GLOBAL DETERMINATION OF SEA LEVEL HEIGHT VIA SATELLITE ALTIMETRY

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Abstract

Global mean sea level (GMSL) is critical for addressing climate change. In 1993, NASA started collecting accurate satellite altimetry measurements of sea surface height. We will describe how NASA collects the data and how to analyze a large NASA data set to determine GMSL. Google Sheets will be used here; however, the task could be downloaded into, and accomplished in, Excel.

Introduction

Global mean sea level (GMSL) is a method that measures absolute sea level change on a global scale. In an earlier presentation, relative sea level was examined using tide gauges to show how to bring a global problem to your backyard (Sinex 2022). This provides a local/regional approach that depends on geological factors that influence tide gauges. Here we will look at satellite altimetry measurements to measure sea surface height. This is a big data project where 36 or 37 orbital satellite tracks per year are processed into an average sea surface height for the whole Earth. We will examine a data set combining four NASA missions over 28 years to see how GMSL is changing. This is a phenomenal engineering accomplishment to accurately measure the height of the sea surface via satellite altimetry with considerable data processing to derive the GMSL.

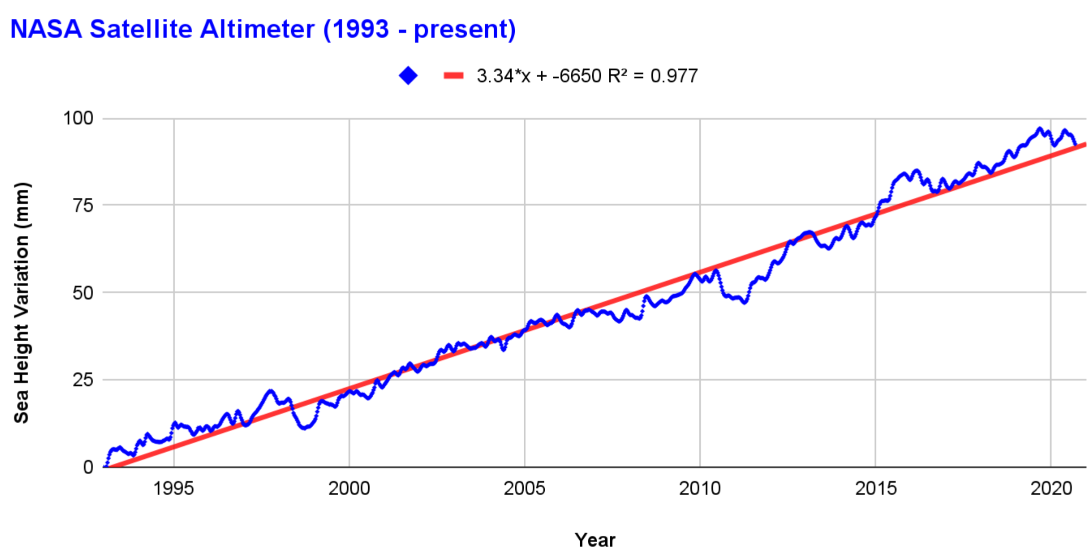
The objectives of this activity, using the processed data from NASA, are given below:

1. Examine the basics of satellite altimetry and how orbit error and altimeter error influenced the determination of sea surface height;
2. Explore an interactive timeline of NASA missions past, present, and future (uses an easy online timeline tool that is constructed through a Google Sheets template);
3. Examine NASA data and perform a number of translations on the data to allow comparisons (nice practical application);
4. Analyze NASA data of sea surface height versus time for four missions by considering linear and quadratic behaviors, goodness-of-fit as judged by r-square and residuals to determine global mean sea level and how it is changing;

5. Estimate the acceleration of sea level rise from the quadratic model; and,
6. Compare GMSL from satellite altimetry with worldwide tide gauge data (optional).

This should not be your students first mathematical modeling experience! Stacking Oreo cookies and examining measurement errors is a great start (Sinex 2013). Students need an exposure to linear regression, judging goodness-of-fit, and examining error. This project extends the concept of dealing with messy data considered by Dorsey (2021) and Gould and others (2014). This can be done as a group project with at least a pair of students.

