What is 'Infinity', where 'Infinity' is and how to get there – A spreadsheet technology approach

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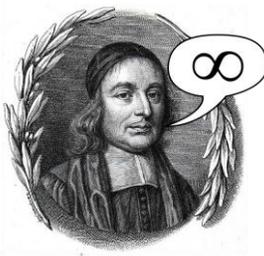Claflin University, Orangeburg, SC 29115

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Abstract

It was observed in teaching calculus classes that for many students the understandings of mathematical operations involving infinite number or infinity (∞), particularly in the limit process, were very confusing. In many mathematical operations inadvertently they considered the infinite number as a real number. This confusion is modestly explained numerically and graphically in this paper using the function capabilities of Microsoft Excel spread sheet. From the table values and graphs of Excel technology students could visualize the approaching techniques towards the infinity numerically and graphically. The graphs of the limiting values of functions make clearly to understand the concept of infinity or a place which doesn't exist in real life situations.

Introduction

Infinity refers to something without any limit, and it is mostly used in Mathematics and Physics. According to Korner[1] Infinity is not a number, and trying to treat it as one tends to be a pretty bad idea. At best you're likely to come away with a headache, at worst with the firm belief that 1 = 0. In Mathematics, "infinity" refers to not a real number i.e., "an infinite number" or a place doesn't exist in real life situations. As Spencer[2] specified *does there **e**xist any number system which, as well as including the familiar numbers we are used to, also includes an "infinity" concept?* The answer is **no**; In areas such as calculus, one often speaks of a sequence like 1, 2, 3, 4, . . . as "converging to infinity". Is this just a convenient phrase, or can there actually exists an object "infinity" that this sequence is converging to? Infinity is both a technical specialized topic in Mathematics. Its Mathematical symbol is ∞, which looks like sleeping eight. This symbol is given by John Wallis[3] (John Wallis 1616 – 1703), which indicates an unending curve. In real analysis, the symbol, "∞" refers to unbound limit. However, we can simply say, infinity is endless or boundless. It has no end. In

other words infinity is unbounded means as keep on going without bound. It is not considered as a real number in mathematics but we predict the outcome after reach the limit until we can.

Problem Statement

After many years of teaching various levels of mathematics classes in a college level, I have noticed that most students particularly in my calculus classes have a misconception of infinity and its operation in that they assume it is a real number. Their misconception answers are $\infty - \infty = 0$, $\infty / \infty = 1$ etc.

Methodology

In order to diminish their misconception, an instructional technology is used to motivate students and to increase their success rate in Calculus and beyond. An important instructional method used in this work is visual learning- regardless of skill level or background. It is based on the fact that in a typical classroom, students vary in their academic abilities, learning styles, personalities, interest, background knowledge and experiences, and level of motivation for learning.

What does Spreadsheet Technology?

Microsoft Excel is a spreadsheet program that comes packaged with the Microsoft Office family of software products, which is mostly available in every computer in the schools and colleges. The new Microsoft Office Mobile apps designed for the iPhone , iPad , and iPad Pro superscript have the familiar look and feel of Office with an intuitive touch experience. Every student now have either one of laptop, I pad, or smart phone. Excel focuses on making it easy for users not only in adding numbers but also for tedious mathematics manipulations. It is a simple matter for the user to specify formulas in the spreadsheet that interact with these numbers in various ways. The spread sheet has 65536 rows and 256 columns. Once you reach the 10000 or 20000 rows in any calculations, you can assume or conclude that in the mathematical application that you are approaching the unique place of infinite number. According to the Oxford Journals[4], electronic spreadsheets make the process of inputting information, retrieving information and performing calculations faster and more efficient. They are also easy to share; even large spreadsheets can be compressed to a few megabytes.

In this paper, two very common problems in differential calculus are discussed, where the x-variable approaches infinity in that the direct substitution method becomes infinities divide each other where the misconception is equal to one.

