Proofs Without Words Demonstrated in Active Videos

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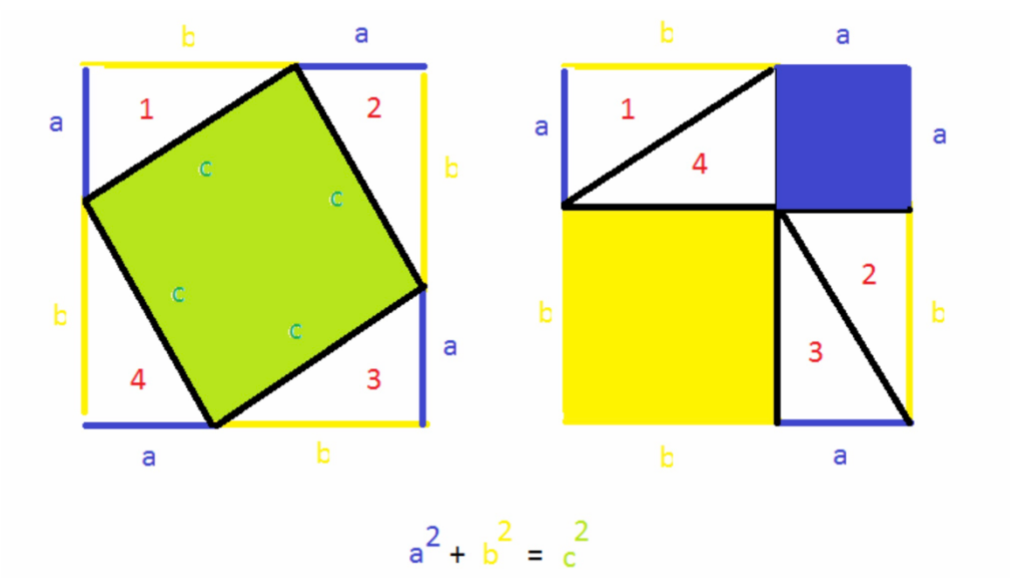
One of the most famous mathematical theorems of all time, has to be the Pythagorean Theorem. Even people who'd self-admittedly never refer to themselves as a "math person", have seen:

$$a^2 + b^2 = c^2$$

Unfortunately, not so many people have seen (or can remember!) a proof of this very important fact. My design for this workshop was to provide a series of three separate activities that one could take back to their students, or math club, where they could be involved in a hands-on verification of the Pythagorean Theorem. These could be recorded in video, posted in YouTube for all to see, and hopefully be a memorable and fun experience… giving the students a better chance to see a proof of the theorem and have some fun in the process!

<u>Activity 1</u>:

The first proof is a classic deconstruction proof, traditionally thought to be the one Pythagoras originally used to prove this now classic result. Here's a visual to demonstrate:

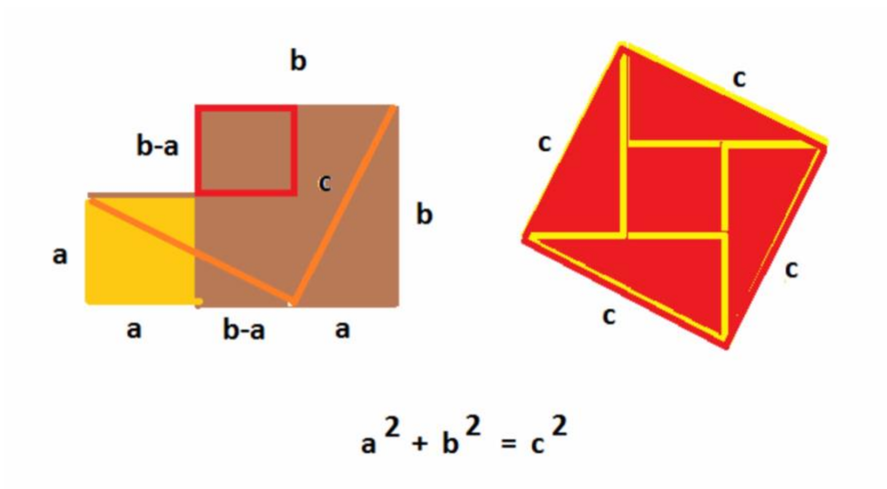


In the left-side figure, we have four equivalent triangles with legs of lengths a and b stacked in a way that gives us a giant square with sides of length $a + b$. This placement also gives a center square with sides of length c. In the right-side figure, we stack the triangles in such a way that also gives an equivalent giant square with sides of length $a + b$, but now leaving two new squares with sides of length a and b, respectively. Since the large square are identical in area, and both contain the same four triangles, the areas of the remaining squares must add up to the same amount. Or, we could say… "yellow + blue = green"!!! At ICTCM 2019, I recorded a YouTube video with the participants of my minicourse illustrating this deconstruction proof.

