



## **Precalculus**

### **Semester A Summary:**

This is the first of two courses that comprise Precalculus. In this course, the student will continue to study higher-level mathematics. The student will begin by reviewing the fundamental concepts in algebra that serve as building blocks for an in-depth study of functions and graphs. Next, the student will explore and analyze polynomial, rational, radical, exponential, logarithmic, and piecewise functions. The student will further delve into quadratics with a unit on the conic sections. Finally, the student will explore sequences and series.

A content thread throughout the course focuses on ways mathematics is applied in the real world and is essential to everyday life. These real-world connections, combined with an emphasis on mathematical reasoning and critical thinking skills, prepare the student for future college and career opportunities.

### **Semester A Outline**

#### **1. Welcome to Precalculus**

1. Introduction to Precalculus
  - Assess personal readiness for studying and learning
  - Discuss strategies for using resources effectively
  - Distinguish between previous mathematics courses and precalculus
  - Identify the various resources that can be used to promote active learning
  - Set goals in the context of mathematical learning
2. Algebraic Expressions and Equations
  - Simplify algebraic expressions
  - Solve algebraic equations
3. Relations and Functions
  - Apply the definition of a relation and a function
  - Determine whether a relation is a function from its graph using the vertical line test
  - Represent relations and functions in tables and graphs
4. The Complex Number System
  - Identify the subsets of complex numbers and their relationships
  - Simplify a number using the imaginary unit,  $i$
  - Graph complex numbers on a complex number plane
5. Operations with Complex Numbers
  - Add, subtract, multiply, and divide complex numbers
  - Show that the set of complex numbers is closed under all four operations
  - Solve equations that have imaginary solutions

#### **2. Function Algebra**

1. Domain and Range of a Function
  - Use set builder and interval notation to represent ranges of numbers
  - Determine the domain of a function from its algebraic expression

- Determine the domain and range of a function from its graph
- 2. Algebra of Functions
  - Compute, graph, and interpret the sum, difference, product, and quotient of two functions
  - Determine the domain and range of the sum, difference, product, and quotient of two functions
- 3. Composition of Functions
  - Compute and interpret the composition of two functions
  - Find the domain of the composition of functions
- 4. Inverse Functions
  - Identify one-to-one functions using either the horizontal line test or the equation
  - Find the inverse of a function
  - Determine the domain and range of a function and its inverse
- 5. Verifying Inverse Functions
  - Determine if two functions are inverses of one another algebraically
- 6. Graphs of Inverse Functions
  - Determine the graph of the inverse function by observing the graph of a one-to-one function
  - Restrict a function's domain to find its inverse function
- 7. Function Algebra Review
  - Review concepts from this unit in preparation for completing the unit test
- 8. Function Algebra Unit Test
  - Determine the domain and range of a function given its algebraic representation or its graph
  - Compute, graph, and interpret the sum, difference, product, quotient, and composition of two functions
  - Determine whether a function has an inverse that is also a function and, if so, find the inverse algebraically or by graphing
  - Use the composition of functions to verify the inverse
  - Restrict a function's domain to find its inverse function

### 3. Graph Behavior

1. Analyzing Functions
  - Find the intervals over which a function is increasing, decreasing, and constant
  - Describe the analysis of the graph of a function
  - Identify the x- and y-intercepts of a function from its graph
2. Even and Odd Functions
  - Determine whether a function is even, odd, or neither
3. Asymptotes and End Behavior
  - Recognize and describe the asymptotes of a function
  - Recognize and describe the end behavior of a function
4. Continuous and Discontinuous Functions
  - Identify continuous and discontinuous functions
  - Identify types of discontinuity
5. Linear, Absolute Value, and Reciprocal Functions
  - Recognize the graphs of the parent functions of the linear, absolute value, and reciprocal functions
6. Power, Root, Exponential, Logarithmic Functions
  - Recognize the graphs of the parent functions of power, root, exponential, and logarithmic functions
7. Transformations of Functions

