PROBLEM POSING AND TECHNOLOGY: A SYNTHESIS OF RESEARCH

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Abstract

What is problem posing (PP), and how can it support depth of understanding of mathematical concepts? Problem posing is generally defined as the creation of new mathematics problems or the reformulation of existing mathematics problems. Problem posing has received increased attention in recent years in mathematics education literature. Researchers and practitioners have identified ways to incorporate problem posing into mathematics instruction and assessment to promote deeper understanding of mathematical content (e.g., Singer, Ellerton, & Cai, 2015; Silver, 1994, 2013. The purpose of this paper is to 1) identify a few PP techniques that can illuminate our students' understandings of content (or lack thereof), and 2) provide a synthesis of the literature on the role of technology in PP. The paper suggests professors can seek to incorporate problem posing with technology into their instructional practice as a means to instruct, engage, and assess students' understandings of mathematical concepts.

Introduction

My journey and interest in problem posing (PP) began early in my teaching career. I did not identify the practice formally until later in my graduate career. I sometimes provided "reversals" of questions and answers for my secondary mathematics students and I was intrigued by their responses. Some prompts seemed "simple" (such as, "The answer is $x = 3$, so what is the question?) or more complex (such as, "Write five valid equations of the form $\log_{A}(B) = 2$ by identifying integers A and B" as opposed to asking "Evaluate the expression $\log_{4}(16) = x$").

In graduate school I registered for a mathematics education course that focused on problem solving. I became familiar with Polya's (1945/2004) famous work, *How To Solve It*. Anecdotal observations engendered a curiosity about how teachers could encourage problem posing in mathematics instructional environments. I became interested in how individuals create mathematics problems and engaged in doctoral dissertation research on the topic of mathematical problem posing in middle grades mathematics (Kitchings, 2014). When I transitioned to the position of a full-time assistant professor of mathematics, I began to wonder how to engage college students in problem posing and I searched for literature to inform my collegiate mathematics instruction in the area of problem posing.

Problem Posing Literature

Calls for Increased Emphasis: What is problem posing, and might it deserve added emphasis in collegiate mathematics curricula? Problem posing is generally defined as the generation of new problems or the reformulation of previously given problems (Duncker, 1945; Silver, 1994). The phrase *problem solving* occurs regularly in discussions of mathematics curricula, especially in early, middle, and secondary settings. Additionally, some universities provide undergraduate and graduate level mathematics or mathematics education courses or seminars that contain *problem solving* in the course title (University of Georgia, Department of Mathematics, MATH 3220). Such considerations can spark debate about the role of problem solving as something integrated into mathematics curricula versus a separate entity or topic inside the world of mathematics.

Perhaps many readers are familiar with George Polya's (1945/2004; 1981) discussions of *problem solving* in mathematics as well. *Problem posing*, on the other hand, has received much less attention in the literature concerning the teaching and learning of mathematics. Polya claimed, "…the mathematical experience of the student is incomplete if he never had an opportunity to solve a problem *invented by himself* (emphasis added)" (p. 68). Singer, Ellerton, and Cai (2013) observed, "the topic of posing problems has largely remained outside the vision and interest of the mathematics education community" (p. 2). Kilpatrick observed, "…the research literature on mathematical problem solving is vast but also incomplete and poorly linked to practice, with little agreement in the field as to how problem posing and problem solving ought to be handled in the mathematics class" (2016, p. 255). Kilpatrick also claimed, "(Problem posing) deserves more consideration by researchers and teachers alike, and *information technologies* (emphasis added) may make that possible" (p. 259). Brown and Walter (2005) proposed, "…the activity of problem posing ought to assume a greater degree of centrality in education" (p. 6). Silver (2013) lamented the absence of a theoretical framework that could help explain "the relationship between problem posing and problem solving" (p. 160).

The idea of problem posing is not new. Nearly a century ago Einstein and Infeld (1938) wrote of problem formulation (problem posing):

> The *formulation of a problem* (emphasis added) is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advance in science. (p. 92)

In light of Einstein and Infeld's proposition, how often do professors of mathematics emphasize the posing of problems as an "essential" component of their mathematics instruction? Do students in the year 2020 and beyond have opportunities in mathematics courses to engage in the creativity Einstein and Infeld proposed? Did they intend such experiences for all levels of mathematics courses? How can professors help current mathematics students advance science as they proposed? The literature seems to suggest the posing of problems may not be a priority (as a whole) among teachers of mathematics at any level, but problem posing can promote additional engagement in mathematics courses at any level.

