1.3 Linear Functions

OBJECTIVES

Upon completing this section, you should be able to

- identify linear and constant functions;
- explain the difference quotient as a slope;
- convert between words, points, graphs, and algebraic representations of linear functions;
- explain when two linear functions are perpendicular;
- interpret slope as a rate of change;
- find the average rate of change of a function.

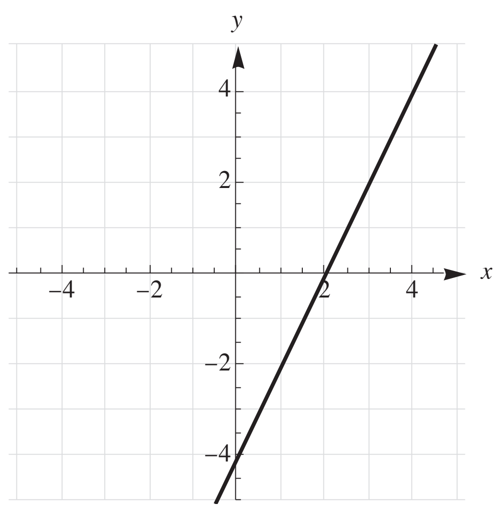
Linear models are used extensively in describing relationships between quantities such as the volume of gas in the tank of a car and the number of miles driven, the distance traveled at a constant speed and the time of travel, the boiling point of water and elevation, or revenue and the number of items sold. You might have experience working with linear functions in algebra. In this section you will expand upon your previous knowledge and work extensively with the defining characteristic of linear behavior: constant additive growth.

Algebra Check »

1. Find the slope of the line passing through the points $(0, -3)$ and $(2, 5)$.

2. Write the point-slope form for the equation of a line with slope m passing through the point (x_1, y_1).

3. Find the slope-intercept form for the equation of the line graphed in the figure.



4. Find the y-intercept of the line passing through the points $(-1, 3)$ and $(2, 2)$.

5. Find the average speed, in miles per hour, for a bicyclist who rides 7 miles in 35 minutes.

Answers »

1. 4

2. $y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$

3. $y = 2x - 4$

4. $\left(0, \dfrac{8}{3}\right)$

5. 12 miles per hour

Linear Functions »

DEFINITION **Linear and Constant Functions**

Any function f that can be written in the form $f(x) = mx + b$ is a **linear function**. The graph of a linear function is a line that is not vertical. A **constant function** has the form $f(x) = b$. The graph of a constant function is a horizontal line.

The defining characteristic of a linear function is that the difference in any two outputs is proportional to the difference in the corresponding inputs. The constant of proportionality is known as the **slope** of the line and is denoted m. Given a linear function with outputs $f(x_1) = y_1$ and $f(x_2) = y_2$, the slope can be written as

$$m = \frac{\overbrace{f(x_2) - f(x_1)}^{\text{difference in outputs}}}{\underbrace{x_2 - x_1}_{\text{difference in inputs}}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x}. \quad \begin{array}{l}\Delta y \text{ is the change in } y\text{-values;}\\ \Delta x \text{ is the change in } x\text{-values.}\end{array}$$

Note »

The capital Greek letter Δ (delta) is used to denote change in a quantity, so Δx is read "change in x."

EXAMPLE 1 Finding the slope of a line

Find the slope of the line representing the linear function that satisfies $f(0) = 2.1$ and $f(3) = 0$.

SOLUTION »

The graph of the linear function f is the line passing through the two points $(0, 2.1)$ and $(3, 0)$. The value of m is

$$m = \frac{\overbrace{f(3) - f(0)}^{\text{difference in outputs}}}{\underbrace{3 - 0}_{\text{difference in inputs}}} = \frac{0 - 2.1}{3 - 0} = -0.7.$$

Related Exercises »

Select show x and Δx in Figure 1.21 and vary the values of x and Δx. Observe that for any value of Δx (change in x), the difference in outputs of f is always $m\,\Delta x$. The difference in outputs is not affected by changes made to the value of b. To verify this analytically, let f be a linear function and analyze the difference in outputs of f associated with inputs $x + \Delta x$ and

x:

$$\begin{aligned}
\underbrace{f(x+\Delta x)}_{\text{output 2}} - \underbrace{f(x)}_{\text{output 1}} &= (m(x+\Delta x)+b)-(mx+b) && f(x)=mx+b\\
&= m(x+\Delta x)-mx && \text{Simplify.}\\
&= m\cdot\left(\underbrace{x+\Delta x}_{\text{input 2}} - \underbrace{x}_{\text{input 1}}\right) && \text{Factor.}\\
&= m\cdot\Delta x. && \text{Simplify.}
\end{aligned}$$

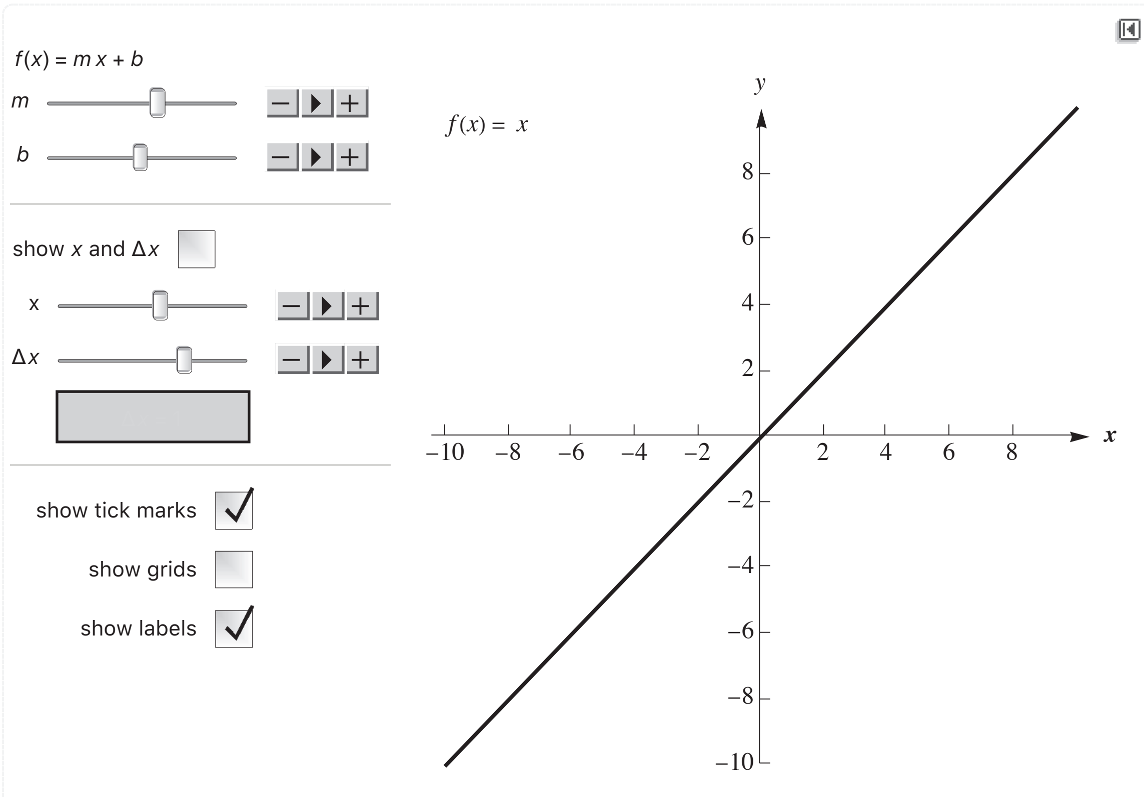


Figure 1.21

The difference in the outputs of f is the product of the slope m and the difference in the corresponding inputs. Furthermore, when the difference in inputs, Δx, is equal to 1, the difference in outputs is equal to the slope, m. In other words, when the independent variable increases by 1, the dependent variable changes by m. This constant change in outputs is evidence that linear functions exhibit **additive growth**. Regardless of the value of the input x, when the input x is increased by 1 to $x+1$, the corresponding output value $f(x+1)$ is the value of $f(x)$ changed by m; that is, for a linear function $f(x)=mx+b$,

$$f(x+1)=f(x)+m.$$

Quick Check 1 Suppose f is a linear function such that $f(3)=5$ and $m=4$. Find $f(4)$.

