AP Calculus AB

Semester A Summary:
In this course, the student will complete the first semester of coursework similar to a first-year college-level calculus course. This course covers the framework, mathematical practices, and learning objectives for an AP® Calculus AB course as recommended by the College Board. This course provides experience with the methods and applications of calculus and effectively prepares the student to take the AP Calculus AB exam in the spring. The overarching topics in this course are limits, continuity, derivatives, methods of finding derivatives, and applications of derivatives. The student will interact with lesson content, multimedia presentations, an online textbook, and a graphing utility to meet learning goals throughout the course. Featured learning strategies in this course include direct instruction, regular checks and practices, discussions, portfolios, and a practice assessment for the AP Calculus AB exam.

Semester A Outline

1. **Course Overview**
    1. Getting Started in AP Calculus AB
        - Analyze the structure and content of the AP Calculus AB course
        - Identify course expectations and how to be successful in the course
        - Identify the materials you will use
        - Determine a plan for preparing for the AP Exam
2. **Prerequisites for Calculus**
    1. Linear Functions
        - Model and solve problems involving linear functions
        - Determine the slope of a line through two points
        - Use point-slope and slope-intercept forms to write and graph linear equations
        - Apply the properties of parallel and perpendicular lines to solve problems
        - Determine solutions for pairs of linear equations simultaneously
    2. Functions and Graphs
        - Use the language, notation, and graphical representation of functions to express relationships between variable quantities
        - Analyze functions and their properties, including domain, range, and symmetry, and even and odd functions
        - Graph functions in the coordinate plane
        - Graph and analyze piecewise-defined and absolute value functions
        - Evaluate and simplify compositions of functions
    3. Exponential Functions
        - Analyze exponential functions and equations graphically, numerically, and algebraically
        - Solve problems involving exponential growth and decay
        - Solve compound interest and continuous compound interest problems
    4. Inverse Functions and Logarithms
        - Analyze relations and their inverses algebraically and graphically
        - Find inverses of one-to-one functions graphically and algebraically

- Analyze logarithmic functions algebraically, graphically, and numerically as inverses of exponential functions
- Use logarithms and their properties to solve exponential equations algebraically

5. Trigonometric Functions
   - Apply transformations of trigonometric functions to solve problems
   - Solve trigonometric equations graphically and algebraically
   - Graph and analyze sinusoidal functions and their properties (domain, range, symmetry, amplitude, period, frequency) with and without technology
   - Evaluate inverse trigonometric functions
   - Evaluate the trigonometric functions for any angle

3. **Limits and Continuity**
   1. Overview of Limits
      - Determine average and instantaneous rates of change using the idea of limits
      - Estimate limits from information given in tables and graphs
   2. Using Properties of Limits
      - Use properties of limits to calculate limits
      - Find right- and left-hand limits
      - Use the Squeeze Theorem or algebraic manipulation to find a limit
   3. Limits Involving Infinity
      - Calculate limits involving infinity
      - Use limits to determine end behavior of functions
   4. Continuity and Discontinuity
      - Find the points at which a function is continuous and discontinuous
      - Determine intervals over which a function is continuous
      - Apply continuity to calculus theorems
   5. Secant Lines and Average Rate of Change
      - Find average rates of change
      - Use secant lines to calculate average rates of change
      - Apply concepts learned to solve real-world problems involving average rates of change and secant lines
   6. Tangent Lines and Instantaneous Rate of Change
      - Find instantaneous rates of change
      - Use tangent lines to calculate instantaneous rates of change
      - Apply concepts learned to solve real-world problems involving instantaneous rates of change and tangent lines
   7. Unit Review
      - Calculate limits of functions at a point and interpret their meaning
      - Calculate limits of functions at infinity and interpret their meaning
      - Calculate infinite limits of functions and interpret their meaning
      - Identify intervals of continuity and points of discontinuity for a function
      - Apply the concept of slope to tangent lines and rates of change
   8. Unit Test
      - Calculate limits of functions at a point and interpret their meaning
      - Calculate limits of functions at infinity and interpret their meaning
      - Calculate infinite limits of functions and interpret their meaning
      - Identify intervals of continuity and points of discontinuity for a function
      - Apply the concept of slope to tangent lines and rates of change

4. **Derivatives**
   1. Using the Definition of a Derivative
      - Use the definition of a derivative to find the derivative of a differentiable function

