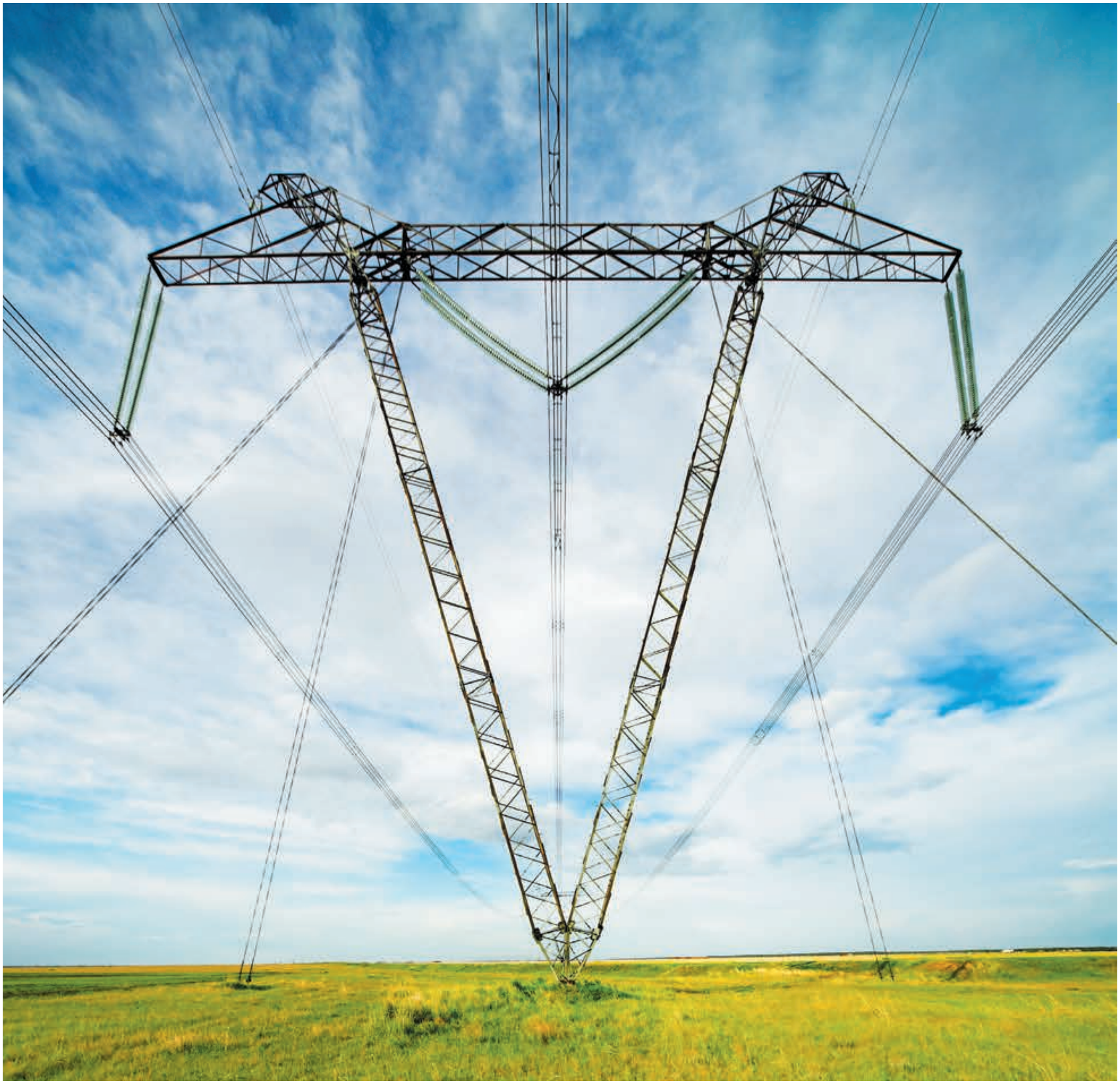
Chapter 2



(© Vasiliy Koval/Fotolia)

This electric transmission tower is stabilized by cables that exert forces on the tower at their points of connection. In this chapter we will show how to express these forces as Cartesian vectors, and then determine their resultant.

Force Vectors

CHAPTER OBJECTIVES

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another.

2.1 Scalars and Vectors

Many physical quantities in engineering mechanics are measured using either scalars or vectors.