The problems discussed are:

(1) $Limit_{x\to\infty} \dfrac{Sin(x)}{x} = 0$

(2) $Limit_{x\to\infty} (1+\frac{1}{x})^x = e$

Problem No. 1:

The proof of $Limit_{x\to\infty} \dfrac{Sin(x)}{x} = 0$ in differential calculus course is demonstrated by the

Squeeze Theorem in that $\frac{-1}{x} \le \frac{Sin(x)}{x} \le \frac{1}{x}$ where $Limit_{x\to\infty} \dfrac{-1}{x} = Limit_{x\to\infty} \frac{1}{x} = 0$. But, most students by direct substitution find the very frequent incorrect answer, 1. This misunderstanding is explained to students, as shown in the Figures 1, 2, & 3, using the Excel functions technology to create the table for the functions sin(x) and sin(x)/x as well as incorporating the graphs which may assist students well to view the behaviors of the functions as x grows larger and larger which we predict that it goes to infinity. Figure 1 shows a table and graph of *sin(x)* and $\dfrac{Sin(x)}{x}$

in that one can view the behavior of the function values when x value goes to a smaller number up to 100, in Figure 2 x value goes to 1000,
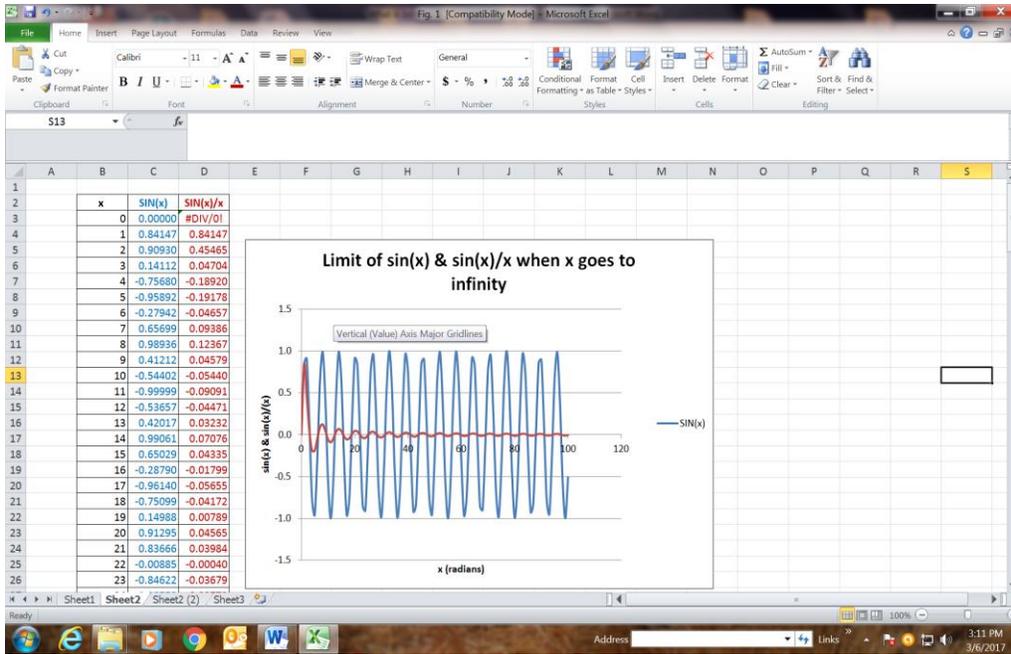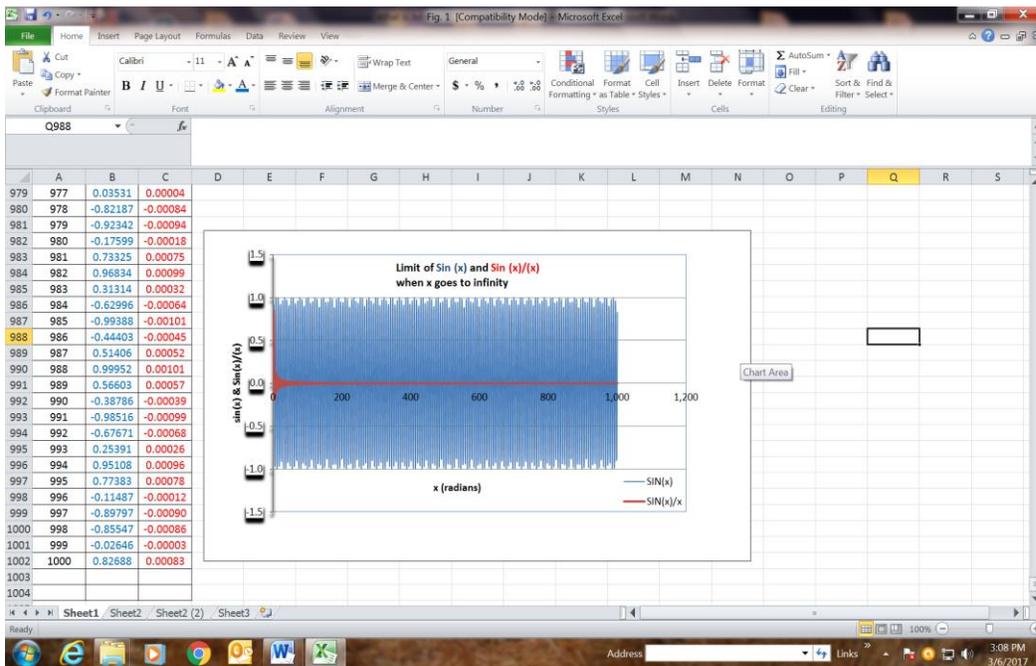
Figure 1