The NASA data will be provided in a [GSheets spreadsheet]() along with a series of questions for students to address. The data of over 1000 points are plotted in Figure 1 below.



Data from: GSFC. 2020. Global Mean Sea Level Trend from Integrated Multi-Mission Ocean Altimeters TOPEX/Poseidon, Jason-1, OSTM/Jason-2 Version 5.0 Ver. 5.0 PO.DAAC, CA, USA. Dataset accessed [2020-12-30] at http://dx.doi.org/10.5067/GMSLM-TJ123.

Figure 1 - NASA Altimetry Data for Global Mean Sea Level

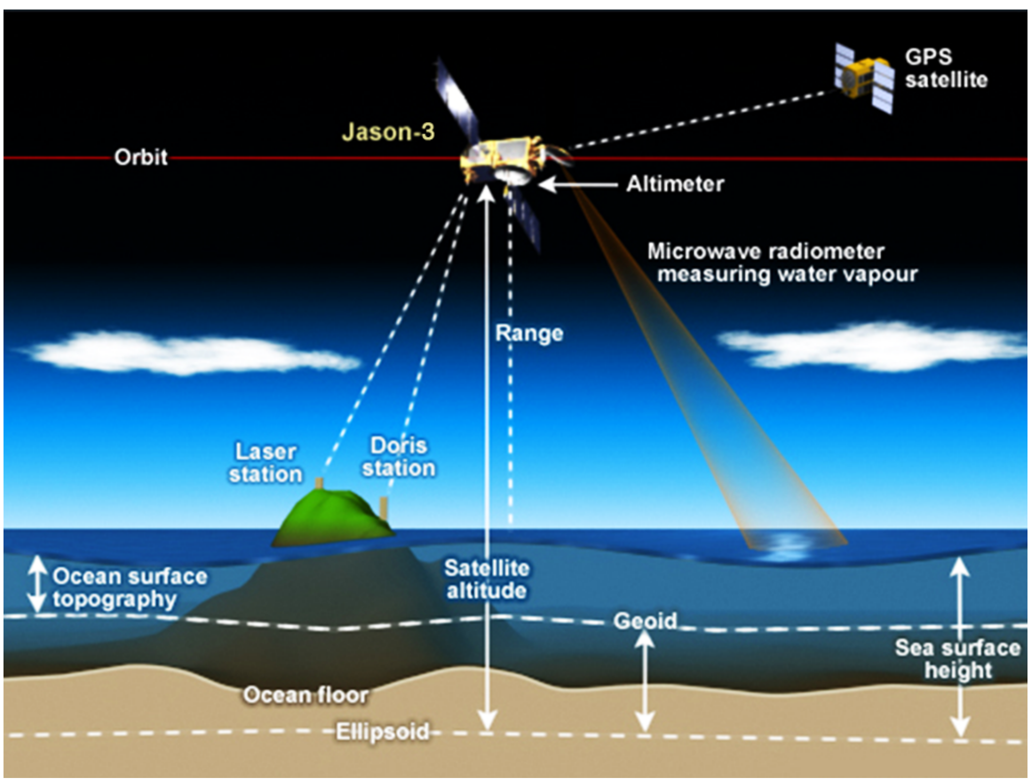
The linear regression model yields an increase in GMSL of 3.34 mm/yr (slope) with a high r-squared value for over 1000 data points. For the linear regression above, the residuals show a slight curvature (data points above, below, and above the regression line). Students may need a little practice at this (Sinex 2005, 2019). This has led to using a quadratic model and determining an acceleration in sea level rise, where acceleration is twice the quadratic coefficient, a. Students can compute results that are consistent with NASA results! The calculation of the acceleration in sea level rise is a debated topic in Earth science at the moment. We will explore a simple interactive spreadsheet model (simulation) based on kinematics (physics of the motion of objects such as the sea surface height). To obtain up-to-date results from NASA, see https://climate.nasa.gov/vital-signs/sea-level/.