- Use the alternate definition of a derivative to find the derivative at a point

2. Graphs of Derivatives
    - Identify the graph of the derivative of a function shown graphically
3. Differentiability at a Point
    - Determine points at which a function is not differentiable
4. Differentiability on an Interval
    - Determine intervals on which a function is differentiable
5. The Intermediate Value Theorem for Derivatives
    - Use the Intermediate Value Theorem to determine differentiability
6. Numerical Derivatives
    - Use numerical derivatives to approximate the derivative at a point
7. Using Differentiation Rules
    - Use differentiation rules to find derivatives
    - Use rules for differentiation to find higher-order derivatives
    - Use rules for differentiation to find slopes and equations of tangent lines at a point
8. Rates of Change
    - Find instantaneous rates of change
    - Solve problems involving rectilinear motion
    - Solve free-fall and projectile motion problems
9. Derivatives of Trigonometric Functions
    - Find derivatives involving trigonometric functions
    - Solve simple harmonic motion problems (speed, acceleration, jerk)
    - Use rules for differentiation to find slopes and equations of tangent lines at points of a sinusoidal function
10. Unit Review
    - Determine the conditions necessary for a function to be differentiable
    - Use the definition of a derivative to find derivatives of differentiable functions and derivatives at a point
    - Apply the Intermediate Value Theorem for derivatives
    - Apply differentiation rules to find derivatives of functions
    - Solve problems involving rates of change
11. Unit Test
    - Determine the conditions necessary for a function to be differentiable
    - Use the definition of a derivative to find derivatives of differentiable functions and derivatives at a point
    - Apply the Intermediate Value Theorem for derivatives
    - Apply differentiation rules to find derivatives of functions
    - Solve problems involving rates of change

5. **Mid-Semester Check**
    1. Mid-Semester Review
        - Calculate limits at a point and at infinity, and calculate infinite limits involving graphs and function equations
        - Identify points and intervals of continuity (including the Intermediate Value Theorem)
        - Determine intervals over which a function is differentiable or nondifferentiable from graphs and function equations
        - Find derivatives of functions and use them to solve problems
        - Solve problems involving slopes of tangent lines and rates of change
    2. Mid-Semester Check
        - Calculate limits at a point, at infinity, and infinite limits involving graphs and function equations

- Identify points and intervals of continuity (including the Intermediate Value Theorem)
- Determine intervals over which a function is differentiable or non-differentiable, from graphs and function equations
- Find derivatives of functions and use them to solve problems
- Solve problems involving slopes of tangent lines and rates of change

6. **More Derivatives**
    1. The Chain Rule
        - Use the Chain Rule to differentiate compositions of two or more functions
        - Use the Chain Rule in conjunction with the Product and Quotient Rules to differentiate functions
    2. Applications of the Chain Rule
        - Find and interpret a derivative using the Chain Rule in the context of a model
        - Find the equation of a tangent or normal line at a point on the graph of a function
    3. Implicit Differentiation
        - Use the Chain Rule to differentiate both sides of an equation that defines a function implicitly
        - Use implicit differentiation to compute the slope of a tangent line to the graph of an equation at a given point
    4. Applications of Implicit Differentiation
        - Use implicit differentiation to find higher-order derivatives of implicitly defined functions
    5. Derivatives of Inverse Trigonometric Functions
        - Find derivatives of invertible functions and their inverses
        - Find derivatives of functions involving inverse trigonometric functions
    6. Derivatives of Exponential Functions
        - Find derivatives of exponential functions
    7. Derivatives of Logarithms
        - Find derivatives of functions involving logarithms
    8. Logarithmic Differentiation
        - Use logarithmic differentiation to find derivatives of functions
    9. Unit Review
        - Use the Chain Rule to find derivatives of composite and implicitly defined functions
        - Determine derivatives of inverse trigonometric functions
        - Determine derivatives of exponential functions
        - Determine derivatives of logarithmic functions
        - Use implicit differentiation to find derivatives of implicitly defined functions and solve problems
    10. Unit Test
        - Use the Chain Rule to find derivatives of composite and implicitly defined functions
        - Determine derivatives of inverse trigonometric functions
        - Determine derivatives of exponential functions
        - Determine derivatives of logarithmic functions
        - Use implicit differentiation to find derivatives of implicitly defined functions and to solve problems
7. **Applications of Derivatives**
    1. Extreme Values of Functions
        - Determine the maximum or minimum value of a function over a given interval using derivatives