A further inspection of the term *problem posing* may also lead one to consider the characteristics of a mathematical *problem* by definition. What constitutes a mathematical *problem*? Such a discussion of a mathematics problem extends beyond the focus this paper. Kitchings (2014) provided a more extensive examination of the notion of the characteristics of a mathematical problem. Problem posing may occur with any of the following three categories of *problems*: routine exercises, traditional problems, and problems that are problematic (Schoenfeld, 1992, pp. 338–340). Additionally, the notion of problem may vary depending on the sophistication of the student involved in the problem-posing episode. Problems exist that are also known problems, and then we also expect problems exist that have yet to be discovered. Getzels (1979) questioned:

> Need problems be *found*? Is not the world already teeming with problems and dilemmas at home and in business, in economics and education, in art and in science? The world is of course teeming with dilemmas. But the dilemmas do not present themselves automatically as *problems* capable of resolution or even sensible contemplation. They must be posed and formulated in fruitful and often radical ways if they are to be moved toward solution. (p. 167)

Problem-posing situations may be classified structured, semi-structured, and free (Stoyanova & Ellerton, 1996) if the teacher is the catalyst of the problem-posing instructional episode. In addition, students may initiate a problem-posing episode in class rather than the instructor (Kitchings, 2014). Kitchings also proposed a framework for classifying the nature of problem posing episodes he observed in 88 middle-grades mathematics courses. The framework could be applied in post-secondary mathematics instructional settings as well.

The role of technology in problem posing: If the literature on problem posing is lacking in general, then the literature base must be even more sparce as regards problem posing and the role of technology. Only a few studies exist concerning the role of technology and problem posing. Even fewer studies occur in university-level mathematics courses beyond teacher-education programs.

Abramovich & Cho (2015) suggested that technology can bridge problem solving and problem posing instead of the two appearing as dichotomous: “The instructional goal of reciprocal problem posing, besides developing and assessing problem-posing skills and encouraging cooperative learning in a technology-rich setting is to highlight problem posing and problem solving not as dichotomized but as closely related mathematical activities” (p. 90). Abramovich and Cho referenced an agent-consumer-amplifier framework for using technology in problem posing (pp. 82–90). In this framework the technology serves as the agent of a mathematical activity in that a need arises for the student to construct a computational environment for solving a problem (p. 82). The technology then becomes the consumer of the activity when similar problems may be posed and solved. Finally, the technology then transitions into an amplifier when it “extends to a new dimension of problem posing, solving, and reformulating activities in a way that is hard to realize without the support of technology" (p. 83). They focused on preservice teachers of mathematics and illustrated how various technologies helped students take ownership of their learning experiences (p. 99).

Santos-Trigo & Moreno-Armella (2016) wrote, "…Digital technologies open the way to fertile reinterpretations of existing concepts, and these new forms of the concepts lead to transformations of the meanings of those concepts" (p. 206). They provided a compelling argument for the use of computer technologies as tools to encourage problem posing in secondary grades. Perhaps some of their findings are applicable in university-level mathematics courses as well, but one may only conjecture about the possible extrapolation beyond secondary mathematics settings. In fact, most other research focuses on problem posing in grades PK–12 or in preservice teacher programs at the university level. Further, most such studies tend to rely heavily on the ways in which dynamic geometry software packages (such as Geogebra or Geometer's Sketchpad) can engender mathematical curiosity and problem posing within a Euclidean geometry framework. While opportunities for rich problem-posing creativity abound using dynamic geometry, the literature base has struggled to extend beyond topics traditionally found in the geometry curriculum in grades PK–12. One exception to this observation is a study by Da Ponte and Henriques (2013) in a numerical analysis course. The study demonstrates the plausibility of using purposeful problem posing in the numerical analysis course, but it did not contain information concerning the role technology may have in encouraging problem posing in that particular context except for a passing reference to the fact that students were allowed to use graphing calculators as an option to complete the task (p. 148). I found no direct research into mathematical problem posing using technology in university-level mathematics courses otherwise.

Problem posing strategies: Brown and Walter (2005) began their discussion of problem posing by asking their readers to consider the equation $x^2 + y^2 = z^2$. They asked readers to write three questions related to the equation. They pointed out the mere presence of the equation itself tends to suggest other questions. What technologies might assist with answering newly written questions? Perhaps the reader could also question how technology might afford additional exploration and creativity to invent questions that might be difficult to frame without the use of technology? Brown and Walter also offered the "What-if-not?" strategy. In this strategy a problem (or scenario or stimulus of some type) is presented and the "What-if-not?" question is posed instead. In other words, what happens if one modifies the givens of the presented situation? For example, a "What-if-not?" strategy applied to the equation $x^2 + y^2 = z^2$ might be to modify the givens to create a new equation such as $x^3 + y^3 = z^3$, which may produce a brand new string of questions to explore. Yet another strategy they suggested is that of switching the givens and unknowns of a given context. If an instructor utilizes appropriate technology along with either the "What-if-not?" or the "reverse the givens and unknowns" of a given situation, then problem posing may occur in a powerful instructional episode.

