FOSTERING IMAGINATION AND CREATIVITY WITH GEOGEBRA IN MATHEMATICS TEACHER EDUCATION

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Abstract

GeoGebra was created in 2001. Now after almost 20 years, GeoGebra has grown to an interactive tool to explore mathematical relationships while teaching mathematics to learners of all ages. Over the years, educators and researchers have created ggb files to simulate math connections with real-world situations. This paper will focus on GeoGebra activities and resources that show the use of creativity and imagination while using GeoGebra to learn mathematics. This paper will show how to find and use such ggb files to encourage students to be involved in their own learning and promote more creativity and imagination while using GeoGebra in the pursuit of learning mathematics.

Keywords: Imagination, Creativity, STEM, Best Practices, GeoGebra, Computers

Introduction

- The true sign of intelligence is not knowledge but imagination.

- Imagination is everything. It is the preview of life's coming attractions.

- I am enough of an artist to draw freely upon my imagination. Imagination is more important than knowledge. Knowledge is limited. Imagination encircles the world.

- The mere formulation of a problem is far more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions,
new possibilities, to regard old problems from a new angle requires creative imagination and marks real advances in science.

-Computers are incredibly fast, accurate and stupid. Human beings are incredibly slow, inaccurate and brilliant. Together they are powerful beyond imagination.

-Albert Einstein 1879-1955

The above quotes from Albert Einstein really focus on how important it is that as educators we focus on fostering creativity and imagination rather than just knowledge in the classroom when teaching young people. When we encourage our young people to explore their creativity and imagination as they learn and problem solve along with pairing that with technology like computers and software, like GeoGebra and other technologies, humans and the technology can be so powerful in advancing us and our understanding which encircles the world for all humanity. In a standards-based assessment driven society we now live in, often creativity and imagination are not encouraged or fostered enough in our schools with our young people. Using technology while teaching mathematics is important and educators need to start emphasizing creative exploration in our schools if we are truly going to excel in the STEM (Science, Technology, Engineering, and Mathematics) fields. Henriksen, Creely & Henderson (2019) argue that the US and Australia are failing in teaching, emphasizing, and supporting creativity in schools today. This can easily be addressed in the math classroom when incorporating GeoGebra during mathematics instruction.

In this paper, the thesis of the concept focuses on the use of GeoGebra software to teach geometric/mathematics concepts to K-12 students while emphasizing creativity and imagination while learning mathematics. Perignat & Katz-Buonincontro (2019) advocate more creativity and imagination used in teaching in the STEAM areas in school to better prepare students. They contend that teachers must take risk in offer creative exploration of students in their classrooms and lessons. When using GeoGebra with middle and high school students, we noticed the ease with which of students followed directions on activity sheets and thought that such activities might be used at the K-5 grade levels as well. We share some activities that we used with second grade students, as well as rationales from research and state standards for using such software with young children. Students starting young can start using Paint and then transition to GeoGebra to advance their creativity.

GeoGebra was created back in 2001. Now after almost 20 years, GeoGebra has grown to an interactive tool to explore mathematical relationships while teaching mathematics to learners at all ages. Over the years, educators and researchers have created ggb files to simulate math connections with real-world situations. The paper will focus on GeoGebra activities and files that show creativity and imagination. This paper will focus on how to find and use such ggb files to encourage students to be involved in their own learning and promote more creativity and imagination while using GeoGebra in the pursuit of learning mathematics.
GeoGebra is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus in one easy-to-use package (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2009). This open-source dynamic mathematics software can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info. GeoGebra has a large international user and developer community with users from 190 countries is now currently translated into 55 different languages.

Fahlberg-Stojanovska & Stojanovski (2009) discovered that using GeoGebra is motivating for students and helps them learn at a higher level while exploring and conjecturing as they draw and measure. In research by Rosen & Hoffman (2009), they established the importance to integrate both concrete and virtual manipulatives into the math classroom, such as representational models like GeoGebra. Furner & Marinas (2007) found that young people can easily transition from the concrete when using manipulatives like geoboards to the abstract when using geometry sketching software like GeoGebra. Although GeoGebra has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in higher education and even now introduced in the elementary math levels.

GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). GeoGebra was also credited with changing teacher habits. Two features were specifically referenced as causing this change: 1) that it is an award winning software system, and therefore has admirable features, and 2) that it provides an effective pedagogical model for teachers. Mathematics teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to rapidly use GeoGebra in their classrooms for math instruction.