- Use the Extreme Value Theorem to analyze functions

2. Mean Value Theorem
    - Describe the behavior of a function over an interval using the Mean Value Theorem
    - Apply the concept of antiderivatives to solve real-world problems
3. Connecting Derivatives of f with the Graph of f
    - Describe functions in terms of characteristics (slope, concavity, inflection points)
    - Analyze the properties of functions using derivatives
4. More Function Analysis Using Derivatives
    - Describe functions in terms of characteristics (slope, concavity, inflection points)
    - Analyze the properties of functions using derivatives
    - Document relationships between graphs of a function, its first derivative, and its second derivatives
5. Modeling and Optimization
    - Develop models of real-world situations expressed in terms of mathematical functions
    - Solve optimization problems using derivatives
6. Linearization, Sensitivity, and Differentials
    - Solve problems involving the slopes of tangent lines
    - Develop linear and quadratic approximations for functions in the vicinity of a given point
    - Evaluate sensitivities using differentials
    - Determine approximate zeros of functions using Newton's Method
7. Related Rates
    - Create equations that relate known rates of change to unknown rates of change
    - Solve problems involving rates of change in applied contexts
8. Unit Review
    - Identify local and global extreme values of a function
    - Identify intervals over which a continuous function is increasing, decreasing, or constant
    - Relate the graph of a function with the graphs of its first and second derivatives
    - Apply the Mean Value Theorem to solve problems
    - Solve problems involving related rates
9. Unit Test
    - Identify local and global extreme values of a function
    - Identify intervals over which a continuous function is increasing, decreasing, or constant
    - Relate the graph of a function with the graphs of its first and second derivatives
    - Apply the Mean Value Theorem to solve problems
    - Solve problems involving related rates

8. **Semester Review and Exam**
    1. Semester Review
        - Use limits at a point, limits at infinity, and limits involving infinity to interpret function behavior
        - Use limits to determine continuity at a point and over intervals in the domain of a function
        - Find derivatives using the definition of derivatives and derivative formulas

- Analyze function behavior using first and higher-order derivatives
- Use derivatives to solve problems involving optimization, related rates, and instantaneous rates of change
- Use the Mean Value Theorem to analyze the behavior of a differentiable function over an interval in its domain
- Find functions that have a given derivative

2. Semester Exam
    - Use limits at a point, limits at infinity, and limits involving infinity to interpret function behavior
    - Use limits to determine continuity at a point and over intervals in the domain of a function
    - Find derivatives using the definition of derivatives and derivative formulas
    - Analyze function behavior using first and higher-order derivatives
    - Use derivatives to solve problems involving optimization, related rates, and instantaneous rates of change
    - Use the Mean Value Theorem to analyze the behavior of a differentiable function over an interval in its domain
    - Find functions that have a given derivative

Semester B Summary:

In this course, students will complete the second semester of coursework similar to a first-year college-level calculus course. This course covers the framework, mathematical practices, and learning objectives for an AP® Calculus AB course as recommended by the College Board. This course provides experience with the methods and applications of calculus and effectively prepares the student to take the AP Calculus AB exam in the spring. The overarching topics in this course are integrals, methods of finding integrals, applications of integrals, differential equations, and mathematical modeling. The student will interact with lesson content, multimedia presentations, an online textbook, and a graphing utility to meet learning goals throughout the course. Featured learning strategies in this course include direct instruction, regular checks and practices, discussions, portfolios, a project that applies concepts from across the course, and a practice assessment for the AP Calculus AB exam.

Semester B Outline

1. **Course Overview**
    1. Getting Started in AP Calculus AB
        - Analyze the structure and content of the AP Calculus AB course
        - Identify course expectations and how to be successful in the course
        - Identify the materials you will use
        - Determine a plan for preparing for the AP Exam
2. **The Definite Integral**
    1. Estimating with Finite Sums
        - Estimate areas under curves using rectangular approximation methods
        - Estimate distances, volumes, and accumulations using finite sums
    2. Definite Integrals and Antiderivatives I
        - Write the limit of a Riemann sum as a definite integral
        - Express the limit of a Riemann sum in integral notation
        - Calculate a definite integral using areas and properties of definite integrals
        - Interpret the meaning of a definite integral within a problem
        - Apply definite integrals to problems involving area and volume
    3. Antiderivatives

