

MATHEMATICAL PROBLEM POSING AND REASONING THINKING OF PROSPECTIVE TEACHERS

Reda Abu Elwan, Ph.D

College of Education, Sultan Qaboos University

abuelwan@squ.edu.om

Abstract

Mathematics Education Program for prospective teachers in College of Education (SQU) has been accredited by NCTM SPA, the program has been established based on NCTM standards, Mathematics courses and Educational preparation courses are prepared to enhance mathematical thinking and a high quality of educational skills. During Teaching practice in schools I did notice that our candidates professional skills still suffering of applying standards indicators related to mathematical problem posing and mathematical reasoning thinking in their teaching. In this research, I did try to diagnose prospective teachers skills of problem posing and mathematical reasoning as they achieved it during their involved in the program. Two instruments has been established to measure their mathematical problem posing and mathematical reasoning thinking, Results of research was discussed based on a designed rubrics. In conclusion, it was concluded that mathematics candidates posed clear and understandable mathematics problems which were compatible with the mathematical reasoning.

Introduction

Mathematics Education program prepared SQU candidates in teaching mathematics for secondary students. The mathematics education program is eight semesters in length, and it requires the completion of at least 125 credit hours. However, most candidates take a preparatory year on campus to develop their English, mathematics and technology skills prior to taking university-level coursework. The inclusion of this Foundation Program makes the college experience five years for most candidates. Program completers are required to demonstrate mathematical knowledge proficiency on the educational field.

Study plan for the program cohort is to have the “Teaching practice” course in the last semester of the candidates’ study plan preceded by four field experience practicum courses called “professional practices” and The candidate spends a full day (6 hours) each week for the whole semester (15 weeks) at relevant hosting field experiences to gain some field experience related to the courses provided simultaneously in relevant semesters. During field experiences hours, candidates use mathematics technology such as graphing calculators, geometry and statistics interactive software with school students. Supervisors hold a weekly clinical settings session for a better use of these technologies in the field and for correcting errors that appear while the use of such technology is used by candidates. NCTM standards is essential of developing the whole program components.

Therefore, while working with our candidates we integrate activities of problem posing and reasoning activities into their method courses and teaching practice. In the present study I do aim at exploring how does the candidates performances on the development of problem posing and reasoning skills.

Research Problem:

The following questions were addressed:

1. What is the level of mathematics problem posing skills of prospective teachers?
2. What are the difficulties that prospective teachers face during problem posing process?
3. How dose mathematics prospective teachers' reasoning skills performance?

Background

School Mathematics should contain problem solving as the main activities in all mathematics aspects; also teachers should offer their students rich problems and reasoning activities, often based in the real world, which would challenge and excite them, because problem solving is an effective way to introduce and explore new areas in mathematics. Through problem solving and posing, the students can develop much of the mathematics for themselves. Candidates are prepared to teach mathematics with a problem solving approach as well with a mathematical reasoning , to help their students in solving mathematical problems. Educational program to do that doesn't reflect their abilities to solve problems and reasoning skills. Abilities to use different problem posing strategies, may affect their problem solving performance.

Problem Posing in mathematics

Problem posing is recognized as an important component of mathematics teaching and learning and it is an important part of problem solving (NCTM, 2000). In order that candidates will gain the knowledge and the required confidence for incorporating Problem Posing activities in their teaching, they have to experience it first, then doing a teaching practice. Candidates might use one or more strategies to formulate new problems or encourage their students in mathematics classes to be good problem posers as well as good problem solvers. Problem solving and posing Strategies could be used depending on the most suitable conditions (mathematics content, students, levels, learning outcomes and mathematical thinking, types). Problem posing situations are classified as free, semi-structured or structured situations.

Free Problem Posing Situations

Situations from daily life (in or outside school) can help a student to generate some questions leading him/her to construct a problem. Learners were asked to pose a problem to encourage them to "make up a simple or difficult problem" or " construct a problem suitable for a mathematics competition (or a test)" or " make up a problem he like." It is more useful if the teacher tries to relate the real life situations to the mathematics content being taught and to ask learners to pose new problems. This will be more effective in developing students' mathematical thinking. Problem posing situations might take these types: everyday life situation, free problem

posing, problem they like, problems for a mathematics competition, problems written for a friend and problems generated for fun.

