GEOGEBRA, PHOTOGRAPHY, AND PICTURE BOOKS: A MEANS TO END MATH ANXIETY

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Abstract

Math anxiety is a real issue and many young people are confronted with this when learning math at many grade levels K-College. This presentation and paper focuses on using photographs inserted into GeoGebra and exploring a wide variety of math objectives related to the Common Core State Mathematics Standards (CCSS) which will motivate students to learn math and minimize math anxiety. The topics will explore the math that surrounds us in the real world thus creating a connection between the abstract math and the life experiences. When math has a purpose, then students are willing to spend time exploring and understanding the math concepts presented. Real-life photographs can easily be inserted into the GeoGebra and can provide the basis to observe relationships with different and similar shapes. Emerging technologies such as GeoGebra can assist in motivating young learners to enjoy learning mathematics while also addressing math anxiety and attitudes towards the subject. The presentation/paper will teach math educators how by inserting photography into the GeoGebra software, math teachers can better explain math concepts and make the teaching and learning of math more real-world and relevant to learners, like using picture books to teach math objectives. In a world where STEM education has become so important, it is critical that we motivate and turn young people onto math through technology like GeoGebra. Online websites and resources for addressing math anxiety and attitudes will also be shared and included in the appendix of the paper.

Key Words:

Teaching Mathematics, GeoGebra, Photography, Technology, Common Core Mathematics Standards, Picture Books, Math Anxiety, Real-world Connections
Introduction

Mathematics teachers can better reach their students and show them how math surrounds us by using photography and GeoGebra while teaching math. In today’s high tech world, students need to be proficient in Science, Technology, Engineering, and Mathematics (STEM) fields. As endorsed by the National Council of Teachers of Mathematics (NCTM, 2000) and stressed in the new Common Core State Standards (CCSS) in Mathematics, it is critical that we teach using technology, address attitudes and anxiety toward math, and make the math that students are learning relevant and meaningful. Often, it may be best to start teaching young people geometry first as opposed to numbers, which are considered more abstract and difficult to learn. Geometry is one of the most concrete branches of mathematics and focusing on this first can benefit students’ whole view of mathematics and their attitudes towards learning it. Today teachers also need to be cognizant and checking for attitudes and dispositions toward learning mathematics, as math anxiety is an issue in today’s classrooms. This paper looks at ideas for teaching mathematics with the use of technology and photography using the free dynamic mathematics software, GeoGebra, to help teachers create mathematically confident young people.

Math Anxiety: The Need to Check for Mathematical Dispositions in our Classrooms

Today math anxiety is a common problem in many mathematics classrooms. Richardson and Suinn (1972) originally defined math anxiety as “a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations” (p. 551). Mathematics anxiety is the "irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations" (Buckley and Ribordy, 1982, p. 1).

There has been much research done on math anxiety since the 1970s (Richardson & Suinn, 1972). Math anxiety still continues to plague our society and affects our young peoples’ success and achievement with the subject (Finlayson, 2014; Quander, 2013). Quander feels that elementary teachers need to help prepare students to be lifelong learners and develop a productive mathematical disposition so that they are prepared for future schooling and eventual careers. Math anxiety can impede not only mathematical performance but also interest and then career choice and many decisions in life (Furner & Berman, 2004). The idea of looking closely at math anxiety levels, motivation to learn mathematics, and using technology like GeoGebra to teach and motivate students is critical today in a world of STEM and also can impact achievement goals of the learners (Gonzalez-DeHass, Furner, Vásquez-Colina, & Morris, 2017; Furner & Marinas, 2016).

As part of the NCTM Standards (1989), the NCTM believe that mathematics teachers need to assess students’ mathematical disposition regularly regarding: checking for
confidence in using math to solve problems, communicate ideas, and reason; flexibility in exploring mathematical ideas and trying a variety of methods when solving problems; willingness to persevere in mathematical tasks; interests, curiosity, and inventiveness in doing math; student ability to reflect and monitor their own thinking and performance while doing math; and value and appreciate math for its real-life application, connections to other disciplines and cultures and as a tool and language. NCTM has set the stage since the late 1980’s in making educators check for dispositions and attitudes toward mathematics part of the assessment of the learner.

