ENHANCING PROBLEM SOLVING BY OFF-LOADING ALGEBRAIC MANIPULATIONS

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Introduction
Over the course of a single academic year, we worked to develop and enact a set of open educational resources [OER] designed for a college-level introductory mathematics modeling course. As the title suggests, one aspect of the curriculum is that it enhances problem solving by offloading algebraic manipulations. The goal of this paper is to share the complete set of instructional materials and our journey in the development of these materials. We present the history of the project and a discussion of the pertinent underlying pedagogical framework. This is followed with a description of the mathematics course and student population for which the design occurred. We present a sample of the curriculum to illustrate connections to the pedagogical framework. The paper concludes with lessons learned; in particular, successes and ongoing challenges. We end with an invitation to critique and review our materials and encourage others to consider creating and using purposeful open educational resources as part of their ongoing professional endeavors.

All textbooks are developed out of pedagogical philosophies and epistemologies that pervade the teaching and learning process. Our previous work (Phipps & Wagner, 2017) researches the implicit messages conveyed to the student in instructional materials. Unfortunately, the textbooks we examined conveyed that mathematics is best learned through a progression of algorithmic procedures, positioning the student as dependent on the instructor for access to mathematics and problem solving. This finding is misaligned with the field's current understandings about how students learn (NCTM, 2014). We committed to develop our own curriculum materials that would reflect these understandings. Coincidentally, our statewide university system (the University System of Georgia) was simultaneously engaging in efforts to reduce the financial burden of course materials to college students. The *Affordable Learning Georgia* [ALG] initiative was designed "to promote student success by supporting the implementation of affordable alternatives to expensive commercial textbooks" (USG, n.d., para. 1). As a part of this initiative, ALG was awarding *Textbook Transformation Grants* to support faculty efforts towards developing and/or implementing no- or low-cost resource materials for college courses. We applied to and were awarded one of these grants to transform our mathematics modeling course by identifying existing OERs that reflect effective teaching and learning practices, developing OER materials when existing resources could not be found, and compiling all the materials into a cohesive course.

Acknowledging the grant is important for two reasons. First, textbook affordability is of serious concern, and the materials we developed are available at no cost to instructors and students. (The course does assume student access to the Internet, a printer, and a laptop, tablet, or smartphone.) Importantly, our materials are governed by a creative commons license, which allows others to adapt the materials to a different setting or population. Secondly, the grant enabled us to initiate the project by reducing our teaching load for a semester while we developed a draft of the materials. The project's sustainability plan ensured multiple iterations or editions. Initiatives like ALG are in place in other states. Federal legislation, namely the Affordable College Textbook Act if passed, would introduce national grant funding opportunities (Xie, 2022). We encourage others to consider the practicality of replicating projects of this scope.

Pedagogical Framework

The National Council of Teachers of Mathematics [NCTM] (2014) has identified eight effective teaching and learning practices. The eight elements of effective teaching and learning, as characterized by NCTM, are: 1) establish mathematical goals to focus learning; 2) implement tasks that promote reasoning and problem solving; 3) use and connect mathematical representations; 4) facilitate meaningful mathematical discourse; 5) pose purposeful questions; 6) build procedural fluency from conceptual understanding; 7) support productive struggle in learning mathematics; and 8) elicit and use evidence of student thinking. One of our goals was to develop materials and pedagogical practices that align with these. In our analyses of our own and others' curricula, we treat the instructional materials as agents capable of conveying messages—to both student and instructor—about student capabilities, what they should learn, and how they should learn it.

We determined that our curriculum would position the teacher as a facilitator and the student as one who must engage in thinking and reasoning in order to learn. We did not want to explain concepts to students through our own writing but rather have students think about the concepts and make conclusions with a facilitator who guides and directs students' endeavors. NCTM promotes "build procedural fluency from conceptual understanding" (NCTM, 2014, p. 42), which positions the learner as capable of meaningful problem solving prior to being taught "efficient" procedures for doing so. From this perspective, students draw upon intuitive strategies and understandings to make sense of and solve problems, albeit often inefficiently. More efficient strategies and procedures are then developed and connected to these initial understandings. According to research, significant and lasting learning experiences are predicated on conceptual understanding (Fuson et al., 2005; Simon, 2011). Our focus, therefore, was making sense of real-world contexts, and we deemphasized procedure with the assistance of technology.

In our experience, when students attempt to problem solve through intuitive methods, they often lack the algebraic skills to fully engage when equations are involved. Many students who attempt algebraic methods end up quitting when they realize they have forgotten the necessary procedures. To facilitate student development of equations to model real-world contexts as an effective strategy, we decided to offload the subsequent algebraic manipulations to technology to enable students to keep their focus on the problem at hand. Although there are many free software resources for this, we decided to use GeoGebra Classic because of its CAS and regression capabilities. It also enables students to view algebraic equations along with the resulting graphs, easily identify special points (y-intercept, extrema, etc.) of graphs, and is user-friendly.