- Compute indefinite integrals

4. Definite Integrals and Antiderivatives II
    - Calculate and apply integrals of piecewise constant and linear functions
    - Determine the average value of a function using integrals
    - Justify and apply the Mean Value Theorem for Definite Integrals
    - Establish and apply the connections between differential and integral calculus
5. Fundamental Theorem of Calculus I
    - Analyze functions defined by an integral
    - Evaluate definite integrals using antiderivatives
6. Fundamental Theorem of Calculus II
    - Determine areas analytically using definite integrals
    - Analyze antiderivatives graphically
7. Trapezoidal Rule
    - Justify and apply trapezoidal approximations to integrals
    - Justify and apply other numerical approximations and compare results to the Trapezoidal Rule
    - Determine the bounded error in the Trapezoidal Rule
8. Unit Review
    - Write the limit of a Riemann sum in integral notation and find definite integrals using area
    - Apply the rules for definite integrals to evaluate definite integrals
    - Apply the Fundamental Theorem of Calculus to find antiderivatives
    - Apply the evaluation part of the Fundamental Theorem of Calculus to evaluate definite integrals
    - Use the Trapezoidal Rule to approximate areas under a curve
9. Unit Test
    - Write the limit of a Reimann sum in integral notation and find definite integrals using area
    - Apply the rules for definite integrals to evaluate definite integrals
    - Apply the Fundamental Theorem of Calculus to find antiderivatives
    - Apply the evaluation part of the Fundamental Theorem of Calculus to evaluate definite integrals
    - Use the Trapezoidal Rule to approximate areas under a curve

3. **Differential Equations and Mathematical Modeling**
    1. Introduction to Differential Equations
        - Determine if a given function is a solution to a differential equation
        - Solve exact differential equations and initial value problems
        - Identify the domain on which a solution to an initial value problem is valid
    2. Slope Fields
        - Use a slope field to approximate the graph of a solution of a differential equation
        - Generate a slope field for a given differential equation
    3. Integration by Substitution
        - Use the method of substitution to find antiderivatives
    4. Other Integration Techniques
        - Evaluate integrals using other integration techniques
        - Use L'Hospital's Rule to evaluate limits in indeterminate form
    5. Separable Differential Equations
        - Solve separable differential equations
    6. Modeling with Differential Equations
        - Use differential equations to model exponential growth and decay
    7. Unit Review

- Determine if a given function is a solution to a differential equation
- Determine general and particular solutions to a differential equation
- Solve initial value problems
- Identify the domain on which a solution to an initial value problem is valid
- Generate a slope field for a given differential equation
- Use a slope field to approximate the graph of a solution of a differential equation
- Compute indefinite integrals

8. Unit Test
    - Use substitution methods to evaluate definite and indefinite integrals
    - Evaluate definite integrals using a calculator
    - Determine general and particular solutions to a differential equation
    - Solve initial value problems
    - Solve separable differential equations and apply them to solving problems relating to exponential growth and decay

4. **Mid-Semester Check**
    1. Mid-Semester Review
        - Apply finite-sum methods to approximate areas under curves and other quantities represented by definite integrals
        - Apply the Fundamental Theorem of Calculus to find antiderivatives and evaluate definite integrals
        - Apply rules for definite integrals and substitution methods to evaluate definite and indefinite integrals
        - Evaluate definite integrals using a calculator
        - Determine general and particular solutions to a differential equation and apply them to solving problems
    2. Mid-Semester Check
        - Apply finite-sum methods to approximate areas under curves and other quantities represented by definite integrals
        - Apply the Fundamental Theorem of Calculus to find antiderivatives and evaluate definite integrals
        - Apply rules for definite integrals and substitution methods to evaluate definite and indefinite integrals
        - Evaluate definite integrals using a calculator
        - Determine general and particular solutions to a differential equation and apply them to solving problems
5. **Applications of Definite Integrals**
    1. Introduction to Accumulation and Net Change
        - Apply definite integrals to linear-motion problems involving displacement and total distance
        - Justify and apply a general strategy for modeling with integrals
    2. Applications of Accumulation and Net Change
        - Use definite integrals to solve problems involving accumulation
        - Find net changes given graphical, tabular, or functional representations of rate of change
        - Determine the total amount of a quantity given a density-vs-location function
        - Determine the net work done given the force as a function of distance
    3. Areas in the Plane I
        - Interpret areas as limits of Riemann sums
        - Apply the definite integral to solve problems involving areas
        - Determine areas between curves over a given x-interval
        - Determine areas enclosed by intersecting curves

