GRAPHING POLAR CURVES USING *EXCEL*

Nadeem Aslam
Mathematics Department
Florida International University
11200 SW 8th Street
Miami, Florida 33199
aslamn@fiu.edu

Polar curves are among the prettiest pictures that we graph, and it is fortunate that they are easy to graph, especially when we use *Excel*. Polar curves are really graphs of parametric equations:

$$x = f(t), \quad y = h(t), \qquad t = \text{parameter}.$$

One good example of parametric equations is the motion of a projectile. The path of a projectile is given by

$$x = (v_0 \cos\ \theta)t, \quad y = -\frac{1}{2}gt^2 + (v_0 sin\theta)t,$$

where *x* and *y* are the coordinates of position of the body, θ is the angle of projection, v_0 is the muzzle velocity, *g* is the acceleration of gravity, and *t* is time. Notice that if we eliminate *t,* we get the equation of the path of the body in *x* and *y*

$$y = -\left(\frac{g}{zv_0{}^2cos^2\theta}\right)x^2 + (tan\theta)x,$$

which graphs as a parabola.

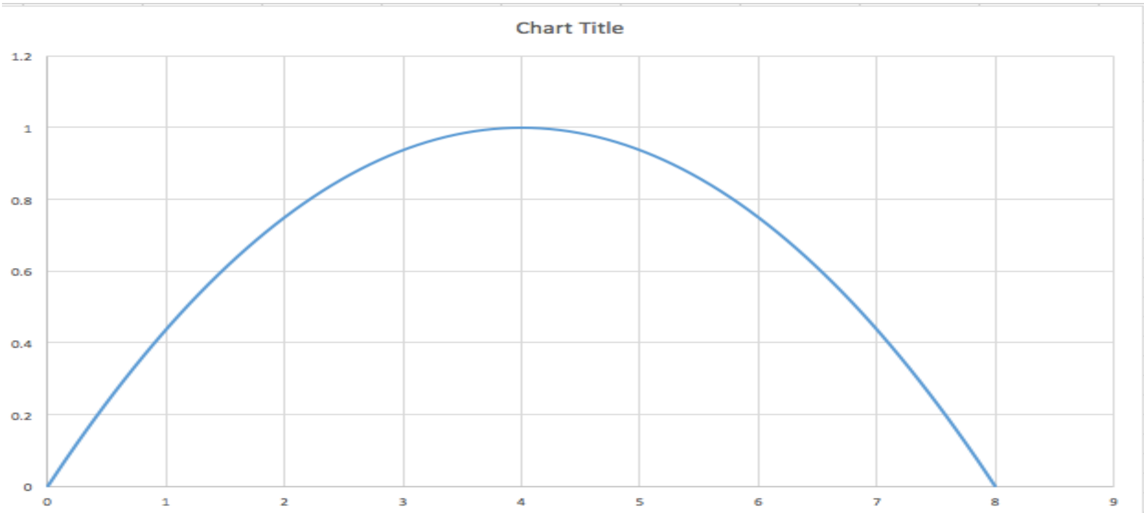


Figure1. Projectile Path

In *Excel* this is done as follows. Calculate *x* and *y,* then we plot them using the *Chart* utility of *Excel*. For a polar curve, $r = r(\theta)$, we calculate *x* and *y* as

$$x = rcos\theta \text{ , } y = rsin\theta$$

and plot them using *Excel.*

Familiar examples:

(1) Straight lines. A horizontal line is: $r = acsc\theta$, or $x = a\cot\theta$, $y = a$.

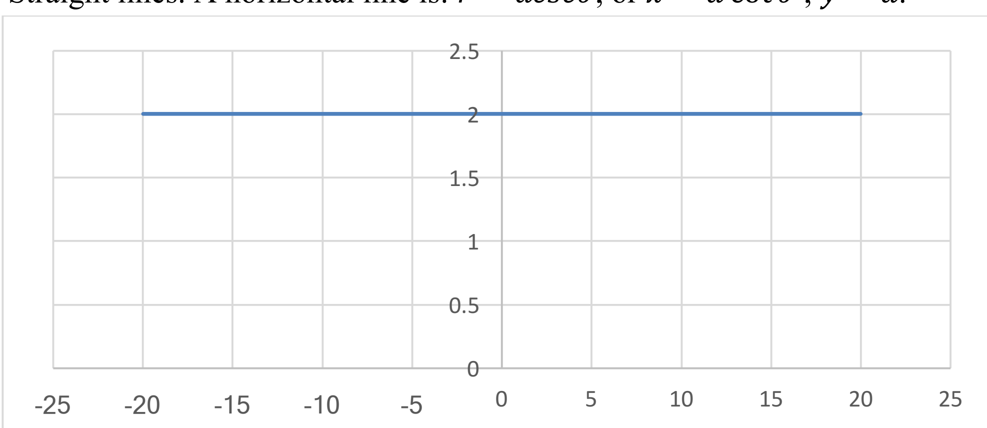


Figure 2. Horizontal line $y = 2$.

(2) A vertical line is: $r = a sec\theta$, or $x = a$, $y = a\tan\theta$.

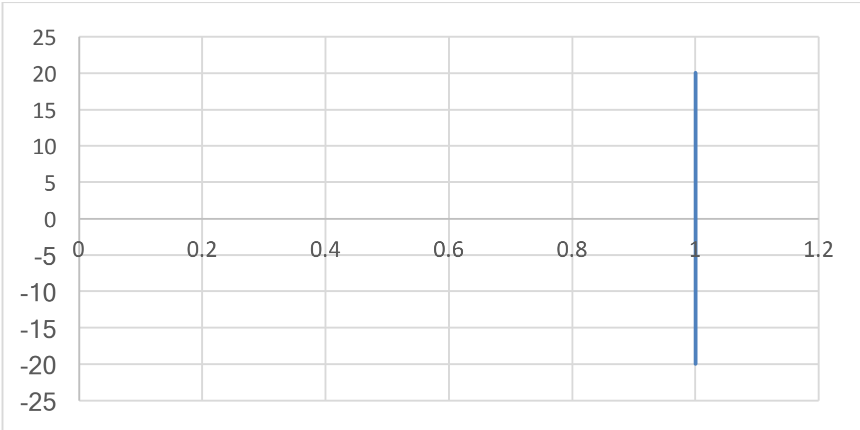


Figure 3. Vertical line $x = 1$.

