

GRAPHING POLAR CURVES USING *EXCEL*

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Polar curves are among the prettiest pictures that we graph, and it is fortunate that they are easy to graph, especially when we use *Excel*. Polar curves are really graphs of parametric equations:

$$x = f(t), \quad y = h(t), \quad t = \text{parameter.}$$

One good example of parametric equations is the motion of a projectile. The path of a projectile is given by

$$x = (v_0 \cos \theta)t, \quad y = -\frac{1}{2}gt^2 + (v_0 \sin \theta)t,$$

where x and y are the coordinates of position of the body, θ is the angle of projection, v_0 is the muzzle velocity, g is the acceleration of gravity, and t is time. Notice that if we eliminate t , we get the equation of the path of the body in x and y

$$y = -\left(\frac{g}{2v_0^2 \cos^2 \theta}\right)x^2 + (\tan \theta)x,$$

which graphs as a parabola.

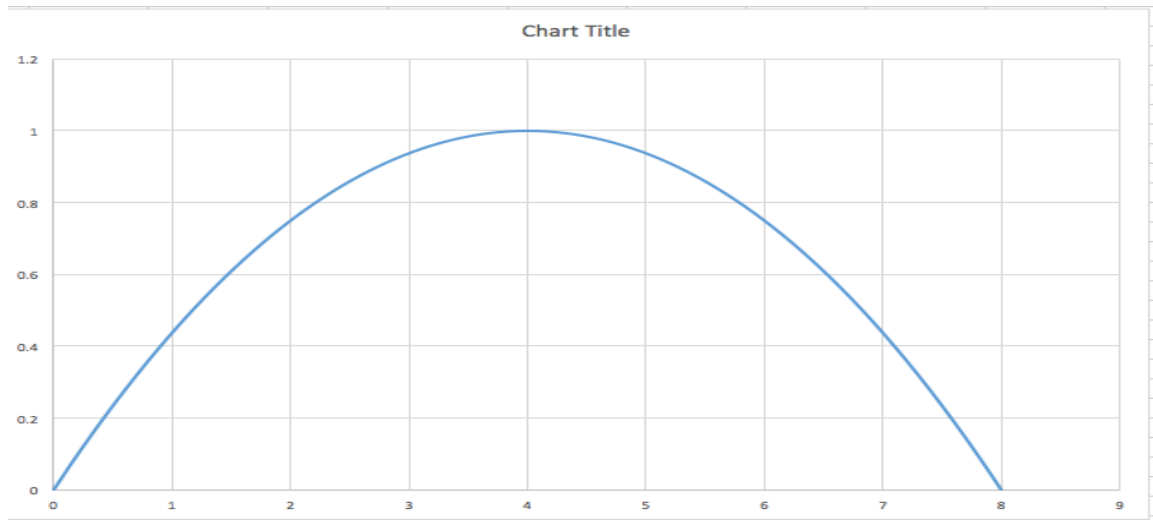


Figure1. Projectile Path

In *Excel* this is done as follows. Calculate x and y , then we plot them using the *Chart* utility of *Excel*. For a polar curve, $r = r(\theta)$, we calculate x and y as

$$x = r \cos \theta, \quad y = r \sin \theta$$

and plot them using *Excel*.

Familiar examples:

(1) Straight lines. A horizontal line is: $r = a \csc \theta$, or $x = a \cot \theta, y = a$.

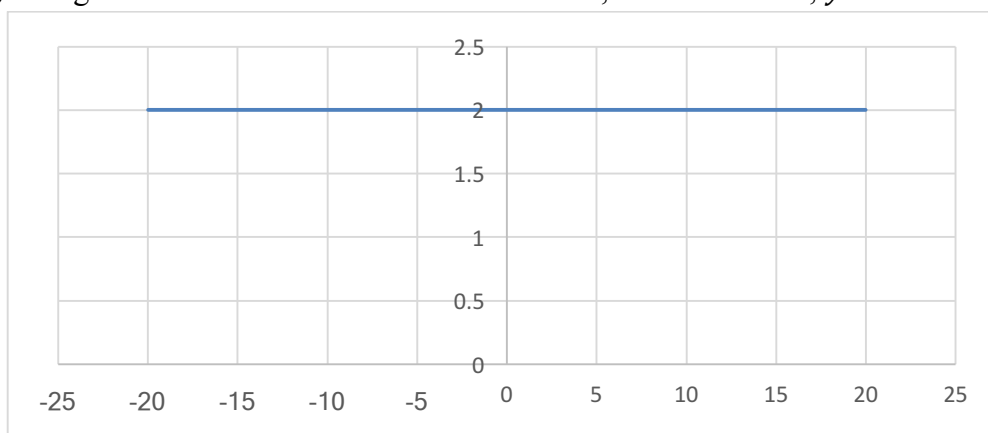


Figure 2. Horizontal line $y = 2$.

(2) A vertical line is: $r = a \sec \theta$, or $x = a$, $y = a \tan \theta$.

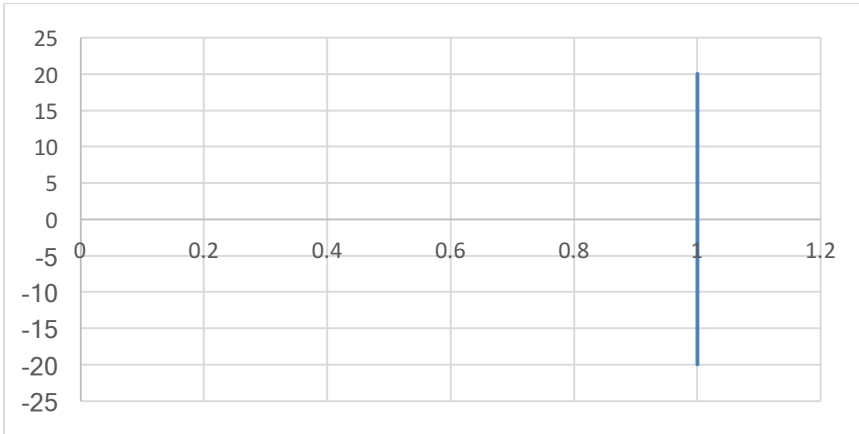


Figure 3. Vertical line $x = 1$.