Semi-Structured Problem Posing Situations

candidates are given on open-ended situation and are invited to explore it using knowledge, skills, concepts and relationships from their previous mathematical experiences and knowledge, this can take the following forms:

Open-ended problems (i.e. mathematical investigation).

Problems similar to a given problems.

Problem with similar situations.

Problems related to specific theorems.

problems derived from a given pictures.

word problems.

candidates can generate problems by omitting the questions from a given situations.

In this study; candidates were encouraged to discuss new strategies of problem posing situations in teaching practice with their learners , and meanwhile they should showed that captivities in final portfolio as a part of their duties.

Structured Problem Posing Situation

Any mathematical problem consists of known and unknown data. Candidate can simply change the known and pose a new problem, or keep the data and change the required. Brown and Walter (1990, 1993) designed an instructional problem formulating approach based on the posing of new problems from already solved problem, but they have also recommended varying the conditions or goals of given problem. This reformulation approach appears to be the most effective method for introducing structured problem posing activities in mathematic classrooms.

In order to create teaching/learning situations that provide a good problem posing situations, Lowrie (1999) recommended the mathematics teacher to:

- 1) encourage students to pose problems for friends who are at or near their own level until they become more competent in generating problems;
- 2) ensure that students work cooperatively in solving the problems so that the problem generator gains feedback on the appropriateness of the problem they have designed;
- 3) ask individuals to indicate the type of understanding and strategies the problem solver will need to use in order to solve the problem successfully before a friend generate a solution;
- 4) encourage problem solving teams to discuss, with one another, the extent to which they found problems to be difficult, confusing, motivating or challenging;

- 5) provide opportunities for less able students to work cooperatively with a peer who challenged the individual to engage in mathematics at a higher level than they were accustomed;
- 6) challenge students to move beyond traditional word problems by designing problems that are open ended and associated with real life experiences ; and
- 7) encourage students to use technology (calculators, CDs, computers) in developing their mathematical thinking skills, so they can use this technology to generate new mathematical situations.

Mathematical reasoning skills:

Mathematical reasoning is the glue that binds together all other mathematical skills. By using inductive and deductive reasoning as students learn mathematical concepts and solve mathematical problems, students come to recognize the extent to which reasoning applies to mathematics and to their world. NCTM vision toward Standard of Mathematical reasoning stated that: All students will develop reasoning ability and will become self-reliant, independent mathematical thinkers. There are various terms used to refer to "reasoning": critical thinking, higher-order thinking, logical reasoning, or simply reasoning. Different subject areas tend to use different terms. Across all of these subject areas, however, there are commonalities. Logical reasoning is defined as the process of using a rational, systematic series of steps based on sound mathematical procedures and given statements to arrive at a conclusion. ([Definition of Logical Reasoning | Chegg.com](#))

It is descriptive as: Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied. Mathematical reasoners are able to reflect on solutions to problems and determine whether or not they make sense. They appreciate the pervasive use and power of reasoning as a part of mathematics.

Algebra is an important area of mathematics and is often seen as the gateway subject to advanced math. While some students may see algebra as a time to memorize equations, good teachers know this is an incredibly important time to make sure students' math reasoning is solid.

In Teaching practice of Algebra in schools, our mathematics candidates does very little telling students how to think, instead they lets their students develop a conjecture that they believe to be true beyond the examples in front of them and requires them to explain why. In a classroom practices that promotes reasoning and problem posing, the candidate begins the class with a rich task designed to give students a chance to explore mathematical concepts by making connections to previous knowledge and then to use various strategies to complete the task. By thinking through what students already know and can use, trying an approach, considering whether

answer is reasonable, and sharing their thinking with classmates, they are developing a new approach of task related to presented problem, In a classroom practices that promotes reasoning and problem posing, the candidate begins the class with a rich task designed to give students a chance to explore mathematical concepts by making connections to previous knowledge and then to use various strategies to complete the task. By thinking through what students already know and can use, trying an approach, considering whether an answer is reasonable, and sharing their thinking with classmates, they are developing a new approach of task related to presented problem. In a classroom practices that promotes reasoning and problem posing, the candidate begins the class with a rich task designed to give students a chance to explore mathematical concepts by making connections to previous knowledge and then to use various strategies to complete the task. By thinking through what students already know and can use, trying an approach, considering whether an answer is reasonable, and sharing their thinking with classmates, they are developing a new approach of task related to presented problem. Such a steps that candidates can take to transform their mathematics classroom teaching practices into an environment that promotes reasoning and problem posing for all students are leading their students to the understand and appreciate the power of mathematics.