The Measurement Basics of Satellite Altimetry

Since the TOPEX/Poseidon satellite in 1992, satellite altitude (1336 km) has been determined from the ellipsoid, a calculated reference surface. See Blumenfeld (2020) for a thorough discussion. The satellite uses a radar altimeter to measure the distance to the sea surface. The height (H) of sea level is measured by the difference as illustrated in Figure 2 of the two measurements:

$$H_{\text{sea level}} = H_{\text{sat. alt.}} - H_{\text{radar range}}$$

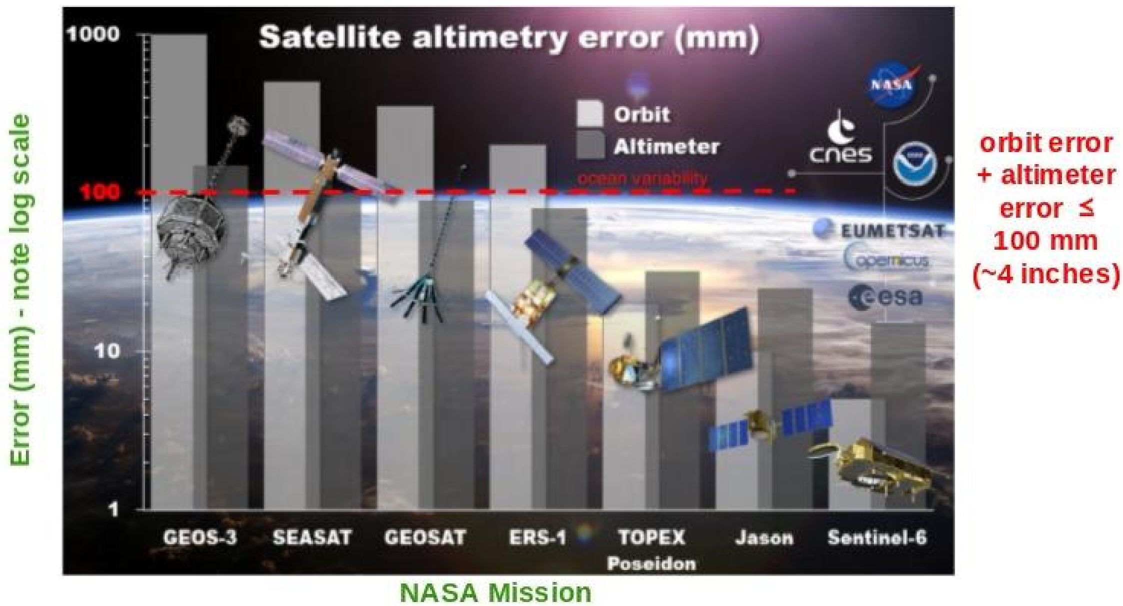
The satellites are in an orbit of 1336 km above the reference ellipsoid.



Source: https://www.nesdis.noaa.gov/jason-3/mission.html

Figure 2 - Measuring Sea Level Height via Satellite Altimetry

To be able to determine the sea level height with any degree of precision, the errors for the satellite altitude and the altimeter measurement must be considered. The sum of the two errors must be less than 100 mm (~4 inches). The bar graph in Figure 3 shows the errors for both over time for a variety of satellites (note the logarithmic scale for the vertical or y-axis). It was not until the TOPEX/Poseidon satellite in 1992 that the less than 100 mm error was achieved and has been improved with each subsequent launch. Notice that the orbital error is now less than the altitude error. Both of these errors have gotten smaller with each new generation of satellite.

