YOU CAN "COUNT" ON ME!
A COUNTING STRATEGY FOR UNDERGRADUATES

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## Introduction

A task that undergraduate students in Liberal Arts Math, Quantitative Reasoning, or Statistics are often required to learn and master is counting the number of ways in which a defined event can occur. One such example that we will solve later is:

*A multiple-choice test has 6 questions, each with 4 possible answers. How many ways are there to mark the answers?*

Students are taught various counting techniques, including:

- Systematic Listing
- Fundamental Counting Principle
- Permutations and Factorials
- Combinations

As an instructor of Liberal Arts Math, I have seen my students struggle with determining which counting technique to apply to solve a given problem. I have developed a one-page decision chart to help students with this determination. In this paper, we will demonstrate the use of this decision chart to solve various types of counting problems that students are often required to solve.

## Review of Basic Counting Methods

Let's start by reviewing the three basic counting methods that will be used in the decision chart.

- Fundamental Counting Principle

  If one task can be done in $m$ ways and a second task can be done in $n$ ways, then the two tasks can be done together in $\boldsymbol{m \cdot n}$ ways (the **product** of $m$ and $n$).

  This principle extends similarly to more than two tasks.

- Permutations and Factorials

  The number of ways that *r* out of *n* distinct objects can be arranged (put in order) is:

$$_n\boldsymbol{P}_r = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$$

  If **all** *n* distinct objects are being arranged, then this formula simplifies to $\boldsymbol{n}!$.

- Combinations

  The number of ways that *r* out of *n* distinct objects can be chosen (order doesn't matter) is:

$$_n\boldsymbol{C}_r = \frac{\boldsymbol{n}!}{\boldsymbol{r}!\,(\boldsymbol{n}-\boldsymbol{r})!}$$

## The Decision Chart

Here is the decision chart I developed to help students determine which of the previous method(s) to use to solve a counting problem:

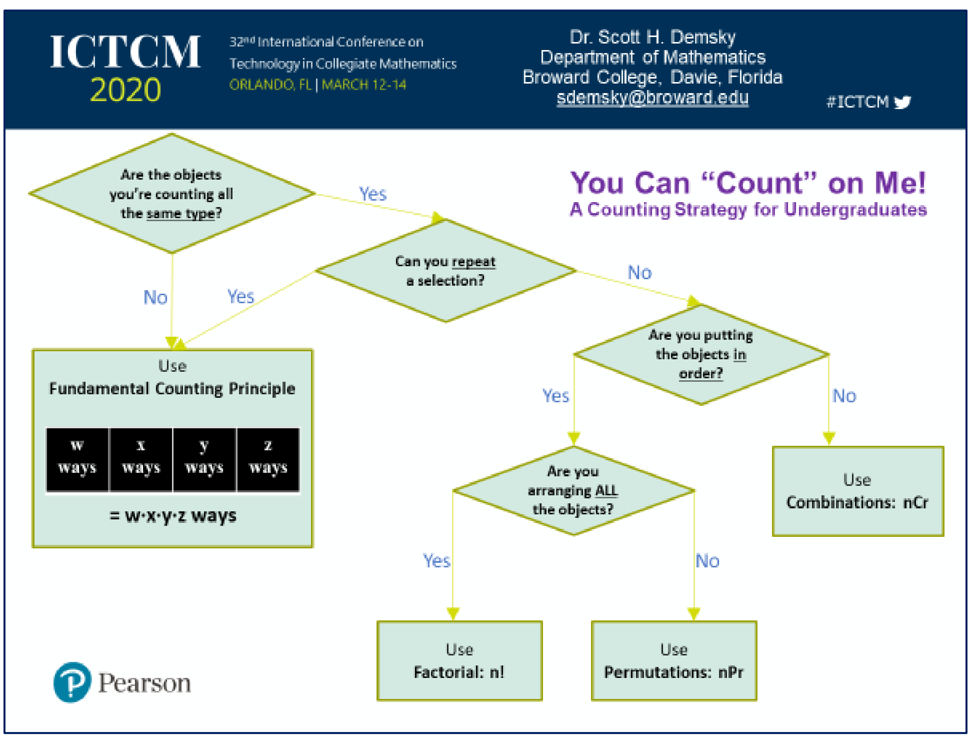


*Figure 1. "A Counting Strategy for Undergraduates" Decision Chart*

Let's see how we can utilize this decision chart to help us solve several different types of counting problems.