A personal example: I do incorporate problem posing in my own mathematics instruction at the collegiate level, and I continue to search for ways to broaden my repertoire of problem-posing techniques. Perhaps the following example is a simple way to create problem posing for entry-level college mathematics students. Consider the following example of a common exchange between a student and an instructor:

Student: (Practices analyzing the characteristics of a quadratic such as $f(x) = 10x^2 + x - 6$ and then asks the instructor, "Can you give me another problem like this to practice?

Instructor: "Can YOU give YOU another problem like this to practice?"

Student: "You mean, I can just make up my own?"

Instructor: "Do you think that is reasonable to ask you to make up your own? What tools do you have at your disposal to check your work? How could you use Desmos or Geogebra?"

The student seemed genuinely surprised that he or she possessed the ability to create his or her own problem in this situation and subsequently created more problems to investigate with technology.

Conclusion

Problem posing has received increased interest in the literature base in recent years. However, much of the problem-posing literature focuses on mathematics instruction in grades PK–12, and studies that occur within university-level instruction largely focus on mathematics courses for prospective mathematics teachers. Little to no research exists on the role of technology in university-level mathematics courses.

Problem posing can lead to creativity in students, but problems are most often stimuli given to students for consumption until the next problem is given from an outside resource such as an instructor or textbook. Students have been indirectly taught that problems come from those who know more than they do, and students rarely suspect that they are capable of creating their own mathematics problems. Perhaps students who engage in purposeful, mathematical problem-posing activities may begin to increase their interests in and engagement with mathematical content. Regardless of how one may define the notion of a mathematics problem, a student may be much more likely to push to solve a problem he or she created as an individual. How many entry-level mathematics professors lament the low levels of observed student engagement in their mathematics courses? Is it possible that creating a culture of problem-posing could increase student engagement in mathematics courses? Consider the potential of teaching a prescribed syllabus, curriculum, or course by incorporating more opportunities for students to pose their own mathematics problems within the scope and context of the prescribed curriculum or syllabus. Further, how might the utilization of various technologies such as Desmos, Geogebra, spreadsheets, and other technologies prompt rich problem-posing engagement surrounding a particular mathematics topic or concept?

Instructors may express hesitancy to formally incorporate problem-posing activities or tasks due to the large amount of content required for a particular course. Problem posing need not hinder the teaching of the published curriculum. Professors can begin to seek ways to incorporate small (or large) problem-posing tasks into their lectures, group discussions, homework assignments, and assessments with online, hybrid, and in-person courses in ways that promote, enhance, or support the already-existing curriculum. Professors can perhaps select as few or as many problem-posing tasks as they are comfortable with presenting to their students in a given semester or quarter. Instruction may begin in small steps, such as asking students to provide a contextualization of a potential real-life application to a routine, non-contextualized equation such as, "Provide a possible contextualization of a word problem for the equation, $3x + 4y = 21$," or "Construct an exponential equation of quadratic-trinomial type that contains $x = \ln 4$ as at a solution." Instructors can give students opportunities to reverse the givens and unknowns and we can ask "What-if-not?" questions such as, "What if we created our own non-real number solution to the equation, $x^2 + 1 = 0$?" as a possible introduction to the complex number system. If studying geometry, we could ask, "What if we do not assume Euclid's parallel postulate?" in efforts to introduce non-Euclidean geometries?

Much more research is necessary at the university level concerning problem posing and the role of technology. Many unanswered questions remain concerning problem-posing at the university level: How can we identify a framework to study the role of technology in the teaching and learning of mathematics at the university level? To what extent do university-level mathematics students view themselves as capable of posing mathematics problems? How might formal instruction in problem posing impact student engagement in university-level mathematics course? What are the differences in how introductory or novice mathematics students view problem posing versus more mature senior-level and graduate-level mathematics students (Schoenfeld, 1982; Borko & Livingston, 1989)? How can problem posing play a role in the process of constructing rigorous mathematical proofs? How might problem posing impact students' problem-solving skills and efficacy? How might problem posing with technology impact the learning of mathematics gateway courses (such as College Algebra or Precalculus)?

Research has been lacking from the past, but problem posing is gaining interest in both the mathematics and mathematics education communities in the present. Perhaps now is an opportune time to begin to experiment with the potential of problem posing in our mathematics instructional planning, especially with the increased availability and accessibility of mathematics technologies.

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