(3) A general straight line Polar equation of a slant striaght line is $r = \frac{c}{acos\theta + b\sin\theta}$, or

$$x = \frac{c\cos\theta}{a\cos\theta + b\sin\theta} \text{ and } y = \frac{c\sin\theta}{a\cos\theta + b\sin\theta}$$

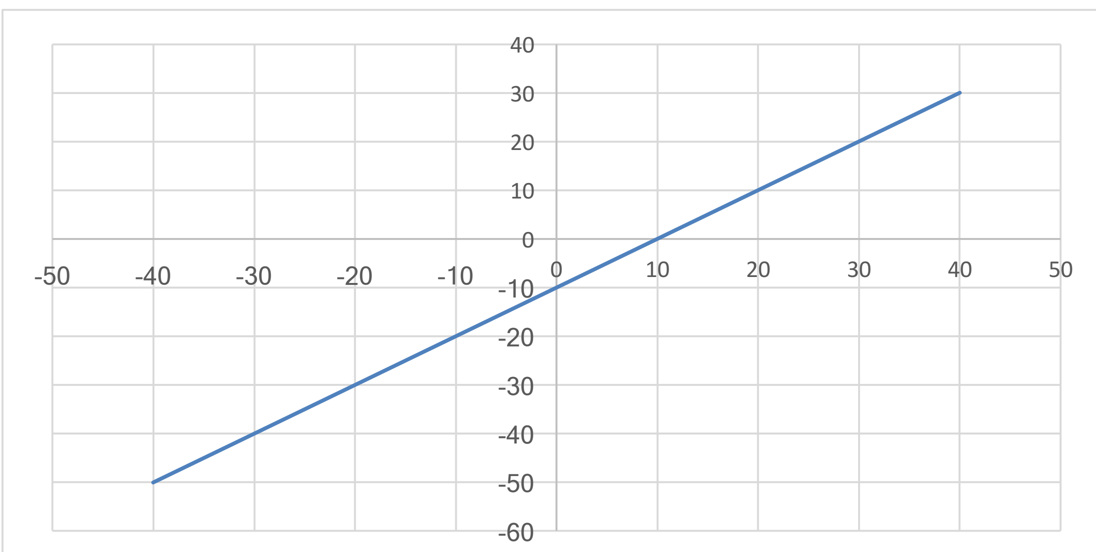


Figure 4. Straight line $r = -\frac{10}{sin\theta - \cos\theta}$.

(4) Circles. (a) Center at origin, radius *a:* $r = a$ or $x = a cos\theta$, $y = a\sin\theta$.

Figure 5a, b, c. Circles

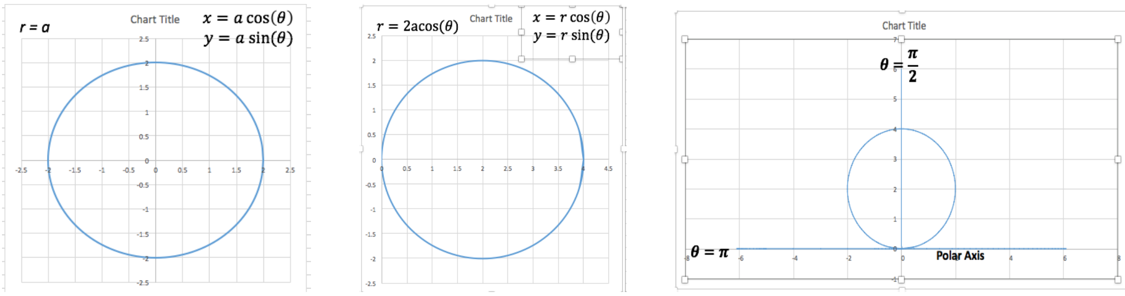
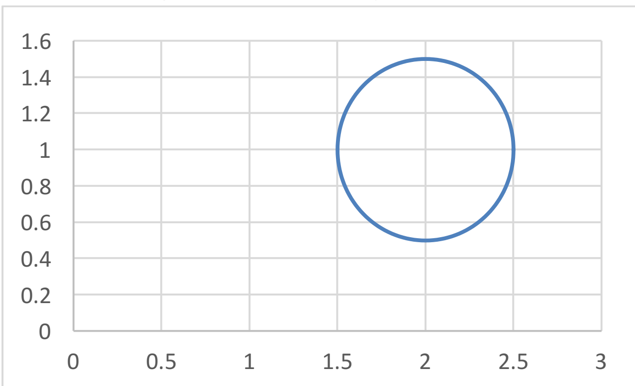


Figure 5d. Arbitrary Circle



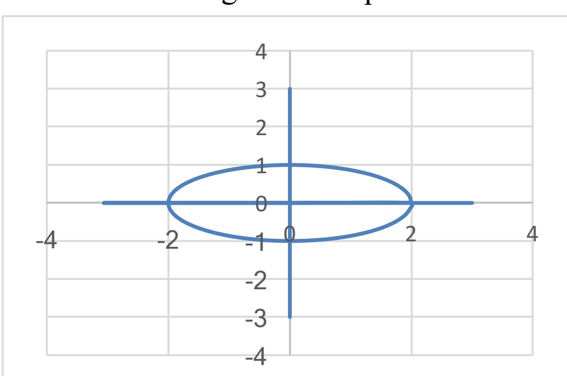
(b) Center at *(a,0)*, radius *a:* $r = 2a\cos\theta$.

(c) Center at *(0,a)*, radius *a:* $r = 2a\sin\theta$.

(d) Center at *(h,k)*, radius *a:* $x = a\cos\theta + h$, $y = a\sin\theta + k$.

(5) Ellipse: Center at (0,0), axes *a, b.*

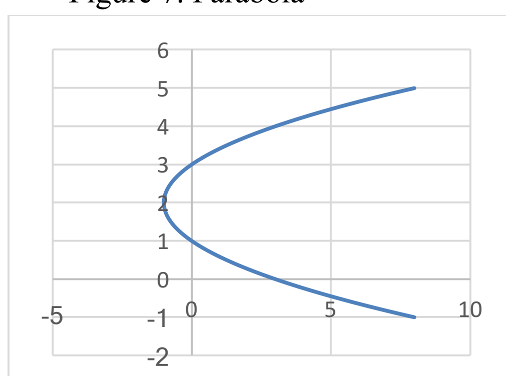
Figure 6. Ellipse



$$x = a cos\theta, \quad y = b sin\theta$$

(6) Parabola: Vertex at (*h, k*)

