

ENHANCING THE GEOMETRY CLASSROOM WITH GEOGEBRA PROJECTS

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Abstract

This paper discusses different discovery-based geometry projects (for Euclidian and non-Euclidian geometries) that I have been successfully using recently. These projects require use of the free GeoGebra math software. Furthermore, students' feedback about these projects are shared. Moreover, it will be discussed how student feedback and my observations have been incorporated in the structure of the class, and how that change has impacted student learning.

Introduction

My home institution, Utah Valley University (UVU), is primarily a teaching institution – the teaching load is 12 credit hours per semester. In addition, UVU is the largest public university in Utah (with almost 40,000 students as of 2019). The structure of the student body we serve is primarily non-traditional and very diverse – 51% of the students work over 21 hours/week, 32% of the students are non-traditional (over 25 years old), and 38% of the students are first generation [10]. As such, one of UVU's emphases is on engaged teaching and learning that prepare students to be successful after graduation. The recent research [1, 5] shows that one effective way to engage students in building their educational knowledge is active learning. As a result, I have been using various active learning approaches in my teaching such as Inquiry Based Learning (IBL), flipped classroom, in-class group work and presentations, individual and group hands-on class projects, etc.

The active learning strategy I discuss in this paper is the use of discovery-based, hands-on class projects. It is my belief that those active strategies improve the student-learning experience and increase their math preparedness, and at the same time benefit me – by enhancing my teaching practices.

In addition, the recent studies indicating the important role of technology in college education [4] prompted my teaching endeavor in using various technology tools to enrich student learning in my classes. Moreover, enhancing the class with technology allows students more flexibility and better engagement with the concepts covered [4, 13]. As a result, in the last decade or so, I have been using the free dynamic mathematics software GeoGebra to enhance the class projects in my college geometry classes.

This paper is structured as follows: First, the paper reviews the advantages of the technology tool I use for my geometry class and then discusses the structure of this class. Next, the discovery-based, hands-on class projects are presented. At the end, the collected feedback about the pros/cons of these discovery projects as well as my personal observations and reflections on the use of these activities are discussed.

Technology – why use GeoGebra?

Technology is an important learning tool to support problem solving and to promote understanding [14], and in addition, recent studies demonstrate that technology plays an important role in college education [4]. As a result, I have used various technology tools to enrich student learning in my classes throughout my teaching career.

When I started teaching college geometry classes (in 2004) I used the Geometer's SketchPad® which was supported by my previous institution and was free for my students. When my free access to the Geometer's SketchPad® was not available anymore, I started looking at other software that would give me the same or similar capability as the Geometer's SketchPad® and be affordable for me. Hence, I started using the dynamic mathematics software GeoGebra.

There were several reasons for using GeoGebra; the most important being that GeoGebra [2] is a free application and accessible to me and my students at school as well as off campus. In addition, GeoGebra is easy to use and is a great tool for creating interactive learning materials; that was the aim I wanted to achieve with my geometry projects. Moreover, it allows dynamic constructions, demonstrations, and explorations in the study of mathematics, especially in geometry. Additionally, GeoGebra can be used to build and investigate mathematical models, objects, figures, diagrams, and graphs, that allow students to do discovery explorations, make conjectures, and generalize statements. Furthermore, it helps with math visualization in a practical, engaging, and fun way, which makes geometry more accessible to students. Note that other similar software (Desmos, Cabri II Geometry®, the Geometer's SketchPad®, etc.) can be used to achieve similar goals with same success. The most convincing reason for me to use GeoGebra was that many of our math education students are already familiar with this software since they use it in other math education classes.

Structure of the UVU Foundations of Geometry class

The Foundations of Geometry class at UVU is a three-credit class, that meets 3 times per week for 50 minutes each. It is a required proof-based class for our math education majors and listed as an elective class for our math majors. Topics covered include: Axiomatic Systems (Finite Geometries, Incidence Geometry), Neutral Geometry, Euclidian Geometry, Non-Euclidian Geometries, and Transformational Geometry. I understand that learning is both more effective and more satisfying when one can be an active participant in the process. That is why I use the following syllabus statement to describe the objectives for my Foundations of Geometry class:

“Students will be introduced to the traditional topics in Euclidian and non-Euclidian geometries, and at the same time they will be given an opportunity to do some discovery lab work and to write proofs using a variety of proof techniques. Also, students will be given an opportunity to increase their fluency in math communication (by expressing themselves orally and in writing using “math language” through the in-class group work, discussions/presentations and through writing proofs).”