- Identify horizontal and vertical shifts by analyzing the equation of the function
  - Identify horizontal and vertical stretches and compressions by analyzing the equation of the function
  - Identify reflections across the x- and y-axes by analyzing the equation of the function
8. Multiple Transformations of Functions
    - Determine the order of transformations of a function
    - Identify multiple transformations of a function by observing the algebraic structure of the function
  9. Graph Behavior Review
    - Review concepts from this unit in preparation for completing the unit test
  10. Graph Behavior Test
    - Identify and interpret significant points on a graph
    - Identify types of discontinuities in a graph
    - Interpret function transformations graphically
    - Identify the parent graphs of common functions
- 4. Polynomial and Rational Functions**
1. Polynomial Functions
    - Identify a polynomial function, its degree, and leading coefficient
    - Determine the end behavior of a function and the number of turning points of its graph
    - Find the local and global extreme values of a polynomial function
  2. Real Zeroes of a Polynomial Function
    - Determine the behavior at the x-intercepts of a polynomial function
    - Find all rational numbers that are possible zeroes of a polynomial function
    - Use the Intermediate Value Theorem to determine if a function has a zero over a closed interval
  3. Dividing Polynomials
    - Use the Remainder Theorem to find the remainder in a division problem
    - Use the Rational Zero Theorem and synthetic division to find the zeroes of a polynomial function
    - Use the Factor Theorem to determine the linear factors of a polynomial
  4. Complex Zeroes of a Polynomial Function
    - Use synthetic division to solve for all zeroes of a polynomial function where two solutions are complex
  5. Graphs of Rational Functions
    - Identify the domain of a rational function and the vertical asymptotes of its graph
    - Identify the horizontal asymptotes of the graph of a rational function
    - Identify the slant asymptotes of the graph of a rational function
  6. Operations with Rational Expressions
    - Add, subtract, multiply, and divide rational expressions
    - Prove that the set of rational expressions is closed under addition, subtraction, multiplication, and division by nonzero values
  7. Polynomial and Rational Functions Review
    - Review concepts from this unit in preparation for completing the unit test
  8. Polynomial and Rational Functions Unit Test
    - Determine the domain and range of polynomial and rational functions from their equations or their graphs
    - Determine the number and nature of the solutions of polynomial equations with real coefficients over the complex numbers

- Use arithmetic operations with complex numbers, polynomial expressions, and rational expressions, and confirm that the set of complex numbers is closed under addition, subtraction, multiplication, and division
- Compare and contrast the polynomial, rational, and radical families of functions and translate among their verbal, tabular, graphical, and symbolic representations
- Apply the Intermediate and Extreme Value Theorems to identify key points on the graph of a function

## 5. **Exponential, Logarithmic, Piecewise Functions**

1. Exponential Functions and Graphs
  - Determine if a function is an exponential function
  - Analyze the graphs of exponential functions
  - Identify and graph transformations of exponential functions
  - Identify the graphs of exponential functions with base  $e$
2. Logarithmic Notation
  - Convert between logarithmic and exponential forms
  - Evaluate a logarithm without a calculator
3. Graphs of Logarithmic Functions
  - Identify and graph transformations of logarithmic functions
  - Compare and contrast the graphs of logarithmic functions containing different bases
  - Graph logarithmic functions
4. Logarithm Rules and Solving Logarithmic Equations
  - Expand a logarithmic expression
  - Rewrite a multi-term logarithmic expression as a single logarithm
  - Solve logarithmic equations
  - Solve logarithmic equations containing composite functions of logarithms
5. Solving Exponential Equations
  - Solve exponential equations, including those containing powers with different bases
  - Solve exponential equations in quadratic form
  - Use the change of base formula to evaluate a logarithm with a calculator
6. Piecewise Functions
  - Evaluate a piecewise function
  - Write the equation of a piecewise function
  - Graph a piecewise function
  - Identify discontinuities in the graph of a piecewise function
7. Exponential, Logarithmic, Piecewise Functions Review
  - Review concepts from the unit in preparation for completing the unit test
8. Exponential, Logarithmic, Piecewise Functions Test
  - Review semester topics, concepts, and processes

## 6. **Conic Sections**

1. Analytic Geometry
  - Apply the distance and midpoint formulas to find the distance and midpoint between two points on the coordinate plane
  - Prove geometric relationships among points and lines using analytical methods
2. Introduction to Conic Sections
  - Recognize circles, ellipses, parabolas, and hyperbolas
  - Use the general equation of a conic section to identify the conic section
3. Circles

