AP Calculus BC

Semester A Summary:

AP Calculus BC is an extension of Calculus AB. Comparable to college and university calculus, this course will help prepare students for the Calculus BC Advanced Placement exam. The course emphasizes broad concepts and applicable methods. The student will describe and analyze functions, limits, and graphs, calculate and apply derivatives, and interpret and apply integrals. The course provides opportunities for the student to apply concepts to real-world situations.

Semester A Outline

1. **Functions**
    1. Getting Started
    2. Unit One Checklist and Pretest
    3. Course Introduction
        - understand the structure of the course
        - understand how to navigate the course
    4. Introduction to Calculus
        - understand the background of calculus
        - identify overall subjects which will be learned throughout the course
    5. Review of Function Terminology and More
        - determine whether a relation is a function
        - determine a function's domain and range
    6. Graphing Calculators
        - use a graphing calculator
        - graph functions
    7. Compositions and Transformations of Functions
        - perform combinations of functions arithmetically or through composition to create new functions
        - perform translations, reflections, and expansions/compressions on functions
        - use technology such as a graphing calculator to experiment with functions
    8. Some Common Functions
        - understand the properties of power functions, polynomial and rational functions, and trigonometric functions
        - use graphing calculators to explore the effects of changing any parameters of various functions on their corresponding graphs
    9. Collaborative Lesson
    10. Unit One Practice Test
    11. Unit One Test
2. **Limits and Continuity**
    1. Unit Two Checklist and Pretest
    2. Introduction to Limits
        - use limits to describe the behavior of a function
        - use a graphing calculator to get numerical approximations for limits

- evaluate the behavior of a function (in terms of limits) graphically (through sketching by hand and by using a graphing calculator) numerically and algebraically
- understand the connection between vertical asymptotes and (infinite) limits and use graphing calculators to support conclusions

3. Properties of Limits
    - evaluate the behavior of a function (in terms of limits) graphically (through sketching by hand and by using a graphing calculator)
    - evaluate the behavior of a function (in terms of limits) numerically
    - evaluate the behavior of a function (in terms of limits) algebraically
4. Limits Involving Infinity
    - use limits to describe the behavior of a function
    - use a graphing calculator to get numerical approximations for limits
    - understand the connection between horizontal asymptotes and the end behavior of a function
    - use a graphing calculator to help explore the connection between horizontal asymptotes and the end behavior of a function
5. Continuity
    - use limits to describe the behavior of a function
    - use a graphing calculator to get numerical approximations for limits
    - understand the connection between limits and continuity
    - use a function's continuity to evaluate its limit at a point
    - be able to determine when a function is continuous (or discontinuous)
    - understand how a graphing calculator can explore further when a function is continuous or discontinuous and know its limitations
    - identify the approximate roots of a function using the Intermediate Value Theorem
6. Applications of Limits
    - After completing this lesson, you will be able to understand various applications of limits, such as instantaneous and average rate of change, as well as motion of an object or particle: displacement, distance, and velocity.
7. Collaborative Lesson
8. Unit Two Practice Test
9. Unit Two Test

3. **Differentiation**
    1. Unit Three Checklist and Pretest
    2. The Derivative
        - understand the definition of the derivative as a local linear approximation and what it implies, as well as differentiability, and use graphs to explore tangent lines
        - understand the different notations for the derivative
        - explore the relationship between the graph of a function and its derivative
        - explore further the characteristics of the graphs of f and f'.
    3. Rules of Differentiation
        - develop derivatives of polynomial functions
        - use the product/quotient rules to find derivatives
    4. Trigonometric, Derivatives, and the Chain Rule
        - develop derivatives of trigonometric functions
        - use the product/quotient rules to find derivatives
        - use the chain rule of Newton's form to find the derivatives of composite functions

- use the chain rule of Leibniz's form to find the derivatives of composite functions

5. Inverse Functions
    - After completing this lesson, you will be able to find the inverse of a function.
6. Exponential and Logarithmic Functions
    - understand the properties of exponential and logarithmic functions
    - use a graphing calculator to explore the effects of changing any parameters of exponential functions on their corresponding graph
    - use a graphing calculator to explore the effects of changing any parameters of logarithmic functions on their corresponding graph
7. Derivatives Exponential/Logarithmic/Inverse Trig.
    - use the product/quotient rules to find derivatives
    - use the chain rule (both Newton and Leibniz's forms) to find the derivatives of composite functions
    - develop derivatives of logarithmic functions
    - develop derivatives of exponential and inverse trigonometric functions
8. Implicit Differentiation
    - use implicit differentiation to find the derivative/slope of a curve that is defined implicitly
    - use logarithmic differentiation to find derivatives
9. Collaborative Lesson
10. Unit Three Practice Test
11. Unit Three Test