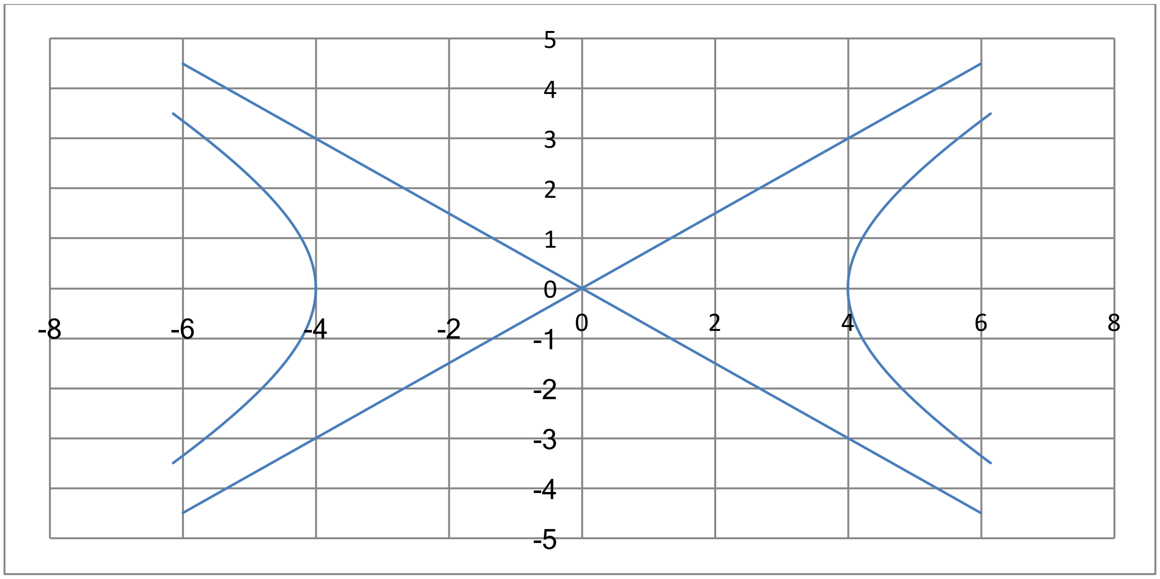
Figure 7. Parabola



$$x = t^2 - 2t, \quad y = t + 1$$

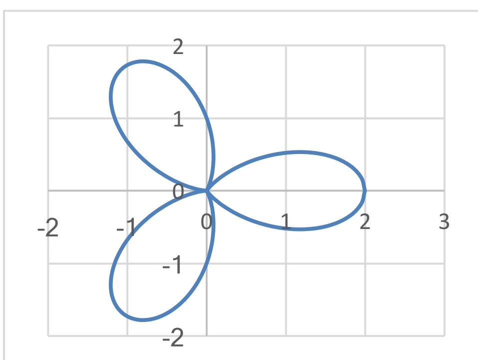
(7) Hyperbola: Center at (0,0), axes *a, b.* : $x = a\sec\theta$, $y = b\tan\theta$

Figure 8. Hyperbola



(8) Roses:

Figure 9a. Roses: $r = a\cos(n\theta)$; Figure 9b. $r = b + a\cos(n\theta)$, n = 3.



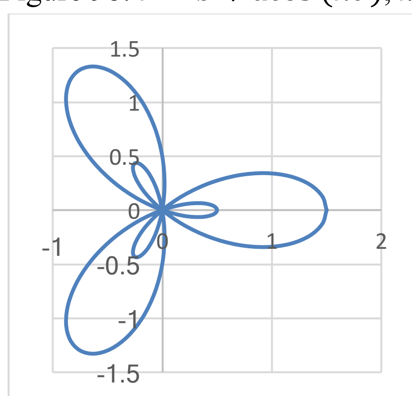
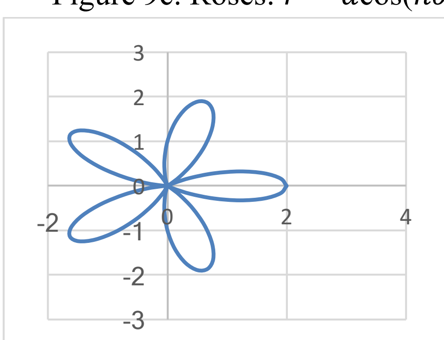
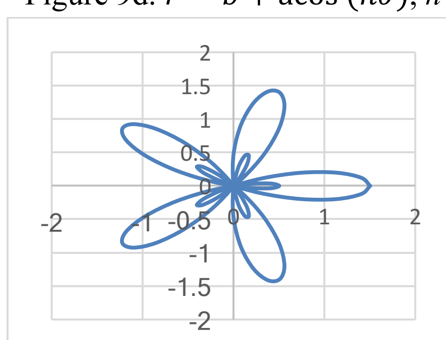


Figure 9c. Roses: $r = a\cos(n\theta)$; Figure 9d. $r = b + a\cos(n\theta)$, n = 5.





(9) Cardioids:

Figure 10a $r = a(1 - cos\theta)$

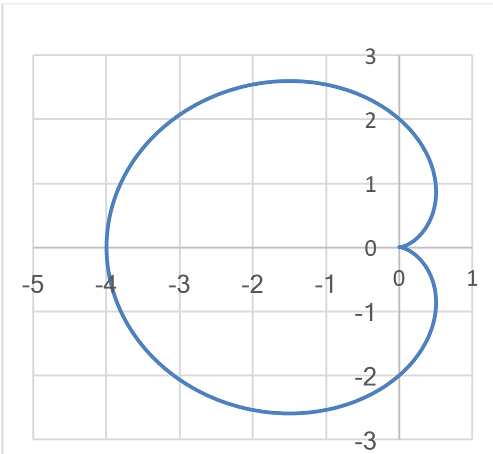


Figure 10b $r = a(1 - sin\theta)$

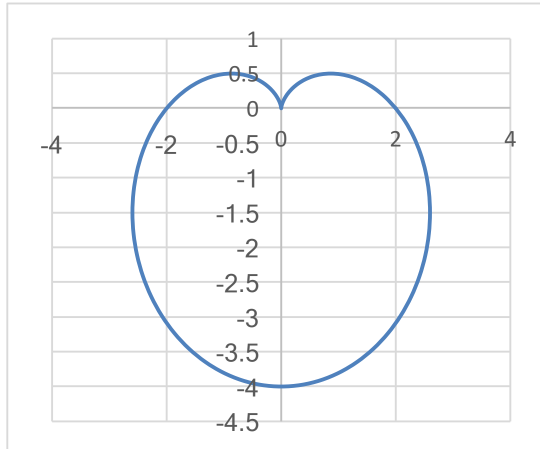


Figure10c $r = b + acos\theta); a > b$

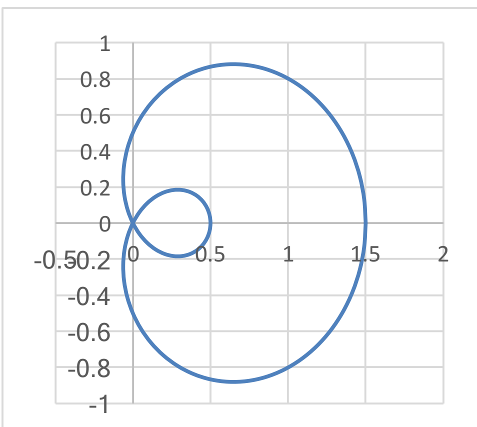


Figure10d $r = b + asin\theta); a > b$

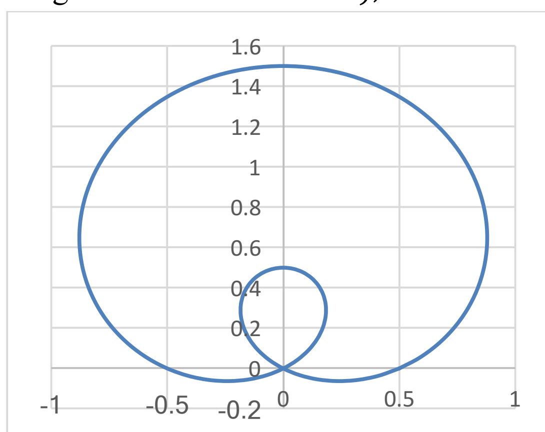
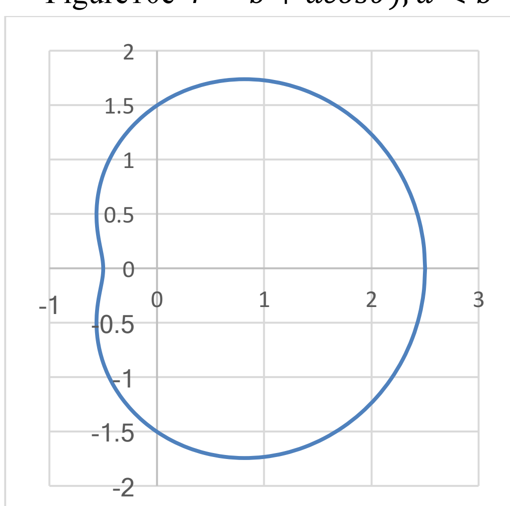


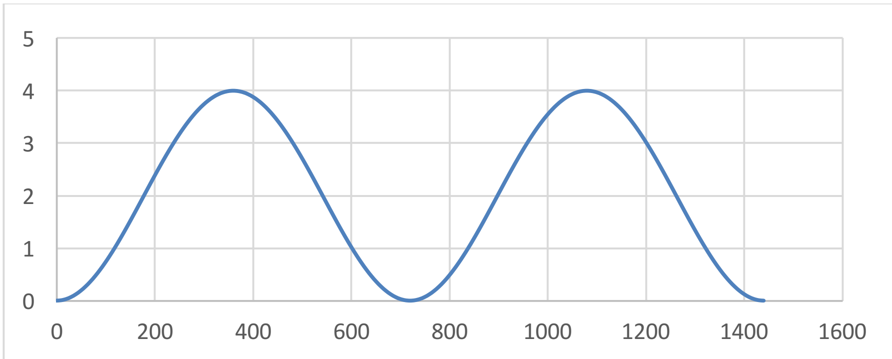
Figure10e $r = b + acos\theta); a < b$



Examples of less familiar polar curves:

(1) Cycloid:

Figure11 $x = 2(a - sin\theta); a > 0, y = 2(1 - cos\theta)$



(2) Archimedean spiral

Figure12a $r = \frac{\theta}{2\pi}$, $[-2\pi, 2\pi]$

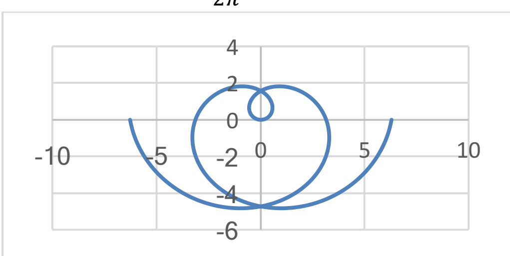
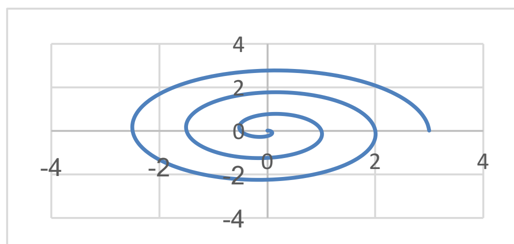
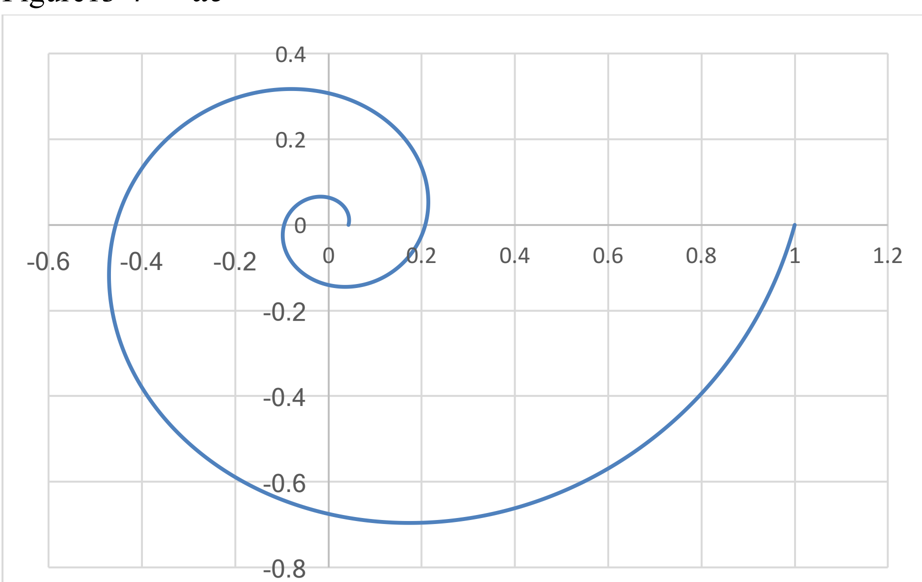


Figure12b $r = \theta$, $[0,2n\pi]; n \geq 1$



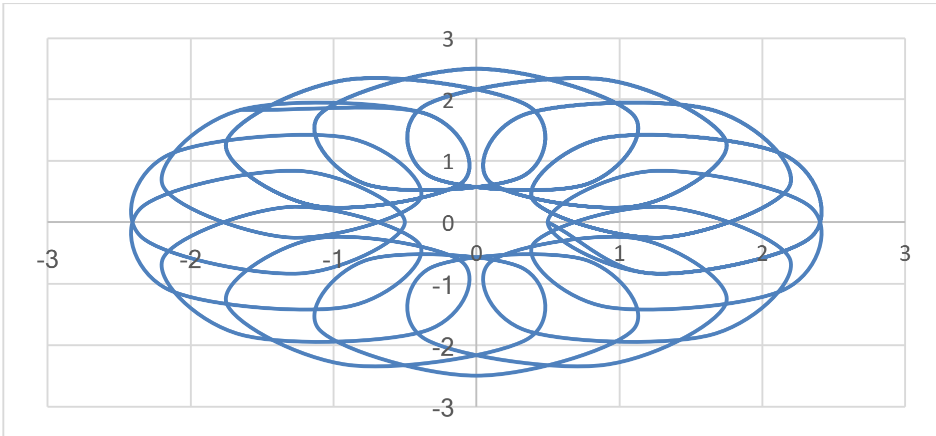
(3) Logarithmic Spiral

Figure13 $r = ae^{b\theta}$



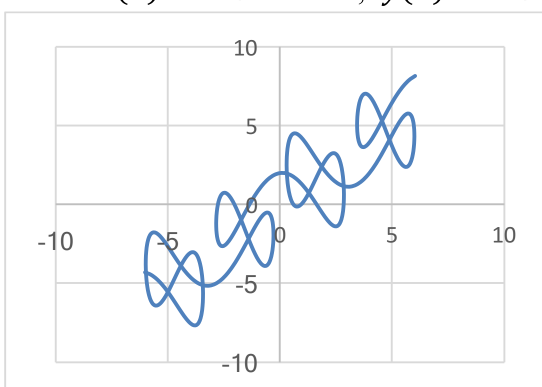
(4) Two Dimensional Torus:

Figure14a $x(\theta) = 1.5cos\theta\text{-}cos15\theta,\ y(\theta) = 1.5sin\theta\text{-}sin15\theta$



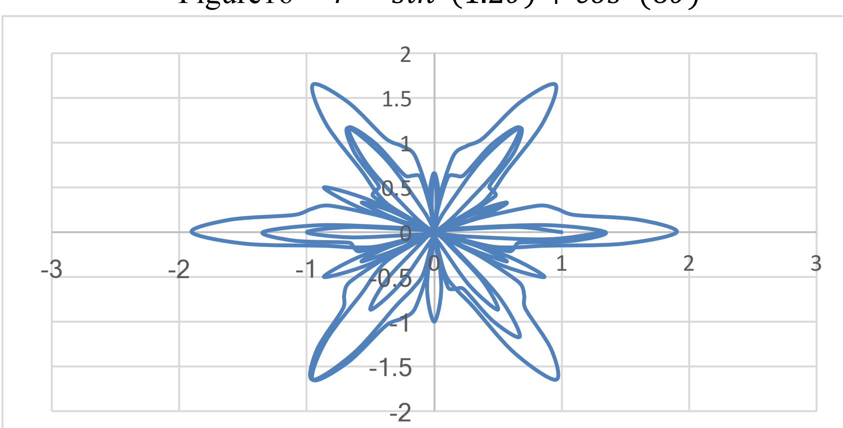
(5) Spring Shaped Curve

Figure15 $x(\theta) = \theta + 2sin2\theta;\ y(\theta) = \theta + 2cos5\theta$



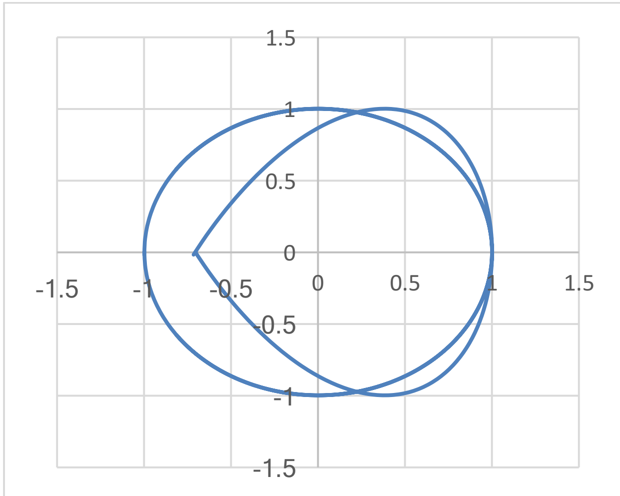
(6) Rose with multi-size petals

Figure16 $r = sin^2(1.2\theta) + cos^3(6\theta)$



(7) Graph of two intersecting curves in same Coordinate system

Figure17 $r_1 = asin^2\theta,\ r_2 = a$



(8) Lissajous curves

Figure18a $x(\theta) = 4sin\left(\frac{12}{13}\theta\right)$

$y(\theta) = 3sin\theta$

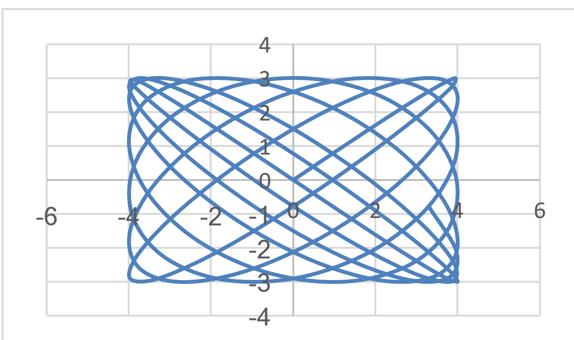
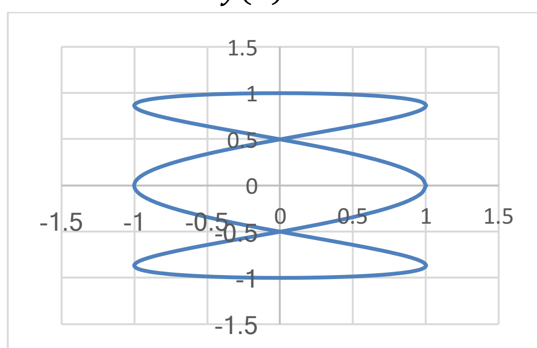


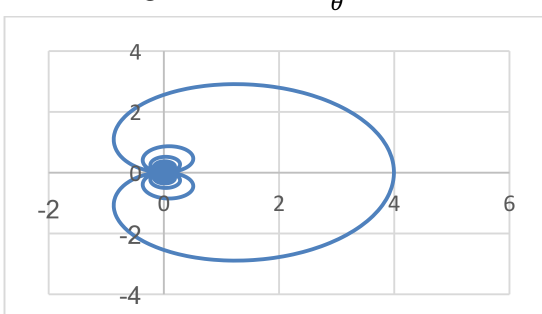
Figure18b $x(\theta) = \sin\left(3\theta + \frac{\pi}{2}\right)$