In research from Jackson and Leffingwell (1999), it was found that in their study only seven percent of the population reported having positive experiences with studying mathematics from kindergarten through college. The study cited that there are numerous covert (veiled or implied) and overt (apparent and definite) behaviors displayed by the math instructor in creating such given math anxiety in students. Behaviors such as difficulty of material, hostile instructor behavior, gender bias, perceptions of uncaring teachers, angry behavior, unrealistic expectations, embarrassing students in front of their peers, poor communication and language barriers, the quality of instruction, and evaluation methods of the teacher. Math instructors' behaviors and teaching methodologies can be hurtful and negative to students learning math. Students often say: "I like the class because of the teacher" because the teacher knows how to present developmentally the subject matter, creates a learning environment conducive to learning with compassion, has high expectations for all students without regard to gender, race, or language barriers, and uses a variety of assessment methods and teaching styles to better reach all students to address math anxiety (Chernoff & Stone, 2014; Dowker, Sarkar, & Looi, 2016).

Research by Furner (2007) in synthesizing math anxiety treatments, it was found that there are two distinctions to math anxiety: prevention and reduction and there are distinct strategies and methods to address each is different ways. It has been found that there are three ways to prevent math anxiety: 1). Using “Best Practice” in mathematics such as: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math, problem-solving approach to instruction, content integration, technology, assessment as an integral part of instruction, etc.; 2). Incorporating the NCTM and State/Common Core Math Standards into the curriculum and/or instruction; and lastly, the importance of discussing feelings, attitudes, and appreciation of mathematics with students. This same research found that there are three methods to reduce math anxiety: 1). Psychological Techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and discussions; 2).Once a student feels less fearful about math, he/she may build their confidence by taking more mathematics classes; and 3). Most research on math anxiety reduction has shown that until a person with math anxiety has confronted this math anxiety by some form of discussion/counseling no “best practices” for teaching mathematics will help to overcome this fear of the subject (Furner & Duffy, 2002).
It can also be very beneficial to provide students with a math attitude surveys at the beginning of each school year or course and also to read the book, *Math Curse* (Scieszka & Smith, 1995), to get students to talk about their true feelings toward math, surveys and biblio-therapy are both effective forms of starting the process of opening up and getting inner feelings out young people may have about mathematics or unpleasant past experiences. It is recommended that mathematics teachers survey their students at the beginning of a school year to check for their students’ dispositions toward mathematics. There are two good online surveys that test for math anxiety and may be useful to classroom teachers as follows are: Mathpower ([http://mathpower.com/anxietytest.htm](http://mathpower.com/anxietytest.htm)) and Mathipedia ([http://www.mathipedia.com/student-math-anxiety-test.html](http://www.mathipedia.com/student-math-anxiety-test.html)), both of these websites offer online tools for teachers and students to be able to take a short survey to assess their overall dispositions toward mathematics.

Math Teachers during the school year while teaching mathematics should use some advantageous instructional methods which are advocated now for teaching mathematics using the **Concrete-Representational-Abstract (CRA)** Model for teaching mathematics as follows: First educators need to start with the Concrete using hands-on manipulatives like Geoboards, then secondly, they must move to Representational models in diagrams (or use Virtual Manipulatives like NLVM at: [http://nlvm.usu.edu/](http://nlvm.usu.edu/)), and lastly, connect to the Abstract symbolism where student understand and function at an abstract level completely (GeoGebra software works well at: [http://www.geogebra.org/cms/en/](http://www.geogebra.org/cms/en/)). The CRA Model is really the bases for the best practices pedagogy for teaching mathematics starting with young people, but should also be used at all levels of math instruction.