- Write the equation of a circle in both standard form and general form and identify its center and radius
  - Graph a circle when given both the standard and general form and identify its center and radius
  - Use circles to model and solve problems in real-world scenarios
4. Ellipses
- Graph an ellipse when given both the standard and general forms, and recognize its center, vertices, and foci
  - Write the equation of an ellipse in both standard form and general form and identify its center, vertices, and foci
  - Write and graph the equation of an ellipse when provided center, vertices, and foci
  - Use ellipses to model and solve problems in real-world scenarios
5. Parabolas
- Write and graph the equation of a parabola when provided the vertex and focus
  - Graph a parabola when given both the standard and general forms of the equation, and recognize its focus and directrix
  - Write the equation of a parabola in both standard form and general form and identify its focus and directrix
  - Use parabolas to model and solve problems in real-world scenarios
6. Hyperbolas
- Graph a hyperbola when given both its standard and general form, and recognize its vertices, foci, and asymptotes
  - Write the equation of a hyperbola in both standard form and general form and identify its center, vertices, foci, and asymptotes
  - Write and graph the equation of an ellipse when its center, vertices, and foci are provided
  - Use hyperbolas to model and solve problems in real-world scenarios
7. Conic Sections Review
- Review concepts from the unit in preparation for completing the unit test
8. Conic Sections Test
- Determine the equation for a conic section from its locus
  - Analyze and graph quadratic equations
  - Model real-world scenarios with conic sections

## 7. Sequences and Series

1. Sequences
- Find the first five terms of a sequence defined explicitly
  - Find the first five terms of a sequence defined recursively
  - Apply factorial notation
2. Arithmetic Sequences
- Determine whether or not a sequence is arithmetic
  - Find a formula for an arithmetic sequence
  - Use an arithmetic sequence to model a real-world scenario
3. Terms of an Arithmetic Sequence
- Find a specific term and find the  $n$ th term of an arithmetic sequence
  - Find the first term of an arithmetic sequence given another term and the common difference
  - Determine the missing terms of an arithmetic sequence using the arithmetic mean
  - Use an arithmetic sequence to model a real-world scenario

4. Finite Arithmetic Series
  - Find the sum of the first  $n$  terms of an arithmetic sequence
5. Arithmetic Series Summation Formulas
  - Use summation formulas to evaluate a partial sum of an arithmetic sequence
  - Express a given arithmetic series in summation notation
  - Identify and use the properties of summations
  - Use summation formulas to evaluate sigma expressions
6. Equations of a Geometric Sequence
  - Determine if a sequence is arithmetic, geometric, or neither
  - Find the first five terms of a geometric sequence
  - Use a geometric sequence to model a real-world scenario
7. Terms of a Geometric Sequence
  - Find the  $n$ th term of a geometric sequence
  - Find the first term and common ratio given any two terms of a geometric sequence
  - Determine the missing terms of a geometric series using the geometric mean
  - Use a geometric sequence to model a real-world scenario
8. Convergent and Divergent Sequences and Series
  - Determine if a series is convergent or divergent
9. Finite Geometric Series
  - Find the sum of the first  $n$  terms of a geometric sequence
10. Infinite Geometric Series
  - Find the sum of an infinite geometric series
11. Sequences and Series Review
  - Review concepts from the unit in preparation for completing the unit test
12. Sequences and Series Test
  - Distinguish between arithmetic and geometric sequences
  - Define sequences and series in terms of functions (notation, range, domain, general term)
  - Apply sigma and factorial notation to a series
  - Determine convergence and divergence of a series and calculate sums of convergent series
  - Model real-world patterns and situations with recursive relationships, sequences, and series
- 8. Semester Review and Exam**
  1. Semester Review
    - Review semester topics, concepts, and processes
  2. Semester Exam

## **Semester B Summary:**

This is the second of two courses that comprise Precalculus. In this course, the student will continue to study higher-level mathematics. The student will expand knowledge of trigonometric concepts, including trigonometric functions and identities, before being introduced to polar coordinates and equations. Next, the student will explore vectors and parametric equations. Finally, the student will examine calculus concepts including limits and derivatives in preparation for studying calculus.

A content thread throughout the course focuses on ways mathematics is applied in the real world and is essential to everyday life. These real-world connections, combined with an emphasis on mathematical reasoning and critical thinking skills, prepare the student for future college and career opportunities.