Such a steps that candidates can take to transform their mathematics classroom teaching practices into an environment that promotes reasoning and problem posing for all students are leading their students to the understand and appreciate the power of mathematics.

Methods

Subjects

Sample size was determined by what the researcher wanted to know, the purpose of the research, the value and worth of the data obtained, The sample size allowed the researcher to explore the variation of responses in depth as well as to maximize information. 28 candidates participated in the study. All of the them were in Final Grade in the College of Education, (Sultan Qaboos University) and their major subjects were mathematics. They were enrolled in the "Teaching Practice" Course. All of them were teaching Mathematics for Grades 7th, 8th and 9th of Basic Education schools, using assessment tools constructed based on NCTM Standards, Problem Posing and Reasoning are part of teaching instructions and assessed using different kind of tools.

Instruments

For that purpose I do employed two evaluative tools – Mathematical problem posing and Mathematical reasoning skills tests; Mathematics Problem Posing Skills Test was designed to determine the candidates' performance in problem posing skills It consists of five problems; each one based on real-life situations containing some mathematical information, but without questions and candidates had to formulate problems at three different levels (easy, moderate, and difficult); the situations were chosen from Benson (1993) and Billstein (2004). easy level corresponds to the development of a new problem with only a few parameters changed but the question is the same in the problem; moderate level corresponds to the addition/removal of parameters leading to development of question; difficult level means the student has to formulate a new creative problem based on the original.

While Mathematics Reasoning skills, the researcher designed a Mathematics Reasoning skills Test related to Algebra of Basic Education for the purpose of the study. Several steps have been done to develop the Test; determined its objectives, determined the mathematics content, choose the effective questions, then measuring the validity and stability of the test. Referees did some comments and changes on test, the researcher did the recommended comments. It consists of five questions, these questions asked students to choose the right answer based on his abilities of reasoning content of the question.

Results and Discussion

Findings and interpretations related to the first research question of “*What is the level of mathematics problem posing skills of prospective teachers?*” are as follows. Students were asked to pose a problem related to the numbers and variables subject suitable for the basic education.

Table 1 showed percentages of posed problems by participants

Table:1 results of problem posing skills test

Problem number	Posed Problems		
	Difficult (3 pt)	Moderate (2 pt)	Easy (1 pt)
1	18 (64%)	5 (18%)	5 (18%)
2	14 (50%)	6 (43%)	8 (29%)
3	5 (18%)	12 (43%)	11 (39%)
4	10 (36%)	8 (29%)	10 (36%)
5	12 (43%)	5 (18%)	11 (39%)

Candidates performance was reasonable as the highest percentage for the difficult posed problems for Q1, Q2, Q5, and Q4. While Q3 was the lowest percentage of difficult posed problems as it seen that 5 participants out of 28 students that participated in the study posed problems.

Interview has been done with only the 23 participants whom failed to pose new hard problems and were asked: “*why you failed to pose new hard problem based on problem# 3?, Is there any relation to your answer related to your knowledge of problem posing skills?, and how is your problem posing skills affected by reasoning skills?*”

Q3 stated: “7 is a prime number, so May 7th is a prime day. In all May has ? prime days”

The difficulties that they mentioned for that, is it was hard to put the question in another contexts. And no further givens in problem to formulate new problems. While some of them posed problems not related to the numbers subject, and therefore these problems were not included in the evaluation. These five problems were evaluated using the “problem posing grading rubric”. The rubric consisted of four skills dimensions (creative problem posing, well structured problem, solvable problem by basic education students, and new problem based on new contexts).

Interview with those who did well on the hard problem posing level, assured that mathematics education program was rich of problem posing and mathematical reasoning skills activates in mathematics courses as well as on the profession courses, that affect their skills to achieve the scores of the difficult problems.