(3) A general straight line Polar equation of a slant straight line is $r = \frac{c}{a \cos \theta + b \sin \theta}$, or

$$x = \frac{c \cos \theta}{a \cos \theta + b \sin \theta} \text{ and } y = \frac{c \sin \theta}{a \cos \theta + b \sin \theta}$$

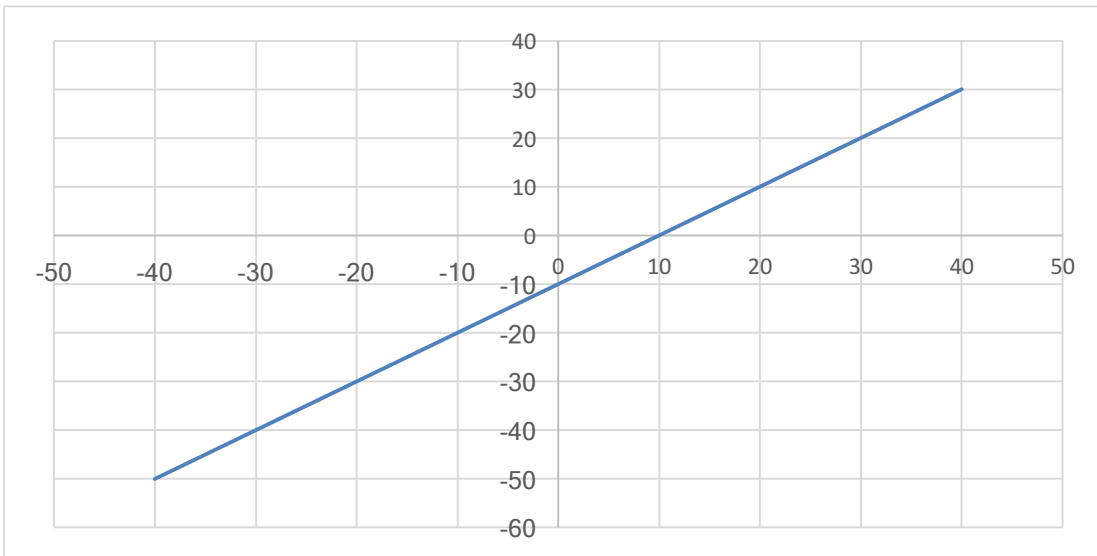


Figure 4. Straight line $r = -\frac{10}{\sin \theta - \cos \theta}$.

(4) Circles. (a) Center at origin, radius a : $r = a$ or $x = a \cos \theta$, $y = a \sin \theta$.

Figure 5a, b, c. Circles

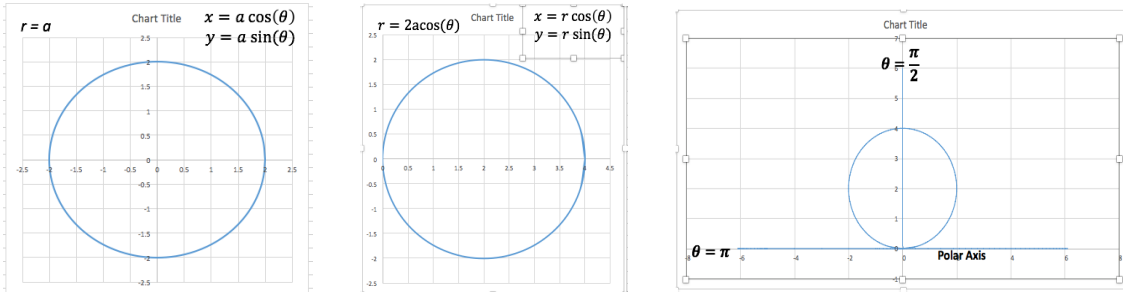
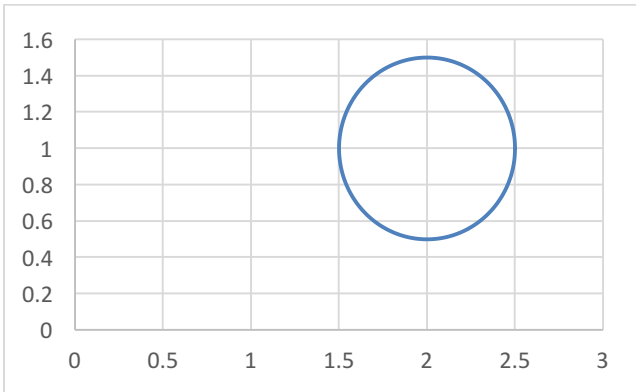


Figure 5d. Arbitrary Circle



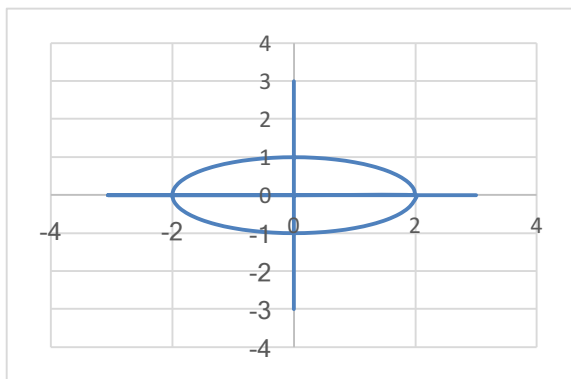
(b) Center at $(a, 0)$, radius a : $r = 2a \cos \theta$.

(c) Center at $(0, a)$, radius a : $r = 2a \sin \theta$.

(d) Center at (h, k) , radius a : $x = a \cos \theta + h$, $y = a \sin \theta + k$.

(5) Ellipse: Center at $(0, 0)$, axes a , b .

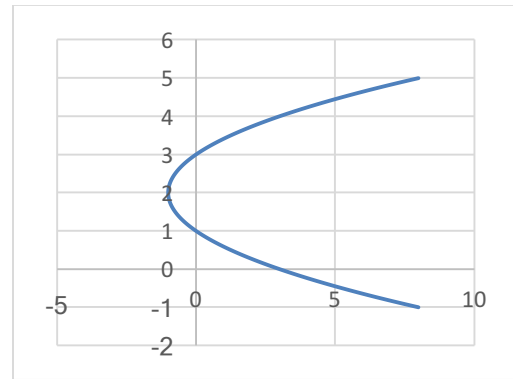
Figure 6. Ellipse



$$x = a \cos \theta, \quad y = b \sin \theta$$

(6) Parabola: Vertex at (h, k)