This video can be seen at: *https://youtu.be/Txd-K2y40M8*

Activity 2:

The second proof is also a deconstruction argument, this one was originally demonstrated by Bhaskara.

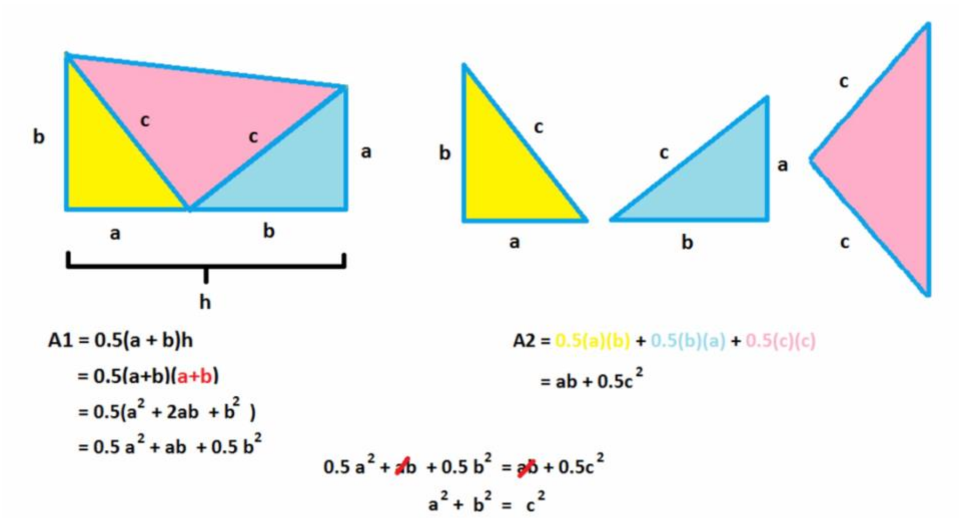


This argument again use four equivalent triangles, with legs of lengths a and b … hypotenuse c. In the figure on the left, two of the triangles are stacked along the bottom edge and another two are stacked along the side. This placement leave a square with sides of length $b - a$. The figure on the right starts with the square having sides of length $b - a$, and then places each of the triangles around its edges using the side of length b. You can see clearly on the left that this gives us two square regions, of area a^2 and b^2 respectively, while on the left we get one square of area c^2. Since both figures are constructed using the same five basic figures, their overall areas must be equal…hence, $a^2 + b^2 = c^2$. At ICTCM 2019, I also recorded a YouTube video with the participants of my minicourse illustrating this deconstruction proof.

This video can be seen at: *https://youtu.be/gkCkjkJ-4L4*

<u>Activity 3</u>:

The third proof is of great historical significance, this one was originally demonstrated by President Garfield. How many other disciplines can claim that a former president of the United States created a new proof of one of their classic theorems? How cool is that! Here's the basic idea of President Garfield's proof:



In the figure on the left, two equivalent triangles with legs of lengths a and b … hypotenuse c, are connected horizontally in "opposite orientations". This leave a gap in the middle, filled by a third triangle having sides of length c. The figure on the right shows this figure "deconstructed" into its three separates triangular pieces. On the left, the area A1 is computed using the known area of a trapezoid formula $\frac{1}{2}(a+b)*h$, while the one on the right just computes the areas of the three individual triangles using $\frac{1}{2}*b*h$. The accompanying simple calculation clearly shows the two areas are equal…giving us our third illustration of the classic Pythagorean Theorem. At ICTCM 2019, I also recorded a YouTube video with the participants of my minicourse illustrating this deconstruction proof.

This video can be seen at: *https://youtu.be/WyhXJpXAgZ8*

I had so much fun with my studies of the historical proofs of this theorem and construction animations verifying the result. I also had a great time with this minicourse and not only showing the proofs to others, but seeing them "give life" to my animations. It was greatly to have a willing and excited set of participants and believe they all had fun during this exercise as we demonstrated these Pythagorean "proofs without words". It is my hope they will take this excitement back to their students and classes, and duplicate these (or many other) proofs without words in active videos!