Answer »

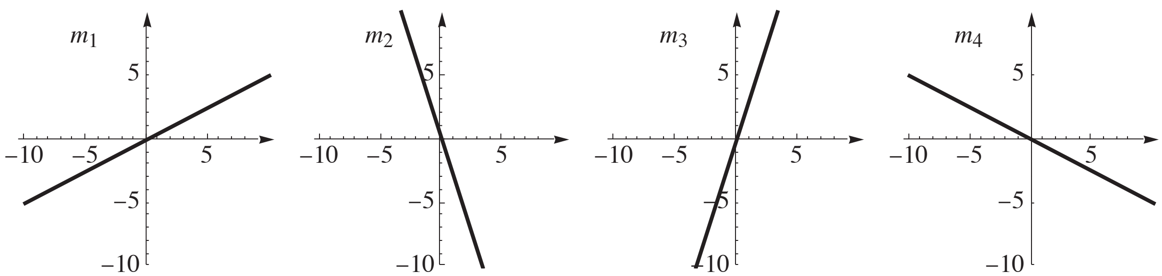
> $f(4)=f(3)+4=5+4=9$

An equivalent way of describing that a function f is linear is to say that the ratio of the difference of any two output values to the difference of the corresponding inputs is constant. Furthermore, this ratio is equal to m from the previous discussion:

$$f(b) - f(a) = m\,(b - a)$$

$$m = \frac{\overbrace{f(b) - f(a)}^{\text{difference in outputs}}}{\underbrace{b - a}_{\text{difference in inputs}}}. \quad \text{Divide by } b - a.$$

Quick Check 2 Order the slopes of the graphed lines from least to greatest.



Answer »

m_2, m_4, m_1, m_3

The ratio $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$ is called a **difference quotient** (the difference quotient was introduced in Section 1.1 using h instead of Δx). It is the familiar slope formula from algebra, $m = \dfrac{\text{rise}}{\text{run}}$, expressed in function notation. In the case where f is a linear function, the difference quotient is constant for all x and nonzero Δx, and its value m is the slope of the line representing the linear function $f(x) = m\,x + b$.

Note »

Other letters are also used to represent slope. The slope is always the coefficient of x in a linear function:

$$f(x) = m\,x + b, \text{ slope is } m$$
$$g(x) = a\,x + b, \text{ slope is } a$$
$$h(x) = a + b\,x, \text{ slope is } b.$$

Quick Check 3 Identify $x + \Delta x$, x, and Δx in each difference quotient calculation where $f(2) = 3$ and $f(-3) = 5$.

a. $m = \dfrac{f(2) - f(-3)}{2 - (-3)} = \dfrac{3 - 5}{5} = -\dfrac{2}{5}$

b. $m = \dfrac{f(-3) - f(2)}{(-3) - 2} = \dfrac{5 - 3}{-5} = -\dfrac{2}{5}$

Answer »

a. $x + \Delta x = 2, x = -3, \Delta x = 5$; **b.** $x + \Delta x = -3, x = 2, \Delta x = -5$

Whenever you are computing the difference quotient for a function, remember that you are just finding the ratio of the change in output values to the corresponding change in input values for a function. For linear functions, the difference quotient (slope) is always constant.

Linear Function Forms

Two common forms for the equation of a line are the slope-intercept form and the point-slope form. Each of these has a counterpart using function notation:

	Equation of a Line	Linear Function	
slope-intercept form	$y = mx + b$	$f(x) = mx + b$	slope-intercept form
point-slope form	$y - y_1 = m(x - x_1)$	$f(x) = m(x - h) + k.$	transformation form

For reasons that will become clear in Section 1.5 on transformations of functions, we also refer to the point-slope form of a linear function as the **transformation form**.

EXAMPLE 2 Finding a linear function

a. Find a transformation form of the linear function f where $f(-2) = 1$ and $f(3) = 4$.

b. Find the slope-intercept form of the linear function g whose graph passes through the points $(0, 5)$ and $(4, 0)$.

SOLUTION »

Finding a linear function

a. The value of m is the ratio of the change in outputs to the change in inputs:

$$m = \frac{4 - 1}{3 - (-2)} = \frac{3}{5}.$$

The two points $(-2, 1)$ and $(3, 4)$ are on the graph of f. Either point can be selected as the point (h, k) in order to obtain the transformation form of the linear function:

$$f(x) = \frac{3}{5}(x - 3) + 4.$$

b. The slope of the line is $m = \frac{5 - 0}{0 - 4} = -\frac{5}{4}$, and the vertical-axis intercept is given directly as the point $(0, 5)$. Thus, $b = 5$ and the slope-intercept form of the linear function is

$$g(x) = -\frac{5}{4}x + 5.$$

Related Exercises »

EXAMPLE 3 Finding a function from its graph

Find the function f represented by the line in Figure 1.22.

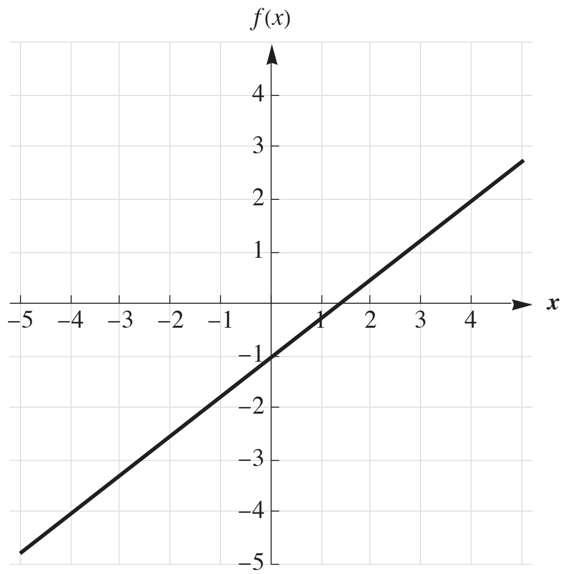


Figure 1.22

SOLUTION »

In slope-intercept form, $f(x) = mx + b$. Since the vertical-axis intercept is -1, $b = -1$. Between points $(0, -1)$ and $(4, 2)$, the line rises 3 for a run of 4, resulting in slope $m = \frac{3}{4}$. The linear function graphed in the figure is

$$f(x) = \frac{3}{4}x - 1.$$

Related Exercises »

EXAMPLE 4 Water in a tank

A tank containing 850 gallons of water is emptied by pumping 2.5 gallons per minute out of the tank.

a. Find a function w giving the volume of water in the tank after t minutes of pumping.

b. Identify the vertical-axis intercept and explain its significance in the context of the water in the tank.

c. Find the time it takes to empty the tank.

d. Find the domain and range of w.

e. Graph w.

SOLUTION »

Water in a tank

a. The volume of water in the tank begins at 850 gallons and is changing at the constant rate of -2.5 gallons per minute:

$$w(t) = 850 - 2.5\,t.$$

b. The vertical-axis intercept is 850 gallons. At the time the pumping begins ($t = 0$), there are 850 gallons of water in the tank.

c. The tank is empty when $w(t) = 0$:

$$\begin{aligned} 0 &= 850 - 2.5\,t \\ 2.5\,t &= 850 \\ t &= 340. \end{aligned}$$

It takes 340 minutes, or 5 hours and 40 minutes, to empty the tank.

d. The domain of w is $[0, 340]$ and the range is $[0, 850]$. Values outside of these do not have meaning in this context.

e. Sketching a line through the points $(0, 850)$ and $(340, 0)$ yields Figure 1.23:

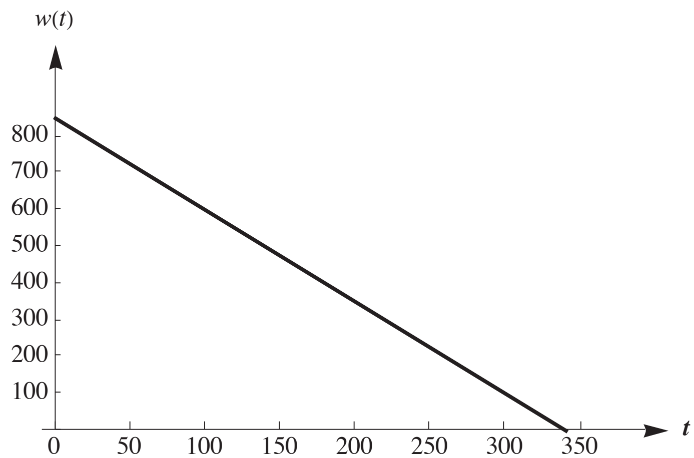


Figure 1.23

Related Exercises »

Graphs »

Two distinct linear functions are said to be **parallel** when their slopes are the same and **perpendicular** when their graphs intersect at a right angle. There are infinitely many functions perpendicular to a given linear function—the lines representing these infinitely many functions are parallel to each other.