$y(\theta) = sin\theta$



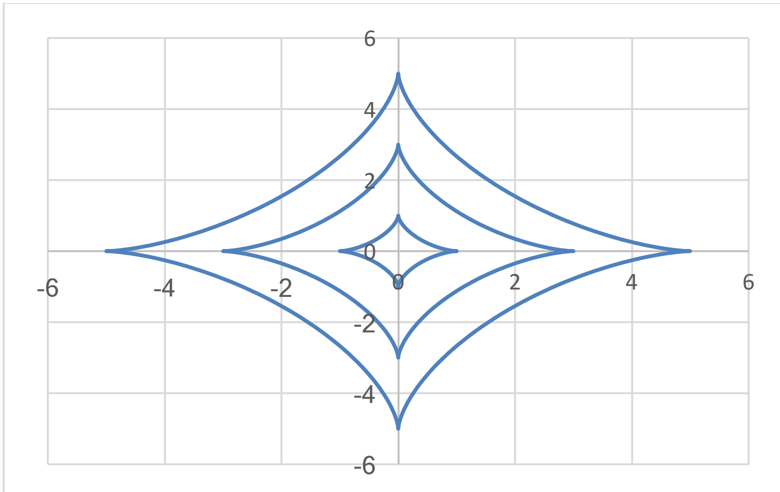
(9) Cochleoids

Figure19 $r = \frac{sin\theta}{\theta}$



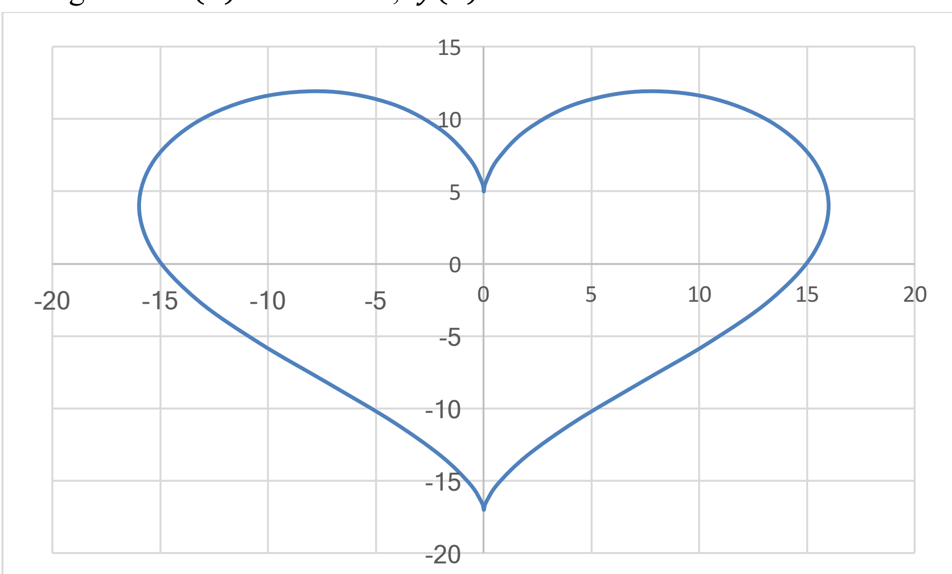
(10) Diamond Shape Family of Curves

Figure20 $x(\theta) = acos^3(\theta);\ y(\theta) = asin^3(\theta)$



(11) Heart shaped curve

Figure21 $x(\theta) = 16sin^3\theta;\ y(\theta) = 13cos\theta\text{-}5cos2\theta\text{-}2cos3\theta\text{-}cos4\theta$

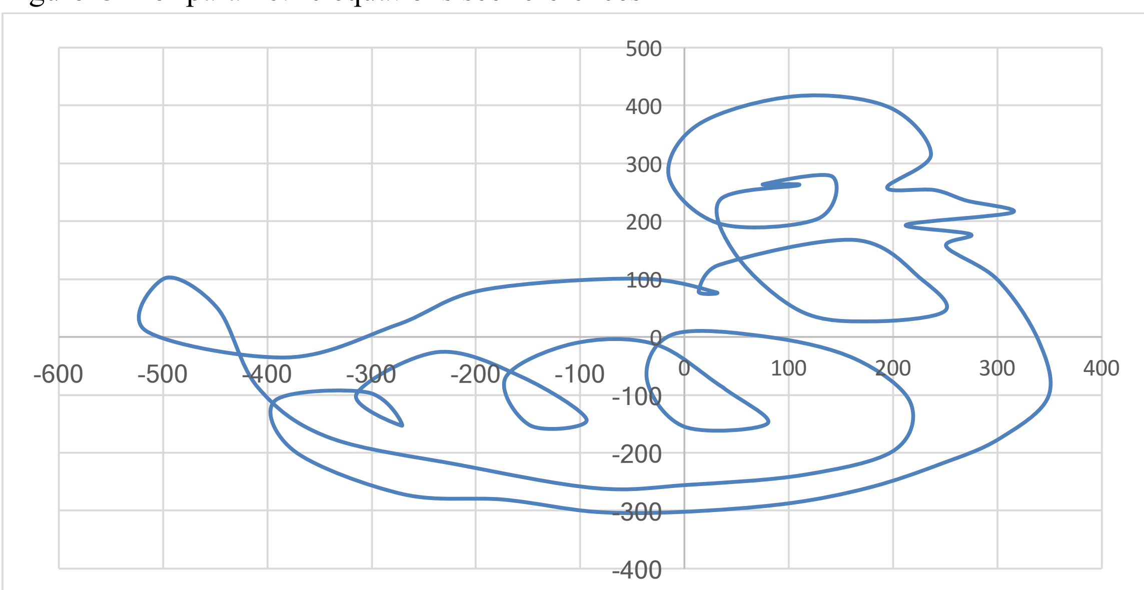


(12) Heart shaped curve with greetings

Figure22 $x(\theta) = 16sin^3\theta;\ y(\theta) = 13cos\theta\text{-}5cos2\theta\text{-}2cos3\theta\text{-}cos4\theta$