Table 2 shows the level of reasoning of each respondent of participants on each of the given mathematical problem. Each problem consists of three parts of reasoning skills: Generalization, induction, and logical proof), all has same weight of 3 points.

Table 2: percentage of reasoning skills test

Problems number	Reasoning skills		
	Generalization achieved (3ps)	Mathematical induction (3ps)	Logical proof (3ps)
1	10 (35%)	5 (18%)	13 (46%)
2	9 (32%)	7 (25%)	12 (43%)
3	11 (39%)	6 (21%)	10 (35%)
4	10 (35%)	5 (18%)	13 (46%)
5	12 (43%)	6 (21%)	10 (35%)
Average Total	52 (37%)	26 (19%)	48 (34%)

The Table indicates that out of the 28 participants 37% was the average of generalization skill, 19% responses from the participants respondents was the average of mathematical induction skills, and 34% of participants among those respondents to logical proof skills. it was Task related to mathematical induction that achieved the lowest parentage among the three parts, which is the hardest skills of reasoning and it needs deep understanding and perception of mathematical relationships

This tallies with the results of the studies made by Yumus (2001) and Bryan (n.d.), where pre-service teachers also demonstrated a low reasoning level when exposed to some mathematical tasks. A summary of how responses were categorized as in the previous table serves only to provide a rough glimpse of the wealth of information about these eleven (28) pre-service teachers' reasoning skills that done by problem posing skills results. To illuminate some of the more general findings in this study, here are some concluded general points:

- Mathematics education program for college of education was effective as the two standards of problem solving and mathematics reasoning were achieved in a reasonable results of the tests.
- Most of participants were exited to showing their skills and abilities of these highly mathematical skills.
- Both standards of problem posing and reasoning in mathematics considered as a high levels of mathematics education and teaching practice as it mentioned by candidates.

Conclusion:

Reasoning ability and problem posing skills could be developed in a multi math course s such as profession courses: Math for teachers I, II, Methods of teaching mathematics, Teaching practices. It has to be developed throughout the entire education program curriculum. Educators know that changes in student outcomes are the result of changes in curriculum and instruction. However, it is apparent that many teachers today are caught in the midst of change for which they have been professionally prepared. Many were educated in classrooms where the role of the

student was to memorize information, conduct well-regulated experiments, perform mathematical calculations using a specific algorithm, and were then tested on their ability to repeat or remember specific facts. Teachers should expect their students to explain and justify their answers, assist them in considering and evaluating several solutions to a problem and create a classroom environment where students feel comfortable questioning, challenging, suspending judgment and demanding reasons and justifications as they deal with mathematical and real-world content. Mathematics will continue on its present course if pre-service teachers do not acquire and learn how to apply reasoning skills and problem posing well and curriculum planners have to respond to the need of developing mathematical reasoning in teacher development programs.

References

- [1] Abu-Elwan, Reda (2007). The Use of WebQuest to enhance the mathematical problem posing skills of pre-service teachers, *The International Journal for Technology in Mathematics Education*, 14(1), 31- 39.
- [2] Bryan, T.J.(2001). The conceptual knowledge of preservice secondary mathematics teachers: how well do they know the subject matter they will teach? Issues in the Undergraduate Mathematics Preparation of School Teachers. www.k-12prep.math.edu/journal/contentknowledge/bryan01/article.pdf
- [3] National Council of Teachers of Mathematics (2000). *Principles and Evaluation Standards for School Mathematics*. Reston, VA: Author
- [4] Sutherland, R. & Balasheff, N. (1999). Didactical complexity of computational environments for the learning of mathematics. *International Journal of Computers for Mathematical Learning*. V. 4, pp. 1-26.
- [5] Walter.M. & Brown, S. (1993). *Problem Posing: Reflections and Applications*. Hilda, New Jersey: Lawrence Erlbaum Associates
- [6] Vincent, J. (2003). A technological environment for promoting mathematical reasoning: A dynamic approach to geometric proof. *Paper presented at ICT and the teacher of the future conference held at St. Hilda's college, the university of Melbourne, 27th -31st January, 2003.*
- [7] www.chegg.com/homework-help/definitions/logical-reasoning-63