Figure 2

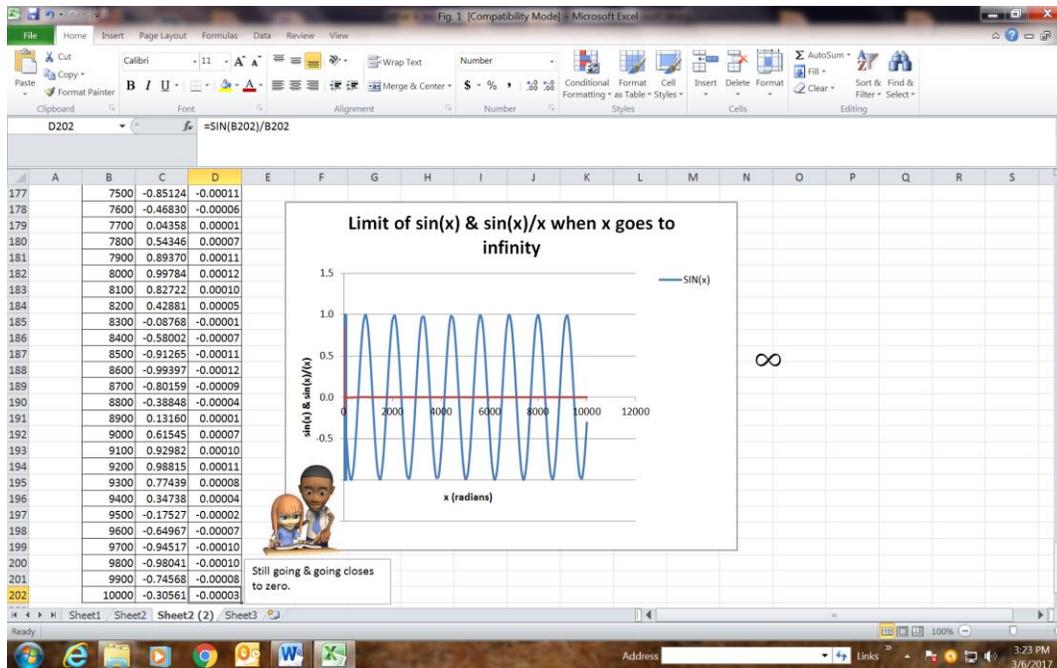and in Figure 3 x goes to the huge number in this case x first goes to 10000, next 20000, next 30000,…60000 so on.

Figure 3

One can see that the *sin(x)* varies nicely between the values of -1 and 1, which is obvious a well known fact in Trigonometry anyway. However, $\dfrac{Sin(x)}{x}$ function value fluctuates closer to 0 and then steady with value 0 for 5 decimal places. Here by doing this simple Excel analysis, students can view the waves of the $\dfrac{Sin(x)}{x}$ converges smoothly for a constant value of 0. Accordingly, the inquiry for where is the infinity and how do you reach the infinity are answered by using the function capabilities of the Excel spreadsheet technology. .

