Background Literature

A picture of what it means to prepare teachers to teach mathematics with technology is emerging in the field, centered around providing preservice mathematics teachers with opportunities to actively engage with technology as mathematics learners (Voogt, Fisser, Pareja Roblin, Tondeur, & van Braak, 2013). However, ensuring that our preservice secondary mathematics teachers (PSMTs) develop a rich understanding of the benefits of incorporating technology into instruction is not a simple matter of exposure (Olive & Leatham, 2000).

Preservice teachers’ beliefs seem to be drawn from previous vivid episodes or events in their lives (Pajares, 1992), events that likely occurred in the mathematics classroom during their years spent as students. Lortie (1975) refers to this phenomenon as an apprenticeship of observation. This foundation of experience plays a pivotal role in how PSMTs engage with their teacher preparation coursework and ultimately their future teaching practice (Pajares, 1992).

In order to influence PSMT’s beliefs about the role technology plays in the classroom, we wanted our students to develop new narratives based on episodes of mathematical learning with technology. It would not be enough to simply orchestrate mathematical events, PSMTs need opportunities to make their experiences solving problems with technology more explicit (Cox & Harper, 2016). Taylor and O’Donnell (2004) ask, “But what if teachers’ perceptions of their teaching are inaccurate?” In the context of teaching preservice secondary mathematics teachers, we, too questioned the accuracy of our students recollections and perceptions and thought to ask, “But what if our students’ perceptions of their learning are inaccurate?”

Accuracy is just one issue, we found that articulation was another. Lesh and Zawojewski (2007) call for research into how to document and assess understandings and abilities related to problem solving, which relies on data that consists of “auditable trails of documentation that problem solvers generate as they go through multiple cycles of expressing, testing, and revising their ways of thinking.” (p. 795). In spite of offering opportunities to solve mathematical problems and to reflect on how technology supported
thinking and communication, our students were often unaware of and unable to reliably articulate their problem solving insights or process. The process of articulating thinking and reasoning after-the-fact is challenging. We needed a way to capture an authentic narrative to facilitate further reflection on the affordances of technology for doing and learning mathematics.

This narrative is, in essence, an ethnographic record of problem solving with technology. Geertz (1973) claims that ethnography is thick description which requires interpretation beyond observation. When collecting data, we must attend not only to what is captured, but also the culture in which it is captured. As Geertz (1973) writes,

> What the ethnographer is in fact faced with...is a multiplicity of complex conceptual structures, many of them superimposed upon or knotted into one another, which are at once strange, irregular, and inexplicit, and which he must contrive somehow first to grasp and then to render (p. 10).

We used the technology of screencasting as a means to capture not only what actions were taken and gestures made on screen, but also capture the complex conceptual structures that underpin that which we could objectively observe. While thick description, synonymous with an ethnographic narrative, was our goal, we use the term thick analysis to refer to the methodological process of braiding together different forms of data as a means to unknot the conceptual structure of an episode of problem solving or posing.

In this study, we set out to create an environment where we could test the capability of technology to capture data rich enough to create a thick description of problem solving behavior for study. Capturing the problem solving process would serve two purposes. First, it would help us study those moments with the intent to better understand how our students use technology to do mathematics. Second, as an artifact of the act of problem solving, such recordings could give PSMTs a personal and explicit sense of the meaning behind their work. To do this, we hypothesized, would require capturing mathematical activity as well as the intention behind that activity. This became a two-fold problem of study: 1. What does it look like when PSMTs engage creatively with a mathematical task, and 2. How can we leverage technology to enable the thick analysis of episodes of authentic problem solving? We have written about the answer to the first question elsewhere (Cox, Harper & Edwards, under review). In this paper, we will focus specifically on the second question, giving an account of how we used thick analysis, with screencasting as a medium, to capture mathematical insights as well as develop a thick description of how students engage with mathematics with technology.