To accomplish this, I structure my class to include various pre-assigned reading/discovery tasks/lab projects, (almost) daily in-class presentations, daily in-class discussions, and frequent in-class group work. In this course, students are required to participate in the in-class discussions, to be active in the in-class group work (if any), and to present the results from the homework or from the group work to the class. In addition, students are assigned out-of-class projects. The evaluation rubric I use for this class is given in Table 1 below.

Assignments	Percent
Three in-class exams	45% (3 x 15%)
In-class participation	15%
Homework	10%
Out-of-class projects	10%
Final exam	20%

Table 1: Class evaluation rubric

The two main motivating factors for adding these types of discovery, hands-on projects (either as pre-assigned or out-of-class projects) to my class structure are the following:

- the struggle the students are facing when they encounter proofs for the first time; and
- the battle they fight for understanding non-Euclidian geometries – topics that put them out of their comfort, Euclidian zone.

Note that for most of the students (especially the math education students) Foundations of Geometry is the first class where they are asked to prove statements; here they learn different proof techniques and, at the same time, use these techniques to prove various geometry theorems. As a teacher, I recognize that the material of this course is more challenging and requires more writing skills than the classes they have taken before. Therefore, when there is an interest, I offer students out-of-class (voluntary) study sessions where we discuss any questions that they may have about the covered material in class as well as some of the solutions of their already-graded homework problems.

Discovery-based, hands-on class projects

As pointed out in the previous section, I use two types of projects in this class:

- pre-assigned reading/discovery tasks/lab projects; and
- out-of-class projects.

I. Pre-assigned reading/discovery tasks/lab projects:

Students regularly receive reading assignments/tasks that they need to finish before coming to class. These assignments can vary in content/type but the goal is the same: to prepare students for the in-class discussion on a particular assigned topic/content or to work in groups in answering the provided quick quiz questions based on the reading assignments.

The discovery lab projects ask students to use GeoGebra to explore a task, conjecture a statement that will be discussed/proved in class, and/or notice and reflect on various observations obtained by discovery. For example, some of these projects are designed to give opportunity to students to discover/explore the statements of some interesting geometry theorems that are then discussed and proved in class (e.g., the Median, Altitudes, and Perpendicular Bisectors Concurrence Theorems, the Euler Line Theorem, Ceva's Theorem, the Theorem of Menelaus, the Nine-Point Circle Theorem, the Nine Point Center Theorem, etc.). I have used these GeoGebra projects to motivate students to come to class prepared and to engage them in class as much as possible. In addition, many times students are better prepared to attack the proof of a theorem in class based on their GeoGebra explorations/observations done before class. Sample pre-assigned discovery questions are provided below [12, 13]:

The Nine-Point Circle Theorem (Figure 1)

1. Construction part:

Construct a triangle ΔABC and its orthocenter.

Construct the following points:

- the midpoint of each of the sides of ΔABC ;
- the feet of the three altitudes of ΔABC ; and
- the midpoints of the segments formed by joining the orthocenter to the vertices of ΔABC .

Construct a circle using any three of the points you constructed in the previous step.

Keep this file – it will be used for the next project.

2. Observation part:

What do you notice about this circle?

Are these relations/observations preserved if different triangles ΔABC are used?

3. Making conjecture(s) part:

Make a conjecture of the observations that you've done so far.

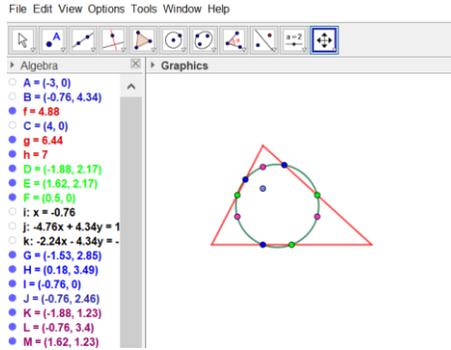


Figure 1: GeoGebra illustration for the Nine-Point Circle Theorem

The Nine-Point Center Theorem (continuation of previous project) (Figure 2)

1. Construction part:

Use the same GeoGebra file from the previous project.

Construct the circumcenter of ΔABC .

Construct the midpoint of the segment joining the orthocenter and the circumcenter of ΔABC .