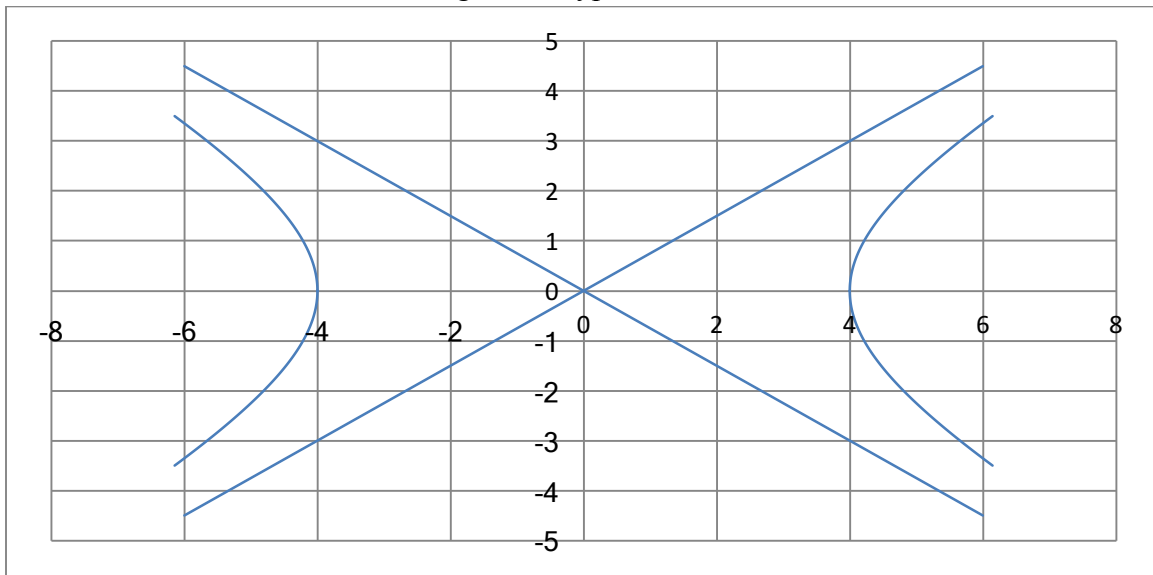
Figure 7. Parabola



$$x = t^2 - 2t, \quad y = t + 1$$

(7) Hyperbola: Center at $(0,0)$, axes a, b : $x = a \sec \theta, y = b \tan \theta$

Figure 8. Hyperbola



(8) Roses:

Figure 9a. Roses: $r = a \cos(n\theta)$; Figure 9b. $r = b + a \cos(n\theta), n = 3$.

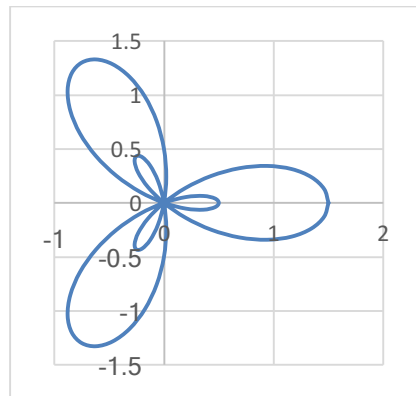
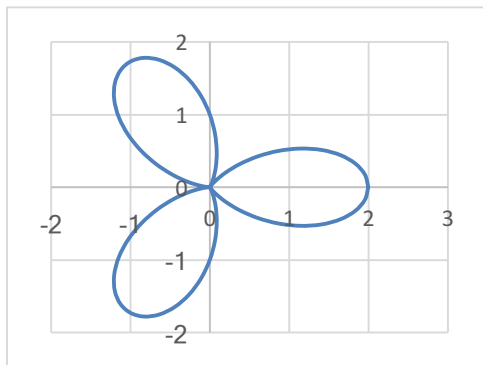
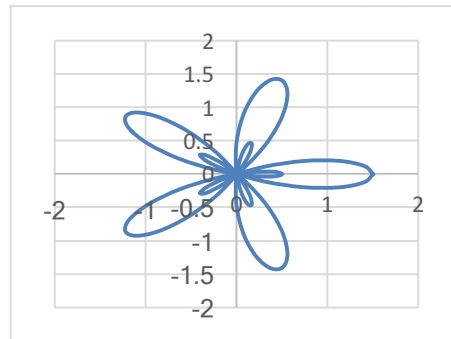
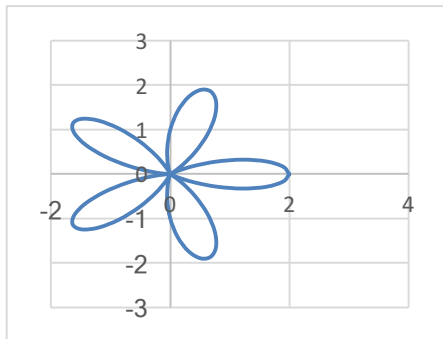


Figure 9c. Roses: $r = a \cos(n\theta)$; Figure 9d. $r = b + a \cos(n\theta), n = 5$.



(9) Cardioids:

Figure 10a $r = a(1 - \cos\theta)$

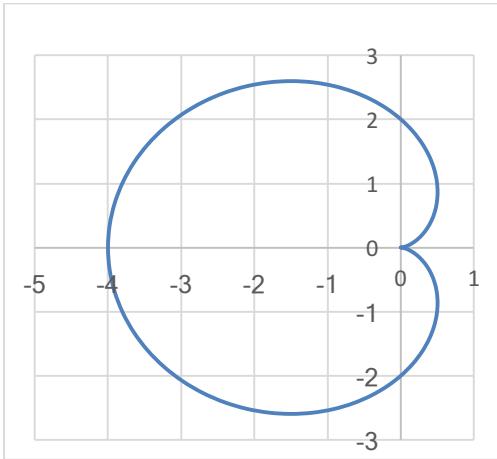


Figure 10b $r = a(1 - \sin\theta)$

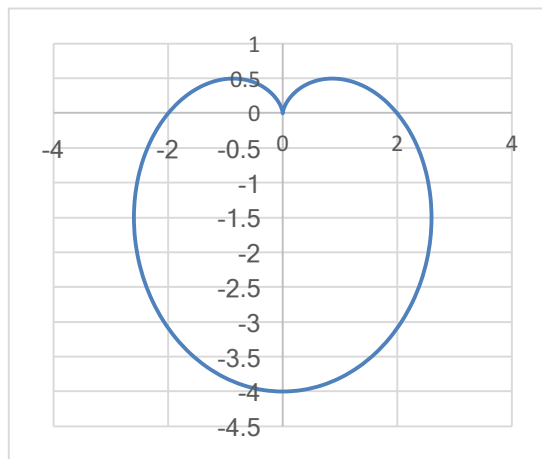


Figure 10c $r = b + a\cos\theta$; $a > b$

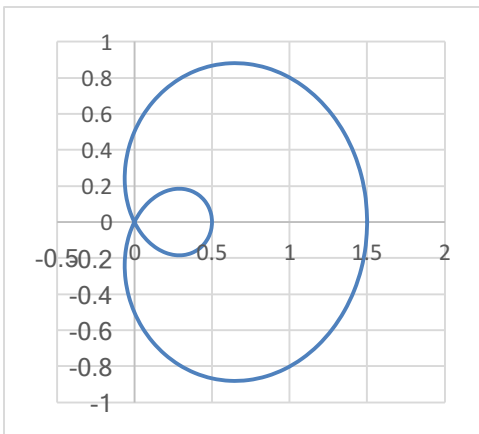


Figure 10d $r = b + a\sin\theta$; $a > b$

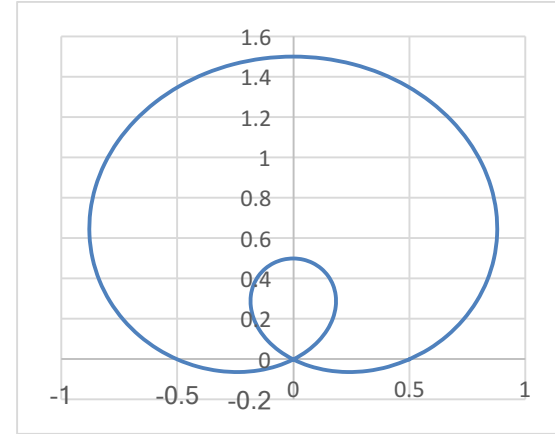
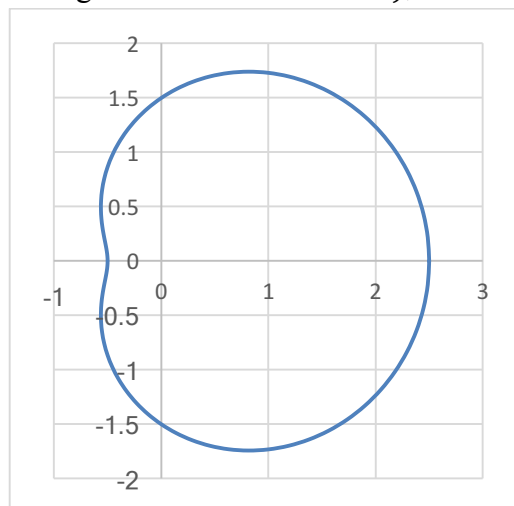


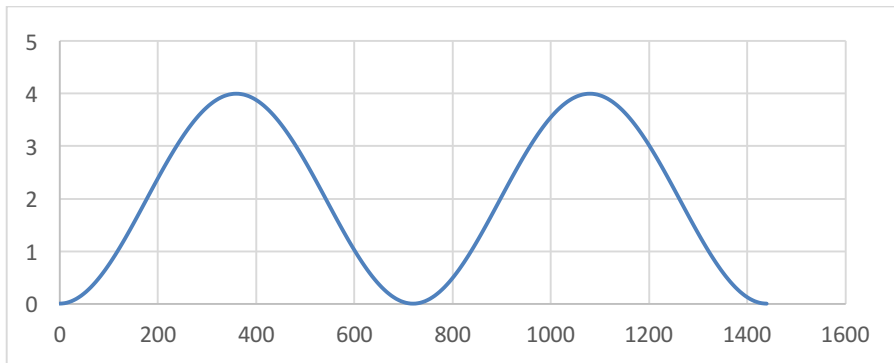
Figure 10e $r = b + a\cos\theta$; $a < b$



Examples of less familiar polar curves:

(1) Cycloid:

Figure11 $x = 2(a - \sin\theta); a > 0, y = 2(1 - \cos\theta)$



(2) Archimedean spiral

Figure12a $r = \frac{\theta}{2\pi}, [-2\pi, 2\pi]$

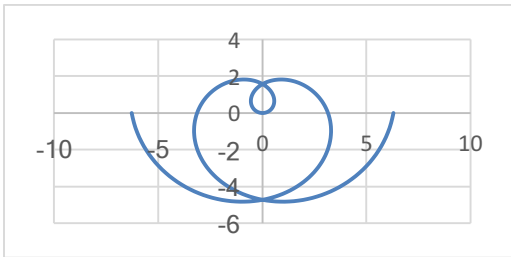
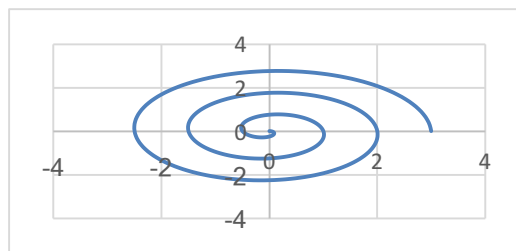
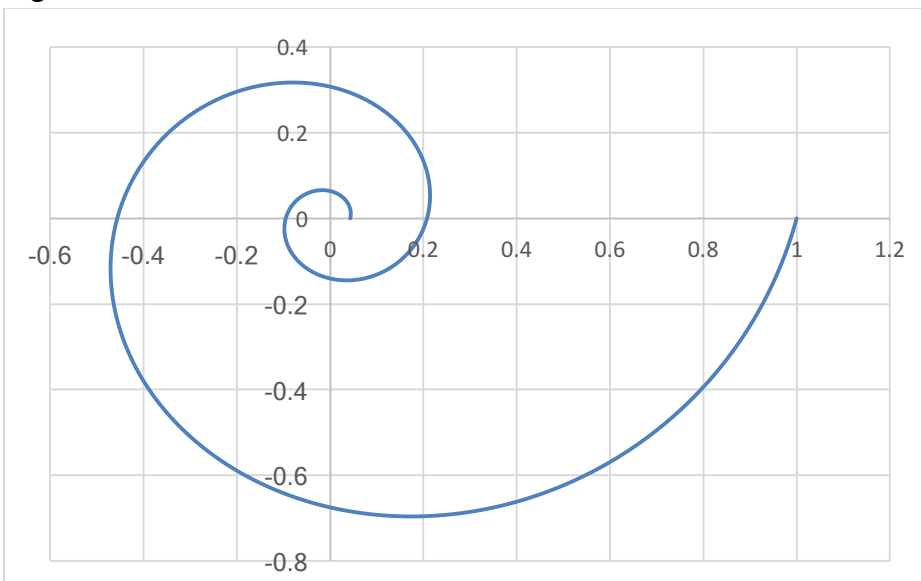


Figure12b $r = \theta, [0, 2n\pi]; n \geq 1$



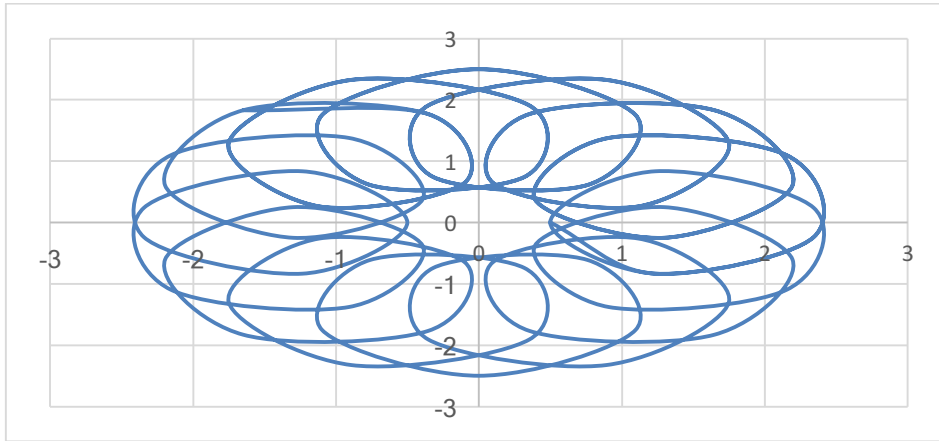
(3) Logarithmic Spiral

Figure13 $r = ae^{b\theta}$



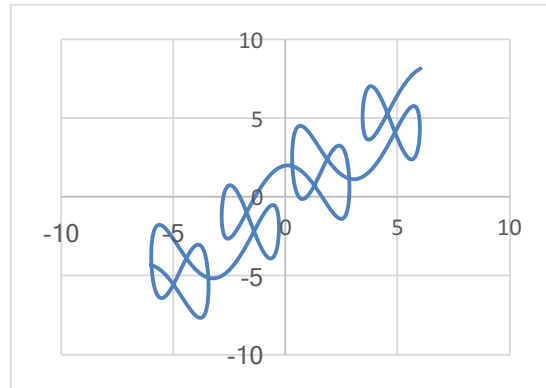
(4) Two Dimensional Torus:

Figure14a $x(\theta) = 1.5\cos\theta - \cos15\theta$, $y(\theta) = 1.5\sin\theta - \sin15\theta$



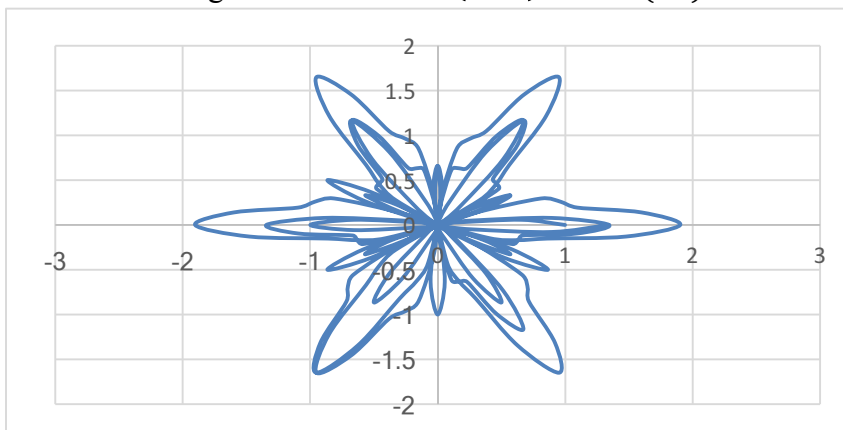
(5) Spring Shaped Curve

Figure15 $x(\theta) = \theta + 2\sin2\theta$; $y(\theta) = \theta + 2\cos5\theta$



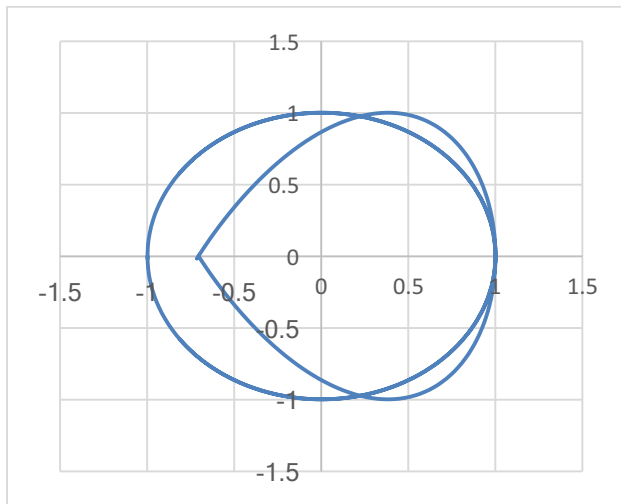
(6) Rose with multi-size petals

Figure16 $r = \sin^2(1.2\theta) + \cos^3(6\theta)$



(7) Graph of two intersecting curves in same Coordinate system

Figure17 $r_1 = a\sin^2\theta$, $r_2 = a$



(8) Lissajous curves

Figure18a $x(\theta) = 4\sin\left(\frac{12}{13}\theta\right)$
 $y(\theta) = 3\sin\theta$

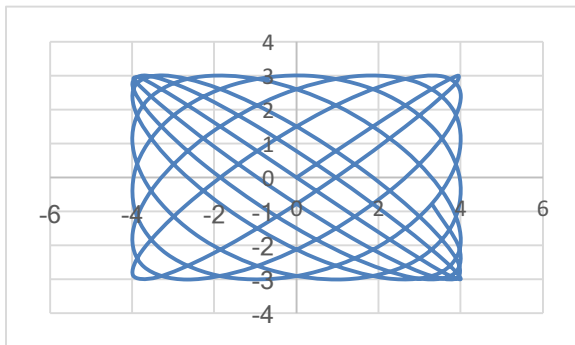
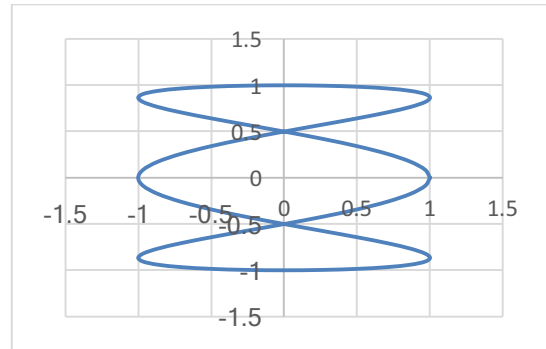
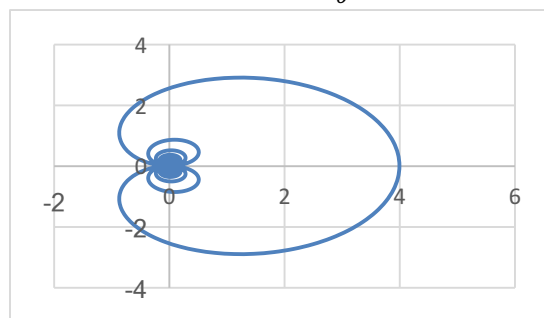


Figure18b $x(\theta) = \sin\left(3\theta + \frac{\pi}{2}\right)$
 $y(\theta) = \sin\theta$