4. **Applications of Derivatives**
    1. Unit Four Checklist and Pretest
    2. Analyzing Functions Part I: Curve Sketching
        - determine the concavity of a function and discuss its implications on the shape of the curve
        - find critical points, find critical points and points of inflection, and points of inflection
        - determine the intervals for where a function is increasing or decreasing; analytically, numerically, and with a graphing calculator
        - sketch the curve of a function based upon information from its first and second derivatives and vice versa
    3. Analyzing Functions Part II: Maximums and Minimums
        - determine the intervals for where a function is increasing or decreasing; analytically, numerically, and with a graphing calculator
        - sketch the curve of a function based upon information from its first and second derivatives and vice versa
        - determine the global or absolute extrema of a function on a closed interval, using both algebraic analytical techniques with the 1st, 2nd, or both derivatives as well as with the use of a graphing calculator
    4. Applied Maximum and Minimum Problems
        - After completing this lesson you will be able to find the optimal values (maximums or minimums) in various application problems.
    5. Distance/Velocity/Acceleration/Rectilinear Motion
        - use derivatives to discuss the motion and rate of change of objects in terms of distance and displacement, velocity, speed, and acceleration
        - use derivatives to discuss rectilinear motion
    6. Related Rates
        - use derivatives to solve related rate problems

- model how the rates of different quantities that depend upon the same parameter, such as time, interact

7. The Mean-Value Theorem & L'Hôpital's Rule
    - use the Mean Value Theorem for Derivatives to make conclusion about a function on certain intervals (and point within those intervals) and explore the results via graphical methods
    - evaluate limits involving indeterminate forms using L'Hôpital's Rule
8. Linearization
    - understand the definition of the derivative as a local linear approximation and what that implies
    - understand differentiability and use graphs to explore tangent lines
    - understand the different notations for the derivative
    - explore the relationship between the graph of a function and its derivative
    - explore the characteristics of the graphs of f, f ', and f ''
    - use local linear approximation or differentials to aid in approximations techniques
9. Collaborative Lesson
10. Unit Four Practice Test
11. Unit Four Test

5. **Integration**
    1. Unit Five Checklist and Pretest
    2. Area Approximation and Riemann Sums
        - understand Integrals with Archimedes' Method of Exhaustion (numerical approximation)
        - understand how Archimedes' Method of Exhaustion leads to the natural use of the rectangle approximation method for the area under a curve
        - represent the area under a curve as a limit using sigma notation
        - sketch the curve of a function based upon information from its first and second derivatives and vice versa
    3. Introduction to the Definite Integral
        - identify the definite integral as a limit of Riemann Sums
        - evaluate definite integrals by interpreting them geometrically
        - understand the differences or similarities (depending) between area--a.k.a. "net signed area"--and the definite integral
        - explore the above-described concept using a graphing calculator
        - explore Riemann Sums and accumulated change from a Rate of Change
    4. The Fundamental Theorem of Calculus
        - use integrals to define functions and explore that relationship
        - take the derivatives of integrals by the Fundamental Theorem of Calculus
        - use the Fundamental Theorem of Calculus to evaluate definite integrals
    5. Integrals and Antiderivatives
        - After completing this lesson, you will be able to evaluate indefinite integrals to find the general antiderivatives of functions.
    6. Integration by Substitution
        - evaluate indefinite integrals by use of the method of substitution
        - evaluate definite integrals by use of the graphing calculator
    7. The Definite Integral
        - evaluate definite integrals by use of the method of substitution
        - evaluate definite integrals by use of the graphing calculator
    8. Collaborative Lesson
    9. Unit Five Practice Test
    10. Unit Five Test