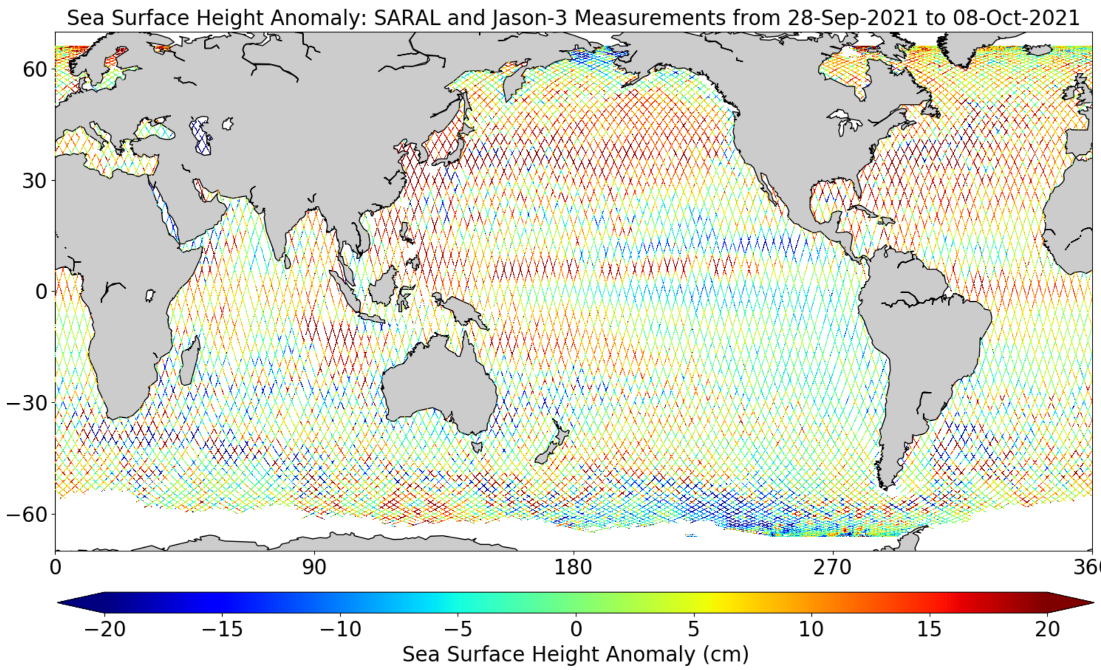


Source: modified from International Altimetry Team (2021)

Figure 3 - The Requirement for Measurement of Sea Level Height to be Viable

Now let's examine the orbital path of the satellite as it flies over the rotating Earth. Figure 4 shows the orbital coverage for Jason-3 over a 10 day period (28 September to 8 October). To get a better idea of the flight path, here is a link to a short NASA video on YouTube - click here. Jason-3 would repeat 36-37 cycles per year, and with each 10-day cycle the satellite collected over 400,000 data points over the area of the path, which gave an average sea level height, and then the position would shift 1 km for the next cycle. The cycle time is reported at the mid-cycle.

The sea level surface shown in Figure 4 is very complex (note the color scale at the bottom of the image) and is influenced by tides, winds, and coastal interactions. In each 10-day cycle over 400,000 data points are reduced to over 300,000 weighted observations and then averaged to a value of global mean sea level. This is big data and accompanying data processing!