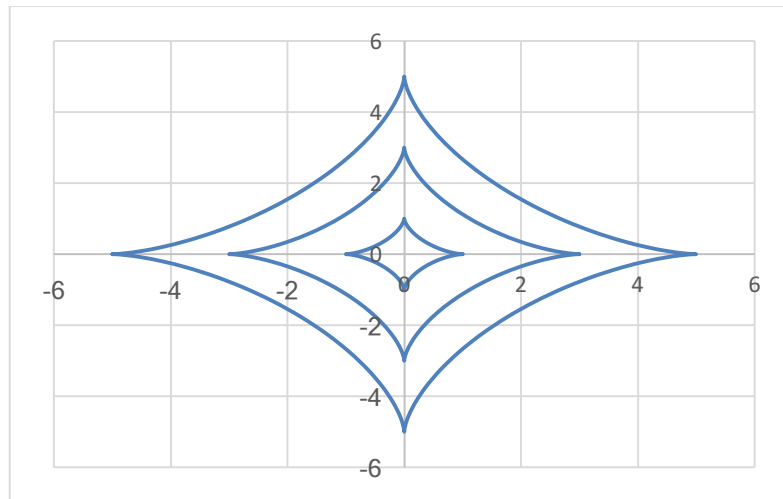
(9) Cochleoids

Figure19 $r = \frac{\sin\theta}{\theta}$



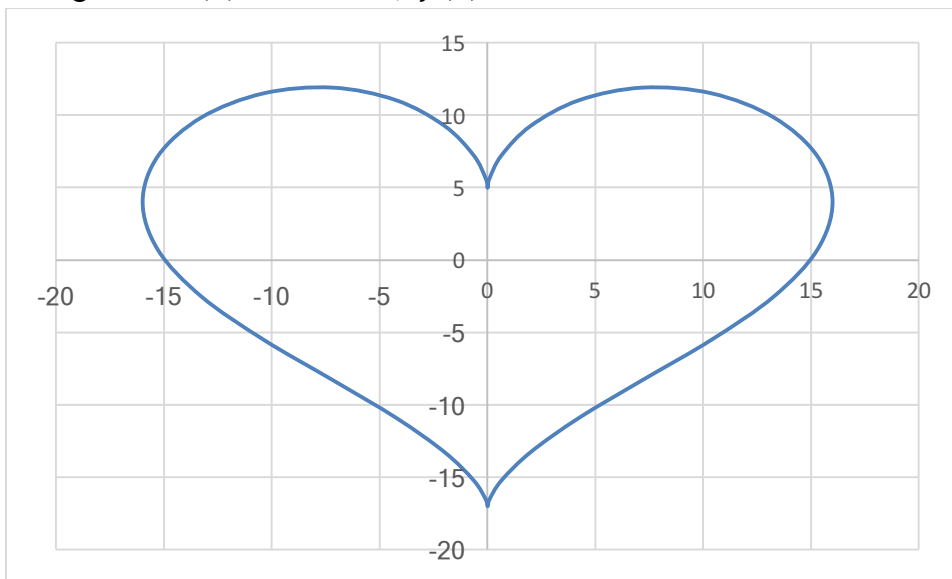
(10) Diamond Shape Family of Curves

Figure20 $x(\theta) = a\cos^3(\theta)$; $y(\theta) = a\sin^3(\theta)$



(11) Heart shaped curve

Figure21 $x(\theta) = 16\sin^3\theta$; $y(\theta) = 13\cos\theta - 5\cos2\theta - 2\cos3\theta - \cos4\theta$



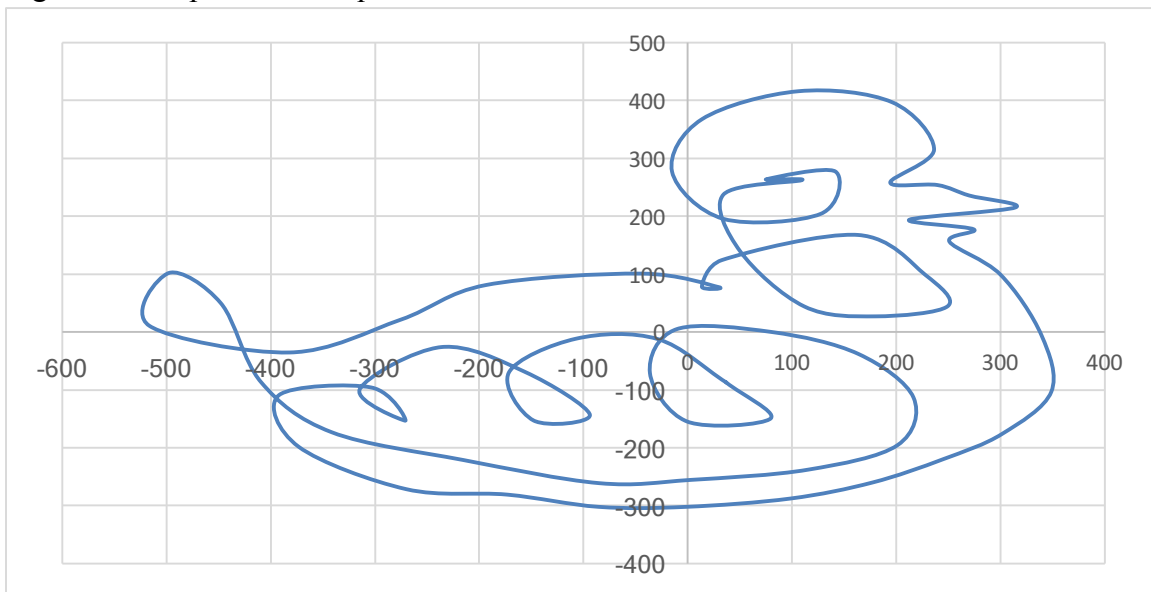
(12) Heart shaped curve with greetings

Figure22 $x(\theta) = 16\sin^3\theta$; $y(\theta) = 13\cos\theta - 5\cos 2\theta - 2\cos 3\theta - \cos 4\theta$