Scalar. A ***scalar*** is any positive or negative physical quantity that can be completely specified by its *magnitude*. Examples of scalar quantities include length, mass, and time.

Vector. A ***vector*** is any physical quantity that requires both a *magnitude* and a *direction* for its complete description. Examples of vectors encountered in statics are force, position, and moment. A vector is shown graphically by an arrow. The length of the arrow represents the *magnitude* of the vector, and the angle θ between the vector and a fixed axis defines the *direction of its line of action*. The head or tip of the arrow indicates the *sense of direction* of the vector, Fig. 2–1.

In print, vector quantities are represented by boldface letters such as **A**, and the magnitude of a vector is italicized, A. For handwritten work, it is often convenient to denote a vector quantity by simply drawing an arrow above it, $\vec{A}$.

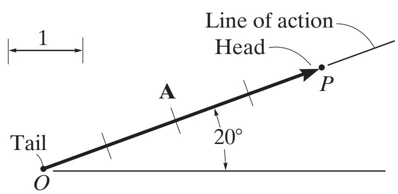
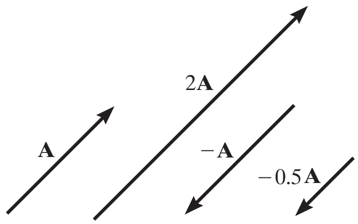


Fig. 2–1

2.2 Vector Operations



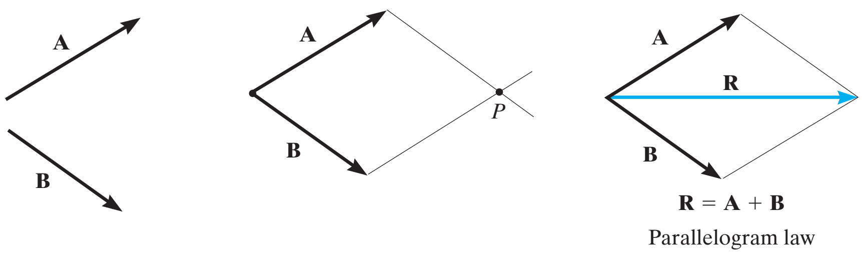
Scalar multiplication and division

Fig. 2–2

Multiplication and Division of a Vector by a Scalar. If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. Multiplying by a negative scalar will also change the directional sense of the vector. Graphic examples of these operations are shown in Fig. 2–2.

Vector Addition. When adding two vectors together it is important to account for both their magnitudes and their directions. To do this we must use the ***parallelogram law of addition***. To illustrate, the two ***component vectors*** **A** and **B** in Fig. 2–3*a* are added to form a ***resultant vector*** **R** = **A** + **B** using the following procedure:

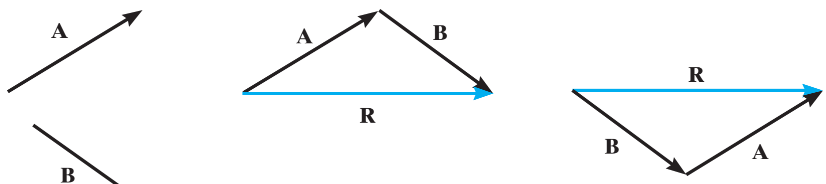
- First join the tails of the components at a point to make them concurrent, Fig. 2–3*b*.
- From the head of **B**, draw a line parallel to **A**. Draw another line from the head of **A** that is parallel to **B**. These two lines intersect at point *P* to form the adjacent sides of a parallelogram.
- The diagonal of this parallelogram that extends to *P* forms **R**, which then represents the resultant vector **R** = **A** + **B**, Fig. 2–3*c*.



(a) (b) (c)

Fig. 2–3

We can also add **B** to **A**, Fig. 2–4*a*, using the ***triangle rule***, which is a special case of the parallelogram law, whereby vector **B** is added to vector **A** in a "head-to-tail" fashion, i.e., by connecting the head of **A** to the tail of **B**, Fig. 2–4*b*. The resultant **R** extends from the tail of **A** to the head of **B**. In a similar manner, **R** can also be obtained by adding **A** to **B**, Fig. 2–4*c*. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e., **R** = **A** + **B** = **B** + **A**.