In Figure 1.24, the graph of $f(x) = m(x - h) + k$ is shown with two points plotted on the line; the difference in x-coordinates of the points is Δx and the difference in y-coordinates is $m\,\Delta x$. Select show $f_\perp$ in the figure to graph a perpendicular function, $f_\perp$, read "f perp." Both functions pass through the black point (h, k). Vary h and k in the figure, watching the slopes of f and $f_\perp$ remain constant.

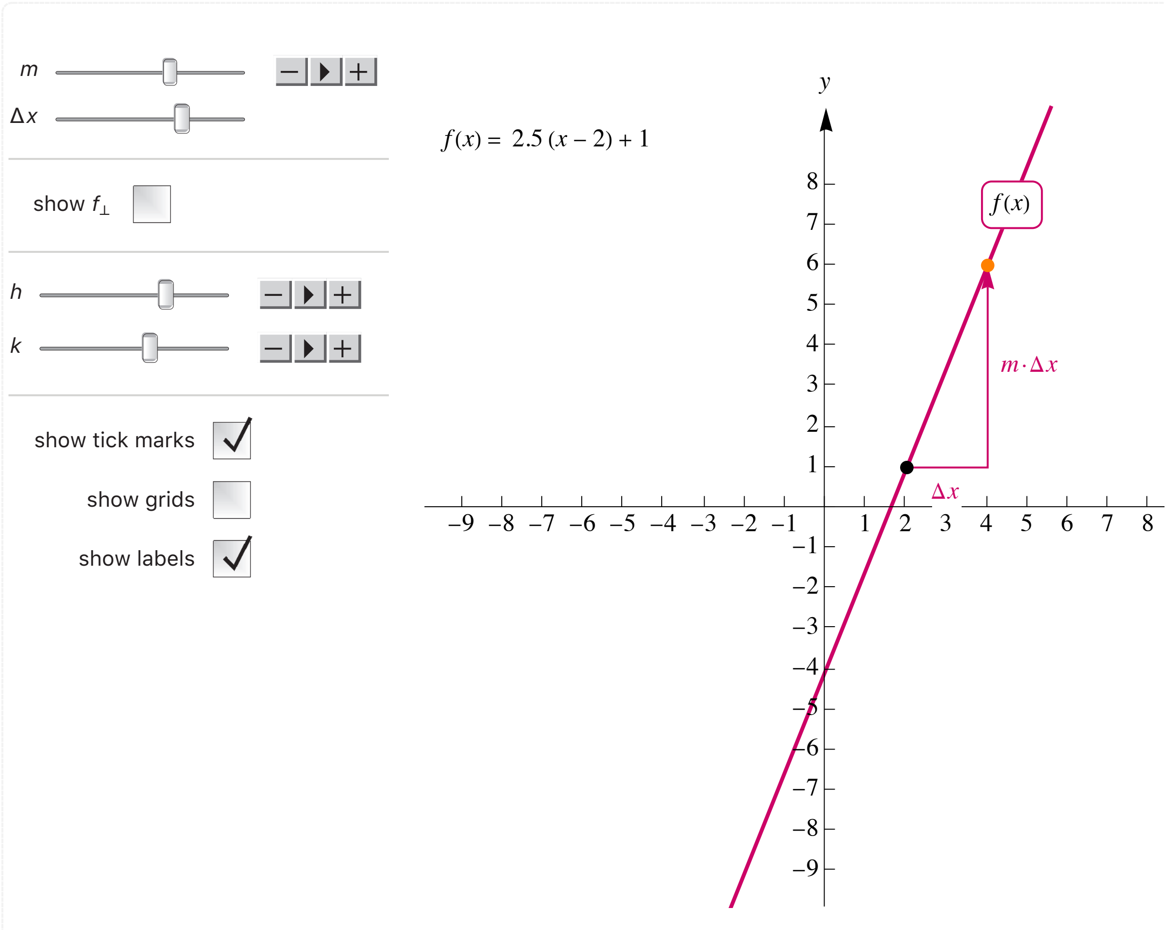


Figure 1.24

Now vary the value of m in Figure 1.24, keeping the graphs of f and $f_\perp$ visible in the figure, and observe the relationship between the slopes of the two lines: Whenever the slope of the solid red line representing f is positive, the slope of the dashed blue line representing $f_\perp$ is negative, and whenever the slope of f is negative, the slope of $f_\perp$ is positive.

Furthermore, the slope of the dashed blue line can be computed using the rise and run quantities shown in the figure. Two points are singled out on each line in the figure: one is the point of intersection between the two lines and the other lies the same distance from the point of intersection along each line. The horizontal run Δx for the two points on the graph of f equals the vertical rise between the two points on the graph of $f_\perp$. The horizontal run between the two points on the graph of $f_\perp$ equals the negative of the vertical rise between the two points on the graph of f. Let $m_\perp$ be the slope of the dashed blue line; then

$$m_\perp = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{-m \cdot \Delta x} = -\frac{1}{m}.$$

The slopes m and $m_\perp$ are negative reciprocals of each other. This is always the case for two linear functions that are perpendicular to each other.

DEFINITION **Perpendicular Linear Functions**

Two linear functions are **perpendicular** if and only if the slopes of their graphs are negative reciprocals of each other. Equivalently, $f(x) = m\,x + b$ and $g(x) = M\,x + B$, where $m \neq 0$ and $M \neq 0$, are perpendicular if and only if $m\,M = -1$.

Quick Check 4 Find a linear function perpendicular to $f(x) = \frac{1}{2}x - 3$.

Answer »

Any linear function perpendicular to f has the form $f_{\perp}(x) = -2x + b$ for some value of b. Selecting $b = 0$ gives $f_{\perp}(x) = -2x$.

EXAMPLE 5 Finding a perpendicular function

Find a linear function perpendicular to $f(x) = \frac{5}{2}x + 1$ that passes through the point $(-4, 3)$.

SOLUTION »

The slope of the line representing f is $\frac{5}{2}$. The slope of a line representing a perpendicular function is the negative reciprocal, $-\frac{2}{5}$. Using the slope-intercept form of a linear function,

$$\begin{aligned} f_{\perp}(x) &= -\frac{2}{5}x + b && m_{\perp} = -\frac{2}{5} \\ 3 &= -\frac{2}{5}(-4) + b && f(-4) = 3 \\ 3 - \frac{8}{5} &= b && \text{Isolate } b. \\ f_{\perp}(x) &= -\frac{2}{5}x + \frac{7}{5}. && \text{Write the function.} \end{aligned}$$

Related Exercises »

Average Rate of Change »

As seen above, the slope of a line representing a linear function describes how the function changes. For instance, if the slope of a line is 2, every increase of 1 in the input value corresponds to an increase of 2 in the output value. Similarly, if the slope of a line is −5, then increasing the input value by 1 results in a decrease of 5 in the output value. The slope of the line is the ratio of the change in output to the change in input and is also known as the **rate of change** of the linear function.

Finding the ratio of the difference in outputs to the difference in inputs can be done for nonlinear functions as well. The ratio, or difference quotient, $\frac{f(x + \Delta x) - f(x)}{\Delta x}$ is called the **average rate of change** and is the slope of the line passing through the points $(x, f(x))$ and $(x + \Delta x, f(x + \Delta x))$. This line, as well as any line passing through two points on the graph of a nonlinear function, is known as a **secant line**.

Figure 1.25 shows the graph of a function f with two of its points plotted in red: $(x, f(x))$ and $(x + \Delta x, f(x + \Delta x))$. The blue line connecting these two points is a secant line to the graph of f. The slope of this secant line,

$$m_{\text{sec}} = \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

is the average rate of change of f for the interval $[x, x + \Delta x]$. By changing x and Δx in the figure, you will see many different secant lines of f with different slopes. In each case, the slope of the secant line is the average rate of change of f for the interval $[x, x + \Delta x]$.

Note »

The notation "m_{sec}" represents the slope of a secant line.