### Example 1

*Don has 5 pairs of shoes, 2 pairs of pants, and 3 shirts. If all items match, how many different outfits can he wear?*

Because the objects we're counting are of different types (shoes, pants, shirts), the chart leads us to apply the Fundamental Counting Principle:

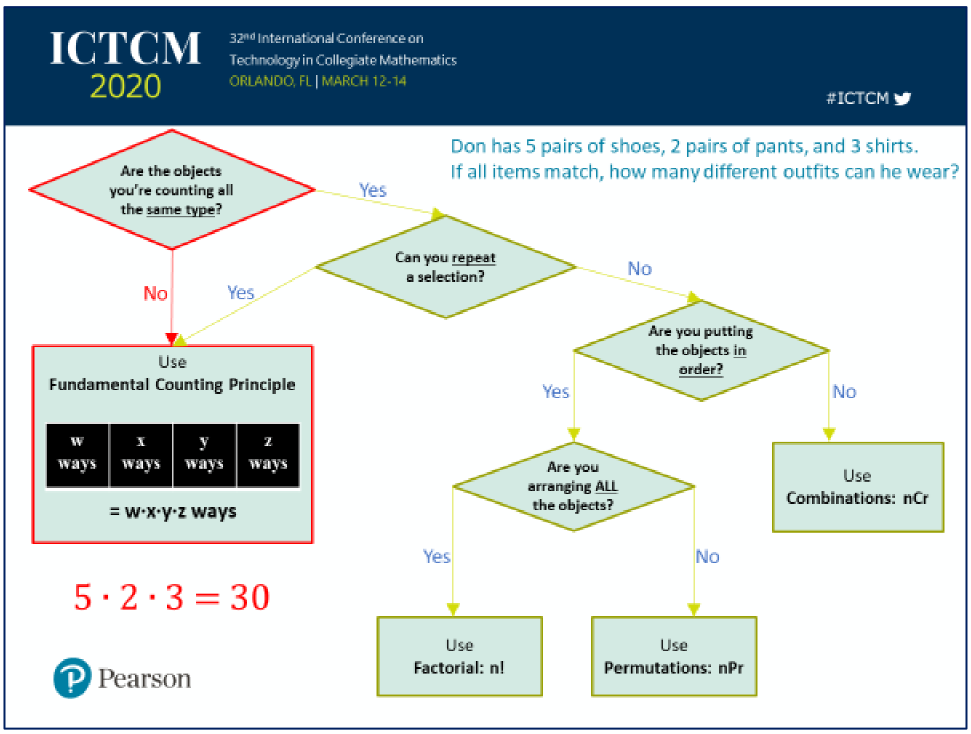


*Figure 2. Use the Fundamental Counting Principle when objects are of different types (Example 1)*

Thus, Don can wear **30** different outfits.