6. **Application of Integrals**
    1. Unit Six Checklist and Pretest
    2. Finding the Area Under and Between Curves
        - find the area between two curves using definite integrals
        - explore the area between two curves using technology
    3. Volume by Discs (Slicing)
        - After completing this lesson, you will be able to use the method of discs/slicing/washers to find the volume of a solid revolution.
    4. Avg. Value of a Function and Rectilinear Motion
        - explore and understand the Mean Value Theorem for Integrals
        - find the average value of a function
        - explore Rectilinear Motion with Integrals
        - explore General Motion of Objects (distance, displacement, velocity, speed, and acceleration)
        - evaluate constants of integration given an initial condition
    5. Collaborative Lesson
    6. Unit Six Practice Test
    7. Unit Six Test
    8. Semester One Practice Exam
    9. Semester One Exam

Semester B Summary:

AP Calculus BC is an extension of Calculus AB. Comparable to college and university calculus, this course will help prepare students for the Calculus BC Advanced Placement exam. The course emphasizes broad concepts and applicable methods. The student will explore differential equations, employ analytic geometry to interpret polar, vector, parametric and planar curves, and study polynomial approximations and series. The course provides opportunities for the student to apply concepts to real-world situations.

Semester B Outline

1. **Differential Equations and More Riemann Sums**
    1. Getting Started
    2. Unit One Checklist and Pretest
    3. Differential Equations—An Introduction
        - evaluate constants of integration given an initial condition
        - solve separable differential equations
        - model various applications using separable differential equations, with particular focus on the study of the equation $y' = ky$ and exponential growth
    4. Initial Value Problems and Slope Fields
        - draw a slope field (or direction field) for a differential equation
        - interpret a slope field when given it
        - interpret the solution curve attached to an initial value
    5. Numerical Approximation Methods and Integrals
        - use numerical approximation techniques to evaluate definite integrals, where appropriate, including the area under a curve
        - use Riemann Sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions through algebraic, graphical, and tabular representation (table of values)
        - discuss error implications of different methods
    6. Collaborative Lesson

7. Unit One Practice Test
8. Unit One Test

2. **Supplemental Topics**
    1. Unit Two Checklist and Pretest
    2. Exploring the Graphs of f, f Prime, and f Double..
        - After completing this lesson, you will be able to describe and explore the relationship and/or characteristics between the graphs of f, f ′, and f″.
    3. Relative Rates of Growth
        - After completing this lesson, you will be able to compare and contrast the relative growth rate functions, including polynomial, exponential and logarithmic growth.
    4. Using Calculus with Data in a Table
        - use integrals to define functions
        - explore the relationship of functions defined by integrals
    5. Functions Defined by Integrals
        - explore accumulated change from a rate of change as an application of integrals
        - use Riemann Sums and trapezoidal sums to approximate definite integrals of functions represented by tables of values
        - explore functions defined by integrals
    6. Integration by Parts
        - use integration by parts to compute antiderivatives and evaluate integrals
        - recognize when using integration by parts to compute antiderivatives will be useful and when it will not be
        - use repeated integration by parts to evaluate integrals
        - include the tabular method for repeated integration by parts
    7. Integration Using Partial Fractions
        - use simple partial fraction decomposition (nonrepeating linear factors only) to compute antiderivatives and evaluate integrals
        - recognize when it will be useful or not to use simple partial fraction decomposition (nonrepeating linear factors only) to compute antiderivatives and evaluate integrals
        - solve applications of partial fraction decomposition such as logistic differential equations, and use them in modeling
    8. Improper Integrals
        - identify integrals as improper and rewrite them as limits of definite integrals
        - determine whether an improper integral converges (to finite limit) or diverges
        - compute definite integrals that are improper in applications including the area under a curve or between curves and volume
    9. Collaborative Lesson
    10. Unit Two Practice Test
    11. Unit Two Test

3. **Analytic Geometry**
    1. Unit Three Checklist and Pretest
    2. Parametric Curves
        - analyze planar curves given in parametric form
        - compute derivatives of parametric functions and their applications such as finding tangent lines
        - find the arc length of a curve described in Cartesian form as well as parametric form

- convert between parametric form and other forms (such as rectangular/Cartesian form)
- graph curves in parametric form, including using technology to do so