Leikin & Pitta-Pantazi (2013) in their paper, Creativity and Mathematics Education: The State of the Art, emphasize the importance of creativity in math education and empowering our educators and research into gearing our instruction toward more creativity emphasized in instruction. Lev-Zamir & Leikin (2013) also found that teachers have many misconceptions about employing and allowing creativity when teaching mathematics. Today with the advent of a standards-based curriculum and emphasis on technology, it is also important that educators also promote and foster more creativity while learning math in our STEM world (Perignat & Katz-Buonincontro, 2019).

Today with such an emphasis on covering and assessing standards in teaching mathematics, often creativity and imagination in learning is neglected during instruction. In a book by Beghetto & Kaufman (2017) Nurturing Creativity in the Classroom, they share many essays on the importance of incorporating more creativity when teaching. Hunter-Doniger (2016) contend that while covering the standards there are creative
approaches to teach while covering the standards and also incorporating more problem solving and also motivating young people by allowing for more creativity in problem solving. Wilke (2006) believes that only when math concepts are taught in an imaginative manner will young learners fully benefit from such experiences and also feels that the teacher's job is to develop a variety of activities that challenge students to think in creative and innovative approaches is important. Hence, with the availability of GeoGebra, teachers are able to make interactive and graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new math concept, students need to think, visualize and explore relationships and patterns and also using the technology can lend itself greatly to allowing for a great deal of creativity in creating and designing as well as imagination in inventing and employing a great deal of creativity in the design of their math work using the software like GeoGebra. Wu and Rau (2019) found that drawing can change the way students interact with the content they are learning and by exploring through drawing can lead to and foster creativity.

While the teachers have the option of creating data files for GeoGebra, some start by learning the basics of the software and then incorporate ggb data files made be other GeoGebra enthusiasts into their methodology. After the initial use of these files, teachers learn how to customize the files for their teaching needs. This builds confidence and leads the instructor to attempt to create their original ideas into a ggb activity. Many premade GeoGebra files, activities, and creations will be shared emphasizing the creativity and imagination one can employ using GeoGebra, see the Figures below.

The goal of this paper is to show educators how to access and select pre-made GeoGebra interactive activities that can help to cover the Common Core Mathematics Standards for teaching grades K-12 mathematics today which employ creativity and imagination in learning so to promote this in their students while teaching mathematics. Students can learn in a Standards-based curriculum, use technologies like GeoGebra, and employ creativity and imagination within the learning scheme.

Technology as a Principle for Teaching Mathematics

The importance of using technology in the teaching of mathematics has been advocated by the National Council of Teachers of Mathematics [NCTM] (1989, 2000, and 2006). Computers are an integral part of life and students need to be prepared to use technology to solve problems and access information as young people and adults. As teachers, we are obliged to follow state standards, curriculum, and best practices in teaching (Common Core, 2011), students are expected to work with two- and three-dimensional shapes; combine and subdivide shapes; use coordinate geometry to locate objects; and describe objects algebraically. In the Measurement Standard, students explore and solve real-world problems by estimation and measurement techniques. These standards can easily be addressed using geometry sketching software like GeoGebra. Yerushalmy (2009) advocates and in this study found that technology can play a key role in allowing students to create and incorporate more imagination and creativity into their learning of
mathematics when using more technology. Kaplan, Özturk & Ertör (2013) found that using technology had the most impact on student success. Their study aimed to compare computer-aided instruction, creative drama and a more traditional teaching approach to teaching of Integers to the seventh grade students and found that the technology had the most impact on student success.

**GeoGebra Software**

GeoGebra was developed by Markus Hohenwarter in 2001/2002 as part of his master's thesis in math education and computer science at the University of Salzburg in Austria. GeoGebra is a dynamic construction and exploration tool that enables students to explore and understand mathematics in ways that are not possible with traditional tools. GeoGebra is also another sketching software where students can construct an object and then explore its mathematical properties by dragging the object using the computer mouse. Another popular piece of software for learning geometry is *Cabri Geometry II* (Texas Instruments). Geometry sketching software brings dynamic power to the study of Euclidean and non-Euclidean geometries, algebra, trigonometry, pre-calculus, and calculus. This type of software can be adapted for use with young learners through college age and lends itself for much exploration and creativity.

GeoGebra has been a growing favorite today among math teachers because it is free software, a multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package (Hohenwarter, Hohenwarter, & Lavicza, 2009). GeoGebra has a large international user and developer community with users from 190 countries. The software is currently translated into 55 languages and attracts close to 300,000 downloads per month. It can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info.

**Geometry Sketching Software Research**

Students today are motivated to learn when activities are presented in a dynamic, hands-on manner that is engaging. GeoGebra software is an interactive tool which allows students to create their own understanding of geometry and mathematical ideas. Almeqdadi (2000) found that children who learned geometry using both a textbook and geometry software had significant gains in achievement over students who just used a textbook without software use. Wu & Rau (2019) advocate the use of drawing to foster creativity and using GeoGebra to draw and do math can certainly enhance and support this fostering of creativity y drawing.