### Example 2

*A multiple-choice test has 6 questions, each with 4 possible answers. How many ways are there to mark the answers?*

Although the objects we're counting (possible answer choices, say a, b, c, d) are the same type, we're allowed to repeat answers from question to question. Therefore, we can repeat a selection and the decision chart again leads us to apply the Fundamental Counting Principle:

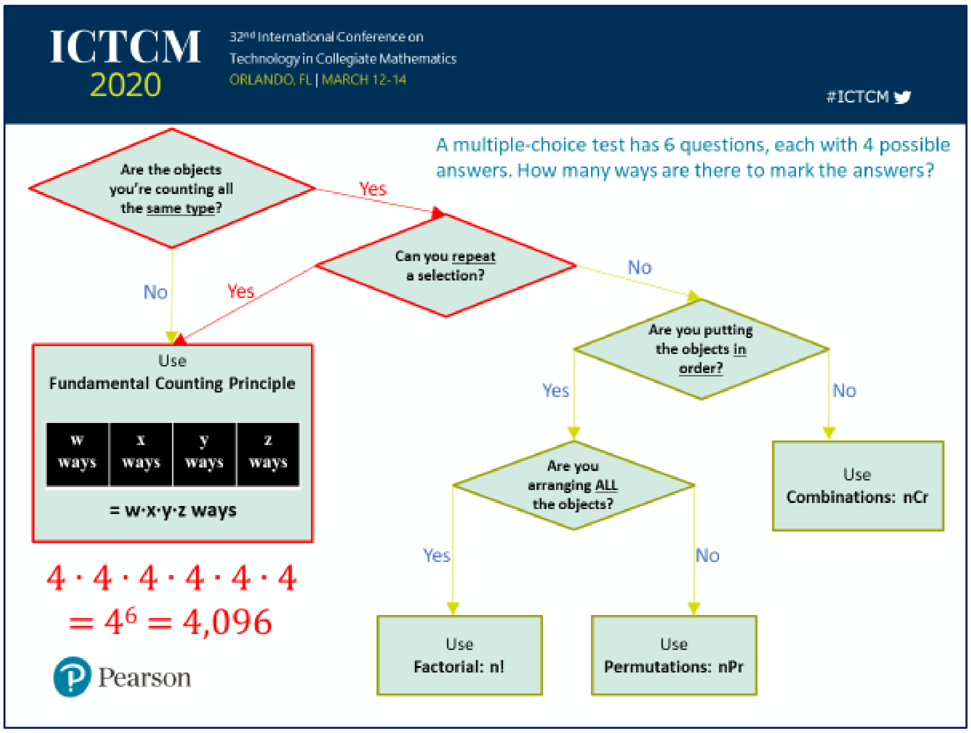


*Figure 3. Use the Fundamental Counting Principle when you can repeat a selection (Example 2)*

Thus, there are **4,096** ways to mark the answers.

### Example 3

*In how many ways can you line up 6 people for a photograph?*

This time, the objects we're counting are *people*, so all the same type. Since we can select a person only once and since we're lining up *all* 6 people (we're putting them in order), the chart leads us to use a Factorial:

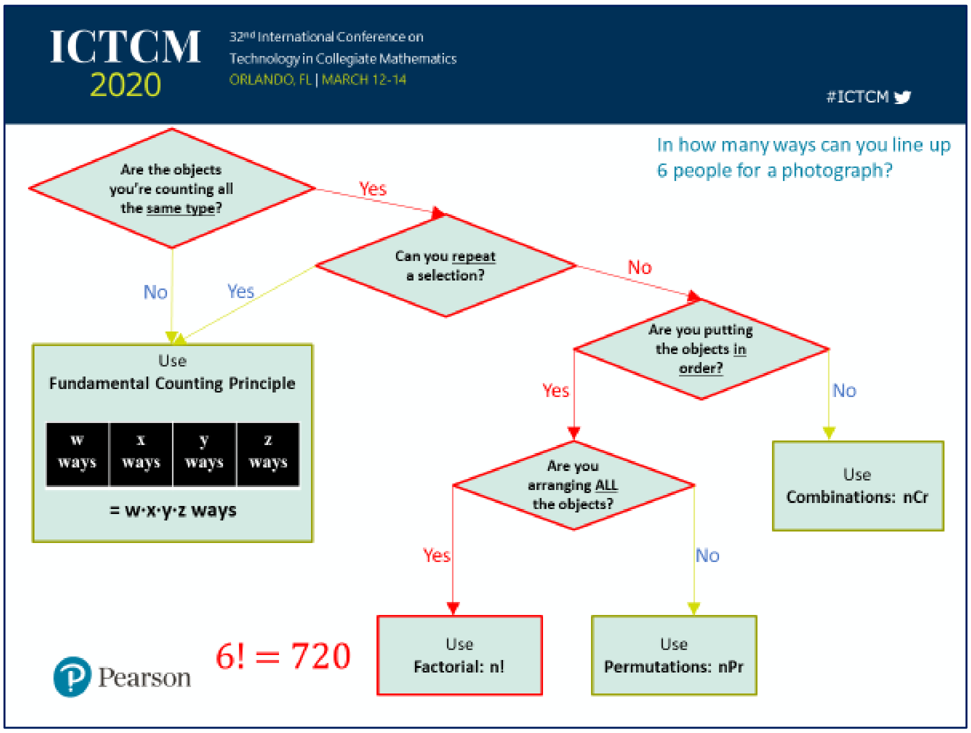

*Figure 4. Use Factorial when arranging all of a set of distinct objects (Example 3)*

Thus, there are **720** ways to line up all 6 people.

### Example 4

*To win the top prize in the Florida Fantasy 5 lottery, you must correctly select 5 different numbers from 1-36 (order doesn't matter). How many selections are possible?*

This time, the objects we're counting are *numbers*, so all the same type, and we cannot select a number more than once. But since order doesn't matter, we're *not* putting the objects in order, and the chart leads us to use Combinations:

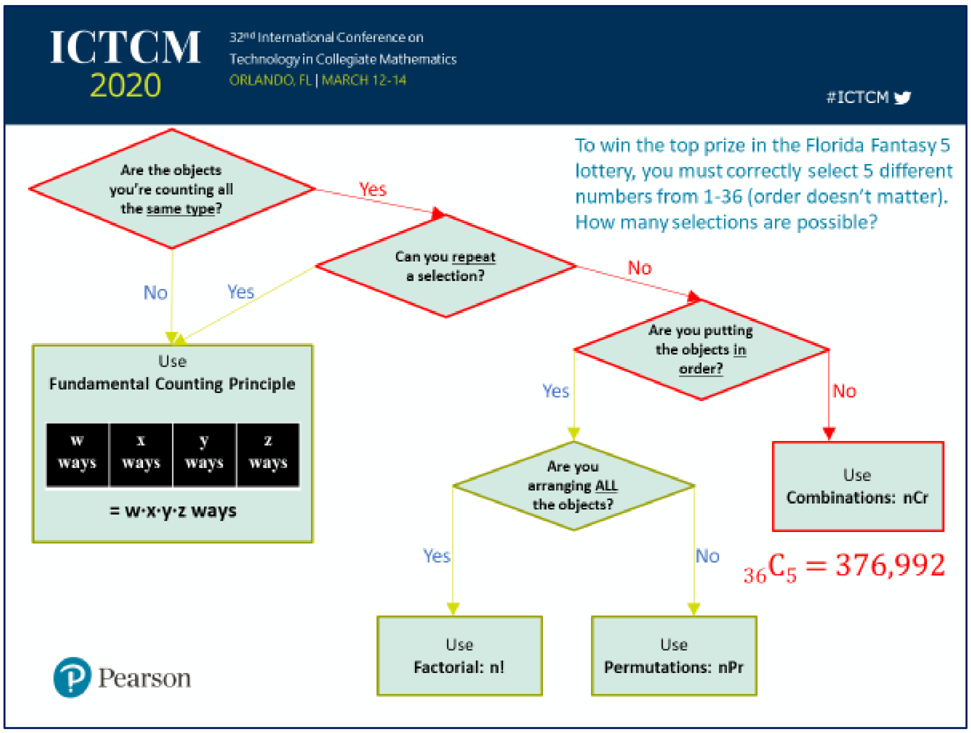


*Figure 5. Use Combinations when choosing from a set of distinct objects when order doesn't matter (Example 4)*

Thus, there are **376,992** possible selections.