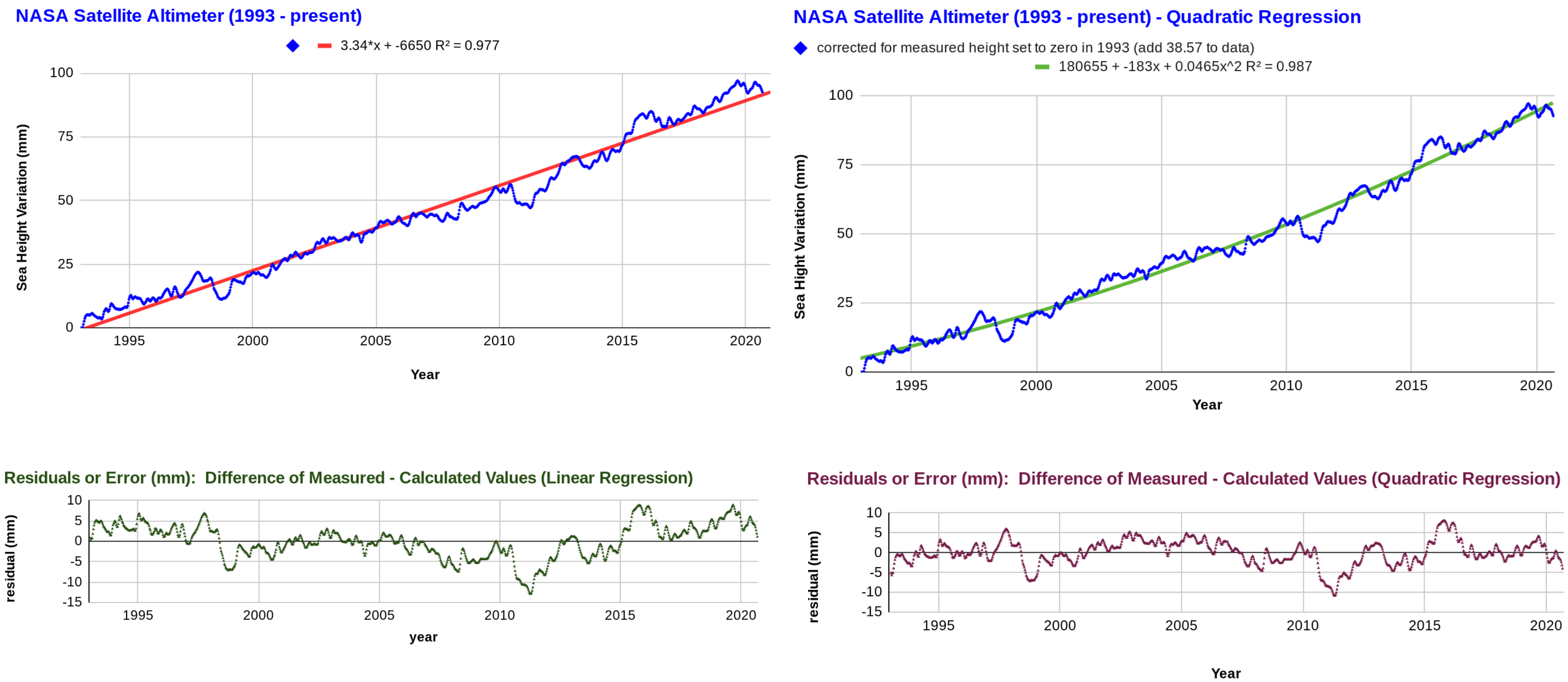


Source: https://sealevel.jpl.nasa.gov/system/nrt_images/24036_SSHA_20211008_010000.png

Figure 4 - Satellite Tracks for 10-days to Cover Earth

Examining the Satellite Altimetry Data

The students are provided the NASA data in the GSheets spreadsheet. Students will translate the sea height data so that the first 1993 datum point will be assigned a height of zero. This is a simple vertical translation. Students plot the sea height as a function of the year plus fraction of a year, and then perform a linear regression where they report the linear equation and value of r-square. The goodness-of-fit, an r-square value of 0.977, is phenomenal for the over 1000 data points and students should recognize the slope is the global mean sea level of 3.34mm/yr or 33.4cm/century. The NASA value is 3.4mm/yr as of May 2021. This is illustrated on the upper left side of Figure 5. Students calculate the height using their regression model and then calculate the residual (measured - calculated). They are asked to consider if the linear model is the best-fit model. Start this as each student giving a reason for their decision and then go to a class discussion. The residuals for the linear model (lower left in Figure 5) show a distinct curved pattern even amidst the scatter.



Linear model and residuals

Quadratic model and residuals

Figure 5 - NASA Satellite Altimeter Measurements of Global Mean Sea Level

The simplest choice to add curvature is the quadratic fit shown in the upper right of Figure 5 with a slightly higher r-square (0.987). The residuals appear to be random (lower right). The quadratic model adds a new variable to the mix. Nerem and others (2018) apply a classic physics kinematic model, which allows the acceleration, a, of sea level to be estimated as a = 2ɑ, where ɑ is the coefficient of the squared term in the standard form of the quadratic equation. This is assumed to be the average acceleration over the time interval. This would increase any predictions for the rate of sea level rise in the future. So, let's play with this model.