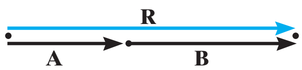
$\mathbf{R} = \mathbf{A} + \mathbf{B}$
Triangle rule
(b)

$\mathbf{R} = \mathbf{B} + \mathbf{A}$
Triangle rule
(c)

(a)

Fig. 2–4

As a special case, if the two vectors **A** and **B** are ***collinear***, i.e., both have the same line of action, the parallelogram law reduces to an *algebraic* or *scalar addition* $R = A + B$, as shown in Fig. 2–5.



$R = A + B$

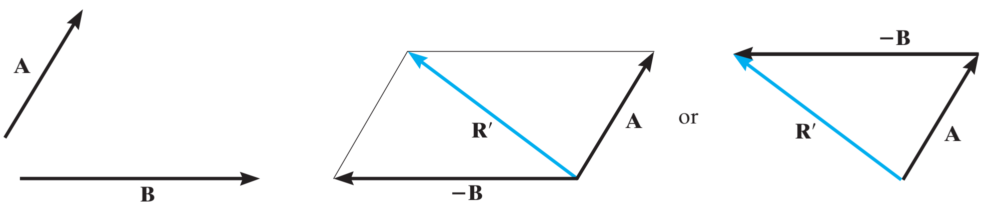
Addition of collinear vectors

Fig. 2–5

Vector Subtraction. The resultant of the *difference* between two vectors **A** and **B** of the same type may be expressed as

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

This vector sum is shown graphically in Fig. 2–6. Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction.



Parallelogram law

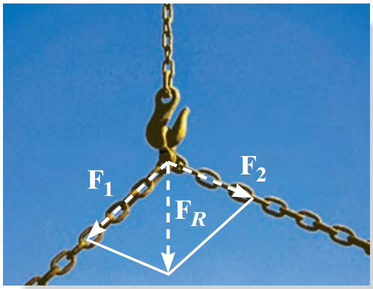
Triangle construction

Vector subtraction

Fig. 2–6

2

2.3 Vector Addition of Forces



The parallelogram law must be used to determine the resultant of the two forces acting on the hook. (© Russell C. Hibbeler)

Experimental evidence has shown that a force is a vector quantity since it has a specified magnitude, direction, and sense and it adds according to the parallelogram law. Two common problems in statics involve either finding the resultant force, knowing its components, or resolving a known force into two components. We will now describe how each of these problems is solved using the parallelogram law.

Finding a Resultant Force. The two component forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the pin in Fig. 2–7*a* can be added together to form the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$, as shown in Fig. 2–7*b*. From this construction, or using the triangle rule, Fig. 2–7*c*, we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.

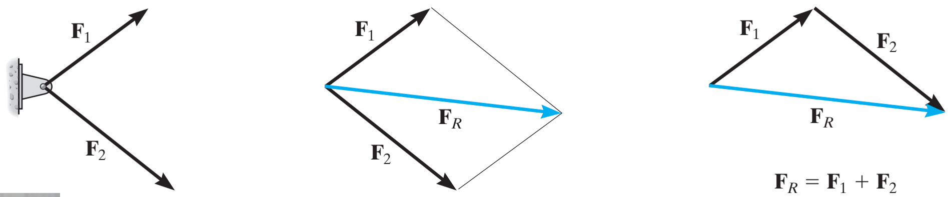


Fig. 2–7

Using the parallelogram law the supporting force $\mathbf{F}$ can be resolved into components acting along the u and v axes. (© Russell C. Hibbeler)

Finding the Components of a Force. Sometimes it is necessary to resolve a force into two *components* in order to study its pulling or pushing effect in two specific directions. For example, in Fig. 2–8*a*, $\mathbf{F}$ is to be resolved into two components along the two members, defined by the u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of $\mathbf{F}$, one line parallel to u, and the other line parallel to v. These lines then intersect with the v and u axes, forming a parallelogram. The force components $\mathbf{F}_u$ and $\mathbf{F}_v$ are then established by simply joining the tail of $\mathbf{F}$ to the intersection points on the u and v axes, Fig. 2–8*b*. This parallelogram can then be reduced to a triangle, which represents the triangle rule, Fig. 2–8*c*. From this, the law of sines can then be applied to determine the unknown magnitudes of the components.