Finally, we determined that our timeline and capabilities limited the amount of new material we could produce. Rather than reinvent the proverbial wheel, we turned to existing OER textbooks, tasks, and resources to incorporate into our curriculum. Like our own materials, most of these OERs' were governed by licenses that allowed us to modify and adapt for our own course and setting. We will now shift our discussion to an explanation of our particular course and students and provide a sample of the curriculum.

The Course and Curriculum

Our materials were designed for an introductory mathematics modeling course at tertiary institutions in the state of Georgia. Sense making of our world is particularly relevant for the mathematical models course. The course description claims an introduction to mathematical techniques to describe and explore real world data and phenomena with an emphasis on the use of elementary functions to investigate and analyze applied problems. The course is populated mostly by non-STEM majors who desire a terminating course to meet general university requirements for a three-hour credit. Many of our students struggled with K-12 mathematics and exhibit poor self-efficacy and dispositions towards mathematics. For these students, the mathematics modeling course represents the last opportunity to affect change in their views towards mathematics.

Rather than provide a bulleted list of standards or learning outcomes, we share a sample task from our materials. This task was chosen to showcase the pedagogical philosophy, style and depth of problem solving, and sense-making that is expected of students. The example is widely known as a Three-Act Task (SFUSD, 2002). Three-act tasks are generally whole-group tasks comprised of three distinct parts. The first act, part 1, purposefully stimulates engagement and perplexing context. It presents, through visual representation or story, a situation that appeals to one's intuition. For example, students may be encouraged to predict what will happen next, describe a pattern or actions, or reflect on what is happening. Most are intended to have multiple entry points (SFUSD, 2002) into the mathematics and the full story is not apparent. The second act, part 2, generates interaction. Students seek more information to apply to their problem. Not all proposed questions or information will be meaningful in the problem context, and it is up to the student to determine what is and is not relevant. The final act, part 3, reveals the answer. Students can reflect on multiple processes and solutions they or their classmates develop. The facilitator can highlight learning outcomes for the lesson through presenting and connecting the multiple strategies.

The Three-Act Task from our curriculum we have selected as an exemplar is *Fry's Ban*k by Dan Meyer (Meyer, n.d.). In this task, students are shown a video clip from the television sitcom Futurama. The main character, Fry, closes a 1,000-year-old bank account that started with a balance of $0.93 and maintained an interest rate of 2.25%. The bank teller announces his current balance, but in act 1, the words are bleeped out. Fry foams at the mouth and faints. Students in our course are left to wonder why Fry had such a dramatic reaction. The obvious question is *what did the bank teller say?* Act two of the task requires students to guess an amount that is too high and too low. This prompts students to engage in the problem so that they become invested in the actual answer. They are then prompted to determine the amount of money in Fry's bank account. It is worth noting that our students do not automatically know common interest formulas, and we do not encourage immediately substituting in values. Our approach is to pose scaffolding questions and point to generalizations. How much would Fry's account be worth in 1 year, 2 years, 3 years, etc.? What does it mean for interest to earn interest? As an extension, when will Fry's account be worth 1 trillion dollars? Act three centers on checking your work and validating appropriate strategies. Sharing within a community of learners is essential in the development of communication skills and self-efficacy.

To see how the Fry's Bank task aligns with NCTM's (2014) effective teaching and learning practices, we will consider each component in turn.

- **Establish mathematical goals to focus learning.** Fry's Bank task is embedded in a week-long lesson plan that was designed to meet several predetermined learning goals. Before selecting this task, we had outlined content goals and process goals. For example, one content goal was, *students will connect the multiplicative rate of change of exponential functions with 1-r, where r represents the percent increase/decrease.* A process goal was *students will justify explanations, interact productively with others to solve problems, and determine reasonableness of their answers to problems.* Additionally, we had identified and articulated why these goals were important, how they were related to prior learning, and where the math ideas were going.
- **Implement tasks that promote reasoning and problem solving.** We selected Fry's Bank task because we believed that it offered opportunity for reasoning and problem solving. In addition, it built on previous learning and was engaging.
- **Use and connect mathematical representations.** Students employ various strategies to determine a solution to this task. By far the most common is the use of a table. It quickly becomes tedious, and students recognize that computing 1000 values is untenable. They are encouraged to look for patterns they can use. Some students try placing points into GeoGebra and experimenting with regression to find a solution. Others identify a pattern in the table and connect it to their previous learning about exponential functions, in which subsequent values are calculated using a common multiplier. In the discussion that follows, these representations are presented and connected. For example, finding 2.25% of a number and then adding it to that number is the same as multiplying by 1.0225, and we can see this is a common multiplier. The growth can be seen with the graph representation. Connections are made between the graph and the table of values that many students started off using.
- **Facilitate meaningful mathematical discourse.** Students typically are actively engaged with each other in this task, discussing various strategies. A whole-class discussion afterwards relies on students sharing their strategies and understandings.
- **Pose purposeful questions.** Our materials anticipate student solutions and misconceptions and provide scaffolding questions facilitators can use to keep student groups working productively.