Simulation of the Acceleration of Sea Level Rise

The GMSL model tab presents an interactive GSheets spreadsheet simulation (Figure 6) based on the kinematic model: $h = h_o + v_o\Delta t + ½\, a(\Delta t)^2$, so a = 2ɑ from the quadratic regression. The simulation is based on the time data that was translated to set the starting year of 1993 to year 0 with a translated height of zero as well. This allows the starting numerical values of adjustable variables of initial height, velocity, and acceleration to appear in the quadratic as found by the regression. This generates perfect data and shows the result for the linear regression as well (a check box allows this to be turned off, not shown).

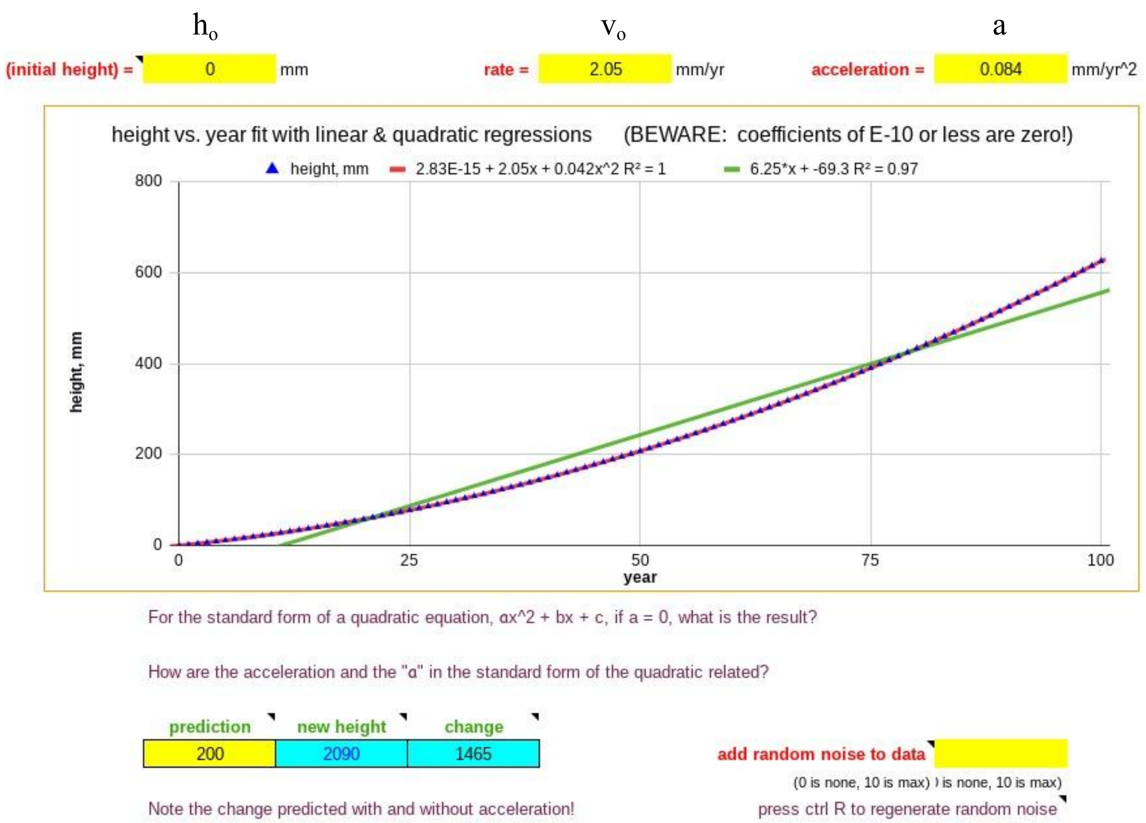


Figure 6 - Spreadsheet Simulation based on Kinematic Model from Physics

The simulation also allows students to make predictions and add random noise (error) to examine behavior.