**Data Collection**

Preservice secondary mathematics teachers pursuing licensure to teach secondary mathematics at our institution take a course, typically in their sophomore year, that
focuses on mathematical problem solving with technology (MPST). Those enrolled in this course have not taken any methods courses or completed any field placements in a clinical or school setting, nor are they expected to take such courses concurrently with this course. All students have completed Calculus I and II, and are typically enrolled in Calculus III and Linear Algebra during the same semester as the MPST course. It is in their junior year, our PSMTs enroll in the majority of their education methods courses.

We assigned 15 PSMTs the task to create a interactive geometry sketch that embodied a "Kaleidoscope". The PSMTs were in the seventh week of exploration at the end of a unit on mathematical problem solving with Interactive Geometry Software (IGS). We instructed them to engage with the task in partnerships (and one triad) using a single computer equipped with IGS. The intent was to give PSMTs a task, tools and environment that pushed for and supported verbal communication, negotiation, and collaborative mathematical problem solving. The task was deliberately ill-defined to allow PSMTs to focus on the varied mathematical facets of a kaleidoscope. There was no idealized product or exemplar for which all were aiming and there were multiple productive paths to take. It would be impossible, given the short time frame (20 minutes), to capture the absolute and true essence of a kaleidoscope. Thus, there was room for decision making and there was potential for diverse thinking. Lastly, the PSMTs were not asked to submit a final product, rather, present the latest draft to the whole class for feedback.

During the construction phase and class presentations, four sources of data were collected. First, we have the completed IGS sketches of kaleidoscopes. Second, we collected construction screencasts documenting on-screen construction activity, including verbal communication, in the IGS environment. In this study, we define screencast as a digital recording of computer screen output, also known as a video screen capture, often containing audio narration. Specifically, PSMTs used Screencast-O-Matic, a freely available, web-based platform, to create their screencasts. The platform works passively in the background and does not interfere with IGS activity. Third, we video recorded presentations and discussions of IGS sketches with the class of PSMTs. The fourth source of data was collected after the class concluded. Individual PSMTs created reflection screencasts where they addressed the problem solving process as well as their completed model.

**Theoretical Framework**

We begin this story at the end, by presenting the theoretical framework that emerged from our analysis. While the framework illustrated in Figure 1 did not become evident until we were in the process of creating a thick description for one partnership, we feel that its inclusion here helps orient the reader and gives a structural overview of the relationship between individual data sources. The construction screencast emerged as a centralized data source because of its linear depiction of the actual episode of problem solving. The remaining data sources supported our interpretation of the recorded events.
The class presentation video, viewed prior to the construction screencast, was an introduction to what was produced by the PSMTs partnership and served as a first-hand explanation and interpretation of what occurred during the episode.

![Diagram of data sources](image)

**Figure 1. Building an authentic record of problem solving practice.**

Once the construction screencast was viewed, the remaining data sources, in addition to a transcription that was made of the construction screencast, helped to interpret the episode and identify key moments of insights. In the next section we will elaborate on what each data source contributed to our analysis as well as the inadequacies of each data source in producing an authentic record of problem solving practice.

**Results**

The data collected in this study supported a thick analysis of episodes of PSMTs problem solving. The IGS atmosphere in conjunction with screencasting generated rich data from which much can be learned about how PSMTs engage with mathematics using technology. In this section, we will describe how each of the data sources contributed to the episode analysis, but also the inadequacies of each source as stand-alone data. The four different data sources worked together to provide an opportunity for this thick analysis leading to ethnography. The data that appears here, excerpted from the larger set, was used to develop an ethnography of one partnership: Olivia and Abby (Cox, Harper & Edwards, under review).

**Class Presentation Video**

We began our process by watching the video of Olivia and Abby’s kaleidoscope presentation. In that presentation, Olivia and Abby demonstrated their sketch, talked about how it worked, and then gave a brief overview of how it embodied, for them, a kaleidoscope including references to polygons, circles, and reflection. This presentation was very much controlled by the PSMTs with some space for audience interaction.
Classmates recognized elements of dilation and randomized motion within the sketch. It was useful in that it enabled us to listen to Olivia and Abby describe first-hand the intentions and emotion behind their creative work, but the interactions were limited to what PSMTs in the room were willing to add or discuss. While it helped orient us as to what we would be seeing in the construction screencast and introduced the final product, it offered very little explanatory power when we began to analyze the episode for problem-solving insight or the relationship of technology to mathematical learning. The PSMTs did not articulate these ideas in their presentation.