## Example 5

*In a race with 8 cars, in how many ways can the cars finish 1st place, 2nd place, and 3rd place? Assume no ties.*

The objects we're counting in this problem are *cars* (same type). Since there are no ties, we must select a different car for each of the first three finishers, so we cannot select a car more than once. Since we're selecting only 3 of the 8 cars and we're putting them in order (1st place, 2nd place, 3rd place), the chart leads us to use Permutations:

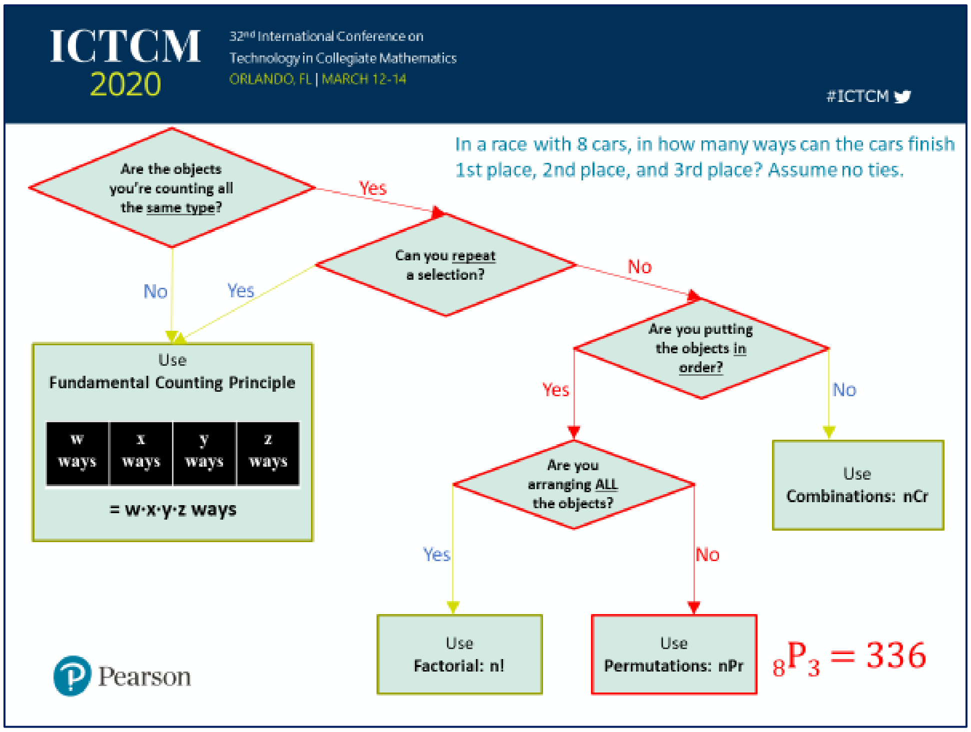


*Figure 6. Use Permutations when putting a subset of distinct objects in order (Example 5)*

### Example 6

*In a certain state, license plates are 3 letters followed by 3 numbers with no repetitions in either group. How many different license plates are there?*

In this example, we'll see that multiple applications of the decision chart will be needed. To begin with, since the objects are of different type (letters and numbers), we must apply the Fundamental Counting Principle:

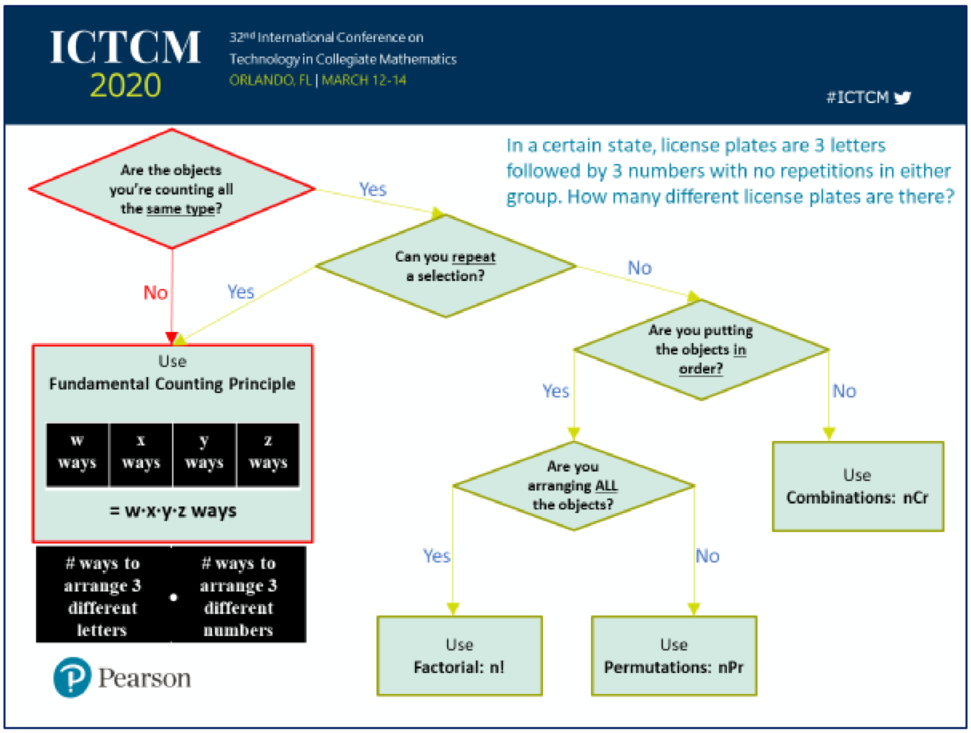


*Figure 7. Use the Fundamental Counting Principle when objects are of different types (Example 6, Step 1)*

We can then use the chart to count the number of ways to arrange 3 out of 26 different letters (Permutations):