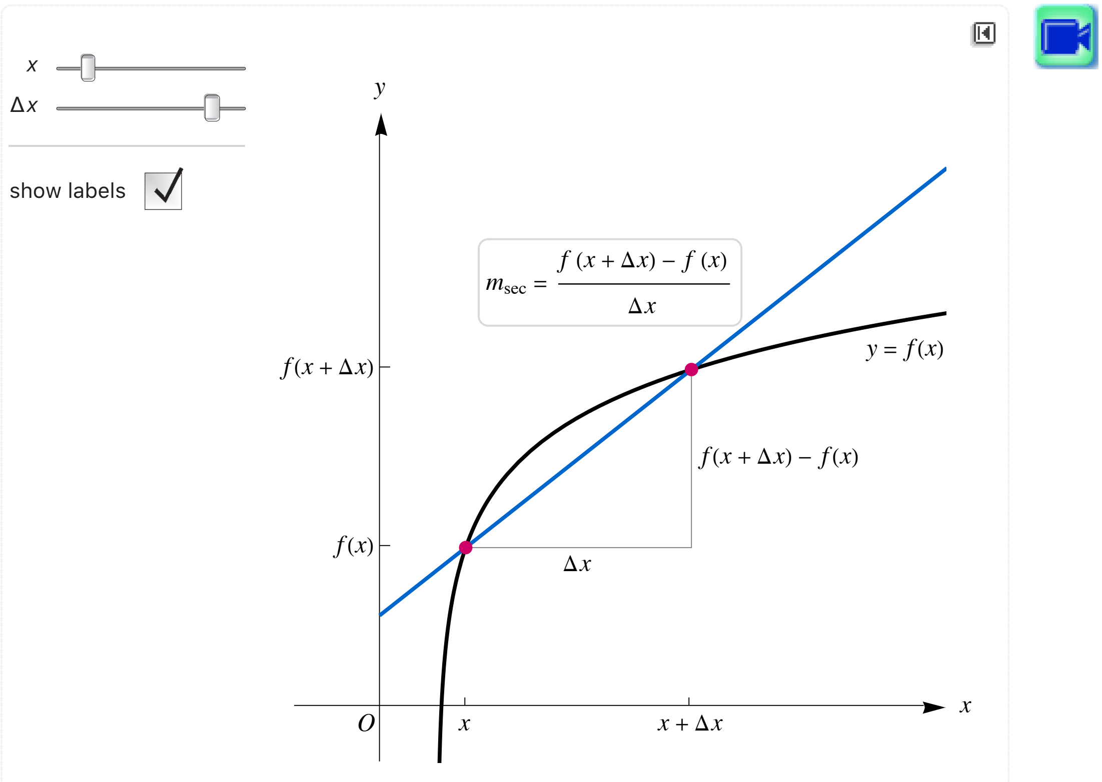


Figure 1.25

We use the average rate of change to calculate average speeds for a car trip in the next example.

EXAMPLE 6 Road trip

You have to drive from Casper, Wyoming, to Fort Collins, Colorado, on Interstate 25. The two cities are 222 miles apart. After traveling for 2.5 hours, you stop in Cheyenne for an hour to have dinner. You then continue on to Fort Collins, arriving a bit more than 4 hours after leaving Casper. A plot of the distance traveled, $d(t)$, is shown in Figure 1.26. The average rate of change in this context is an average speed:

$$\frac{d(t+\Delta t) - d(t)}{\Delta t} = \frac{\text{change in distance}}{\text{change in time}} = \text{average speed.}$$

a. Find the average speed for the drive from Casper to Cheyenne.

b. Find the average speed for the drive from Casper to Fort Collins.

c. Find the average speed for the time period from when you arrived in Cheyenne until the end of the trip.

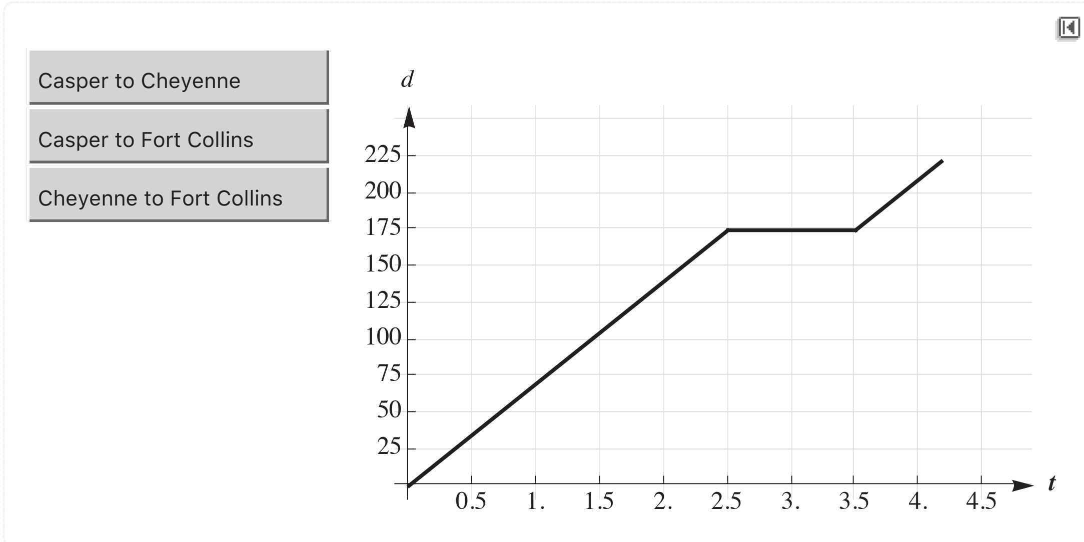


Figure 1.26

SOLUTION »

a. The average speed for the trip from Casper to Cheyenne is the slope of the line through (0, 0) and (2.5, 175):

$$m_{\text{sec}} = \frac{175 - 0}{2.5 - 0} = 70\ \frac{\text{miles}}{\text{hour}}. \quad \text{See Figure 1.26.}$$

b. Use Figure 1.26 to estimate that the time it takes for the trip from Casper to Fort Collins is approximately 4.2 hours. The average speed for the entire trip is the slope of the line through (0, 0) and (4.2, 222):

$$m_{\text{sec}} \approx \frac{222 - 0}{4.2 - 0} \approx 52.9\ \frac{\text{miles}}{\text{hour}}. \quad \text{See Figure 1.26.}$$

c.

$$m_{\text{sec}} \approx \frac{222 - 175}{4.2 - 2.5} \approx 27.6\ \frac{\text{miles}}{\text{hour}} \quad \text{See Figure 1.26.}$$

Related Exercises »

Quick Quiz »

1. If a function is linear, then the ratio of the

- ◯ **(a)** inputs to the corresponding outputs is constant.
- ◯ **(b)** outputs to the corresponding inputs is constant.
- ◯ **(c)** difference in outputs to the difference in corresponding inputs is constant.

2. Linear functions exhibit

- ◯ **(a)** additive growth.
- ◯ **(b)** multiplicative growth.
- ◯ **(c)** neither.

3. The graph of a linear function of the form $f(x) = ax + b$ is a line with

- **(a)** slope m and y-intercept b.
- **(b)** slope a and y-intercept b.
- **(c)** slope b and y-intercept a.

4. A linear function f with slope m and for which $f(h) = k$ is

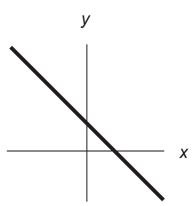
- **(a)** $f(x) = m(x+h) - k$
- **(b)** $f(x) = m(x-h) + k$
- **(c)** $f(x) = m(x+h) + k$

5. According to the function $f(x) = 5x - 3$, if x increases by 1,

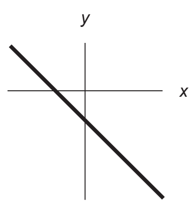
- **(a)** $f(x)$ decreases by 3.
- **(b)** $f(x)$ increases by 2.
- **(c)** $f(x)$ increases by 5.

6. Select the graph of $h(x) = mx + b$ for which $m < 0$ and $b > 0$.

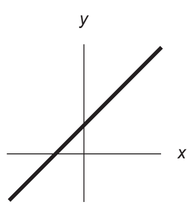
(a)



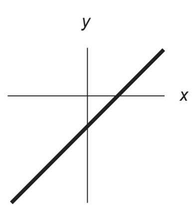
(b)



(c)



7. The graph



represents a function $f(x) = mx + b$ with

- **(a)** $m < 0$ and $b < 0$.
- **(b)** $m > 0$ and $b < 0$.
- **(c)** $m > 0$ and $b > 0$.

8. A linear function perpendicular to $g(x) = \frac{1}{3}x + 2$ is

◯ **(a)** $g_{\perp}(x) = -3x + 1$.

◯ **(b)** $g_{\perp}(x) = -\frac{1}{3}x + 1$.

◯ **(c)** $g_{\perp}(x) = 3x + 1$.

9. The average rate of change of a function f is

◯ **(a)** $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$.

◯ **(b)** the slope of a secant line through the graph of f.

◯ **(c)** both (a) and (b).

10. If $f(1) = 34$ and $f(4) = 10$, the average rate of change of f on the interval $[1, 4]$ is

◯ **(a)** 8.

◯ **(b)** −8.

◯ **(c)** −24.