Research suggests that elementary students should learn shape recognition through hands-on manipulatives (Reys, Lindquist, Lambdin, Smith, & Suydam; 2015). In Pre-K and first grade, students should experience activities that involve shape recognition with real-life examples. They should recognize that the shape of many table-tops is a rectangle and that the shape of a pizza is often a circle.
It is important for students to move gradually into the computer environment by relating hands-on manipulatives to two-dimensional computer shapes. By introducing students to these activities at an early age, they will be able to proceed to abstract mathematical concepts in the upper elementary grades and beyond (McClintock, Jiang, & July, 2002). When primary age children are learning mathematics concepts, computer simulations provide a smooth transition from concrete manipulation of objects to their abstract understanding (Berlin and White, 1986).

Aktümen & Yildiz (2016) in their paper, GeoGebra as an Artist’s Paintbrush, found from their study that participants used GeoGebra to create original designs when they used and explored with GeoGebra and the participants spent more time on the significance of visuals and patterns and so many groups made various creative tasks when asked to explore with this software, creativity is a key factor and while using GeoGebra, participants can create and use their own imagination to design and were encouraged to do so. Olsson (2017) also found that while investigating students’ reasoning when the students were working in pairs, solving mathematics problems using the dynamic software, GeoGebra that they also employed creative reasoning. This study found in the first part of the study that when students used GeoGebra in collaborative situations where they shared their individual reasoning to each other; that GeoGebra provided the students with feedback that, to some extent, became a basis for their creative reasoning as they used the software to check their work and use things like the algebra view. The study showed that students who are arguing predictively for their strategies are more likely to use feedback from software elaborately. They found that GeoGebra helps in guiding creative reasoning and provides feedback both for verification and elaboration when solving problems and that there is not just one right way to solve problems. Olsson felt that creativity is fostered when using the GeoGebra software during instruction.
Exploring, Learning, and Creating with GeoGebra

Math Teacher Trish Smith at Cardinal Newman High School in West Palm Beach, FL uses GeoGebra to get students familiar with certain aspects of groups of graphs (See Figure 1). While doing this, she noticed that it would be fairly simple to create "pictures" on the GeoGebra software and so it began. At Christmas time, she gives her students the basic shape of a tree and they have to "decorate" it with functions that her students are learning about in class. At Easter time, she gave her students the basic shape of an egg and they had to "decorate" it with functions that they are learning at that time. For the "extra creative" students, Trish encourages them to use multiple functions to create a new picture within the tree or egg. GeoGebra is a great tech tool to turn students on to math, help them learn, and let them create.

Figure 1. Sample GeoGebra Creative Graphs while decorating Egg Shapes
Figure 2. Using GeoGebra and graphs to decorate egg shapes
Second-Graders Experiences with MS Paint and GeoGebra Activities

Young children are easily motivated by geometry and dynamic computer software like GeoGebra. The second graders who participated in the activities (see worksheets in the Appendices) commented on the fun and ease of use of such software and compared it to MS Paint. They enjoyed constructing shapes like rectangles, triangles, and circles. The young learners also enjoyed measuring lines and angles and liked using the animation tool to make their figures move around on the screen. The tools for both pieces of software were easy to use. Today's young children who are born into a world full of technology are excited by the dynamic software that exists for learning. Many children have prior use of using MS Paint before coming to school and working with it easily; GeoGebra is more sophisticated. Paint can serve as a transition or bridge to GeoGebra because GeoGebra has many more advanced features such as measuring lengths, constructing figures, etc. As we do not want to discourage children when using the technology, the teacher needs to be available to help students who may not know how to use a function of the GeoGebra software. Students typically show no fear or anxiety about using the software and feel confident using the computer for learning.
Daniel Mentrard has published some very creative GeoGebra creations on YouTube, (See Figures 4 and 5), these are also wonderful examples of how by using this free software, people can be very creative in creating interactive and artistic and appealing images and designs creating while applying imagination and creativity to their work and investigations.