Construct a circle with center – the midpoint you just constructed – and any other of the nine points you constructed in the previous project.

Find the length of the radius of this circle.

Find the length of the circumradius of ΔABC .

2. Observation part:

What do you notice about the circle you just constructed?

Compare the lengths of the radius of this circle and the circumradius. What do you notice?

Are these relations/observations preserved if different triangles ΔABC are used?

3. Making conjecture(s) part:

Make a conjecture of the observations that you've done so far.

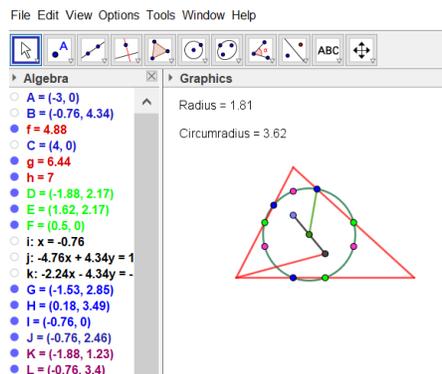


Figure 2: GeoGebra illustration for the Nine-Point Center Theorem

II. Out-of-class projects:

There are five to six out-of-class projects required for this class:

a) Origami geometry projects

One or two of the out-of-class projects are connected with Origami:

- The first one introduces the Origami geometry axioms and does comparison with the Straight Edge and a collapsible Compass (SE&C) construction axioms. In addition, it requires students to trisect an angle using Origami geometry (which is impossible to do with SE&C constructions) [3, 6, 7];
- The second project (if given) uses Origami and trigonometric functions to construct angles [8].

b) GeoGebra Euclidian geometry lab projects

Three of the out-of-class projects require use of GeoGebra to discover and explore some of the most famous geometry theorems in advanced Euclidian geometry and make conjectures of some amazing geometry facts (e.g., Napoleon's Theorem, Feuerbach's Theorem, Miquel's Theorem, Pascal's Mystic Hexagram, Morley's Theorem, etc.). These projects allow students to become (more) familiar with GeoGebra, discover and observe various relationships that exist among triangles' and circles' elements, and then to come to understand them for themselves.

In each of these GeoGebra lab projects students are guided to construct geometry object(s) and then observe various relationships between those objects, and at the end to make conjecture about their findings (similarly as it is done in the pre-assigned discovery lab projects). After they have submitted their project findings, during the class meeting, as a class we discuss what their discoveries were, and reveal the Theorem(s) they just discovered (without proving them – which is out of the scope of this class). Sample project questions are provided below [12].

GeoGebra Lab Project – Napoleon's Theorem (Figure 3)

1. Construction part:

Construct an equilateral triangle on each side of the original triangle ΔABC (either on the outside or inside of the original triangle).

Construct the centroid of the equilateral triangles $\Delta A'BC$, $\Delta AB'C$, and $\Delta ABC'$ (say U , V , and W respectfully).

2. Observation part:

Observe what type of triangle is ΔUVW ?

Consider the lines \overrightarrow{AU} , \overrightarrow{BV} , and \overrightarrow{CW} . What can you conclude about them?

Are these relations/observations preserved if different triangles ΔABC are used?

Are those observations still valid regardless of whether the equilateral triangles are constructed outside or inside the original triangle?

3. Making conjecture(s) part:
Make a conjecture of the observations that you've done so far.

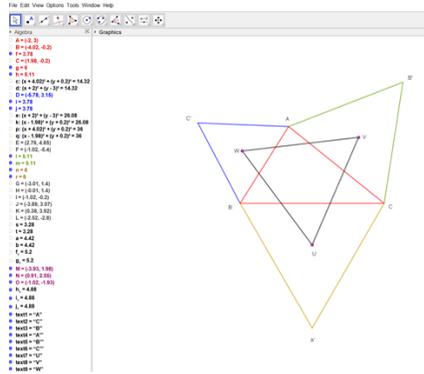


Figure 3: GeoGebra illustration for part of Napoleon's Theorem

GeoGebra Lab Project – Feuerbach's Theorem (Figure 4)

1. Construction part:
Construct the Nine-Point Circle for the original triangle ΔABC .
Construct the incircle and the three excircles of ΔABC .
2. Observation part:
Observe the relationship between the Nine-Point Circle and the four other circles.
Is this relation/observation preserved if different triangles ΔABC are used?
3. Making conjecture(s) part:
Make a conjecture of the observations that you've done so far.