Exercises

Getting Started

1. Describe the graphs of functions of the form $f(x) = mx + b$ where $m < 0$ and $b = 0$. Sketch an example.

2. Describe the graphs of functions of the form $f(x) = mx + b$ where $m = 0$ and $b < 0$. Sketch an example.

3. Write the slope-intercept form of a linear function.

4. Write the transformation form of a linear function.

5. Find the slope of any line that is perpendicular to the graph of $f(x) = -\frac{2}{5}x + 1$.

6. Given two linear functions, how can you determine whether or not they are perpendicular?

7. What is a secant line?

8. Use Figure 1.25 to explain why the slope of a secant line is $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$.

9. Explain why the average rate of change $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$ is constant for a linear function.

10. The maximum heart rate for a healthy adult during physical activity can be estimated by subtracting the individual's age from 220. Find a function m giving the estimated maximum heart rate of an individual t years old. (*Source:* Centers for Disease Control and Prevention, U.S. Department of Health & Human Services)

Practice

11–14. Slope of a line *Find the slope of the line representing the linear function whose graph passes through each pair of points.*

11. (0, 6) and (2, 2)

12. (4, 0) and (8, 2)

13. (4, 6) and (2, 20)

14. (−5, 2) and (10, 32)

15–20. Transformation form *Find a transformation form of the linear function f whose graph passes through the following pairs of points.*

15. (−2, −1) and (10, 12)

16. (4, −3) and (−5, 2)

17. $\left(\frac{1}{2}, \frac{3}{4}\right)$ and $\left(3, \frac{9}{2}\right)$

18. $\left(\frac{4}{3}, \frac{1}{5}\right)$ and $\left(-\frac{5}{3}, 2\right)$

19. (0.38, 1.94) and (0.75, 3.45)

20. (21.4, 32.5) and (59.6, 71.2)

21–24. Slope-intercept form *Find the slope-intercept form of the linear function g whose graph passes through the following pairs of points.*

21. (−3, 0) and (0, 10)

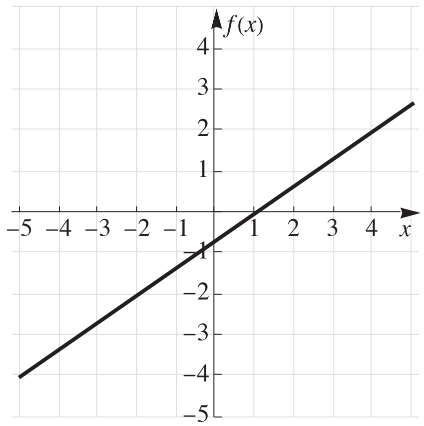
22. (0, −2) and (1, 5)

23. (2, 3) and (5, 1)

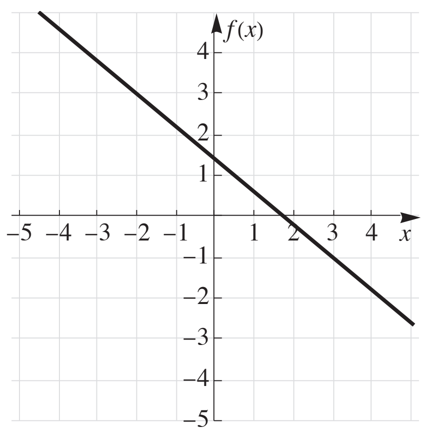
24. (−4, 3) and (1, −6)

25–30. Function from a graph *Find the function represented by each of the following lines.*

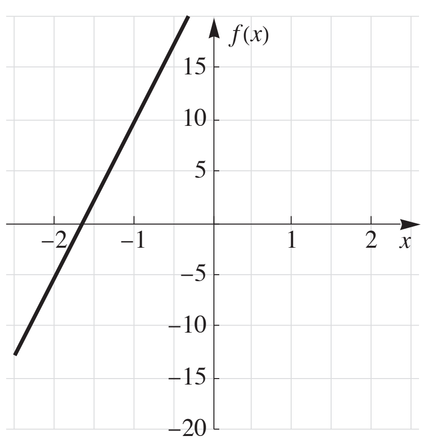
25.


f(x)
x
4
3
2
1
–5 –4 –3 –2 –1
1 2 3 4
–1
–2
–3
–4
–5

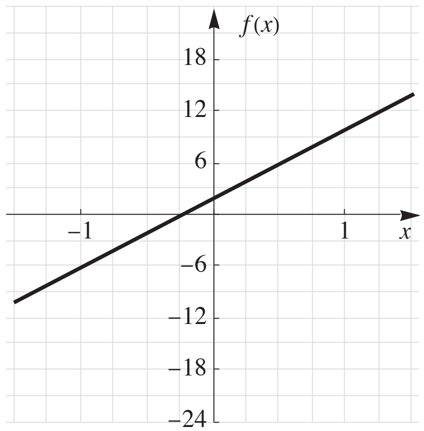
26.


f(x)
x
4
3
2
1
–5 –4 –3 –2 –1
1 2 3 4
–1
–2
–3
–4
–5

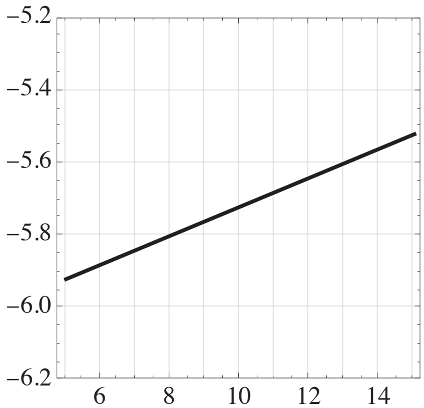
27.


f(x)
x
15
10
5
–2 –1
1 2
–5
–10
–15
–20

28.



29.



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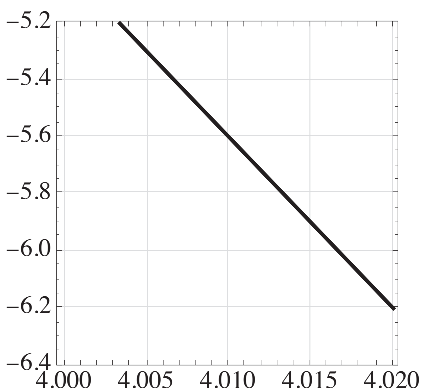


31. Filling a hot tub A hot tub can contain at most 785 gallons of water. A hose delivering 4.2 gallons per minute is used to fill an empty hot tub.

a. Find a function w giving the volume of water in the hot tub after t minutes.

b. Find the domain and range of w.

c. Graph w.

32. Emptying a hot tub A hot tub containing 785 gallons of water is emptied using a pump rated at 3.5 gallons per minute.

a. Find a function w giving the volume of water remaining in the hot tub after t minutes of pumping.

b. Find the domain and range of w.

c. Graph w.

d. Identify the horizontal-axis intercept and explain its significance in the context of the water in the hot tub.

33. **Fueling a yacht** A 70-foot yacht can carry 2500 gallons of diesel onboard and the fuel tanks must be filled before the yacht heads out to sea. The tanks contain 400 gallons of fuel when the yacht arrives at the refueling dock. Diesel is pumped into the tanks at 40 gallons per minute.

a. Find a function d giving the total volume of diesel in the tanks after t minutes of pumping fuel.

b. Find the time it takes to fill the tanks.

c. Find the domain and range of d.

d. Graph d.

e. Let $p(t)$ be the total price of the diesel pumped after t minutes. Find $p(t)$ assuming the price of diesel at the fuel dock is \$4.89 per gallon.

34. **Fueling a semi** An 18-wheeler carries 150 gallons of diesel and the fuel tanks must be filled before the driver leaves on a cross-country trip. The tanks contain 20 gallons of fuel when the semi arrives at a gas station. Diesel is pumped into the tanks at 18 gallons per minute.

a. Find a function d giving the volume of diesel in the tanks after t minutes of pumping fuel.

b. Find the time it takes to fill the truck's diesel tanks.

c. Find the domain and range of d.

d. Graph d.

e. Let $p(t)$ be the total price of the diesel pumped after t minutes. Find $p(t)$ assuming the price of diesel at the station is \$4.39 per gallon.