Using the satellite altimetry data with appropriate translations, the regression results are given in Table 1. Can students see the relationship between acceleration and the “a” values in the standard form of the quadratic equation? Have them change the acceleration values a number of times. The next thing to notice is the big difference in the velocity or rate (3.34 mm/yr for linear model and 2.05 mm/yr for quadratic model).

Table 1 - Regression Results for Satellite Altimetry Data

Regression Model	Regression Equation	R-square value
Linear	$h = 3.34t - 0.851$	0.977
Quadratic	$h = 5.09 + 2.05t + 0.0465t^2$	0.987

The quadratic model also increases the predicted height, h, values over the linear model. Yes Virginia, it can get worse!

Experimenting with one noise setting (set = 1) and press control R to regenerate the noise will give a taste of how results can change!

Then again, maybe it’s not a quadratic model, but a combination of piecewise linear functions or linear and a decelerating quadratic as illustrated in Figure 7!

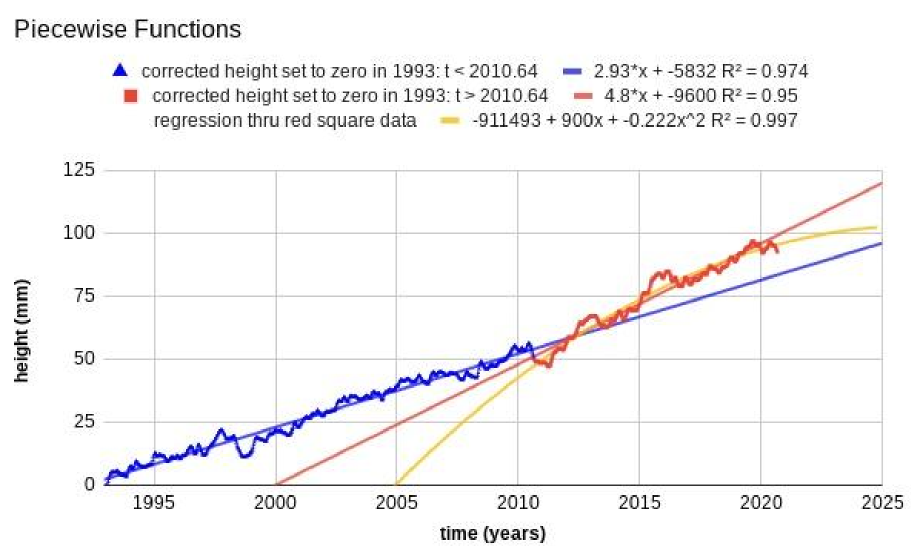


Figure 7 - Using Piecewise Functions

Comparing Satellite Altimetry with Worldwide Tide Gauge (optional)

In an earlier paper (Sinex 2022), the use of tide gauge data was discussed for determining relative sea level in your local areas. Here we examine tide gauge data that was corrected for geological changes so it could be compared with the satellite altimetry data (Jevrejeva & others 2014). Tide gauge data was culled to use data starting from 1935, since this data had lower error and had a starting height of zero. When the satellite altimetry data is vertically translated to match the height of the tide gauge data, the overlap from 1993 to 2011 is remarkable (Figure 8); however, comparing the actual differences between the two data sets is hindered by the time scales not matching.

