USE OF TECHNOLOGY IN MATHEMATICAL MODELING ACROSS THE CURRICULUM

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Abstract
In this paper we will introduce the audience to a proposed program, "Mathematics Modelling across the Curriculum" (**M²AC**), at Texas Lutheran University. The goal of this program is to help students and faculty across the curriculum to become comfortable with using mathematical modeling in their discipline. We will demonstrate the role of technology through several modeling projects in non-STEM disciplines.

Keywords: Math/Stat Modelling, CAS, Maple. Math Web resources

Introduction
M²AC is a proposed program at Texas Lutheran University which encourages and facilitates the use of mathematical and statistical modeling across disciplines. The goal of this program is to engage faculty and students from history, political science, business, and other disciplines from outside of mathematics in the creation of meaningful quantitative applications in their own discipline. We want to encourage students and faculty from non-STEM fields to use these quantitative modules in their classes. To that end we intend to create an on-line **M²AC** archive of peer reviewed modeling projects, available to TLU faculty and faculty across the USA. Modules in this archive will be meaningful and "turn-key" so as to minimize the work required of a faculty member who wishes to introduce a quantitative application into a class that has not been quantitative. The projects will be produced by small teams typically consist of two to three students led by a faculty member from an appropriate discipline (natural sciences, social sciences, or applied studies) with a math faculty member serving as a resource. The mathematics/statistics faculty will supply encouragement, guidance, and support but will not be the builders of the modules. Because of the use of technology it is not necessary that the faculty and students be deeply versed in mathematics. Tools such as Excel, Maple or Desmos remove the drudgery of long calculations and helps the practitioner build confidence, thus encouraging their repeated explorations into quantitative applications.

Of particular interest to us are modeling problems which utilize technology and are relevant to current events and affect society as a whole such as sustainability, climate change, voting patterns and sports. There is a strong role for technology such as Maple and Statistical software packages and online resources such as WolframAlpha. These tools facilitate the completion of the project by non-STEM faculty and students. An example of a typical project: Does a high star-rating of a high school football player translate into future success (college and NFL)? Team activities will include collecting data and developing the statistical model which predicts the probability of an NFL draft based on the star–rating of a player.

TLU has a modest history of faculty and student summer cooperation on innovative projects. Both authors have an extensive record of presentations and publication in the use of technology in mathematical modeling [1], [2]. Reza Abbasian has utilized internal grants to conduct summer modeling projects with students. John Sieben has created several differential equation modeling projects which are available at the SIMIODE (Systematic Initiative for Modelling Investigations and Opportunities with Differential Equations) web site [3]. Both authors are members of SIMIODE which is a community of math educators dedicated to a modeling approach to the teaching of differential equations. There has been similar projects in the past such as the 2003 MATC project at Dartmouth College [4], although that program did not emphasis modelling or use of technology. In the following sections, we will present examples of the modelling projects which are either already completed or soon to be completed as well as proposed projects from various disciplines. Our paper is intended for mathematics and/or statistics educators with interest in using CAS and web resources in their classroom teaching.

Examples of Modelling Projects

Example 1: What does the data say about a Mercy Rule for college football?

Motivation for the project:

- Football is the leading cause of catastrophic injuries (brain or spinal cord) in College Sports.
- NCCSIR data for Last 31 years: Among 11 sports, Football accounted for 62% of major injuries in college and 58% in High School
- College football does not implement mercy rules

Goal: Implement a mercy rule in college football games based on score difference and thus reduce the number of major injuries.

Following is a short list of the current state of the mercy rule in college and International Sports:

- NCAA /International baseball: Game ends if ahead by 10 runs after 7 innings except for tournament play
- NCAA softball: Game ends if ahead by 8 runs after 5 innings.
- Amateur boxing: Fight stops if a boxer leads by 20 points.
- Basketball: No mercy rule in college. Most states have “continuous clock” rule, game stops after a large lead (ex: 35 points in Iowa) in second half
- NCAA wrestling: Win by technical fall if ahead by 15 points.

- International free style and Greco-Roman wrestling: Win by technical superiority if ahead by 10 points.

Currently there are no “official” mercy rules in college football. However, one could find many examples of high school and college football games where the games were shortened by mutual agreement and unofficially allows “continuous clock”. For example in 2003, Oklahoma sooners were beating Texas A&M 59-0 at the half time. Both coaches agreed to use a “continuous clock”. Oklahoma won the game 77-0. In 2013,University of North Carolina was beating Old Dominion by a score of 80-20 before the fourth quarter was shortened to 10 minutes. In our project we utilized the data from 10464 division-I games [5] which were played between 2003 to 2015. Initially we used a logistic regression model to determine the probability of winning at the end of each quarter as a function of the score difference.