35–40. Perpendicular functions *Find a linear function $f_\perp$ perpendicular to f and passing through the given point.*

35. $f(x) = 3x + 2$; $(0, -5)$

36. $f(x) = 2 - 4x$; $(0, -3)$

37. $f(x) = -\frac{3}{2}x - \frac{4}{5}$; $(2, 4)$

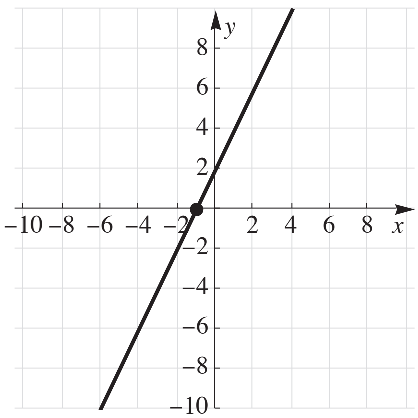
38. $f(x) = \frac{7}{3}x + 1$; $(-6, 2)$

39. $f(x) = 1.34x + 19.76$; $(52.3, 87.1)$

40. $f(x) = 101.28 - 0.29x$; $(29.4, 153.2)$

41–42. Graphing perpendicular functions *Find the linear function $f_\perp$ perpendicular to f and passing through the plotted point, and graph $f_\perp$.*

41.



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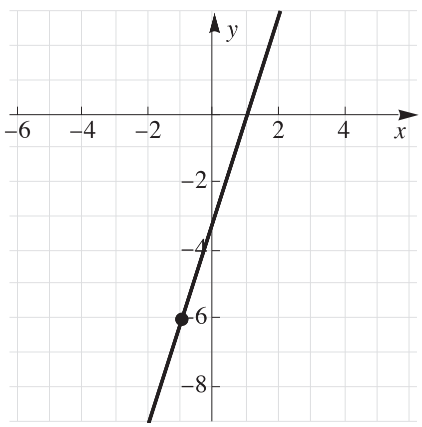
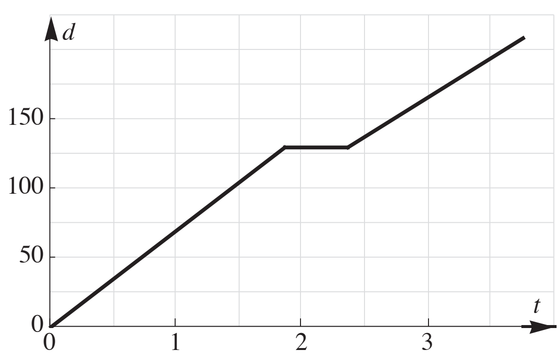


43. You have to drive 213 miles from Nashville, Tennessee, to Memphis on Interstate 40. After traveling for 130 miles at 70 miles per hour, you stop in Jackson for a half-hour to eat lunch. You then continue on to Memphis, arriving 3 hours and 45 minutes after leaving Nashville.

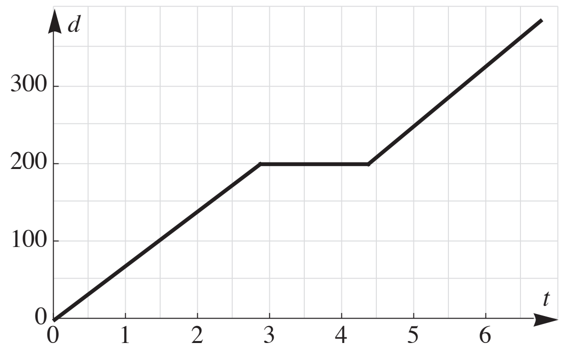


a. Find the average speed for the drive from Jackson to Memphis.

b. Find the average speed for the drive from Nashville to Memphis.

c. Find the average speed for the time period from when you left Nashville until you left Jackson.

44. You have to drive 384 miles from Los Angeles, California, to Sacramento on Interstate 5. After traveling for 200 miles at 70 miles per hour, you stop at a rest area for 1.5 hours. You then continue on to Sacramento, arriving 6 hours and 45 minutes after leaving Los Angeles.



a. Find the average speed for the drive from the rest area to Sacramento.

b. Find the average speed for the drive from Los Angeles to Sacramento.

c. Find the average speed for the time period from when you left Los Angeles until you left the rest area.

45. Explain why or why not Determine whether the following statements are true, and give an explanation or counterexample.

a. The line $y = -3x + 6$ has an x-intercept $(4, 0)$.

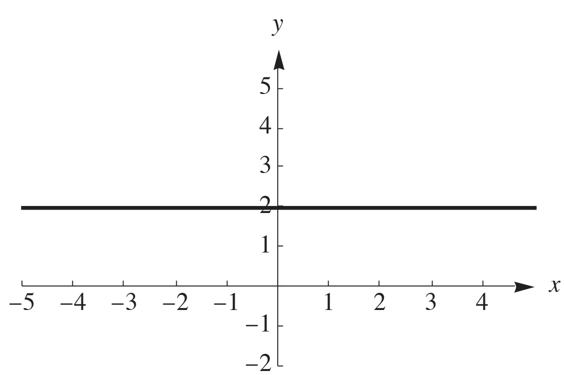
b. The line described by the equation $2y = 3x + 8$ has a y-intercept $(0, 8)$.

c. The lines $y = 4x - b$ and $y = -\frac{1}{4}x + c$ are perpendicular for all values of b and c.

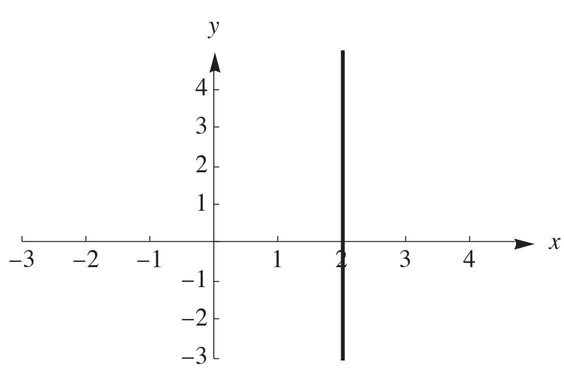
d. The lines $y = 2$ and $x = -2$ are perpendicular.

46. Horizontal and vertical lines Explain why the following lines do not represent linear functions.

a.



b.



47. Linear behavior Determine whether the values in the tables could be linear function values.

a.

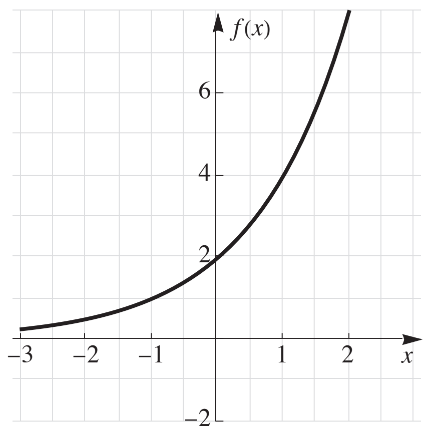
x	-2	0	2	4
$f(x)$	6	1	-4	-9

b.

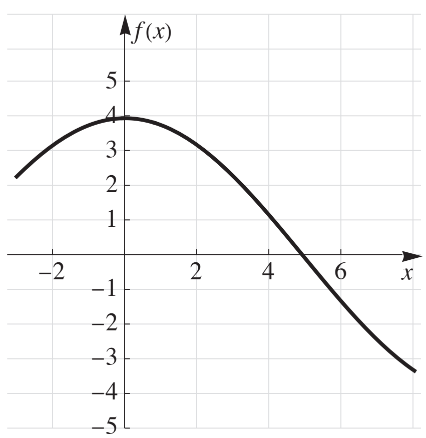
x	-1	0	2	4
$f(x)$	1	5	25	125

48–51. Average rate of change *Find the average rate of change of the following functions on the given intervals.*

48. $[-1, 1]$



49. $[0, 5]$



50. $f(x) = \dfrac{1}{x}$; $[1, 10]$

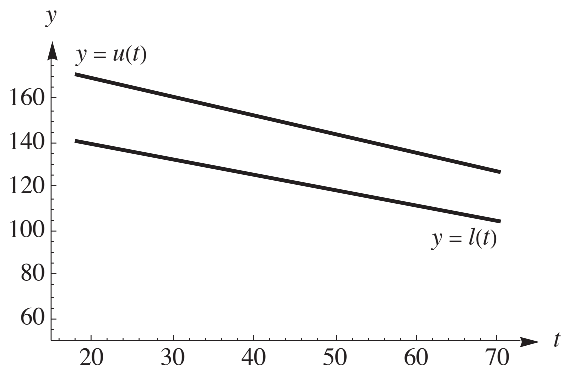
51. $f(x) = \sqrt{x - 2}$; $[2, 6]$

52. Vigorous physical activity The heart rate of a healthy adult exercising vigorously should remain between 70% and 85% of their maximum heart rate. The maximum heart rate for an individual, measured in beats per minute, is estimated by subtracting his or her age from 220. (*Source:* Centers for Disease Control and Prevention, U.S. Department of Health & Human Services)

a. Find a function l giving the lower limit for the heart rate of a t-year-old adult exercising vigorously.

b. Find a function u giving the upper limit for the heart rate of a t-year-old adult exercising vigorously.

c. Graph l and u for $18 \le t \le 70$.