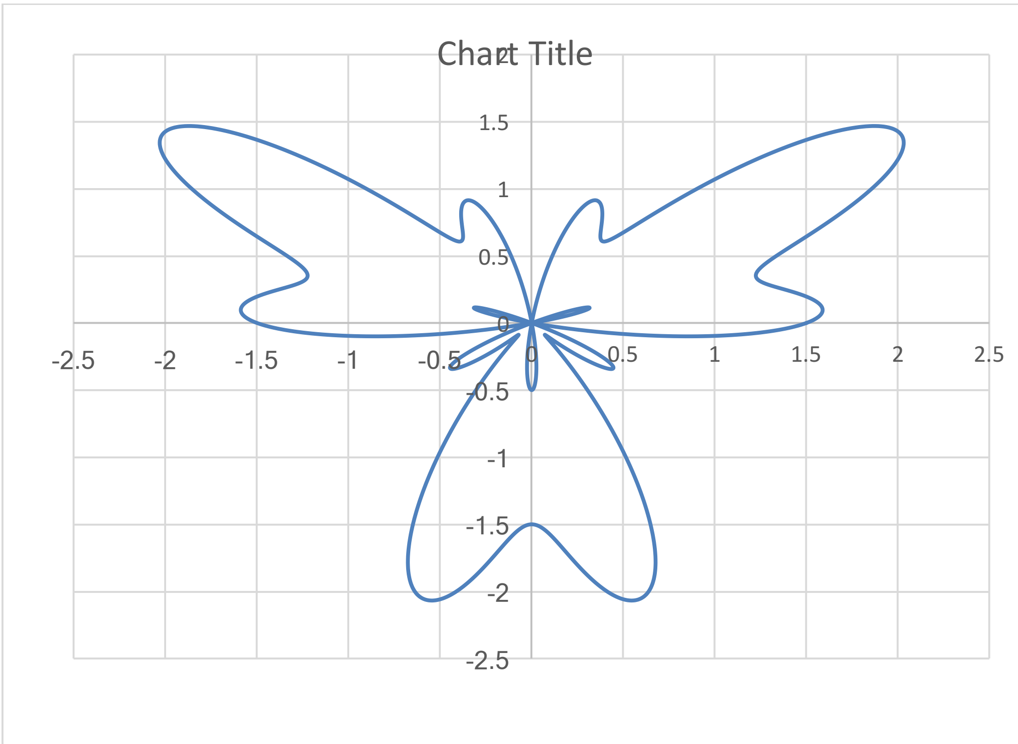
(13) Rubber Ducky

Figure23 For parametric equations see references



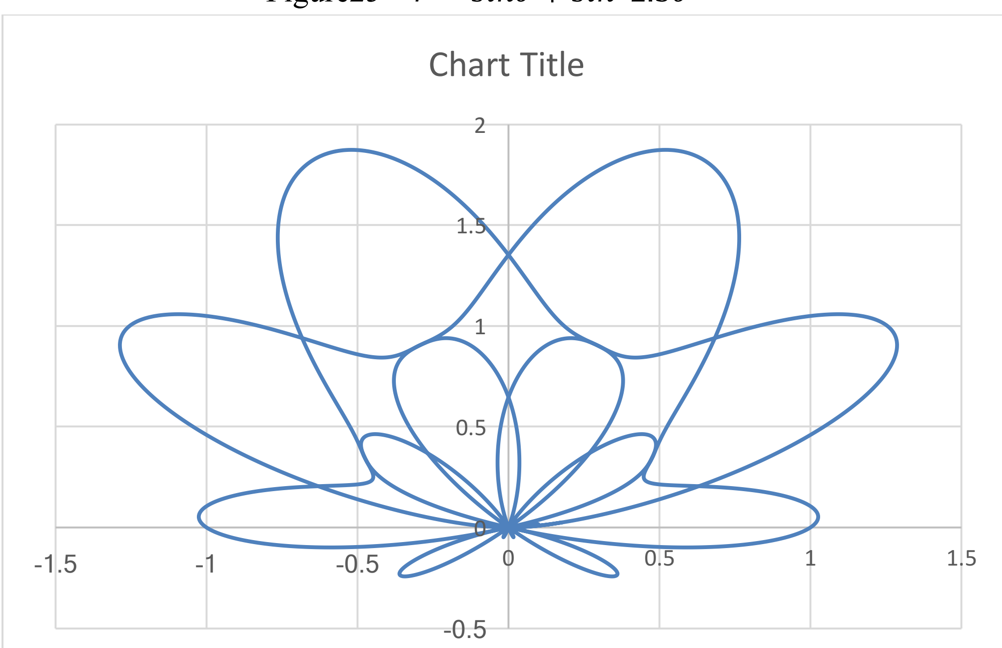
(14) Walking Butterfly Like curve

Figure24 $r = cos^2 5\theta + sin3\theta + 0.5$



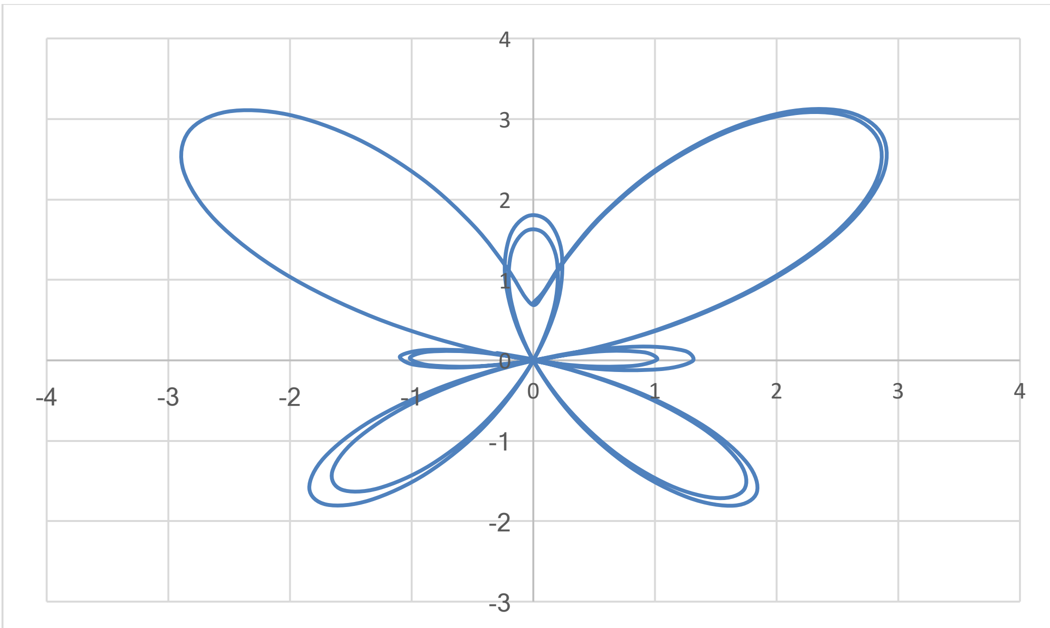
(15) Butterfly in the sitting position like curve

Figure25 $r = sin\theta + sin^3 2.5\theta$



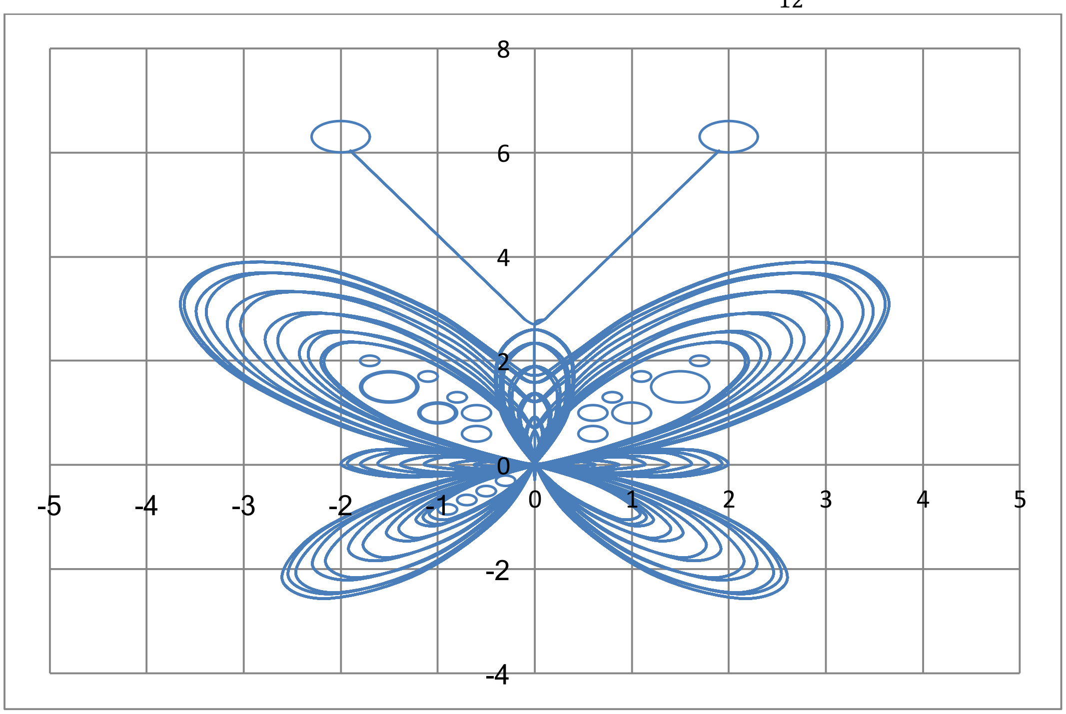
(16) Butterfly without Antennas

Figure26 $r = e^{cos\theta}$-$2cos4\theta + \sin\left(\frac{\theta}{12}\right)^5$