**Construction Screencast**

Screencasts were created to document the entire problem-solving episode. These screencasts link the PSMTs exploration of the task in the technological environment with their verbalized intentions about their constructions. We provided for readers, an excerpt of the creation **screencast** for examination. The screencast excerpt begins after about three minutes and thirty seconds of discussion and after some initial IGS construction.

Figure 2. Transcript of the excerpted construction screencast.
There are two key stories embedded within this excerpt. One story is Olivia’s study of “the points on the outside circle” (0:28) (see Figure 2). The second story is Olivia’s haphazard dragging of one key point elicited motion that Abby felt would make their model behave in a more realistic way (around 01:40).

As we watch the construction screencast, we have access to mouse movement and the ordering of the construction. We can see on the screen the movements that provoked speech and then examine the meaning behind shorthand speaking and stutters. Voice inflection and timing help to define this meaning. The sketch is dynamic in the sense that the girls can interact with it, but static from our perspective as we are limited to their interactions with the sketch and we do not have freedom to verify or explore ourselves. We are also limited to those intentions that the girls are able to verbalize or otherwise communicate through gesture. Interpretation is still key and needs support from other data sources.

**Construction Screencast Transcript**

In order to better interpret the construction screencast excerpt, we first turned to transcription as a tool for data analysis. The transcription along with a screenshot depicting the kaleidoscopic sketch can be found in Figure 2. In it, we were able to create an audio-only record of the screencast with timestamps to link the transcription to the video recording. As a stand-alone representation of the episode, the transcript was inadequate for analysis. The PSMTs often moved into a shorthand method of speaking that relied on spoken words, gestures, and technological activity (such as the moment a PSMT animates the kaleidoscope for the first time).

When Abby notices Olivia moving the sketch between 01:39 and 01:46, a moment of insight is provoked by unarticulated action taken within the IGS environment. The lack of articulation renders the linked video record essential in documenting the nature of this insight and its provocation. Furthermore, it is only through watching the screencast that a full analysis of either moment is possible, the transcript leaving far too much to be interpreted.

**Reflection Screencast and Transcript**

After the class presentation, individual PSMTs created *reflection screencasts* where they addressed, from an individual standpoint, questions similar to those posed in the whole class discussion, but including additional prompts regarding the construction process: 1) the process by which they created their model, 2) the facets of a kaleidoscope they had hoped to model, 3) what they had hoped to accomplish but could not and the limitations they perceived, and 4) insights they had along the way and the impact of those insights on their model.
A screencast was chosen as the medium for this reflection over a written response because it enabled PSMTs to use their sketches as communication tools in static as well as dynamic ways. For example, Abby manipulated the mouse cursor on screen to gesture to various parts of the static sketch when talking about the creation process. She animated the sketch when she talked about the facets of a kaleidoscope that they modeled, “Another thing that I think really makes ours a kaleidoscope is the fact that there’s dilation, but also rotation. It’s really kinda neat. And there’s also like never the same shape at the same...like all the time. So like, there’s always something different each time the dots move.”

From the reflection screencast we gain a much richer sense of how Abby was seeing mathematics within her sketch. Analytically, these screencasts uncovered some of the intention and meaning of the experience through the first-person eyes of PSMTs. The quotes in Figure 3 convey some of the technological and mathematical goals of the partnership. Neither Abby nor Olivia identified any important insights that they had when solving the task in their reflection screencasts. While Abby ignored the prompt entirely, Olivia went so far as to deny that insight occurred, which stands in contradiction to our initial analysis of the construction screencast which indicated that major insights were reached.

**Olivia:** One of the things we were trying to accomplish is maybe to have the entire kaleidoscope rotate in a circle not just have the points rotating around the kaleidoscope, but we couldn’t figure out a way to do that without having the shape...with the shapes because we couldn’t get them to stay fixed.