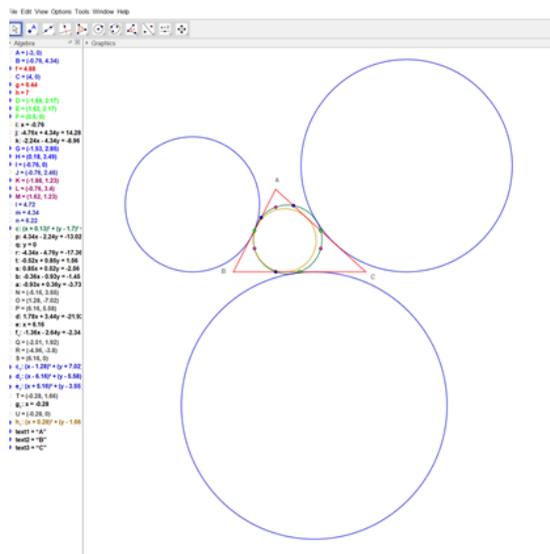


Figure 4: GeoGebra illustration of Feuerbach's Theorem

GeoGebra Lab Project – Miquel’s Theorem (Figure 5)

1. Construction part:
Construct points D , E , and F in the interiors of the sides \overline{BC} , \overline{AC} , and \overline{AB} of the original triangle $\triangle ABC$.
Construct the circumcircles for $\triangle AEF$, $\triangle BDF$, and $\triangle CDE$.
2. Observation part:
Observe the relationship between the three circumcircles.
Is this relation/observation preserved if different triangles $\triangle ABC$ are used?
3. Making conjecture(s) part:
Make a conjecture of the observations that you’ve done so far.

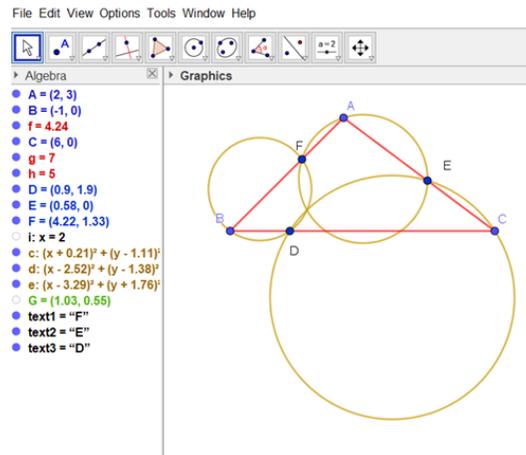


Figure 5: GeoGebra illustration of Miquel’s Theorem

GeoGebra Lab Project – Pascal’s Mystic Hexagram (Figure 6)

1. Construction part:
Construct a circle and choose six points A, B, C, D, E , and F in cyclic order on the circle. Adjust these points such that no pair of opposite sides of the hexagon $ABCDEF$ are parallel.
Construct the lines through the sides and the corresponding intersection points.
2. Observation part:
Observe the relationship between the three intersection points.
Is this relation/observation preserved if different six points on the circle are used?
Is this relation/observation preserved if different circles are used?
3. Making conjecture(s) part:
Make a conjecture of the observations that you’ve done so far.