Problem No. 2:

$\lim_{n\to\infty}(1+\frac{1}{n})^n$ = e where e = 2.71828183…an irrational number. Students usually simply the function as $(\frac{n+1}{n})^n$ and by direct substitution they ended up as $\frac{\infty}{\infty}$, then their answer becomes as 1. The concept of approaching infinity stills a challenging for the first year undergraduate students. In upper level Calculus classes L'Hopital's rule with logarithm manipulation is implied to analyze the limiting value of this sequence. As shown in Figure 4 below, using the Excel spread sheet one can create a table for larger x values as much as on go to find the one-to-one function values to analyze the concept of e.
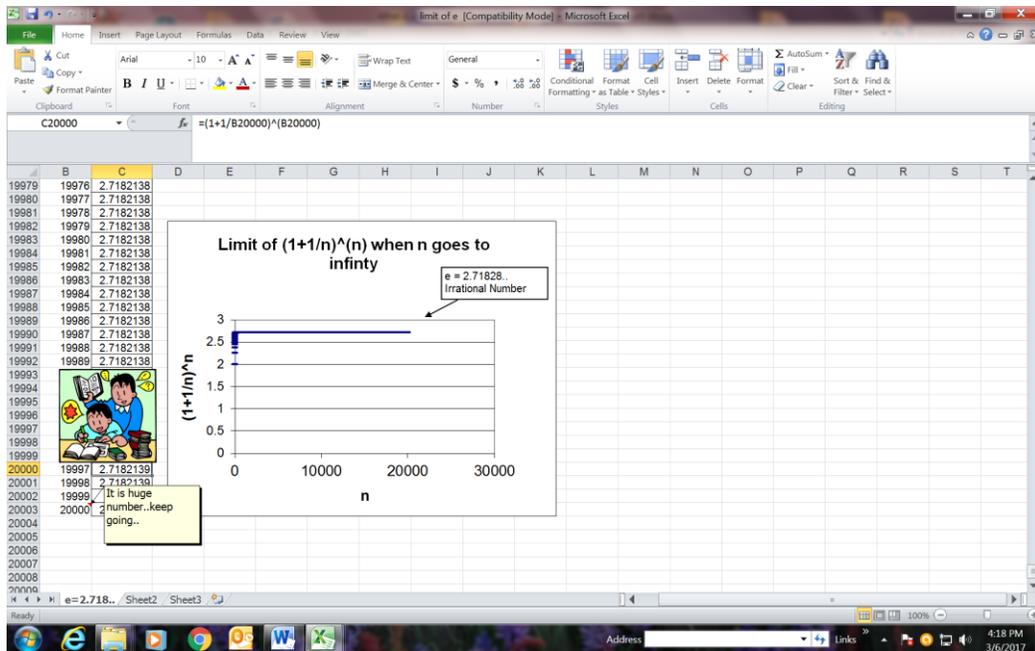
Figure 4

As one said that picture makes 1000 words, here the picture is in spreadsheet in that n value goes up to 20,000. Students can be observed that for the nth values between 10,000 and 20,000 the function value is equal to a constant number, 2.71828, for five decimal places. This makes an assumption that when n goes to infinity the function, $\lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n$, gets the magic number, e ($\approx$ 2.71828…).

CONCLUSION

Infinity exists only as a means of description like found in Mathematics, or any other thing that exists only in the abstract. Like an example of social effects as the love of mother to their children is unlimited and it is not possible to count, Infinity is in another word abundant (∞). Using Excel one can approach the state of abundant along the rows where the number of rows are closer to 65,000. Students' understanding about the infinity is overwhelming.

References

1. NRICH is part of the family of activities in the Millennium Mathematics Project, University of Cambridge, United Kingdom, 1997 -2017

 2. http://www.math.toronto.edu/mathnet/answers/infinity.html

3. http://platonicrealms.com/minitexts/Infinity-You-Cant-Get-There-From-Here/

4. https://academic.oup.com/journals