COMPUTER AIDED MATHEMATICS IN CALCULUS I

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1. Introduction

At Tarleton State University one of the learning objectives for the mathematics program is the use of technology. This learning objective is introduced in Calculus I, reinforced in Calculus II and mastered in both Calculus III and Differential Equations. The technology learning objective covers the use of both graphing calculators and computer algebra systems (CAS) software. This report describes a set of labs, using *Mathematica*, that partially addresses the technology learning objective.

2. Overview of Calculus I at Tarleton State University

Calculus I is taught on a five day a week, 15 week semester. The students are predominately freshman and sophomore mathematics, engineering, or science majors. The course is designed as an early transcendental calculus course using the first five chapters of Stewart [1] or the equivalent material in Openstax [2]. Through the Texas A&M system, the students have access to the CAS *Mathematica* in all open access computer labs as well as being able to install the software on their personal computers.

Students enter Calculus I having moderate skill levels with the graphing calculator, but are unskilled with the use of CAS software. The course design varies based on the instructor, but in general the mathematical concepts are introduced in the classroom (lecture) with the aid of graphing calculators and CAS software, then homework is assigned. The homework incorporates the use of both the calculator and CAS software.

The course uses teaching computer labs to reinforce the mathematical concepts with *Mathematica* assignments. The technology labs usually include:

- A review of the mathematical concept(s).
- Example(s) of how to use a CAS to aid in the computations and visualizations.
- Assigned targeted problems for the students to complete using the CAS.

Ten such labs are presented for this version of the course and are assigned evenly throughout the semester.

3. Technology Lab Topics

The following technology labs may be found on the authors website [6] along with a syllabus and course calendar. The syllabus and calendar correspond to the most recent semester the course was taught by this author. The labs are updated as new versions of the software become available.

- **Lab 01 Introduction** - This notebook is a brief overview of the elements of the CAS *Mathematica*. The *Mathematica* interface and command structure is explored while reviewing expressions, functions, basic operations and graphing.
- **Lab 02 Limits** - One and two sided limits are explored in this notebook. The relationship between graphs of functions and limits is strengthened using continuous functions as well as functions with jump discontinuities and vertical asymptotes.
- **Lab 03 Derivatives Part 1** - A review of the limit definition of a derivative and an introduction of the *Mathematica* commands for computing derivatives of both expressions and functions.
- **Lab 04 Derivatives Part 2** - Uses the CAS to compute the derivatives of the commonly used functions: powers, trigonometric, exponential and logarithmic functions. Composite functions and higher order derivatives are also explored.
- **Lab 05 Implicit Graphs** - The graphs of families of implicitly defined functions are explored using animation (the Manipulate command [3]).
- **Lab 06 Linear Approximations** - The CAS is used to graph a function and it's tangent line near a point of interest. The tangent line is then used to approximate the value of the function near the point of tangency.
- **Lab 07 Local Maximum/Minimum Values** - The CAS is used to explore absolute and local extreme of a function on closed intervals.
- **Lab 08 Graphs** - The First and Second Derivative Tests, intervals where functions increase/decrease, concavity of functions and graphing are investigated.
- **Lab 09 Riemann Sums** - "Area Under a Curve" using approximating rectangles is explored. A complimentary notebook is provided that includes subroutines (Modules [4]) that implement right-, left-, and mid-point methods for approximating the area under a curve.
- **Lab 10 Introduction to Integrals** - The CAS is used to compute definite and indefinite integrals.

4. Impact of Technology Labs and Conclusion

The use of technology labs gives the students in Calculus I the opportunity to spend more time on topic, explore more complicated and real world problems as well as becoming familiar with professional level software used in industry. Based on two decades of student observations, the author has noted that there is a correlation between successfully completing the technology labs and success in Calculus I as well as the remainder of the calculus sequence.

These technology labs may be viewed using the Wolfram player [5]. All technology labs on the authors website [6] may be freely used. Suggestions for improvements are welcome. The author would like to thank the mathematics faculty at Tarleton State University for their input over the years that have contributed to these labs.

References

[1] James Stewart, *Single Variable Calculus: Early Transcendentals*, seventh edition, Brooks/Cole, 2012.
[2] Openstax, `http://openstax.org/`
[3] Wolfram *Mathematica 11*, Documentation Center, `http://reference.wolfram.com/language/` **Manipulate** command.
[4] Wolfram *Mathematica 11*, Documentation Center, `http://reference.wolfram.com/language/` **Module** command.
[5] Wolfram *Mathematica 11*, Document player, `https://www.wolfram.com/player/`
[6] Peter White, `http://faculty.tarleton.edu/white/`