**Abby:** I think our goal was that we knew we had to have symmetry and another thing was that we wanted to make sure that the points on ...er...we wanted to make sure there was rotation. Olivia and I both remember that when we were younger we would have little kaleidoscopes made out of paper towel tubes. When you would rotate them you’d see new shapes. We thought that was so cool. We wanted to make sure that was included.

Figure 3. Expressions of technological and mathematical goals.

The reflection screencasts provided a limited view of the episode and illustrate that a first person assessment of the experience fails to capture it in an authentic way, in spite of it being a first-hand account. Although the PSMTs were more articulate in presenting a polished review of their process, they cannot articulate the creative process, nor fully elaborate insights about mathematics, problem solving, and technology. From an analytical standpoint, the PSMTs controlled what they presented and in that way limited our view of and understanding of their process to those parts of the story of which they were aware. A comparison to the construction screencast exposed blind spots in their analysis.
Interactive Geometric Sketch

During our analysis, we were provoked by the necessity of interpreting a portion of the construction screencast that was not clear using the transcripts or screencasts. Specifically, we had difficulty understanding Olivia’s concern, “… if they (the points) are allowed to move between the segments. If they are allowed to move beyond …” (Figure 2, 00:34 timestamp). The interactive geometric sketch was a powerful piece of data since it enabled us to carefully document different facets of a kaleidoscope that PSMTs attended to during their construction, as well as the relationship of these facets to mathematical constructions within the created models.

It is only because of the nature of the interactive geometric sketch that a post-hoc analysis of the mathematical model was possible. The sketch allowed us to test conjectures about the construction mechanics and design of the kaleidoscope. This type of analysis was not supported by the construction screencast. During the construction screencasts, the PSMTs were in control the movements on the screen and what they wanted the researchers or one another to see. Because the data was an electronic file, we could revisit and investigate the sketch further than what the PSMTs showed in their exploration. In this way the researchers had access to the potential of the sketch and not just what was realized, verbalized, or otherwise apparent to the PSMTs. We were free to examine objects in ways the students chose not to or took for granted.

Conclusion

We were able to develop a comprehensive ethnography of episodes of PSMTs problem solving using multiple data sources that worked together to provide an opportunity for thick analysis. Additionally, Olivia made herself available for a brief interview after the course had concluded. In this interview she provided additional insights into the construction of their kaleidoscope model. Olivia was then given an opportunity to engage in member check (Lincoln & Guba, 1985) to strengthen the validity of our interpretations. Our analysis matched her experience, and made her aware for the first time the depth of her mathematical insights within the episode.

Findings

The construction screencasts were used as an anchor in the analysis of episodes of mathematical problem solving with technology. In this paper, we have provided a sense of how we used screencasting as a medium through which to capture mathematical insights as well as other data sources that we used to thicken our analysis and interpret the construction screencast. We conclude our paper with two major findings.

First, an authentic record of problem solving with technology relied on multiple data sources working in tandem to interpret a linear progression of events. The construction screencast operated as a central data source interpreted with support from additional data.
By enacting the task in a collaborative atmosphere, no additional prompts were needed to help PSMTs elaborate on their process in the moment; the conversation, on-screen gesture and activity were natural behaviors provoked by the task itself. The simultaneous recording of on screen activity with verbalized thinking provided a rich and authentic portrait of collaborative problem solving and the articulation of mathematics (Cox, Harper & Edwards, under review).

The other data sources were essential to interpretation. The presentation videos as well as the reflection screencasts provided access to first-hand accounts of the episode and its meaning. It was possible to get a more accurate sense of the intentions behind gestures or unarticulated ideas when revisited in a more polished or formal format. In their reflections, Olivia and Abby conveyed their perceptions of what a kaleidoscope does and the facets they strove to represent mathematically and technologically. We conclude from the construction screencast that they are wrestling with questions that might be technical and mathematical at the same time. In the reflections, corroborated by the final member check, we conclude that they were not be aware of the origins of these questions or how solving the task in the IGS environment influenced not only their strategies and methods of construction (problem solving), but in the genesis of questions to be answered along the way (problem posing). Recovering these lost insights could have a significant impact on the way Olivia and Abby, and PSMTs in general, come to view the impact of technology on the teaching and learning of mathematics. This is a central tenet in the preparation of mathematics teachers.