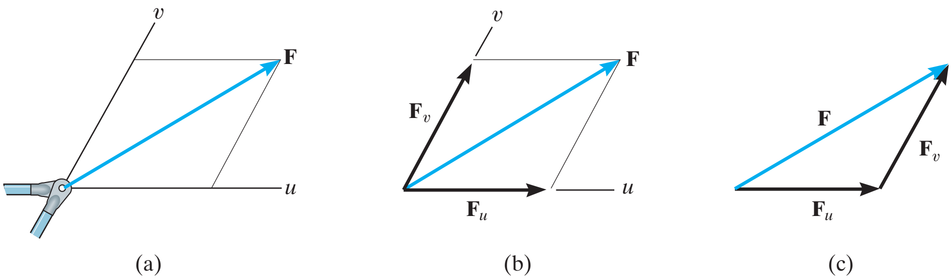


Fig. 2–8

Addition of Several Forces.

If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force. For example, if three forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ act at a point O, Fig. 2–9, the resultant of any two of the forces is found, say, $\mathbf{F}_1 + \mathbf{F}_2$—and then this resultant is added to the third force, yielding the resultant of all three forces; i.e., $\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$. Using the parallelogram law to add more than two forces, as shown here, often requires extensive geometric and trigonometric calculation to determine the numerical values for the magnitude and direction of the resultant. Instead, problems of this type are easily solved by using the "rectangular-component method," which is explained in Sec. 2.4.

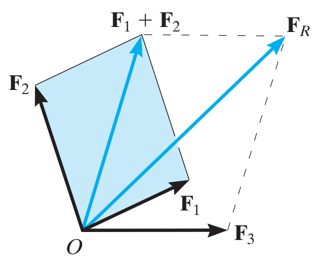
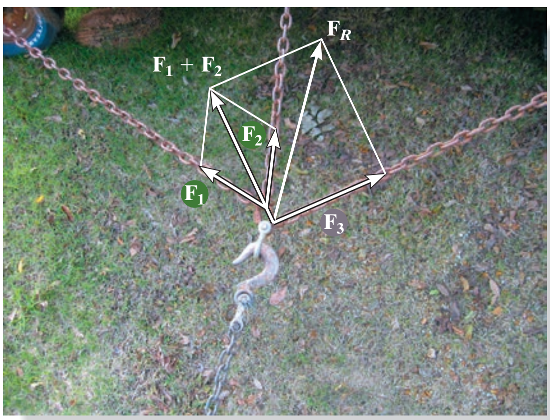


Fig. 2–9



The resultant force $\mathbf{F}_R$ on the hook requires the addition of $\mathbf{F}_1 + \mathbf{F}_2$, then this resultant is added to $\mathbf{F}_3$. (© Russell C. Hibbeler)

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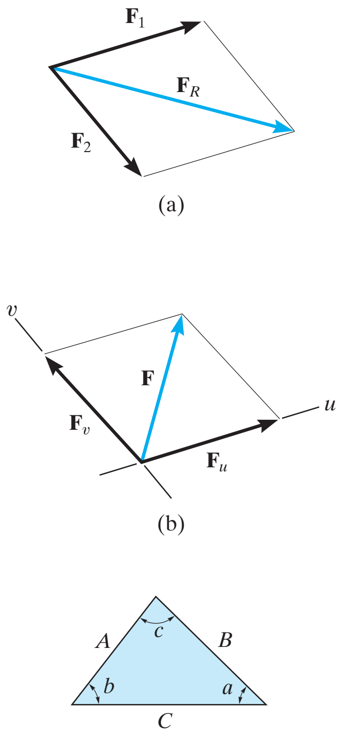
Important Points

- A scalar is a positive or negative number.
- A vector is a quantity that has a magnitude, direction, and sense.
- Multiplication or division of a vector by a scalar will change the magnitude of the vector. The sense of the vector will change if the scalar is negative.
- As a special case, if the vectors are collinear, the resultant is formed by an algebraic or scalar addition.



Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

(c)

Fig. 2–10

Procedure for Analysis

Problems that involve the addition of two forces can be solved as follows:

Parallelogram Law.

