will go to Applets > Resampling > Randomization test for two proportions. They will enter the appropriate counts and totals in the applet and then run the simulation for at least 30,000 results. Based on these simulations, each student will determine the probability of getting a sample difference of 25% or more and compare their results with others in the class. A final class discussion will follow.

2.2 Yellow – White Exams: Was the Yellow Exam More Difficult? – Randomization Tests

This activity is adapted from an example available in StatCrunch™.

Exercise: In a statistics class of twenty students, students were randomly assigned either a yellow or white version of the exam in such a way that 10 students had a yellow exam and 10 students had a white exam. After the exam, both groups of students complained about the exam saying their version was harder. The mean for the white exam scores was 71% and the median was 75% while the mean for the yellow exam scores was 77.3% and the median was 78.5%. Is there significance evidence that the typical student will score lower on one version of the exam than the other OR is this sort of difference due to chance alone?

Walk: Each student will be given 20 notecards containing the original 20 exam scores. They will shuffle their notecards and deal them into two groups, one representing a yellow exam score and one representing a white exam score. This process is equivalent to the notion that the student would get the same exam score regardless of the exam they take. Each student will calculate the mean of each group and compute the difference between the two means.

Jog: Each student will then take the "Yellow/White Exam" survey shared on the StatCrunch™ group page recording the sample means and the sample difference. As a class, a histogram of the distribution of the sample mean differences is created. The mean, median, and shape of the distribution is discussed and then the probability of getting the sample difference of 6.3% or more extreme is determined. Based on the class simulation and the probability, it is then determined if there is evidence that one of the exams was more difficult than the other.

Run: What we really want to know is if we conducted this experiment many, many times, how likely is it to obtain a sample difference in means of as extreme, or more extreme than 6.3 points that was observed if there is no difference in the difficulty of the two exams. Students will open the Yellow –White Exam Data in StatCrunch™, and then go to Applets > Resampling > Randomization test for two means. Each student will run the simulation for at least 30,000 results. Based on these simulations, each student will determine the probability of getting a sample difference of 6.3% or more extreme and compare their results with others in the class. A final class discussion will follow.

3. Virtual Reality

Virtual Reality activities start with a computer simulation. Although some of these activities can take up to 30 minutes, many can take as little as 5 – 10 minutes. Hence, these activities can be incorporated as a much shorter part of a lesson. Since these activities often do not take most of the class meeting, I incorporate quite a few in lessons throughout the semester and am also always looking for new ideas.

Virtual Reality activities do require that students have a computer or some other electronic device to run the simulation and then possibly take a survey to collect the data from all students in the class. Unlike the Walk – Jog – Run simulations, the students have no hands-on physical simulation to compare to the computer-generated simulations. Without this comparison, I thought the students would think the computer simulations were rigged and would question the results. I have not found this to be true at all. Students have no difficulty believing and understanding the computer simulations.

The following section explains one example of a Virtual Reality simulation. Also note that although not presented here, Virtual Reality activities are excellent to use to introduce probability and the Law of Large numbers. Years ago, I had students roll dice and toss coins. Dice and coins would be flying everywhere. It is certainly easier to simulate rolling a die and tossing a coin on the computer.

3.1 Is the Die Fair? – Chi-Squared Goodness of Fit Test

This is an applet available in an open spreadsheet in StatCrunch™.

Exercise: Suppose we wanted to determine if a standard 6-sided die was fair. In a perfect world, if the die was fair, the distribution of outcomes would be uniform. On this exercise, an experiment will be run to determine if a die rolled manufactured by one of six manufacturers is "fair". There are six manufacturers of dice to investigate: Luckytown Dice Company, Dice 'R' Us, High Rollers, Inc., Dice, Dice, Baby!, Pips and Dots, and Slice 'n' Dice

Virtual Simulation: In an Open StatCrunch™ spreadsheet, each student will go to Applets > Games > Fair Die? Each student has been assigned one of the six die manufacturers. They will each be rolling their die 630 times and reporting the number of rolls that result in 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, and 6 dots. After running their experiment, the students will select Analyze Rolls. They will go to Stat > Goodness-of-fit > Chi-Square Test. After each student has run the test, a class discussion will be conducted regarding which of the companies create a "fair" die and which companies do not.

4. Old School with a Twist

Old School with a Twist activities start with data gathering by hand. Then software is used for gathering the students' data with a survey and running the statistical analysis. These activities may take 30 – 45 minutes, so, as with a Walk – Jog – Run activity, proper placement in the curriculum and a well-designed lesson plan is essential to the success of these activities.

Old School activities have been included in my courses prior to teaching in a computer lab. Moving into a computer lab added the Twist component to the activity. In a computer lab, students can take a survey to collect the class data. The data is then shared with the students in the class for the analysis. When I was limited to only an instructor computer, manual collection of data occurred, and the data was analyzed together as a whole class. Thus, students do not necessarily need their own computer or electronic device to participate, but for data collection and sharing, it is nice.

The following sections introduce two Old School with a Twist activities. Both activities presented involve M&Ms and are very popular with students. There are many statistical activities that involve M&Ms. Each semester, I try to include at least one or two.