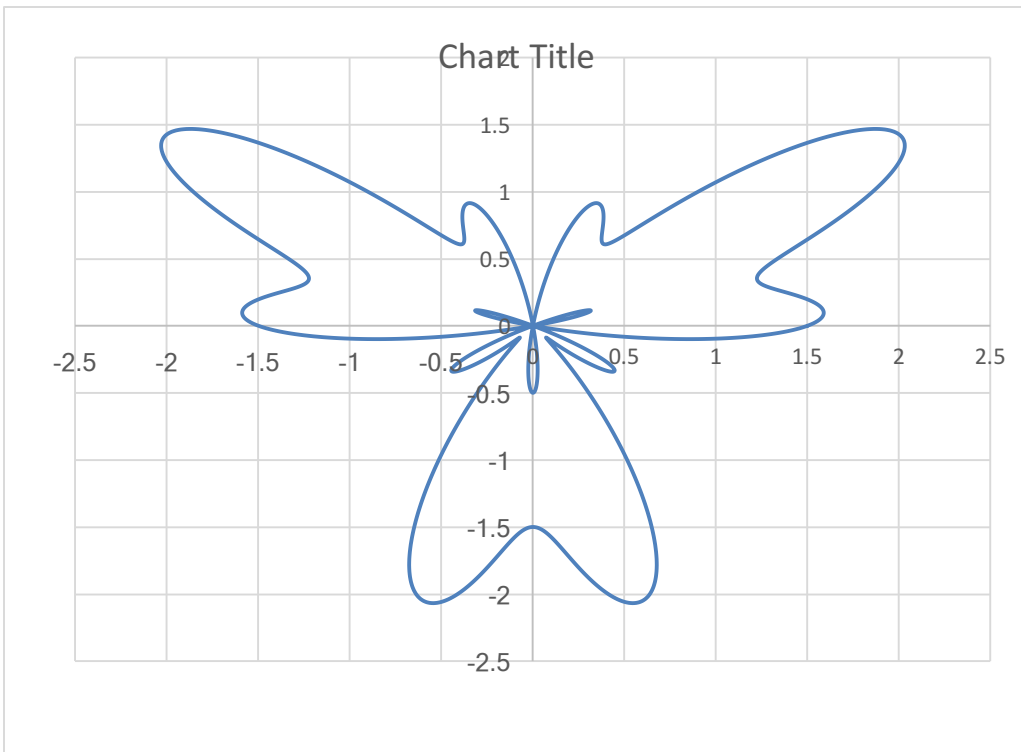
(13) Rubber Ducky

Figure23 For parametric equations see references



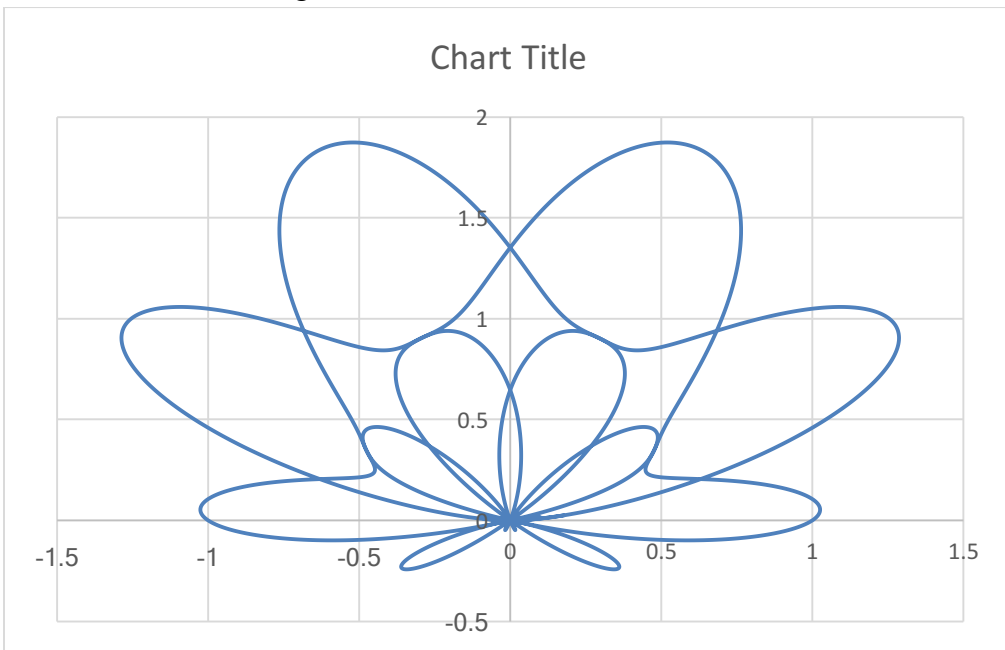
(14) Walking Butterfly Like curve

Figure24 $r = \cos^2 5\theta + \sin 3\theta + 0.5$



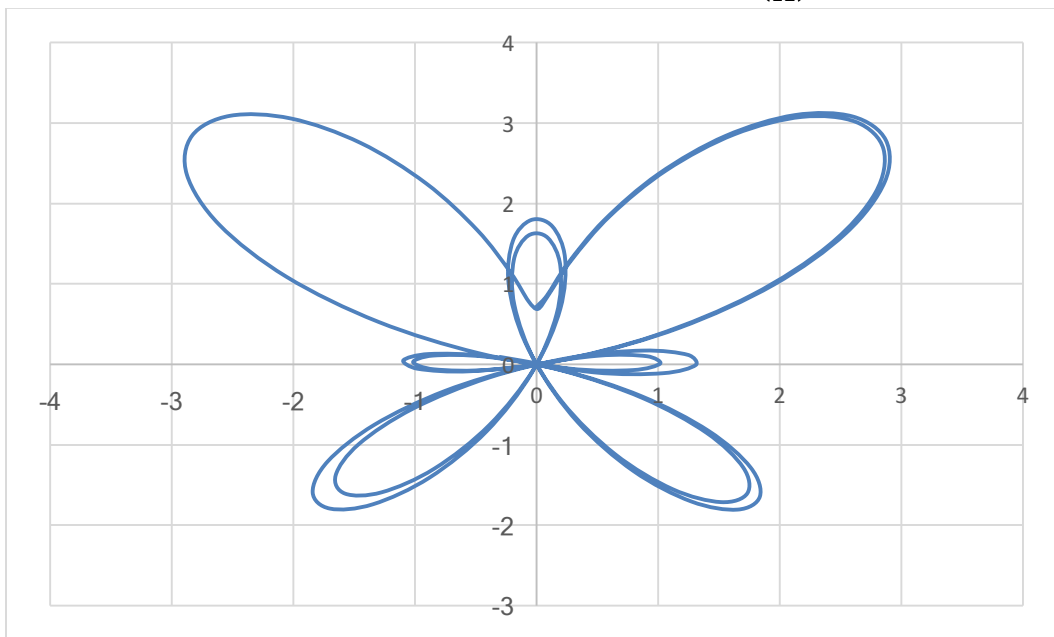
(15) Butterfly in the sitting position like curve

Figure25 $r = \sin\theta + \sin^3 2.5\theta$



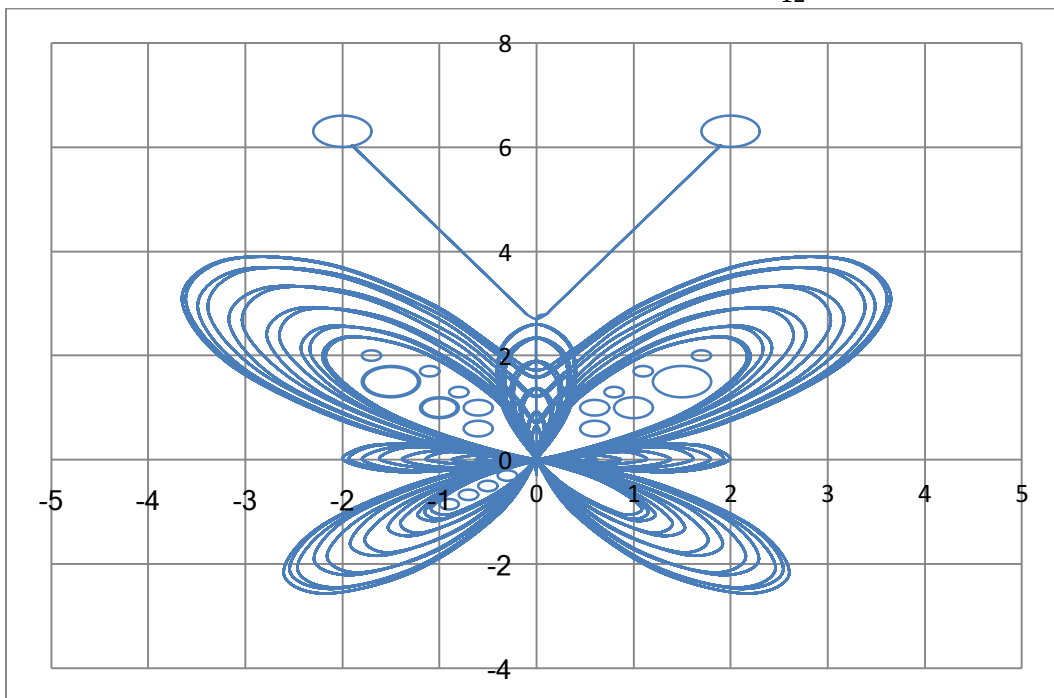
(16) Butterfly without Antennas

Figure26 $r = e^{\cos\theta} - 2\cos 4\theta + \sin\left(\frac{\theta}{12}\right)^5$