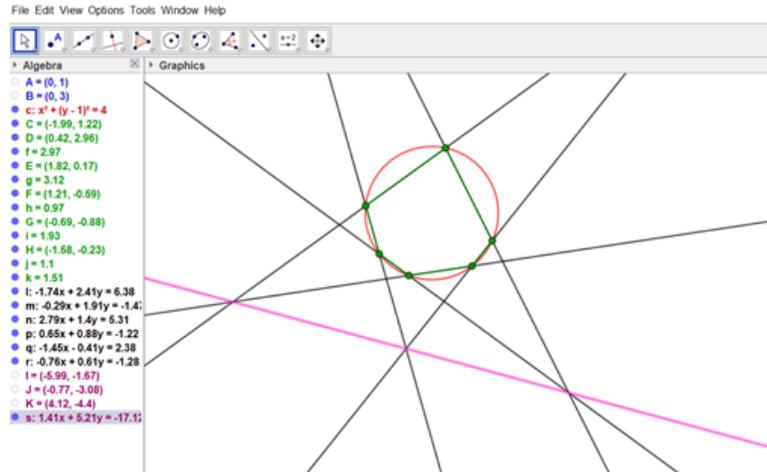


Figure 6: GeoGebra illustration of Pascal's Mystic Hexagram

c. GeoGebra non-Euclidian geometry lab project – Hyperbolic geometry

Understanding non-Euclidian geometries has been a huge struggle for my geometry students. I have been constantly trying to find ways to overcome this struggle. This involves usually bringing/making hands-on activities that can help visualize and experience/demonstrate the results we are discussing in these unfamiliar geometries. For example, when discussing Elliptic geometry, I bring stress balls and use their surfaces as a spherical model for the double elliptic geometry. In addition, we use rubber bands as models for lines in this spherical model. Then we use the stress ball and the rubber bands to construct spherical polygons (triangles and quadrilaterals) for further demonstration and explorations (Figure 7). After that, we discuss various results corresponding to the objects constructed (for example, sum of angles in triangle, quadrilateral, etc.).



Figure 7: Hands-on explorations of Elliptic Geometry

To provide hands-on experiences in Hyperbolic geometry, I recently started implementing a project that asks students to use GeoGebra to perform geometric constructions in the Poincaré disc model for the hyperbolic plane. Students are first asked to construct the model and then discover and explore some of the properties of the hyperbolic plane using that model.

The project consists of three parts. In Part I, students are first assigned a reading task: *Section 6.6: Showing consistency: A model for hyperbolic geometry* [13]. Then, they are

asked to use the knowledge they gained from *Section 6.6* [13] to construct a Poincaré disc model using GeoGebra. Note that I specifically ask them to construct their own GeoGebra Poincaré disc model, not use templates made by others.

In Part II of this project, students are asked to explore various properties of polygons (triangles and quadrilaterals) using GeoGebra and the model they constructed in Part I (Figure 8). As in the Euclidian geometry lab projects, this part of the project asks for construction, observation, generalization, and making conclusions/conjectures. Sample project questions pertaining properties and results about triangles and quadrilaterals (*Section 6.10: laboratory Activities using Dynamic geometry Software* [13]):

1. Determine if Pythagorean theorem is valid.
2. Find the defect of a given triangle, generalize, and draw conclusions.
3. Find the perimeter of a given triangle and quadrilateral.
4. Determine if the given quadrilateral is a parallelogram/rectangle.
5. Find the sum of the measures of the interior angles of the given quadrilateral and draw conclusions.
6. Explore properties of Saccheri and Lambert quadrilaterals:
 - a) Are they parallelograms?
 - b) Compare the length of opposite sides.
 - c) Compare the lengths found in b) with the length of the midpoint segment.
 - d) Make observations and state conjectures.
 - e) Compare the observations you made in d) with the corresponding results in the Euclidian plane.

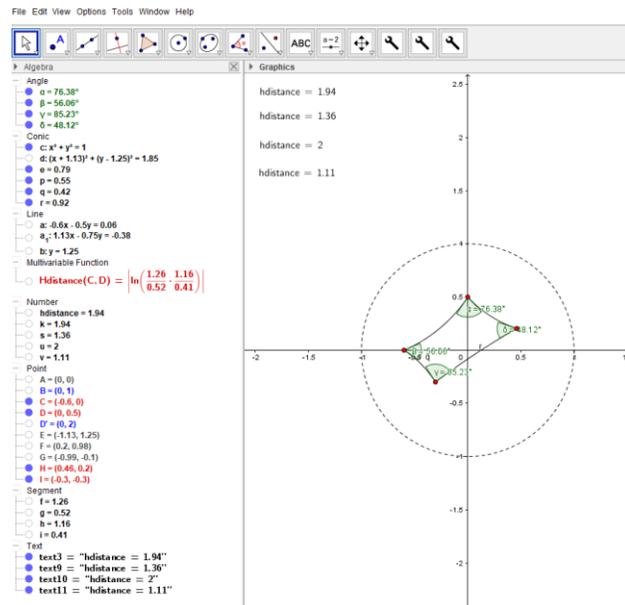


Figure 8: GeoGebra explorations in Poincaré dick model

Part III of this project involves demonstration/presentations of students' results and class discussion about Hyperbolic geometry.

The project does not require students to make any Macros, but I do strongly recommend that they do so, because it makes their explorations less tedious and time consuming.