53. Commission on sales Suppose a salesperson working an 8-hour shift sold \$300 of merchandise and earned \$95 for the day. During a different 8-hour shift, the salesperson sold \$700 of merchandise and earned \$115 for the day. Assume the relationship between sales and daily wage is linear.

a. Find the linear function $y = w(s)$ that gives the salesperson's daily wage for s dollars of merchandise sold.

b. Find the percent commission the salesperson earns on sales.

c. Find the minimum wage the salesperson can earn for an 8-hour shift.

54. Lease versus purchase A car dealer offers you a purchase option and a lease option on a new car. You can buy the car outright for \$22,000 or you can lease it for a start-up fee of \$2000 plus monthly payments of \$300.

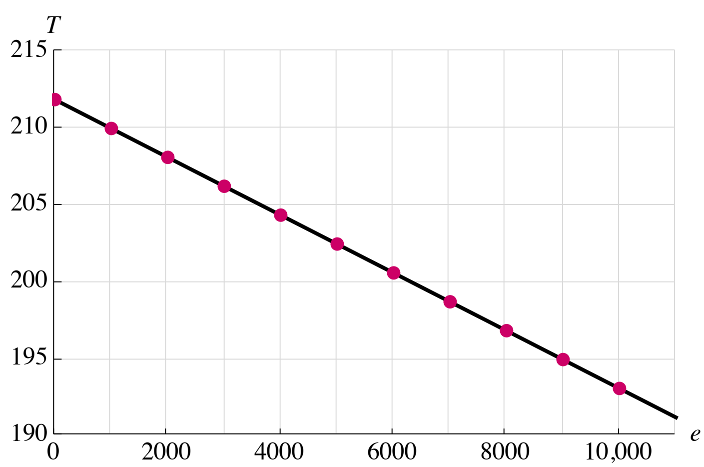
a. Find the linear function $y = f(m)$ that gives the total amount you will have paid on the lease option after m months.

b. Graph the lease function in part (a) for $0 \le m \le 48$.

c. How much will you have you paid on the lease after 48 months?

d. At the end of a 48-month lease period, the car has a residual value of \$10,000, which is the amount that you could pay the dealer to buy the car. Given the result of part (c) and assuming no other costs, should you buy or lease the car? Explain.

55. Boiling point of water At sea level, water boils at a temperature of 212°F. However, the boiling point of water decreases linearly with elevation according to the graph shown below. We let T represent the boiling point temperature in degrees Fahrenheit and e represent elevation above sea level in feet.



a. Using the data points (0, 212) and (10,000, 193), find the slope of the line that fits the boiling point data.

b. Find the equation of the line that fits the boiling point data.

c. Assuming the same relationship between boiling point and elevation is valid at higher elevations, what is the boiling point of water at 20,000 feet?

d. Complete the following sentence: For every 1000 feet of elevation gain, the boiling point of water decreases by approximately _____ degrees.

56. Printing and selling t-shirts Many manufacturing processes involve a setup cost and a per-item cost. For example, suppose that printing a batch of t-shirts involves a one-time setup cost of \$120 after which the cost of the shirt and printing is \$7 per shirt. Suppose also that the shirts can be sold at a retail price of \$11.

a. Find the linear function $C = f(n)$ that gives the cost in dollars of printing n shirts.

b. Find the linear function $R = g(n)$ that gives the revenue generated by selling n shirts.

c. The profit in making and selling n shirts is revenue minus costs, or $P(n) = R(n) - C(n)$. What is the profit if 40 shirts are printed and sold?

d. Graph the cost, revenue, and profit functions on the same set of axes.

e. What is the fewest number of shirts that can be printed and sold to make a (positive) profit?

57. Linear demand The demand for an item is the amount of that item that can be sold at a given price. The owner of a small gas station examines his sales records over the past year and estimates that if gas is priced at \$3.80 per gallon, then the demand is 700 gallons of gas per day. If the price is \$4.00 per gallon, then the demand is 600 gallons per day. Let $D = f(p)$ be the number of gallons of gas that can be sold at a price p and assume that the demand function is linear.

a. Find the equation of the line that passes through the points $(p, D) = (3.80, 700)$ and $(p, D) = (4.00, 600)$.

b. Graph the demand function in part (a).

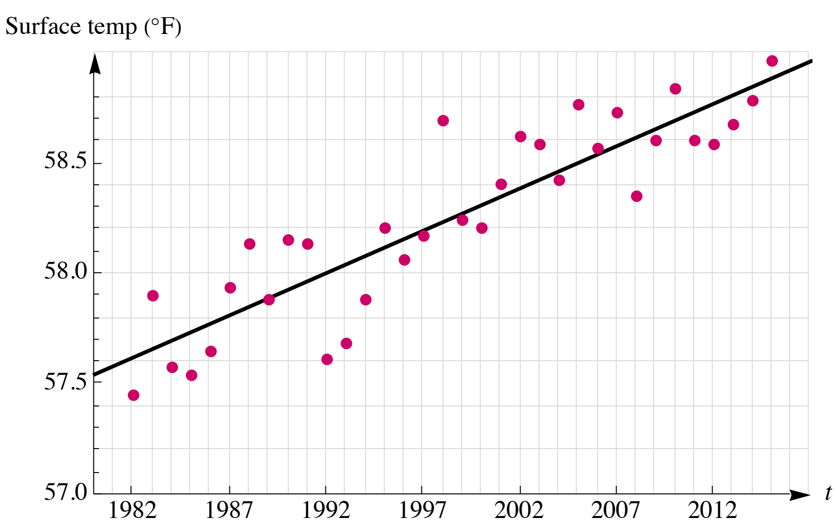
c. How many gallons can be sold at a price of \$4.20? at a price of \$3.50?

d. According to this model, what is the price above which the demand is zero?

e. If the owner sets the price at p, then he expects to sell $D(p)$ gallons at p dollars per gallon, which gives him revenue of $R(p) = p\,D(p)$ dollars. Graph the revenue function.

f. At what price can the owner expect to maximize his revenue?

58. Mean surface temperature The graph shows data for the annual mean surface temperature of Earth for the years from 1982 to 2015. The graph also shows the line that gives the best fit to the data. (*Source: GISS Surface Temperature Analysis*, National Aeronautics and Space Administration)

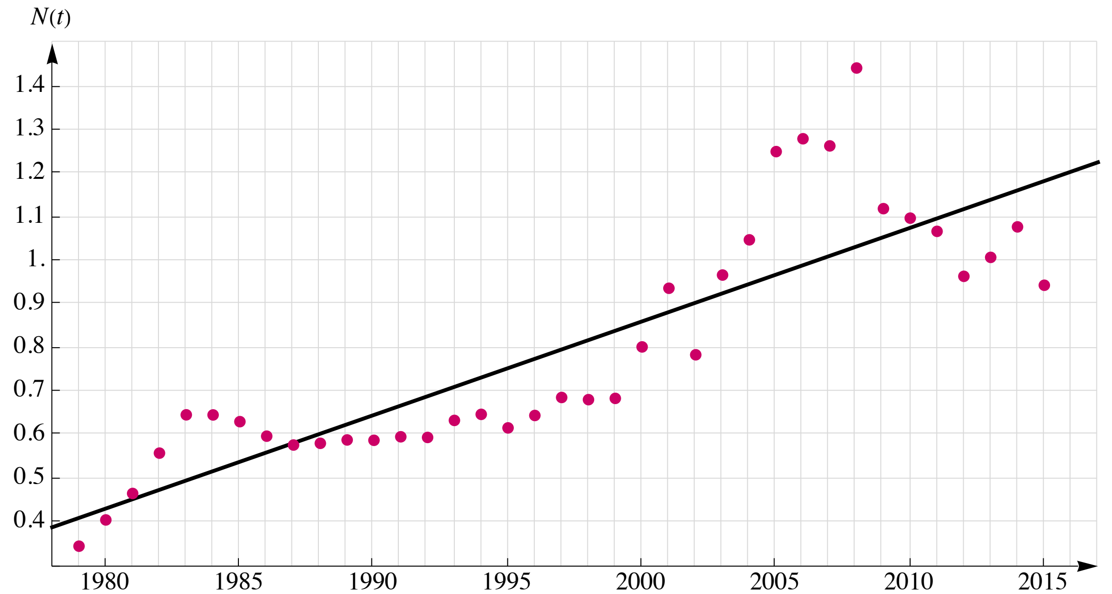


a. Find the slope of the best-fit line shown in the graph.

b. Explain what the slope of the line describes for this data.

c. Find the linear function T whose graph is the best-fit line.