(17) Butterfly with Antennas

Figure27 $r = e^{cos\theta}$-$2cos4\theta + \sin\frac{5\theta}{12}$



Conclusions:

- A polar equation $r = r(\theta)$ can be written in parametric form $x = rcos\theta$ and $y = rsin\theta$.
- We can make use of Excel to graph polar equations by writing the polar equation in parametric form.
- Parametric equations are easy to graph, especially using *Excel*. We use the subtype chart "scatter with smooth lines" facility available in Charts group in *Excel* to plot graphs of the polar curves using their parametric form.
- Examples of polar curves, now can be displayed using *Excel.*
- Even complicated polar curves can be readily graphed when already expressed in parametric form, and sometimes the they look very pretty.
- Polar equation is very useful way of representing space as a function of direction, and with the use of parametric equations we can get a clear, pretty and understandable picture of this relationship.

References:

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8 WolframAlpha https://www.wolframalpha.com/examples/PopularCurves.html
Rubber Ducky parametric equations on the next page

$$x(t) = -\frac{8}{5}\sin\left(\frac{11}{10} - 43t\right) - \frac{6}{5}\sin\left(\frac{6}{5} - 41t\right) - \frac{29}{7}\sin\left(\frac{4}{3} - 39t\right) - \frac{10}{3}\sin\left(\frac{7}{5} - 36t\right) - \frac{41}{10}\sin\left(\frac{2}{5} - 34t\right) - \frac{11}{5}\sin\left(\frac{23}{22} - 30t\right) - \frac{33}{5}\sin\left(\frac{2}{3} - 28t\right) - \frac{10}{3}\sin\left(\frac{5}{6} - 25t\right) - \frac{44}{5}\sin\left(\frac{1}{3} - 21t\right) - \frac{83}{7}\sin\left(\frac{8}{7} - 12t\right) - \frac{276}{5}\sin\left(\frac{6}{5} - 4t\right) + \frac{2}{3}\sin(42t) + \frac{852}{5}\sin\left(t + \frac{21}{5}\right) + 125\sin\left(2t + \frac{3}{5}\right) + \frac{994}{5}\sin\left(3t + \frac{4}{5}\right) + \frac{592}{7}\sin\left(5t + \frac{19}{5}\right) + 34\sin\left(6t + \frac{11}{3}\right) + 20\sin\left(7t + \frac{8}{7}\right) + \frac{102}{5}\sin(8t + 2) + \frac{67}{3}\sin\left(9t + \frac{1}{3}\right) + \frac{122}{5}\sin\left(10t + \frac{9}{4}\right) + \frac{95}{3}\sin\left(11t + \frac{8}{9}\right) + \frac{37}{2}\sin\left(13t + \frac{6}{7}\right) + \frac{25}{2}\sin\left(14t + \frac{4}{3}\right) + \frac{47}{5}\sin\left(15t + \frac{1}{4}\right) + \frac{46}{3}\sin\left(16t + \frac{9}{4}\right) + \frac{15}{4}\sin\left(17t + \frac{22}{5}\right) + \frac{127}{14}\sin\left(18t + \frac{11}{3}\right) + \frac{11}{2}\sin\left(19t + \frac{85}{21}\right) + \frac{28}{3}\sin\left(20t + \frac{1}{5}\right) + \frac{23}{3}\sin\left(22t + \frac{17}{4}\right) + \frac{19}{3}\sin\left(23t + \frac{5}{4}\right) + \frac{12}{5}\sin\left(24t + \frac{15}{4}\right) + \frac{17}{6}\sin\left(26t + \frac{14}{3}\right) + \frac{25}{12}\sin\left(27t + \frac{5}{4}\right) + \frac{28}{5}\sin\left(29t + \frac{10}{9}\right) + \frac{5}{4}\sin\left(31t + \frac{17}{7}\right) + \frac{21}{5}\sin\left(32t + \frac{4}{3}\right) + 3\sin\left(33t + \frac{11}{3}\right) + 3\sin\left(35t + \frac{13}{4}\right) + \sin\left(37t + \frac{2}{5}\right) + \frac{39}{19}\sin\left(38t + \frac{10}{7}\right) + \frac{9}{5}\sin\left(40t + \frac{12}{5}\right)$$

$$y(t) = -\frac{7}{4}\sin\left(\frac{1}{9} - 39t\right) - \frac{2}{5}\sin\left(\frac{4}{5} - 35t\right) - \frac{11}{4}\sin\left(\frac{5}{4} - 33t\right) - 2\sin\left(\frac{2}{5} - 26t\right) - \frac{13}{2}\sin\left(\frac{3}{7} - 24t\right) - \frac{153}{7}\sin\left(\frac{1}{7} - 6t\right) - \frac{231}{5}\sin\left(\frac{3}{7} - 5t\right) - \frac{1023}{5}\sin\left(\frac{7}{5} - t\right) + \frac{116}{9}\sin(4t) + 2\sin(41t) + \frac{562}{5}\sin\left(2t + \frac{4}{3}\right) + \frac{367}{4}\sin\left(3t + \frac{11}{4}\right) + \frac{38}{3}\sin\left(7t + \frac{17}{4}\right) + \frac{104}{3}\sin\left(8t + \frac{53}{13}\right) + \frac{401}{10}\sin\left(9t + \frac{9}{4}\right) + \frac{119}{8}\sin\left(10t + \frac{17}{4}\right) + \frac{161}{10}\sin\left(11t + \frac{7}{3}\right) + \frac{44}{5}\sin\left(12t + \frac{17}{4}\right) + \frac{111}{4}\sin\left(13t + \frac{14}{5}\right) + \frac{83}{5}\sin\left(14t + \frac{8}{3}\right) + 5\sin\left(15t + \frac{1}{3}\right) + \frac{119}{6}\sin\left(16t + \frac{29}{7}\right) + \frac{32}{5}\sin\left(17t + \frac{25}{6}\right) + 4\sin\left(18t + \frac{1}{4}\right) + 9\sin\left(19t + \frac{2}{5}\right) + \frac{13}{5}\sin\left(20t + \frac{1}{3}\right) + \frac{44}{9}\sin(21t + 2) + \frac{14}{5}\sin\left(22t + \frac{46}{15}\right) + \frac{17}{3}\sin\left(23t + \frac{9}{4}\right) + \frac{11}{6}\sin\left(25t + \frac{17}{4}\right) + \frac{23}{3}$$