Today learning connections are often made when we teach math using such things as technology like GeoGebra and photography. Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors, although the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The photography project had been used in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. The project found student interest and motivation were peaked when photography was part of the instructional strategies to teach new material while making meaningful connections to the math using the photography. Jones (2012) also in her book, *Visualizing Mathematics*, discusses how teachers need to help students visualize and create representations of their math understanding so to turn them on to the subject. Beilock & Willingham (2014) in their research have found that math teachers can help to address and reduce math anxiety. The author believes by using technology like GeoGebra along with the photography teachers can make better connections and students are going to be more highly motivated to learn math (Furner & Marinas, 2016; Marinas, Furner, & Escuder, 2016).
Common Core State Standards (CCSS) as They Relate to using GeoGebra

Most schools and states today are adhering to the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) which can be found at: http://www.corestandards.org/ When math teachers relate real-world problems through the use of dynamic technology like GeoGebra and connecting them to photography to make important connections in math, our learners recognize that geometry and shapes/mathematics surround us. GeoGebra is an ideal piece of math software to use to teach many of the Common Core or State Standards today. Furner and Marinas (2016) offer many premade GeoGebra activities that math up with the Common Core Math Standards and with many more at www.matharoundus.com.

Using Technology in the Teaching of Mathematics in Today’s Classrooms

Using technological tools is critical in today’s world. Our students need to learn to excel at higher levels of generalization, model and solve complex problems, and focus on decision-making and reasoning (National Council of Teachers of Mathematics (NCTM) 1989, 2000, 2006). NCTM believes that mathematical power can arise from technology, which includes increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future. The President’s Council of Advisors on Science and Technology (PCAST) (Holdren, Lander, & Varmus, 2010) published an executive report with specific recommendations to government leaders given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades. One major recommendation is to recruit and train upwards of 100,000 new STEM middle and high school mathematics teachers over the next decade that are able to prepare and inspire students to have strong majors in STEM fields and strong STEM content-specific pedagogical preparation for such fields. PCAST believes that teachers are the most important factor to address in ensuring excellence in STEM education of future young people for our nation. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Educational Technology Standards for Teachers [NETS*T], 2008), teachers’ ineffective use of technology has been reported in the literature. A major reason that is frequently mentioned is that teachers are not trained in utilizing and employing emerging technologies in the classroom within the subject context on a regular and consistent fashion. Hwang, Su, Huang, & Dong, (2009) found that by combining virtual manipulatives and software like GeoGebra along with whiteboard, educators can better model problems, help students understand and solve the problems while reaching higher levels in the teaching of many mathematical ideas in the math curriculum.
Using GeoGebra

The software, GeoGebra, is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus in one easy-to-use package (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2009). This open-source dynamic mathematics software that can be downloaded free and accessed immediately at: http://www.geogebra.org/cms/en/info. There are no licensing issues associated with using GeoGebra, this then allows students and teachers the freedom to use it both within the classroom and while at home or on the go. GeoGebra has a large international user and developer community with users from 190+ countries it has currently been translated into 55 different languages.

Some research by Fahlberg-Stojanovska, & Stojanovski (2009), they discovered that using GeoGebra is motivating for students and helps them learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) established the importance to integrate both concrete and virtual manipulatives into the math classroom, such as representational models like GeoGebra. Furner & Marinas (2007) found that young people could easily transition from the concrete when using manipulatives like geoboards to the abstract when using geometry sketching software like GeoGebra. GeoGebra may have been primarily intended for mathematics instruction at the secondary and college levels; today it is even now introduced at the elementary math levels as well. The Appendix A provides online websites on resources related to GeoGebra.

GeoGebra may be used to show how mathematical equations can be applied to everyday objects. Aydin & Monaghan (2011) in their research feel that math teachers need to explore the potential for students to "see" mathematics in the real world through "marking" mathematical features of digital images using a dynamic geometry system like GeoGebra. Mathematics teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to quickly use GeoGebra in their classrooms.

Research using GeoGebra has been described as raising the enthusiasm for the effective and thoughtful application of technology to the mathematics teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). Observations of participants in schools and during the summer workshops are also cited as evidence for providing such motivation for learning. GeoGebra was also credited with changing math teacher instructional habits. Two specific features were referenced as creating this change: 1) that it is an award winning software system, and therefore has admirable features, and 2) that it provides an effective pedagogical model for math teachers.