Figure 4 Daniel Mentrard’s GeoGebra designs showing creativity found at: https://www.youtube.com/watch?v=dioctXkvBrs

Figure 5 Daniel Mentrard’s GeoGebra design found at: https://www.youtube.com/watch?v=g5-L_J9lK3o
Figure 6. GeoGebra copies from photos drawn in GeoGebra

Figure 7. Creative Drawings with GeoGebra found on Google Images
Research on Fostering Students Creativity and Imagination

Leikin & Pitta-Pantazi (2013) in their paper, *Creativity and Mathematics Education: The State of the Art*, emphasize the importance of creativity in math education and empowering our educators and research into gearing our instruction toward more creativity emphasized in instruction. Lev-Zamir & Leikin (2013) also found that teachers have many misconceptions about employing and allowing creativity when teaching mathematics. Schoevers, Leseman, Slot, Bakker, Keijzer & Krosbergen (2019) found that creating an open climate in the classroom during open interdisciplinary lessons in math are certainly linked to student mathematical creative expressions when learning and it also can help students engage in more mathematical interchanges and discussion in which mathematical concepts were questioned as they created which also involved mathematical learning while creating and exploring.

Today with such an emphasis on covering and assessing standards in teaching mathematics, often creativity and imagination in learning is neglected. In a book by Beghetto & Kaufman (2017) *Nurturing Creativity in the Classroom*, they share many essays on the importance of incorporating more creativity when teaching. Hunter-Doniger (2016) contend that while covering the standards there are creative approaches to teach while covering the standards and also incorporating more problem solving and also motivating young people by allowing for more creativity in problem solving. Wilke (2006) contends that only when math concepts are taught in an imaginative manner will young learners fully benefit from such experiences and feels that the teacher's job is to develop a variety of activities that challenge students to think in creative and innovative approaches. Hence, with the availability of GeoGebra, teachers are able to make interactive and graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize and explore relationships and patterns and also using the technology can lend itself greatly to allowing for a great deal of creativity in creating and designing as well as imagination in inventing and employing a great deal of creativity in the design of their math work using the software like GeoGebra.

Rodrigues (2012) advocates innovative practices and procedures in a learner-centered approach to learning mathematics. This author feels that creativity and hands-on learning proved to be beneficial not only to tactile and kinesthetic learners, but to others in the class also. The author believes that using best practices in teaching math whether to the math anxious student or others, like gifted, and using all such best practices in teaching mathematics, even like using technology, are all characteristics of effective teachers and stresses the importance of emphasizing having students express their creativity in teaching.
Summary

Geometry is an important topic in the math curriculum and can easily be taught in an exciting manner using hands-on activities and dynamic geometry software like GeoGebra. Teachers can excite young people about geometry while having them use sophisticated software like GeoGebra early to prepare for advanced ideas in geometry later. We would like to encourage elementary teachers to use GeoGebra and MS Paint when teaching geometry. Not only do the young students like the activities and learn the math/geometry, but they also get practice with using the various pieces of drawing software. These pieces of software can serve as a bridge in the classroom for connecting concrete geometric ideas to abstract ideas in geometry as a child ages. Einstein once said, “Imagination is more important than knowledge. Knowledge is limited.” This idea is so true today; we need to focus more on our students’ creativity, imagination, and exploration while teaching young people mathematics. When using the GeoGebra software, students can use and employ so much math yet be creative and employ imagination in their work while learning the math we need to teach them in today’s STEM world. It is critical we foster more creativity and imagination in the learning of mathematics today while incorporating emerging technologies like GeoGebra.

References


Appendix A

Activities for Creative Exploration in Mathematics

Worksheet 1: Animal Faces

*MS Paint*- with younger children you may want to start using Paint instead of GeoGebra initially as it may be easier for your students to use, but you can then transition them into using GeoGebra to draw, explore, and create many creative expressions while having them include math ideas in their creations.

1. Go to *Paint*.

2. Draw a bunny using the Pencil and Oval Tools.

3. Color the inside of the bunny grey by using the Paint Bucket.

4. Color the eyes white and black as shown using the Paint Bucket.

5. Use the TEXT TOOL if you want to add your name or put my bunny!
Worksheet 2: Sailboat

MS Paint or GeoGebra Activity

1. Go to GeoGebra or Paint.
2. Use the Line Segment or polygon and circle tools to draw a sailboat and sun.
3. Use the tool to fill and color your sailboat and sun.
4. Use the pen tool to make waves.
5. Be as creative as you want to with GeoGebra!
Website Resources for Incorporating Creativity using GeoGebra

Creative Animations with GeoGebra may be found on YouTube at:

https://www.youtube.com/watch?v=3Z9_lmXMYgI

Dysleemar Carrillo and others have creations done in GeoGebra at:

https://www.geogebra.org/m/NKy44jTt

The Hong Kong Academy for Gifted Education offers an online course with a certificate in “Creative Geometry with GeoGebra” at:


GeoGebra also offers as part of its website of resources something called GeoGebra Creative Works at:

https://www.geogebra.org/m/udMG2PWU