4.1 Is the Color Distribution of M&Ms Independent of the Type of M&Ms – Chi-Squared Test of Independence

Exercise: Is the color distribution of plain and peanut M&Ms the same?

Old School: Give each student two bags of M&Ms; one plain and one peanut. The students will then open each bag and count the number of M&Ms in each bag based on color; recording the results in a table.

Twist: The students will then take the StatCrunch™ survey "M&Ms – Chi-Squared" available on the StatCrunch™ group site to collect the number of plain and peanut M&Ms of each color in their bags. Once all students have completed the survey, the data is shared on the StatCrunch™ group site. The students will open the class data and go to Stat > Tables > Contingency > With Data and under Hypothesis Tests: Select Chi-Square test for independence to analyze the data. A final class discussion will follow.

Is the Mean Number of M&Ms in Fun Size the same for different types of M&Ms? – ANOVA

Exercise: Is the mean number of M&Ms in the sharing size the same for all the different types of M&Ms?

Old School: Give each student one or more bags of M&Ms. Make sure there are at least three or more types of M&Ms to investigate. The students will then open each bag and count the number of M&Ms in each bag; recording the results in a table.

Twist: The students will then take the StatCrunch™ survey "M&Ms – ANOVA" available on the StatCrunch™ group site to collect the type of M&Ms and the number of M&Ms in their bags. Once all students have completed the survey, the data will be shared on the StatCrunch™ group site. The students will open the class data and go to Stat > ANOVA > One Way. Students will run the analysis letting the Responses be the number of M&Ms in the bag and the Factor be the type of M&Ms. A final class discussion will follow.

5. The Big Screen

Big Screen simulations are conducted as a full class. Only one instructor computer is needed. These activities do not take considerable time. Typically, 5 – 10 minutes is sufficient to complete one of these activities. Since these simulations involve the whole class, they can be incorporated in a more traditional class setting and can be used in almost all class meetings.

Active discovery-based learning is the key to the simulations and activities outlined in this paper. Big Screen simulations also need to incorporate active learning and student interaction. Otherwise, a Big Screen simulation will mimic a lecture. One technique that I have found to be effective in promoting active learning is to use groups. As we are working through the simulation, I will ask questions and give the groups time to discuss their answer. I then ask a group and not an individual student to share their answer. Using groups allows the students the chance to discuss and reflect on ideas prior to presenting their ideas to the whole class.

The following two sections give examples of Big Screen simulations. There are many more that I use in class and many others available that I discover each semester to add to the curriculum. Searching the internet and StatCrunch™ always results in innovative ideas for Big Screen simulations.

5.1 Can You Guess the Correlation? – Correlation Coefficient

This is an applet available in an open spreadsheet in StatCrunch™.

<u>Objective</u>: Understand the properties of the correlation coefficient.

<u>The Big Screen</u>: In an open StatCrunch™ spreadsheet, go to Applets > Correlation by eye > Compute. Explain to the class, that they need to select a number between -1 and 1 that represents the scatterplot. No other directions are given. Students work in groups to guess a number and the properties of the correlation coefficient slowly emerge through proper direction by the instructor.

5.2 How Many Distributions Can You Create? – Probability Distributions

This is an applet available in an open spreadsheet in StatCrunch™.

<u>Objective</u>: Understand the properties of the different probability distributions.

<u>The Big Screen</u>: In an open StatCrunch™ spreadsheet, go to Applets > Distribution demos > Select a distribution > Compute. Explain to the class that they need to determine what happens to the distribution as the parameters are changed. No other directions are given. Students work in groups to discuss what happens as the graph is being changed and the conceptual understanding of the relationship between the parameters and the distribution is explored.

6. Conclusions

Simulations and activities should be an integral part of an introductory statistics course. As demonstrated in this paper, regardless of the classroom set up, there are ways to integrate simulations effectively into the course.

If you have never used simulations in your classroom, start by integrating some Big Screen simulations. These activities can be used in most classrooms and take less prep work and less time in class. When you are ready to include even more active learning techniques in your course, add a few Virtual Reality simulations. If only one instructor computer is in the classroom, some activities that I have designated as Virtual Reality can be run as a Big Screen activity. It is important, though, to continue to support student interaction, discussion, and reflection when running a Big Screen simulation. Otherwise, the activity will not meet the learning objective set.

Although more time consuming in class, the next natural addition to a course would be an Old School with a Twist activity. Recall, these activities can also be run without a Twist and use minimal technology. These activities were the first type I integrated in my classes years ago. You do need some extra preparation time to use class time effectively. Otherwise, you can lose your audience while you are setting up the activity.

The only simulations that require students in a computer lab or on an electronic device are the Walk – Jog – Run simulations. These simulations should not be overlooked if you do not teach in a computer lab though. Students could also complete these activities as a homework or lab assignment outside of class. Careful planning is necessary for student success regardless of the setting the simulations will be run. Furthermore, these activities could be adapted to one of the other types of simulations as necessary to accommodate the course structure.

Flexibility is truly the key when it comes to adopting an alternate path to instruction in the classroom. There are many simulations available to enhance an introductory statistics course. There also many different formats for proper use of these activities. The key to successfully integrating these simulations into the classroom instruction is to first determine the objective you want to achieve and then design an activity that can be carried out effectively based on the resources in the classroom and the time available.

References

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