My own observations and reflections, students' feedback, and survey results (IRB Approval # 01244)

At the end of each semester, I give a short survey to my students to assess the usefulness of the printed lecture notes I provide for this class, as well as the projects I give to make the student learning better and more interactive. Those are volunteer surveys, so students can choose to respond or not respond to them. There are no benefits to students who choose to answer the survey questions, except that the survey feedback I receive help me to evaluate the class and the activities undertaken. In this paper, I will share the survey responses collected during the most recent geometry classes I taught (Fall 2014 and Spring 2017), as well as my observations about the effect of the projects on student learning and understanding of the material.

Before I address the survey results, I want to point out that the class sizes were very small, so the number of responses is not statistically significant. Yet, the feedback was very helpful and a good indication for me as a teacher to continue improving and using these active learning strategies. I hope that this feedback and observations will also be helpful and beneficial for anyone who wants to implement them in their geometry classes. The class I taught in the fall of 2014 had a total of eight students (just half of them took the survey) and the one in the spring of 2017 had six students (all of them took the survey).

The survey results presented in Table 2 show that students thought GeoGebra projects were fun, interesting, and gave them opportunities to learn to/become better in use(ing) GeoGebra. In addition, students (strongly) agreed that the GeoGebra projects gave them opportunities to deepen their geometry knowledge by exploring and learning about some of the most interesting theorems in geometry. Also, my observation was that the interactive GeoGebra projects were a great tool for students who struggled to see the results discussed as well as for students who just wanted to enrich their knowledge through new, interesting, and fun facts. The survey results also showed that students (strongly) recommend using these GeoGebra projects in future classes.

	Semester and year taught	Fall 2014	Spring 2017
	# of students that took survey	4	6
#	Survey Statement		
5	The GeoGebra lab projects were fun and interesting.	4.75	4.83
6	The lab projects gave me opportunity to learn to/become better in use(ing) GeoGebra	4.75	4.66
7	The Lab projects gave me opportunity to conjecture and learn interesting theorems from modern geometry.	4.75	4.33
9	I recommend this type of lab projects to be given in Foundations of Geometry class	4.75	5

5-strongly agree; 4-agree; 3-neither; 2-disagree; 1-strongly disagree

Table 2: Survey results (IRB # 01244)

Note that I take very seriously the feedback my students provide on these surveys and most of the modifications I implement come from the helpful students' remarks. For example, one of the questions on the survey asked:

What would you change or make it differently if you were to give these types of projects to students?

Most of the students left this question blank, but the few students who addressed it provided very valuable feedback. Namely, one of the responses was "I might give more of them." And I would love to be able to give more of those projects without overwhelming the students. But keeping in mind that they take other classes, have regular homework and reading assignments for this class, and most of them have jobs outside of school, maintaining time balance for my students is very important for me. On the other hand, I did have a few students commenting that the projects took a lot of time to do. However, I would like to try and implement more projects next time I teach this class and take the students' feedback in account to weight on this.

Another student commented:

"I think these projects might be a good opportunity for collaborative work. I say that as someone who hates group projects, but seriously, I think this allows people with different strengths to have an opportunity to stand out."

Frequently, I do assign group projects in my classes, but sometimes I receive feedback from students that it is hard for them to work on a group project mostly because of time constraints. But I do plan to ask my students for their preference of individual vs. group projects next time I teach this class.

On the Spring 2017 survey I received a couple of comments that the wording on some of the projects was not as clear as they would have liked it (that apparently caused some confusion of what the project was asking them to do). I plan on rewording the questions and providing clearer instructions next time I teach this class.

The survey contains a section that ask students to provide overall comments about the projects: did they think if the projects were fun, interesting, difficult, not worthy of their time, etc.; and to provide any particular advantages/disadvantages in doing these types of projects. This section has proven to be very beneficial in improving these projects as they are today. Students are very open and honest in providing constructive criticism in their written feedback, and I truly appreciate that. For example, on the Fall 2014 survey, students indicated that the second Origami project was not beneficial to them, so I did not assign that project to my Spring 2017 class. Below are some sample comments (IRB # 01244) that reflect why students liked the lab projects:

“I definitely enjoyed the GeoGebra projects...[especially] enjoyed seeing the outcome... they were good because they let us do something a little different...[and] let us have fun with math.”

“[Projects] make geometry more interesting. They definitely provide more involvement.”

“The advantage [of these projects] was seeing geometry via a different medium, which was fun and helpful.”