## **Semester B Outline**

### **1. Introduction to Trigonometry**

1. Right Triangle Trigonometry
  - Identify trigonometric ratios on a given right triangle
  - Apply trigonometric ratios to find missing side lengths of right triangles
  - Use complementary angles to demonstrate the relationship between a trigonometric function and its cofunction
2. Solving Right Triangles
  - Apply trigonometric ratios to find all missing sides and angles of a right triangle
3. Angle Measurements
  - Differentiate between degree and radian measures
  - Convert angle measurements between degrees and radians
  - Use the concept of arc length to develop the relationship between degree and radian measures
4. The Unit Circle
  - Identify the standard position of an angle on the coordinate plane
  - Use the unit circle and right triangles to define and evaluate trigonometric functions of real numbers
5. Special Right Triangles
  - Identify relationships of side lengths in special triangles
  - Develop the values of trigonometric functions at special angles in both radians and degrees
6. Trigonometric Functions in Quadrants I–IV
  - Identify the reference angle of an angle in standard position on the unit circle
  - Identify the relationship between the values of trigonometric functions in each quadrant of the unit circle
  - Evaluate trigonometric functions of quadrantal angles in any position
7. Introduction to Trigonometry Review
  - Review concepts from unit in preparation for completing the unit test
8. Introduction to Trigonometry Test
  - Use the unit circle and right triangles to define and evaluate trigonometric functions of real numbers.
  - Use degree and radian measures for angles and convert between them.
  - Develop the values of trigonometric functions at special angles in both radians and degrees.

### **2. Trigonometric Functions**

1. Graphs of Trigonometric Functions
  - Graph trigonometric functions on the coordinate plane using a table of values
2. Domain and Range of Trigonometric Functions
  - Determine the domain and range of trigonometric functions
3. Behavior of Trigonometric Functions
  - Describe the behavior of the trigonometric functions, including periodicity, amplitude, zeroes, asymptotes, and symmetries
  - Use periodic and even and odd identities to find values of trigonometric functions
4. Properties of Trigonometric Functions
  - Identify and interpret amplitude, frequency, period, and vertical and phase shift
5. Graphing Transformations of Trigonometric Function
  - Graph transformations of trigonometric functions

- Compare and contrast characteristics of trigonometric functions with transformations of other families of functions
  - 6. Writing Equations of Trigonometric Functions
    - Write the equation of a trigonometric function by identifying the properties and behavior of its graph
  - 7. Modeling with Trigonometric Functions
    - Write and graph a trigonometric function that accurately models a real-world scenario
  - 8. Inverse Trigonometric Functions
    - Use domain restriction to make trigonometric functions invertible
    - Graph inverse trigonometric functions on the coordinate plane
    - Evaluate inverse trigonometric functions at real-number values
  - 9. Composition of Trigonometric Functions
    - Compose two given trigonometric functions
    - Use composition of functions to verify inverse trigonometric functions
  - 10. Applications of Trigonometric Functions
    - Solve real-world problems with trigonometric functions
  - 11. Trigonometric Functions Review
    - Review concepts from this unit in preparation for completing the unit test
  - 12. Trigonometric Functions Test
- 3. Trigonometric Identities and Applications**
1. Trigonometric Identities
    - Identify reciprocal, quotient, and Pythagorean identities
    - Use the fundamental identities to verify other identities and to simplify trigonometric expressions
  2. Sum and Difference Formulas
    - Develop and apply the sum and difference formulas to simplify trigonometric expressions
    - Use the sum and difference formulas to find exact values of trigonometric expressions
    - Use the sum and difference formulas to verify trigonometric identities
  3. Double-Angle Formulas
    - Develop and apply double-angle formulas to simplify trigonometric expressions, find exact values, and verify trigonometric identities
  4. Law of Sines
    - Prove and apply the Law of Sines
    - Identify conditions necessary for zero, one, or two triangles
  5. Law of Cosines
    - Prove and apply the Law of Cosines
  6. Trigonometric Equations
    - Solve trigonometric equations and interpret them graphically.
  7. Trigonometric Identities and Applications Review
    - Review concepts from this unit in preparation for completing the unit test
  8. Trigonometric Identities and Applications Test
- 4. Polar Coordinates and Functions**
1. Polar Coordinates
    - Locate and plot points in the polar coordinate system
    - Identify additional representations of a point in polar form
    - Convert coordinates between rectangular and polar forms
  2. Polar Equations
    - Convert equations between rectangular and polar form
  3. Graphs of Polar Equations