A summary of the results is shown in the following table:

QUARTER	PROBABILITY OF WINNING				
	95%	96%	97%	98%	99%
1	17	18	20	23	27
2	16	17	18	21	24
3	13	14	15	17	20

Table 1: probability of winning based on score difference

If we assume 99% as the winning threshold, our proposed mercy rule would end a game at the end of first quarter if the score difference is 27, at the end of second quarter if the score difference is 24 and at the end of third quarter if the score difference is 20 points. We did not consider the difference in ranking in our calculations. One could use a multi-variable logistic regression model by using both the score difference and the ranking difference as the two independent variables.

Example 2: Using M&M candies to model a population growth/decay death and immigration rates. This project was prepared as a module for SIMIODE. Brian Winkel must be credited with the first SIMIODE model of birth and death using M&M candies it remains a popular model both in differential equation classes and elsewhere. Recently we used it with a “Teacher Circle” group where it was a very popular exercise. The basic activity is thus:

A starting number of Plain M&Ms (say P_0) are placed in a cup, gently shaken and rolled out onto a plate. The M&Ms with the letter “m” face up are deemed dead or emigrated and are removed from the population. Then some number (say k) M&Ms immigrate into the population, simulated by adding an additional “k” M&Ms to the plate. Repeat the process. What is the long term population result: Extinction, cyclic highs and lows, stability at some level, explosive growth?

For **M²AC** this experiment has a particular attraction in that it can be solved on several levels. Specifically,

- Physical simulation (Shake and roll M&Ms)
- Iterative equation:
  $P_{n+1} = P_n * \frac{1}{2} + k$
- Closed form equation:
  $P_n = P_0 * \left(\frac{1}{2}\right)^n + \frac{2^n - 1}{2^{n-1}} * k$.
- Solution to a Differential Equation:
  $P(t) = (P_0 + 2k)e^{-\frac{1}{2}t} + 2k$.
- Stochastic Simulation

The output from a stochastic simulation that was built using Maple suggest that in the long run the population will be stable at a value close to twice the annual immigration value.

```
candy(50, 10, 10)

The initial population is 50. Immigration policy is add 10. Executing 10 iterations
Iteration 2 Population is 39.
Iteration 3 Population is 37.
Iteration 4 Population is 29.
Iteration 5 Population is 22.
Iteration 6 Population is 20.
Iteration 7 Population is 21.
Iteration 8 Population is 19.
Iteration 9 Population is 19.
Iteration 10 Population is 17.
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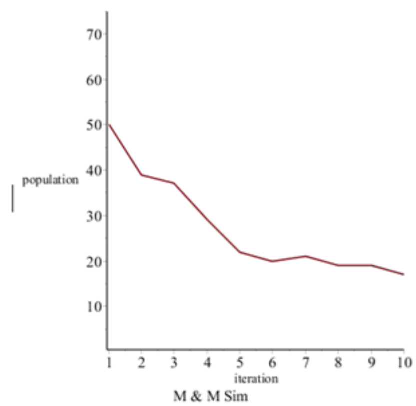

Proposed Projects in Other Disciplines

Following is a list of proposed projects suitable for the non-STEM disciplines:

- Sociology: Model of interactions between police, perpetrators, and victims of crime in a given area.
- Sustainability: Model the impact of several sustainability initiatives.
- Political Science: How to do an exit poll.

- Public Health: Models of transmission of disease.
- Political Science: Investigate voting fraud/malfunction of voting machines: Did the electronic scan machines malfunction during the Comal county (Texas) election in 2002?
- Sociology: Use of probability in determining how to best assist motorists involved in an accident. How should the DPS deploy their officers: parked midway between nearest cities or continuously patrol the highway?

  Various Fields: Spread of rumor: Use of probability and simple first order O.D.E to determine the percentage that have heard the rumor after certain amount of time has passed.

Conclusions

Over the years we have been disappointed in the number of non-STEM students who are taking their required math and/or statistics courses in their senior year. They should be using these skills throughout their undergraduate education. We hope that creating projects with faculty and students from disciplines outside STEM will encourage them to utilize their mathematics and statistics skills early on their academic career to model problems in their specific discipline.

Acknowledgements

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References

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[3] https://www.simiode.org/.

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