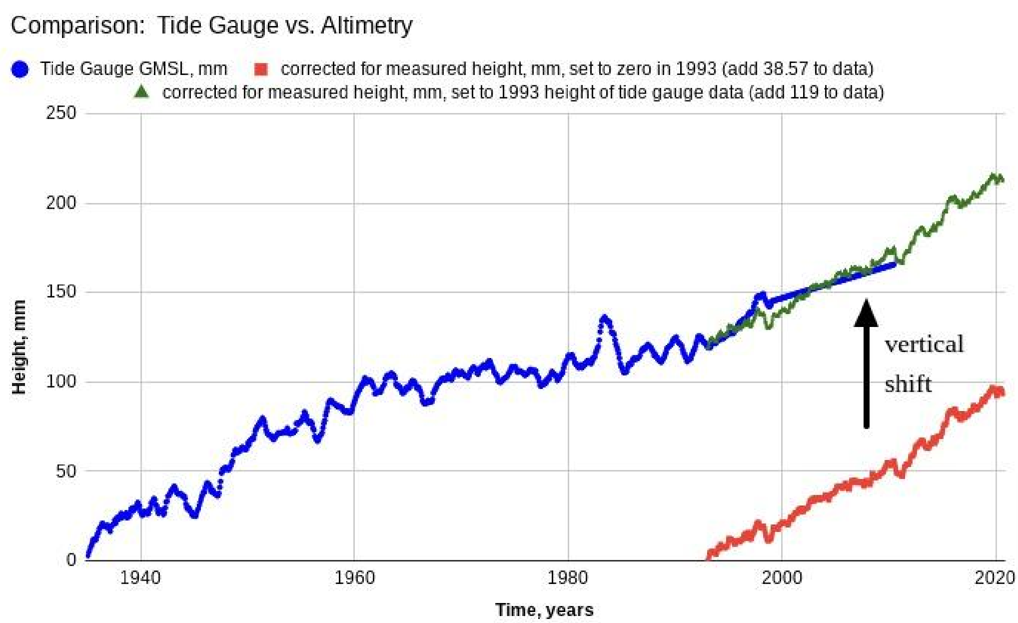


Figure 8 - Comparing Tide Gauge with Satellite Altimetry Data

Some Final Thoughts

What do students get out of this? Here is a list of outcomes.

- Dealing with a LARGE authentic data set - Big Data!
- Translations, both horizontal and vertical, to allow data comparisons
- Linear modeling and judging good-of fit by residuals (the high r-square value is deceiving)
- See scatter in data (messy data)
- Examine a quadratic model & its residuals
- Explore a spreadsheet simulation & possible acceleration of sea level rise using a simple kinematic model from physics

- Compare to actual NASA results, go to Sea Level | Vital Signs – Climate Change: Vital Signs of the Planet
- (optional) Use corrected tide gauge data to derive GMSL and compare to satellite altimetry
- Students examine a modern day environmental problem - sea level rise due to climate change

What next? NASA has a wealth of satellite data that can allow the exploration of the causes of sea level rise and examine global warming! Keep an eye out at Sea Level Change: Real-world Data Analysis and Mathematical Modeling Spreadsheet Projects, https://sites.google.com/view/ssinex/home/sea-level-change for new projects.

Acknowledgements

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S.A. Sinex (2022) Relative Sea Level Change: A Global Problem in Your Backyard, **Electronic Proceedings of the ICTCM Conference,** Vol. 34, 8pp.

<u>Classroom Resources</u>

Sea Level Change: Real-world Data Analysis and Mathematical Modeling Spreadsheet Projects, https://sites.google.com/view/ssinex/home/sea-level-change

Satellite Tracks, NASA movie (YouTube) - https://youtu.be/Y5PFl-PR88k

NASA satelite altimetry data in GSheets at https://docs.google.com/spreadsheets/d/1gjWIe3-t5DidDUfxaBazlVLWaIVsytaNV719HXvWyQs/edit?usp=sharing (Go to File > Make a Copy… to be able to manipulate data, i.e. edit)

S.A. Sinex (2021) NASA Satellite Altimetry for Measuring Sea Surface Height - Timeline at https://cdn.knightlab.com/libs/timeline3/latest/embed/index.html?source=1pTkcmsN6Gb05l2X_ehtExC3qQtKR9cDMQPfBDlXIqKA&font=Default&lang=en&initial_zoom=2&height=650

NASA Global Climate Change - Sea Level at https://climate.nasa.gov/vital-signs/sea-level (For updated data, you will need to create a free Earthdata account to access NASA's sea level data)