- Graph a polar equation using point plotting
  - Identify graphs of polar equations of circles
  - Write polar equations of a circle given its graph
  - Use polar symmetry tests to determine if the graph is symmetric about the x-axis, the y-axis, or the origin
4. Conic Sections in Polar Coordinates
    - Identify and graph the polar equations of conic sections
  5. Limaçons
    - Identify the graphs of limaçons
    - Identify and graph the polar equations of limaçons
  6. Rose Curves and Lemniscates
    - Identify the graph and equation of a rose curve expressed in polar form
    - Identify the graph and equation of a lemniscate curve in polar form
  7. Complex Numbers in Polar Form
    - Convert complex numbers between rectangular and polar forms
  8. Operations of Complex Numbers in Polar Form
    - Find the product of complex numbers in polar form
    - Find the quotient of complex numbers in polar form
  9. DeMoivre's Theorem
    - Prove and apply DeMoivre's Theorem
    - Find powers of complex numbers in polar and rectangular form
    - Find roots of complex numbers in polar and rectangular form
  10. Polar Coordinates and Functions Review
    - Review concepts from the unit in preparation for completing the unit test
  11. Polar Coordinates and Functions Test

## 5. **Vectors**

1. Representing Vectors
  - Represent vectors geometrically and in component form
  - Determine the components of a vector
  - Find the magnitude of a vector
  - Determine whether two vectors are equal
2. Operations with Vectors
  - Represent operations on vectors geometrically
  - Perform vector addition and subtraction and find the resultant vector
  - Perform scalar multiplication
3. Unit Vectors
  - Represent vectors in the rectangular coordinate system in terms of  $i$  and  $j$
  - Find a unit vector
4. Direction Angle
  - Find the direction angle of a vector
  - Represent a vector in terms of magnitude and direction
5. Dot Product
  - Find the dot product of two vectors
  - Apply the properties of the dot product
6. Angle Between Two Vectors
  - Use the alternative form of the dot product to find the angle between two vectors
  - Use the dot product to determine if two vectors are orthogonal, parallel, or neither
  - Use the dot product to solve real-world problems
7. Vectors Review
  - Review concepts from this unit in preparation for completing the unit test

## 8. Vectors Test

### 6. Parametric Functions

1. Plane Curves and Parametric Functions
  - Analyze plane curves in parametric form
2. Plotting Points and Eliminating the Parameter
  - Graph plane curves by plotting points
  - Graph plane curves by eliminating the parameter
  - Graph plane curves by solving for  $x$  or  $y$
3. Using Trigonometric Identities
  - Graph plane curves by using trigonometric identities
  - Identify advantages of using parametric equations
4. Finding Parametric Equations
  - Find parametric equations for functions
  - Analyze different parametric equations for the same functions
5. Polar and Parametric Equations
  - Convert between polar and parametric forms
6. Modeling Linear and Projectile Motion
  - Model linear and projectile motion using parametric equations
7. Parametric Functions Review
  - Review concepts from the unit in preparation for completing the unit test
8. Parametric Functions Test

### 7. Looking Ahead to Calculus

1. Limit of a Function
  - Apply the definition of a limit of a function to specific examples
  - Determine the limit of a function for a given value using various methods
2. Properties of Limits
  - Use the properties of limits to find limits of functions
3. Continuity of Functions and Limits
  - Use the criteria for continuity to determine if a function,  $f$ , is continuous at  $a$
4. Rate of Change
  - Define the instantaneous rate of change of a function at a point
  - Find the slope of a tangent line at a given point on a curve
  - Write the equation of a tangent line
  - Define the derivative of a function,  $f$ , at  $x = a$ .
5. Applications of the Derivative
  - Solve real-world problems involving the derivatives of functions
  - Find average and instantaneous velocity of an object in motion
  - Use the velocity of an object to find its acceleration

### 8. Semester Review and Exam

1. Semester Review
  - Review semester topics, concepts, and processes
2. Semester Exam