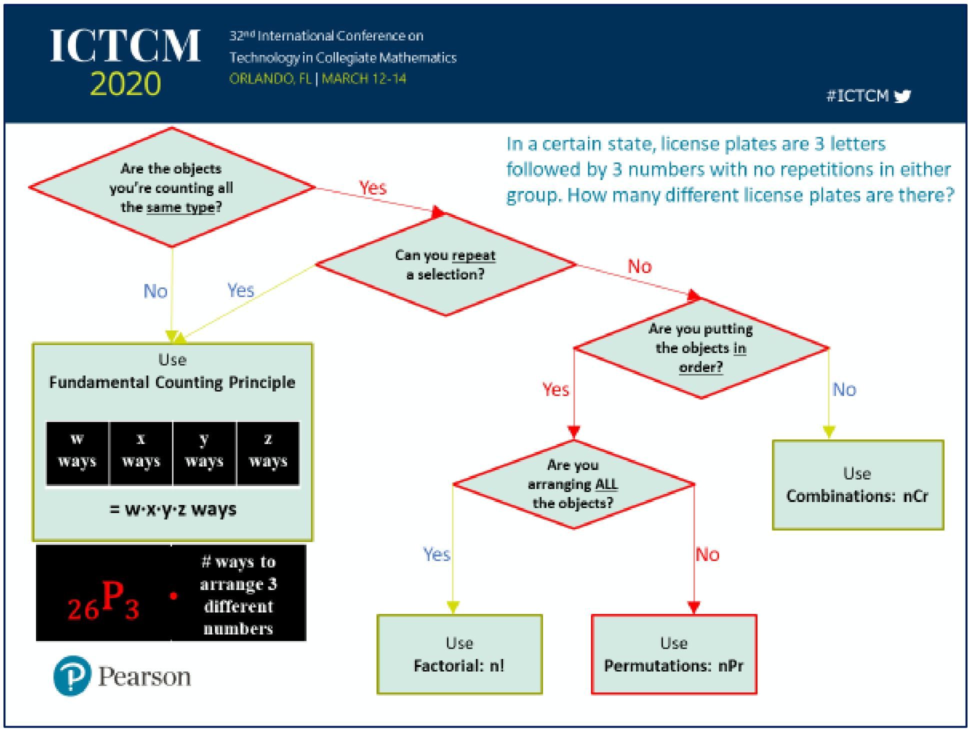


*Figure 8. Use Permutations when putting a subset of distinct objects in order (Example 6, Step 2)*

and then to count 3 out of 10 different numbers (Permutations):

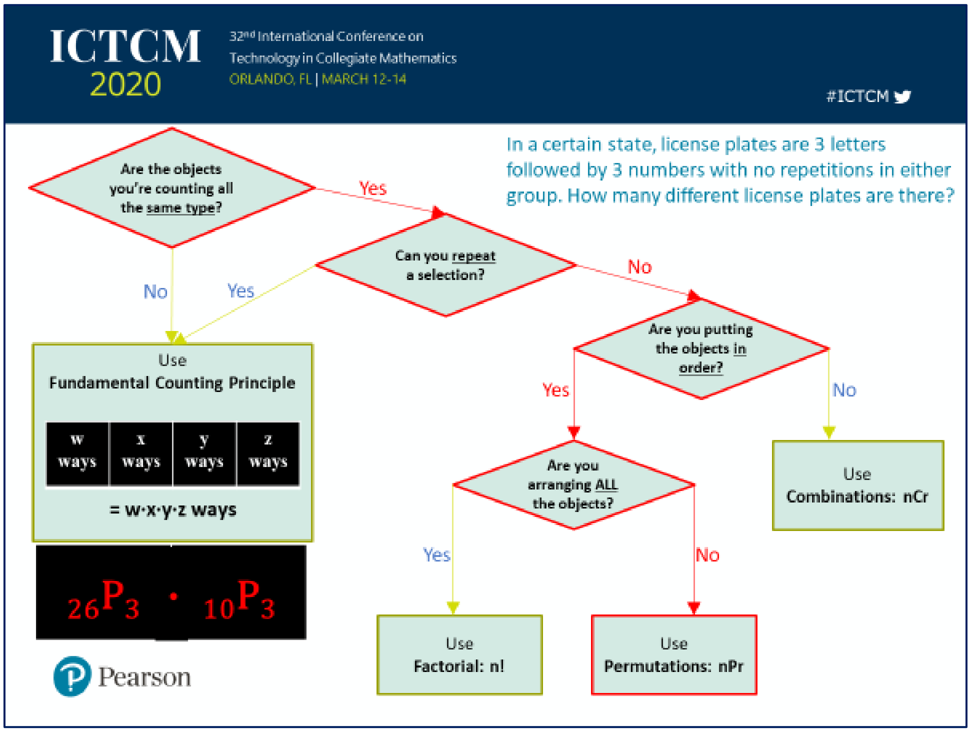


*Figure 9. Use Permutations when putting a subset of distinct objects in order (Example 6, Step 3)*

From this, we conclude that there are ${}_{26}P_3 \cdot {}_{10}P_3 = \mathbf{11,232,000}$ different license plates.

## Conclusion

For the last several semesters, I have used this decision chart in my Liberal Arts Math classes to help students solve counting problems. Feedback from students has been positive, and results on student assessments have been significantly improved. I encourage my students to use the decision chart on in-class and take-home assignments, and I allow them to use the chart on in-class exams. I believe the chart has enabled my students to be more successful in solving counting problems because it helps them organize their thoughts and ask the right questions. By making this chart available in the ICTCM Proceedings, I hope to enable the success of an even wider range of students.