Build procedural fluency from conceptual understanding. Importantly, students are not provided an “equation” prior to working this task. If they had been, it would become a less than five-minute task. Although it can be argued that finding the solution to problems in an efficient manner is desirable, in our experience students usually forget these equations and are therefore at a loss in situations that require them. By presenting the task in the way we do, students develop conceptual understandings by building upon their previous knowledge. In the follow-up to the task, students apply this understanding to develop a procedure that will work in any similar situation.

Support productive struggle in learning mathematics. Students often have been preconditioned with a message that there is a “right” way to solve math problems (Hopkins, 2014) and can become frustrated when a problem takes a long time to solve. We choose tasks like Fry’s Bank and implement them in the way we do to counter this damaging narrative. The entertaining nature of the task serves to ease the tension.

Elicit and use evidence of student thinking. Because students use their own strategies and intuitions to approach the task, facilitators are able to assess student thinking and build the culminating discussion with that in mind.

Reflections and Lessons Learned

Five years after our first draft of the curriculum, we continue to revise and refine the materials. Throughout this process, we have noted successes, hurdles, and continuing challenges. We share some of our lessons learned to inform those who are interested in developing course materials.

Developing a course is a time-consuming task. Our initial draft took months to produce, even with a grant that enabled us to devote a greater amount of time to this endeavor. Attending to requirements of the Americans with Disabilities Act [ADA] required us to learn such things as how to add closed captioning to videos, the functionality of different heading styles, and adding alternative text to PowerPoint images. It was also important to educate ourselves about copyright law and open-source licensing to be sure we were using nonoriginal content appropriately.

In our experience, offloading algebraic manipulations to GeoGebra has had a positive impact on student learning and perhaps their mathematical dispositions. Students have used the technology in creative ways to solve problems, even uncovering and using software commands of which we had been unaware. More commonly than not, students in our classes adopt a collaborative classroom community that organically evolves into groups of students discovering and sharing effective strategies and ways to interact with and use GeoGebra.

GeoGebra is available across a variety of platforms: personal computers, laptops, tablets, and smartphones. A challenge we encountered early on was that the commands and capabilities varied, depending on the type of device being used. Tablet and smartphone users had different capabilities than did those using laptops. We finally determined that the variation came from whether the user was accessing the program via web browser or an app. We now caution students against downloading the apps, accessing the program only via web browser. This has enabled us to standardize instructions for using GeoGebra and has simplified the process to a single web site, whereas multiple apps had previously been necessary to include all the capabilities we use in our course.

A recent challenge has emerged. In previous years, attendance was robust and consistent. However, this academic year, after the resumption of face-to-face instruction, we have noticed an increase in absence rates in our classes. This does not appear to be unique to our course; we have heard similar concerns from colleagues and instructors of other courses. It is unclear whether this phenomenon is due to COVID-related quarantines, a new mindset of students who have become accustomed to "learning" from home, or some other reason. It is also unclear whether this will be a continuing issue. However, spotty attendance can impact the development of a classroom community as well as students' opportunity to learn.

Completing a first draft of curriculum was just the beginning of course development. We still uncover errors in our work that must be addressed in a new edition. We also continue to learn more about how our students develop understandings of the content. These understandings then inform changes to the materials. We are coming to realize that our curriculum will never be fully complete, as there will always be ways to improve upon it. For this reason, we continue to seek feedback on our course and materials. Unfortunately, continuous updating is not possible. We can adjust materials with the production of a new edition, and we would like each edition to reflect meaningful changes. We therefore invite readers to access, adapt, implement, and evaluate our curriculum. Feedback may be directed to either author at the email addresses provided in the heading of this paper. We thank you in advance for your interest. The materials may be accessed here: https://oer.galileo.usg.edu/mathematics-ancillary/17/

References

Affordable Learning Georgia. (n.d.). *About us.* Retrieved from https://www.affordablelearninggeorgia.org/about/about_us

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. *How students learn: History, mathematics, and science in the classroom*, 217-256.

Hopkins, E. (2014). Math: Not Just One Answer and One Way. Kids Included Together. Retrieved from https://www.kit.org/math-not-just-one-answer-and-one-way/

Meyer, Dan. "Dan Meyer's Three-Act Tasks." http://threeacts.mrmeyer.com/frysbank/. Accessed 5 May 2022.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

Phipps, M. & Wagner, P. (2017). The hidden curriculum in higher education mathematics modeling textbooks. In T. A. Olson & L. Venenciano (Eds.), *Proceedings of the 44th annual meeting of the Research Council on Mathematics Learning* (pp. 137–144). Fort Worth, TX.

San Francisco Unified School District (n.d.). 3 Act Tasks. Retrieved from https://www.sfusdmath.org/3-act-tasks.html

Simon, M. A. (2011). Studying mathematics conceptual learning: Student learning through their mathematical acitivty. In L. R. Weist & T. Lamberg (Eds.), *Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 31–43). University of Nevada, Reno.

Xie, Dan. (2022, March). Statement-New Federal Legislation to Make College Textbooks More Affordable. Student Pirgs. https://studentpirgs.org/2022/03/10/statement-new-federal-legislation-to-make-college-textbooks-more-affordable/