“I enjoyed the lab projects. It made it easier to visualize what we were learning.”

“I really liked the GeoGebra projects and knowing how to use GeoGebra helped me in my HW.”

“They were good because they let us do something a little different, which is nice if you feel overwhelmed by the HW. I think the projects let us have a little fun w/ the math.”

Even students who did not really like the projects (mostly because of time constraints) still thought that they were worth doing and saw benefits from them:

“I don’t love them, but I can see how other people would and they would make geometry more interesting to them.... So even though I might have preferred not doing them I think it is important to have learned them, so I can incorporate similar projects when teaching.”

“They were interesting and fun. Because of time constraint, I couldn’t really enjoy them... They were worthy of my time, just not much time to give.”

My observation was that in general almost all students really enjoyed doing the projects, not just because they were fun, engaging, and hands-on, but also because students saw that they benefited from doing the projects in many ways:

“They give insight to writing proofs by showing these constructions.”

“The lab projects were really fun and due to how precise this software is, made it really easy to see results as opposed to trying to draw a diagram for a theorem, for example.”

In both classes I observed the same advantage: the pre-assigned GeoGebra lab projects prepared students much better for proving the theorems they were exploring before class. In previous classes where I did not use these pre-assigned GeoGebra projects, students struggled to ‘see’ how the proof of a particular theorem proceeds. The students who have done these pre-assigned projects, were much more alert and had much better visualization of how to proceed with proving the theorem, were able to be much more actively involved in class and in the in-class discussions, and used their GeoGebra explorations with more confidence to contribute toward the proofs done in class.

The sixth project (GeoGebra non-Euclidian geometry one) in the 2014 class was slightly different than the one I gave in the 2017 class. Namely, all students were assigned the same reading section (*Section 6.6* [13]), but not all of them had the same parts of the described project in previous section. In the 2014 project, I allowed students to choose which model of hyperbolic plane they want to construct: crocheted model [9], a triangular model [11], a soccer ball model [11], or the Poincaré disk model described in the previous section. Then they were asked to demonstrate and explore various properties of the hyperbolic polygons on their constructed model, and at the end to present about their finding. They all did an excellent job and truly enjoyed working on this project – and they were all proud to showcase their models at the end of the semester during the presentations (Figure 9).

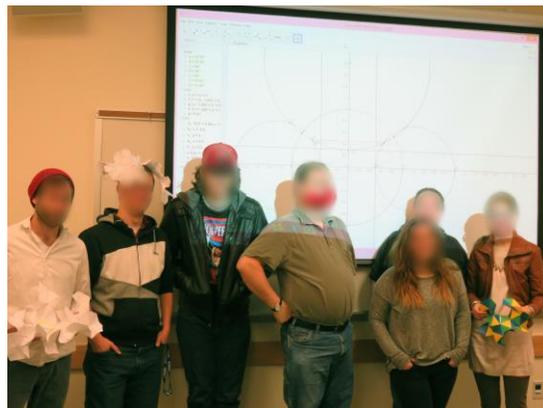


Figure 9: End of semester showcase of various models of hyperbolic plane – 2014

My observation was that students who read about the Poincaré disk model in the assigned reading section, but worked with non-GeoGebra models for hyperbolic plane, did not fully understand the material covered and have difficulty during the in-class work with Hyperbolic geometry as well as with the exam questions pertaining to this material. In addition, I compared the students that were working with Poincaré disk models: the ones that used already existing templates vs. the ones that made their model from ‘scratch.’ The latter ones demonstrated much better understanding of the model, showed better ability to discuss the explored results on their model, and expressed greater confidence during their presentation than the former ones. These observations led me to change the project in the 2017 class by requiring all students to use GeoGebra and construct the Poincaré disk model from scratch, which consequently led students to understanding the model much better.

Most of the written feedback and my observations, reflect similar conclusions: almost all students liked the GeoGebra projects because they helped them with the material, their homework, obtaining better visualizations, as well as the in-class proof constructions.

Conclusion

The lab projects have had a great impact on my students: they are better prepared for class after doing the pre-assigned discovery GeoGebra projects, and have fun hands-on time doing the exploration of more advanced geometry theorems using GeoGebra. Students become more communicative and confident when working during the in-class activities. I would strongly recommend to everyone interested in integrating technology in their classes, to try implementing similar lab projects and find the ones that work best for their students, material covered, as well as the ones that work best with their teaching strategy.

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