59. Price of natural gas The graph shows data for the average yearly price in dollars of 1 therm of natural gas for the years from 1978 to 2015 in the United States. The graph also shows the line that gives the best fit to the data. (*Source:* Bureau of Labor Statistics, U.S. Department of Labor)



a. Find the average rate of change for the price of natural gas from the year 2003 to the year 2012.

b. Find the average rate of change for the price of natural gas from 2008 to 2015.

c. Find the slope of the best-fit line shown in the graph.

d. Explain what the slope of the line describes for this data.

e. Find the linear function N whose graph is the best-fit line.

Explorations and Challenges

60. Natural gas versus electricity The energy in 1 therm of natural gas is equivalent to the energy in 29.3 kilowatt-hours of electricity. When natural gas is burned to generate heat for a home, not all of the energy in the natural gas is converted to heat energy to warm the home. The efficiency rating of a natural gas furnace is the percent of the energy in natural gas that is converted to heat energy and used to warm a home. The average price in the United States for 1 kilowatt-hour of electricity in 2015 was \$0.138 and the average price for 1 therm of natural gas in the same year was \$0.945.

a. Assume a furnace burns 1 therm of natural gas and then uses the converted heat energy to warm a home. Find a function f that takes as input the efficiency rating r of the furnace, and gives as output the cost of the electrical energy equal to the heat energy extracted from the natural gas.

b. Graph f for $0 < r < 1$.

c. For what efficiency ratings r is natural gas a more economical source of energy than electricity for heating a home?

61. Difference quotients Consider the linear function $f(x) = mx + b$, where m and b are real numbers, $m \neq 0$. Show that $\dfrac{f(x+h) - f(x)}{h}$ is constant for all values of x and nonzero values of h. What is the value of the constant?

T **62. Lines and parabolas** Let $f(x) = x^2$ and let $P(1, 1)$ be a point on the graph of f.

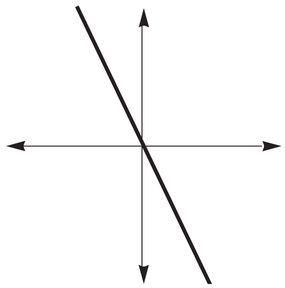
a. Find the equation of the line L that passes through P with slope $m \neq 0$.

b. Graph f and the lines L with slopes $m = 1, 1.5, 2, 2.5$.

c. Which of the lines in part (b) intersect the graph of f exactly once?

Answers to Odd Exercises

1. The graphs are lines going down from left to right and passing through the origin.



3. $f(x) = mx + b$

5. $5/2$

7. A secant line is a line passing through two points on the graph of a function.

9. The average rate of change is the slope of the line through two points on the graph. A linear function has constant slope; thus, the average rate of change remains constant and equal to the slope of the linear function.

11. $m = -2$

13. $m = -7$

15. $f(x) = \frac{13}{12}(x - 10) + 12$ or $f(x) = \frac{13}{12}(x + 2) - 1$

17. $f(x) = \frac{3}{2}\left(x - \frac{1}{2}\right) + \frac{3}{4}$ or $f(x) = \frac{3}{2}(x - 3) + \frac{9}{2}$

19. $f(x) = 4.081\,(x - 0.38) + 1.94$ or $f(x) = 4.081\,(x - 0.75) + 3.45$

21. $g(x) = \frac{10}{3}x + 10$

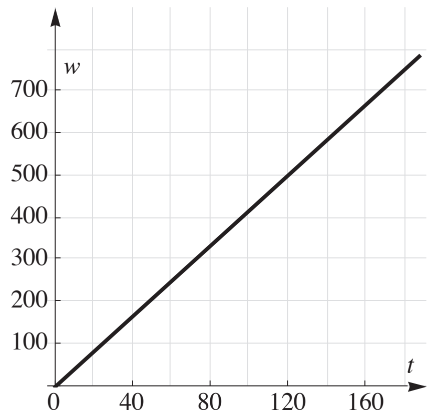
23. $g(x) = \frac{13}{3} - \frac{2}{3}x$

25. $f(x) = \frac{2}{3}x - \frac{2}{3}$

27. $f(x) = 15x + 25$

29. $f(x) = 0.04x - 6.12$

31.
a. $w(t) = 4.2t$
b. The domain is $[0, 186.9]$ and the range is $[0, 785]$.
c.



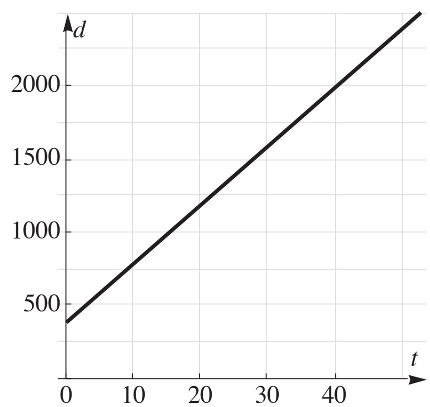
33.

a. $d(t) = 400 + 40\,t$

b. Solve $d(t) = 2500$ to find it takes 52.5 minutes to fill the tanks.

c. The domain is [0, 52.5] and the range is [400, 2500].

d.



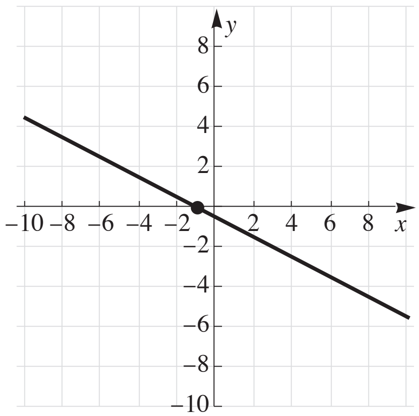
e. $p(t) = 4.89\,(40\,t) = 195.6\,t$ dollars

35. $f_{\perp}(x) = -\dfrac{1}{3}x - 5$

37. $f_{\perp}(x) = \dfrac{2}{3}x + \dfrac{8}{3}$

39. $f_{\perp}(x) \approx 126.13 - 0.75\,x$

41. $f_{\perp}(x) = -\dfrac{1}{2}x - \dfrac{1}{2}$



43.
a. 59.58 miles per hour; b. 56.8 miles per hour; c. 55.2 miles per hour

45. a. False; b. False; c. True; d. True

47. a. Yes; b. No

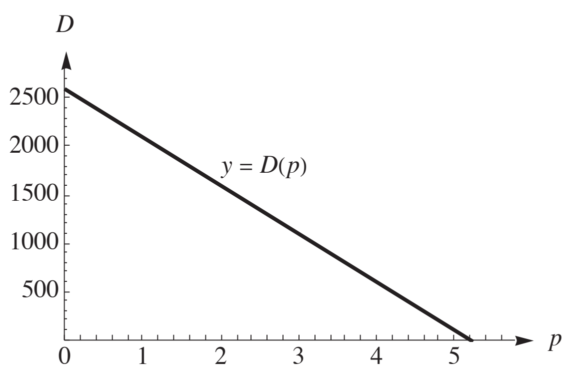
49. $m_{\text{sec}} = -\frac{4}{5}$

51. $m_{\text{sec}} = \frac{1}{2}$

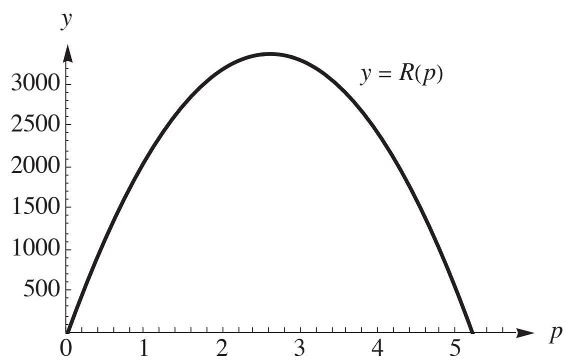
53. a. $w(s) = \frac{1}{20}s + 80$; b. 5%; c. \$80

55. a. −0.0019 degree per foot; b. $T = 212 - 0.0019\,e$; c. $T = 174$; d. 2

57. a. $D = -500\,p + 2600$
b.



c. $D(4.20) = 500$, $D(3.50) = 850$; d. \$5.20
e.



f. \$2.60

59. a. 0; b. –0.07; c. 0.022; d. The slope of the line is the average annual increase in the price of 1 therm of natural gas for the years from 1979 to 2015; e. $N(t) = 0.022\,t - 43.144$

61. m