The sketches were essential in that they gave us, as outside observers, a chance to question and verify what we saw in the construction screencast beyond what the PSMTs explored together. Olivia’s interest in how the points along the outside circle moved was put on hold when Abby changed the direction of the conversation toward using those points to turn the kaleidoscope. While it was possible that the two PSMTs were exploring the same idea, it was only in our exploration of the IGS sketch that we were able to fully comprehend Olivia’s concerns as distinct from Abby’s idea. As the partnership did not return to these concerns, they went unexplored in the construction screencast and there was not enough data to determine their legitimacy.

As a second finding of this study, we posit that identifying moments of creative insight is not intuitive or natural work; urging PSMTs to reconstruct their thinking process is fraught. Screencasting and IGS create an environment where PSMTs can not only engage freely in creative mathematics, but can create artifacts and records of practice such as screencasts and sketches for use in presenting their work and also for individual or group reflection. Without access to the construction screencast, we would have to rely on PSMTs memory and interpretation of problem solving events after-the-fact. The reflection screencasts verify that insights would have gone uncaptured because they were so subtle as to fail to register as important to the PSMTs.
Implications and Conclusion

While the National Council of Teachers of Mathematics (2000) and Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) have framed a professional conversation around the role of problem solving and problem posing in mathematics education, more needs to be done to define what experiences PSMTs need with respect to each and conduct research into making these experiences and the supporting mathematical practices more explicit (Cox & Harper, 2016). More can be done to create “auditable trails” (and from them thick description) to use as examples for the field and cases for use in the preparation of future teachers.

In spite of offering opportunities to solve mathematical problems and to reflect on how technology supported thinking and communication, our PSMTs were often unaware of and unable to reliably articulate mathematical processes or claims they made along the way. Articulating mathematical processes or the role technology plays in them is not intuitive or natural work and helping PSMTs reconstruct their thinking process after-the-fact is fraught.

Screencasting and IGS create an environment where PSMTs can not only engage in creative mathematics, but can create artifacts and records of practice such as creation screencasts and IGS files for use in presenting their work. These records can also be used for individual or group reflection focused on identifying and describing episodes of problem solving and posing and the roles technology plays in those episodes.

We would go even further, however, and say that making these processes explicit is the only way to change deeply-held beliefs (Phillip, 2007) about the nature of mathematics and instruction. Tharp, Fitzsimmons and Ayers (1997) found that that philosophical beliefs about the nature of mathematics and what it means to learn mathematics influenced teachers recognition of the value of technology for instruction. We feel that creating thick descriptions has the potential to impact not only views about the role of technology in the classroom, but also the potential of technology to spur and support mathematical insights for students.

The creation screencast is as valuable in research as it is in the mathematics classroom and in the preparation of PSMTs. We see a great deal of possibility in pursuing research projects that utilize the screencast to document episodes of collaborative problem solving and posing in a IGS environment. In this paper, we have presented one thick description, that illustrates the power that this tool has in preserving the authenticity of those moments. The power is also in creating an auditable trail, one that increases both accuracy and articulation, to which both students and teachers might refer when making learning processes explicit. Without access to the creation screencast, we would have to rely on Abby and Olivia’s memory and interpretation of problem solving events after-the-fact, and creative insights may have gone uncaptured simply because they were
so subtle as to fail even to register as important.

Recollection and reflection alone are not enough to ensure the articulation of mathematical processes and the role technology plays to support and ignite that insight. Technology, in the form of screencasting and transferable IGS sketches has the power to overcome the necessity of recreating these experiences as post-hoc analysis. By creating a real-time record of practice, rather than a description of the journey taken, we have the opportunity to recreate the mathematical landscape and make it available for study and reflection.

**References**