(17) Butterfly with Antennas

Figure27 $r = e^{\cos\theta} - 2\cos 4\theta + \sin\frac{5\theta}{12}$



Conclusions:

- A polar equation $r = r(\theta)$ can be written in parametric form $x = r\cos\theta$ and $y = r\sin\theta$.
- We can make use of Excel to graph polar equations by writing the polar equation in parametric form.
- Parametric equations are easy to graph, especially using *Excel*. We use the subtype chart “scatter with smooth lines” facility available in Charts group in *Excel* to plot graphs of the polar curves using their parametric form.
- Examples of polar curves, now can be displayed using *Excel*.
- Even complicated polar curves can be readily graphed when already expressed in parametric form, and sometimes they look very pretty.
- Polar equation is very useful way of representing space as a function of direction, and with the use of parametric equations we can get a clear, pretty and understandable picture of this relationship.

References:

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- 8 WolframAlpha <https://www.wolframalpha.com/examples/PopularCurves.html>
Rubber Ducky parametric equations on the next page

$$\begin{aligned}
x(t) = & -\frac{8}{5} \sin\left(\frac{11}{10} - 43t\right) - \frac{6}{5} \sin\left(\frac{6}{5} - 41t\right) - \frac{29}{7} \sin\left(\frac{4}{3} - 39t\right) - \frac{10}{3} \sin\left(\frac{7}{5} - 36t\right) - \\
& \frac{41}{10} \sin\left(\frac{2}{5} - 34t\right) - \frac{11}{5} \sin\left(\frac{23}{22} - 30t\right) - \frac{33}{5} \sin\left(\frac{2}{3} - 28t\right) - \frac{10}{3} \sin\left(\frac{5}{6} - 25t\right) - \frac{44}{5} \\
& \sin\left(\frac{1}{3} - 21t\right) - \frac{83}{7} \sin\left(\frac{8}{7} - 12t\right) - \frac{276}{5} \sin\left(\frac{6}{5} - 4t\right) + \frac{2}{3} \sin(42t) + \frac{852}{5} \sin\left(t + \frac{21}{5}\right) \\
& + 125 \sin\left(2t + \frac{3}{5}\right) + \frac{994}{5} \sin\left(3t + \frac{4}{5}\right) + \frac{592}{7} \sin\left(5t + \frac{19}{5}\right) + 34 \sin\left(6t + \frac{11}{3}\right) + \\
& 20 \sin\left(7t + \frac{8}{7}\right) + \frac{102}{5} \sin(8t + 2) + \frac{67}{3} \sin\left(9t + \frac{1}{3}\right) + \frac{122}{5} \sin\left(10t + \frac{9}{4}\right) + \frac{95}{3} \sin\left(11t + \frac{8}{9}\right) \\
& + \frac{37}{2} \sin\left(13t + \frac{6}{7}\right) + \frac{25}{2} \sin\left(14t + \frac{4}{3}\right) + \frac{47}{5} \sin\left(15t + \frac{1}{4}\right) + \frac{46}{3} \sin\left(16t + \frac{9}{4}\right) \\
& + \frac{15}{4} \sin\left(17t + \frac{22}{5}\right) + \frac{127}{14} \sin\left(18t + \frac{11}{3}\right) + \frac{11}{2} \sin\left(19t + \frac{85}{21}\right) + \frac{28}{3} \sin\left(20t + \frac{1}{5}\right) \\
& + \frac{23}{3} \sin\left(22t + \frac{17}{4}\right) + \frac{19}{3} \sin\left(23t + \frac{5}{4}\right) + \frac{12}{5} \sin\left(24t + \frac{15}{4}\right) + \frac{17}{6} \sin\left(26t + \frac{14}{3}\right) \\
& + \frac{25}{12} \sin\left(27t + \frac{5}{4}\right) + \frac{28}{5} \sin\left(29t + \frac{10}{9}\right) + \frac{5}{4} \sin\left(31t + \frac{17}{7}\right) + \frac{21}{5} \sin\left(32t + \frac{4}{3}\right) + \\
& 3 \sin\left(33t + \frac{11}{3}\right) + 3 \sin\left(35t + \frac{13}{4}\right) + \sin\left(37t + \frac{2}{5}\right) + \frac{39}{19} \sin\left(38t + \frac{10}{7}\right) + \frac{9}{5} \sin\left(40t + \frac{12}{5}\right)
\end{aligned}$$

$$\begin{aligned}
y(t) = & -\frac{7}{4} \sin\left(\frac{1}{9} - 39t\right) - \frac{2}{5} \sin\left(\frac{4}{5} - 35t\right) - \frac{11}{4} \sin\left(\frac{5}{4} - 33t\right) - 2 \sin\left(\frac{2}{5} - 26t\right) - \frac{13}{2} \\
& \sin\left(\frac{3}{7} - 24t\right) - \frac{153}{7} \sin\left(\frac{1}{7} - 6t\right) - \frac{231}{5} \sin\left(\frac{3}{7} - 5t\right) - \frac{1023}{5} \sin\left(\frac{7}{5} - t\right) + \frac{116}{9} \sin(4t) \\
& + 2 \sin(41t) + \frac{562}{5} \sin\left(2t + \frac{4}{3}\right) + \frac{367}{4} \sin\left(3t + \frac{11}{4}\right) + \frac{38}{3} \sin\left(7t + \frac{17}{4}\right) + \frac{104}{3} \sin\left(8t + \frac{53}{13}\right) \\
& + \frac{401}{10} \sin\left(9t + \frac{9}{4}\right) + \frac{119}{8} \sin\left(10t + \frac{17}{4}\right) + \frac{161}{10} \sin\left(11t + \frac{7}{3}\right) + \frac{44}{5} \sin\left(12t + \frac{17}{4}\right) \\
& + \frac{111}{4} \sin\left(13t + \frac{14}{5}\right) + \frac{83}{5} \sin\left(14t + \frac{8}{3}\right) + 5 \sin\left(15t + \frac{1}{3}\right) + \frac{119}{6} \sin\left(16t + \frac{29}{7}\right) \\
& + \frac{32}{5} \sin\left(17t + \frac{25}{6}\right) + 4 \sin\left(18t + \frac{1}{4}\right) + 9 \sin\left(19t + \frac{2}{5}\right) + \frac{13}{5} \sin\left(20t + \frac{1}{3}\right) + \\
& \frac{44}{9} \sin(21t + 2) + \frac{14}{5} \sin\left(22t + \frac{46}{15}\right) + \frac{17}{3} \sin\left(23t + \frac{9}{4}\right) + \frac{11}{6} \sin\left(25t + \frac{17}{4}\right) + \frac{23}{3}
\end{aligned}$$