3. Polar Curves
    - analyze planar curves given in polar form
    - compute derivatives of polar functions and their applications such as finding tangent lines
    - find the area between polar curves
    - convert between polar form and other forms (such as rectangular/Cartesian form)
    - graph curves in polar form, including using technology to do so
4. Vector Curves
    - analyze planar curves given in vector form
    - compute derivatives of vector functions, including their application in rectilinear motion and calculating related properties such as distance, displacement, velocity, acceleration, jerk, etc.
5. Length of Planar Curves
    - After completing this lesson, you will be able to analyze planar curves given in parametric form; be able to compute derivatives of parametric functions and their applications such as finding tangent lines; find the arc length of a curve described in Cartesian form as well as parametric form; convert between parametric form and other forms (such as rectangular/Cartesian form); graph curves in parametric form, including using Technology to do so.
6. Area of Planar Curves (Polar Curves Only)
    - After completing this lesson, you will be able to analyze planar curves given in polar form; be able to compute derivatives of polar functions and their applications such as finding tangent lines; find the area between polar curves; convert between polar form and other forms (such as rectangular/Cartesian form); graph curves in polar form, including using technology to do so.
7. Collaborative Lesson
8. Unit Three Practice Test
9. Unit Three Test

4. **Series and Convergence**
    1. Unit Four Checklist and Pretest
    2. Series
        - review familiar series including arithmetic and geometric series
        - compute, using familiar series, partial sums as well as infinite sums if they exist
        - evaluate if a familiar series is convergent—particularly using geometric series
        - find the general term of a series and identify whether it is arithmetic, geometric or neither
    3. Convergence
        - use graphs and technology to explore convergence and divergence
        - determine whether a sequence converges (to a finite value) or diverges
        - explore examples that involve the application of geometric series including repeating decimal expansion
        - view terms of series as areas of rectangles and their relationship to integrals
    4. Tests for Convergence Part I
        - After completing this lesson, you will be able to test a series for convergence or divergence using the:
        - p-Series test

- integral test
- ratio test
- comparison tests

5. Tests for Convergence Part II
    - After completing this lesson, you will be able to test a series for convergence or divergence using the:
    - ratio test
    - alternating series test
6. Error Bound
    - identify common series such as alternating series and the harmonic series
    - explore and compute the error bound of alternating series
7. Collaborative Lesson
8. Unit Four Practice Test
9. Unit Four Test

5. **Polynomial Series and Approximations**
    1. Unit Five Checklist and Pretest
    2. Maclaurin Series
        - calculate the nth order Maclaurin polynomial approximation for a given function
        - be familiar with the Maclaurin series for the functions ex, sinx, cosx, $\frac{1}{1-x}$
        - approximate a function with an appropriate Maclaurin series
    3. Taylor Series and Error Bound
        - calculate the nth order Taylor polynomial approximation for a given function centered at x = a
        - approximate a function or value (such as In3) with an appropriate Taylor series
        - explore and compute the Lagrange error bound for Taylor polynomial approximations to functions with graphical demonstration of convergence, including use of appropriate technology to do so
        - use Taylor approximations in real-life applications such as modeling and calculating differential equations
    4. Power Series
        - explore functions defined by power series
        - be able to find the power series of a given function by formally manipulating (and computing with) known power series
        - use techniques such as substitution differentiation and integration with the power series of a given function
    5. Radius and Interval of Convergence of Power Series
        - calculate the interval and radius of convergence of a given power series
        - be familiar with the interval of convergence for the common power series (Maclaurin series) for ex, sinx, cosx, $\frac{1}{1-x}$
    6. Applications of Polynomial Series
        - use polynomial series to approximate irrational numbers
        - use polynomial series to solve differential equations
        - use polynomial series to Do rectilinear motion
        - use polynomial series to model real-life behavior
        - use polynomial series to evaluate limits
    7. Collaborative Lesson
    8. Unit Five Practice Test
    9. Unit Five Test
    10. Semester Two Practice Exam
    11. Semester Two Exam

6. **Getting Ready for the Exam**
    1. Unit Six Checklist
    2. Test Format—MC Part A
        - understand the format of the AP Exam
        - understand test taking techniques for taking the multiple-choice part of the exam
        - understand question types to expect on the multiple-choice part of the exam
    3. Using a Calculator—MC Part B
        - learn which calculator models may be used on the exam
        - understand which calculator operations are permitted
        - understand the degree of accuracy required on the exam
    4. The Free Response Section
        - understand the format of free response questions on the exam
        - review a list of topics frequently tested in the FRQ section
    5. Common Mistakes. How Is the Exam Scored?
        - review a list of common mistakes made on the exam
        - understand how the BC score is computed
        - understand how the AB sub-score is computed