Mishra and Koehler (2006) found that math teachers need to know how to represent math content while using technology in the teaching of mathematics. They coined the term:
Technological Pedagogical Content Knowledge (TPCK) which they feel is the basis of good teaching while using technology and that this requires not only content knowledge and/or pedagogical knowledge, but also an understanding of the representation of concepts using such technologies, how to teach these math concepts using technology, knowledge on the challenges their students may face when presented with this new pedagogy, and how technology can be used to build on existing knowledge and develop new mathematical knowledge. Research from Scandrett (2008) found that math teachers need to always start by using concrete models in geometry using manipulatives like geoboards, which provide a concrete model of understanding. Rosen & Hoffman (2009) have found that teachers need to connect students understanding from the concrete to abstract and using virtual manipulatives and software like GeoGebra better help make those connections to representational models connecting the concrete using geoboards to something even more abstract in understanding. With the availability of GeoGebra, teachers are able to make graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize and explore relationships and patterns. This is consistent with the CRA (Concrete, Representational, and Abstract) Model for teaching math which is currently the most advocated and employed model in today’s classrooms for reaching students as they learn and understand mathematical concepts. Technology makes all of this possible for learners quickly.

So Why Should Math Teachers use GeoGebra as part of their instruction?

In reviewing the research on teaching using GeoGebra, there are many reasons to use GeoGebra some of which are: that it is free to download and use from GeoGebra.org; it is an up and coming dynamic teaching tool in our schools today, dynamic for learners; it is user-friendly for students and teachers; it lends itself well to connection from the hands-on Geoboards to virtual Geoboards to something even more abstract; it is a software that provides many resources and teaching tools at its wiki for educators at: [http://www.GeoGebra.org/en/wiki/index.php/Main_Page](http://www.GeoGebra.org/en/wiki/index.php/Main_Page); GeoGebra may be used for primary-aged students through college: and lastly it is fun, easy to use, and students learn a lot about geometry, algebra, measurement and beyond by using this dynamic learning tool.

Math educators may ask why it is important to make connections and excite students about learning math while using GeoGebra? To answer this educators will find that when using GeoGebra, educators will be able to: show a purpose for math; develop relationships between math concepts and shapes and ideas; the software will show practical applications to math in life; it employs innovative teaching in the classroom; it stimulates through photography/modeling; it employs emerging technologies in math with the real world application; and it can address math anxiety so students feel confident for any STEM field when they graduate from school.
Additional help with math anxiety and its research can be found at Professor Freedman Provides Math Help at: [http://www.mathpower.com](http://www.mathpower.com) and Mathitudes Online website at: [http://www.coe.fau.edu/centersandprograms/mathitudes/](http://www.coe.fau.edu/centersandprograms/mathitudes/)

A famous quote from W. V. Williams (1988) is a reminder of how critical it is to teach for understanding making things as hands-on and real world as possible: “Tell me mathematics, and I will forget; show me mathematics and I may remember; involve me...and I will understand mathematics. If I understand mathematics, I will be less likely to have math anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math-anxious students for generations to come” (p. 101). Today math teachers need to break the cycle of math anxiety and address it, and by using GeoGebra and making connections with photography, teachers can better connect the math they teach to students and their understanding while using emerging technologies like GeoGebra.

Furner (1999) also made these related observations as they relate to the importance of mathematics confidence: “If math teachers do something about helping their students to develop their confidence and ability to do math, we can impact their lives in a positive way forever.” In addition, “Our students’ careers and ultimately many of their decisions they will make in life could rest upon how we decide to teach math. We must make the difference for the future of our kids in an ever growing, high-tech, competitive, global world which depends so heavily on mathematics.”

In Conclusion

Young learners intrigued by technology will construct and investigate geometric shapes and many math ideas with GeoGebra and will start enjoying math and have less math anxiety in our STEM World that we now live in. By using emerging technology like GeoGebra and photographs, our young learners who are often intrigued by technology will construct and investigate geometric shapes with GeoGebra and start liking and enjoying math more and will optimistically be less math anxious in the years to come so to pursue any STEM field of their liking. There are many free resources for math teachers Grades K-12 to download which are in Appendix A.

References


Furner, J. M., & Berman, B. T. (2004). Confidence in their ability to do mathematics: The need to eradicate math anxiety so our future students can successfully compete in a high-tech globally competitive world. Philosophy of Mathematics Education Journal, 18 (1), Pages 1 of 33.


National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) (2010). *Common core state standards*


### Appendix A: GeoGebra and Math Anxiety Websites and Resources for the Math Classroom

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