4. Areas in the Plane II
    - Determine areas for regions bounded by more than one function
    - Determine areas of regions more easily described by functions of y than x
    - Determine areas of regions in cases where geometry formulas can be used to expedite the process
5. Introduction to Volumes
    - Interpret volumes as limits of Riemann sums
    - Apply the definite integral to solve problems involving volumes
    - Determine volumes of solids with known cross sections
6. Circular Cross-Sections
    - Determine volumes of solids of revolution using disks and washers
7. Cylindrical Shells
    - Determine volumes of solids of revolution using cylindrical shells
8. Unit Review
    - Solve problems involving accumulation and net change
    - Apply definite integrals to determine areas in the coordinate plane
    - Apply definite integrals to determining volume of three-dimensional solids
    - Integrate with respect to y to solve area and volume problems determined by functions of y
9. Unit Test
    - Solve problems involving accumulation and net change
    - Apply definite integrals to determine areas in the coordinate plane
    - Apply definite integrals to determining volume of three-dimensional solids
    - Integrate with respect to y to solve area and volume problems determined by functions of y

6. **Review and Full-Length Practice Exam**
    1. Limits and Continuity
        - Review the interpretation, estimation, and determination of limits of function values
        - Review the interpretation, estimation, and determination of infinite limits and limits at infinity
        - Review the analysis of functions to find intervals of continuity and points of continuity, and the determination of the applicability of the Intermediate Value Theorem
    2. Rates of Change, Tangent Lines, and Derivatives
        - Review the use of limits to determine instantaneous rates of change, slopes of tangent lines, and sensitivity to change
        - Review the computation of the derivative of a function at a point using both forms of limit definitions, and explain its relationship to slope
        - Review the analysis of the differentiability of functions and the rules for differentiation
        - Review the interpretation of derivatives as representing rates of change
    3. Using Differentiation Rules
        - Review the differentiation of composite functions using the Chain Rule
        - Review the determination of derivatives of implicitly defined functions
        - Review the derivatives of inverse trigonometric, exponential, and logarithmic functions
        - Review the determination of various properties of functions using the Extreme Value Theorem and Mean Value Theorem
        - Review the use of derivatives to solve optimization problems and problems involving the slope of the tangent line
    4. Integrals and Differential Equations

- Review estimation of distances, areas, volumes, and accumulations using finite sums
- Review notations related to and relationships between Riemann sums, integrals, and antiderivatives
- Review the analysis of functions defined by an integral and the evaluation of definite integrals based on the Fundamental Theorem of Calculus
- Review techniques for solving differential equations and relate them to mathematical models describing a particular context

5. Full-Length Practice Test
    - Interpret and determine limits of function values, including infinite limits and limits at infinity, and use them to determine instantaneous rates of change, slopes of tangent lines, and sensitivities to change
    - Analyze, determine, and discuss intervals of continuity, points of discontinuity, and the differentiability of functions, apply rules of differentiation to determine derivatives of functions, and apply the results to rate-of-change problems
    - Perform differentiation of an expanded set of functions using the Chain Rule, implicit differentiation, and other techniques, and use the results to solve applied problems involving modeling, optimization, linearization, and sensitivity analysis
    - Interpret and determine integrals of functions, both in terms of Riemann sums and their limits, and via relationships between definite integrals and antiderivatives described by the Fundamental Theorem of Calculus
    - Interpret and solve integrals and differential equations used to model applied problems in a real-world context

7. **Semester Project: Response to a Letter**
    1. Letter from a Client
        - Analyze a real-world problem that has more than one solution
    2. Research Solutions
        - Analyze a real-world problem that has more than one solution
    3. Presentation of Findings
        - Analyze a real-world problem that has more than one solution
8. **Semester Review and Exam**
    1. Semester Review
        - Use Riemann sums to approximate areas under a curve and, as limiting cases, find definite integrals
        - Apply the Fundamental Theorem of Calculus to find antiderivatives and evaluate definite integrals
        - Determine general and particular solutions to a differential equation
        - Apply differential equations and their solutions to exponential growth and decay problems
        - Apply definite integrals to determining accumulations, net changes, areas, and volumes
    2. Semester Exam
        - Use Riemann sums to approximate areas under a curve and, as limiting cases, find definite integrals
        - Apply the Fundamental Theorem of Calculus to find antiderivatives and evaluate definite integrals
        - Determine general and particular solutions to a differential equation
        - Apply differential equations and their solutions to exponential growth and decay problems

- Apply definite integrals to determining accumulations, net changes, areas, and volumes