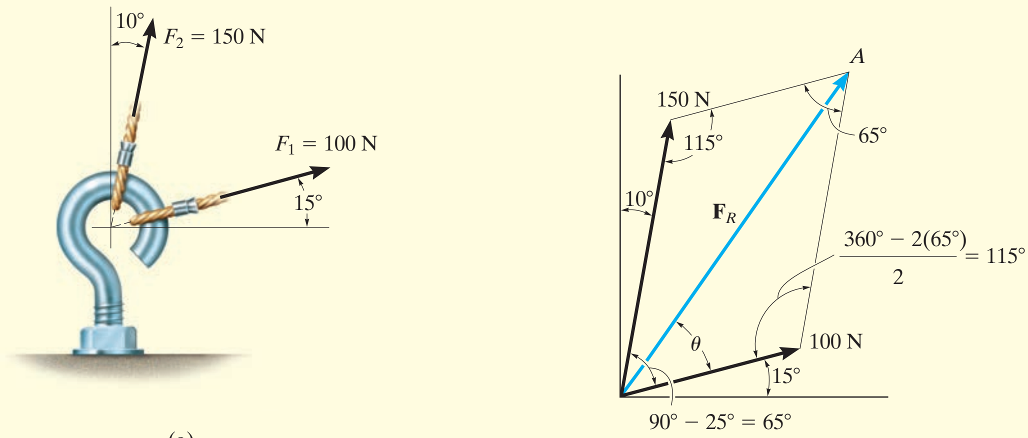
- Two "component" forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 2–10*a* add according to the parallelogram law, yielding a *resultant* force $\mathbf{F}_R$ that forms the diagonal of the parallelogram.
- If a force $\mathbf{F}$ is to be resolved into *components* along two axes u and v, Fig. 2–10*b*, then start at the head of force $\mathbf{F}$ and construct lines parallel to the axes, thereby forming the parallelogram. The sides of the parallelogram represent the components, $\mathbf{F}_u$ and $\mathbf{F}_v$.
- Label all the known and unknown force magnitudes and the angles on the sketch and identify the two unknowns as the magnitude and direction of $\mathbf{F}_R$, or the magnitudes of its components.

Trigonometry.

- Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.
- From this triangle, the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig. 2–10*c*.

EXAMPLE 2.1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a) (b)

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point A, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

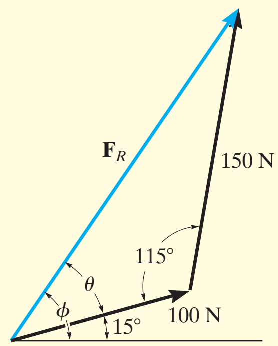
Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$
$$\theta = 39.8^\circ$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ \qquad \textit{Ans.}$$

NOTE: The results seem reasonable, since Fig. 2–11*b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them.



(c)

Fig. 2–11

EXAMPLE 2.2

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.

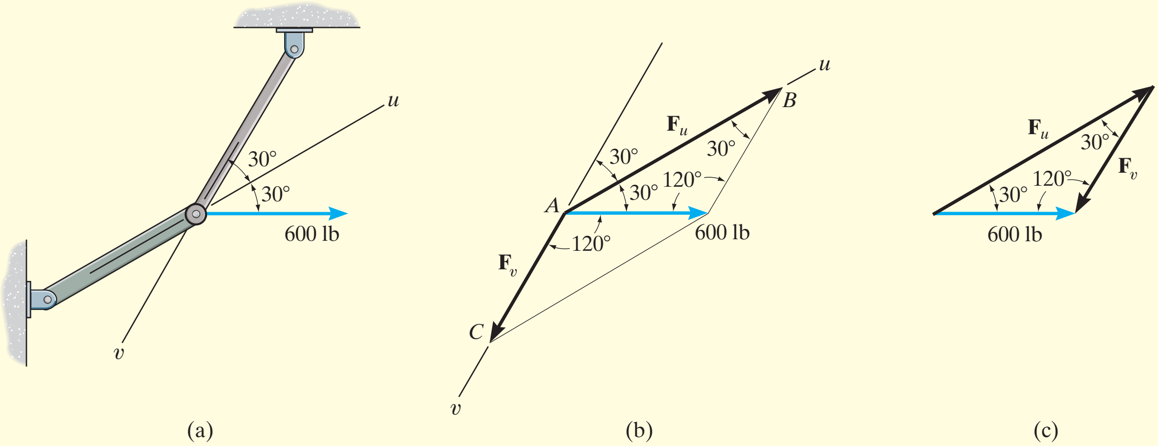


Fig. 2–12

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the *v* axis until it intersects the *u* axis at point *B*, Fig. 2–12*b*. The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the *v* axis at point *C*, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$

NOTE: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

EXAMPLE 2.3

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

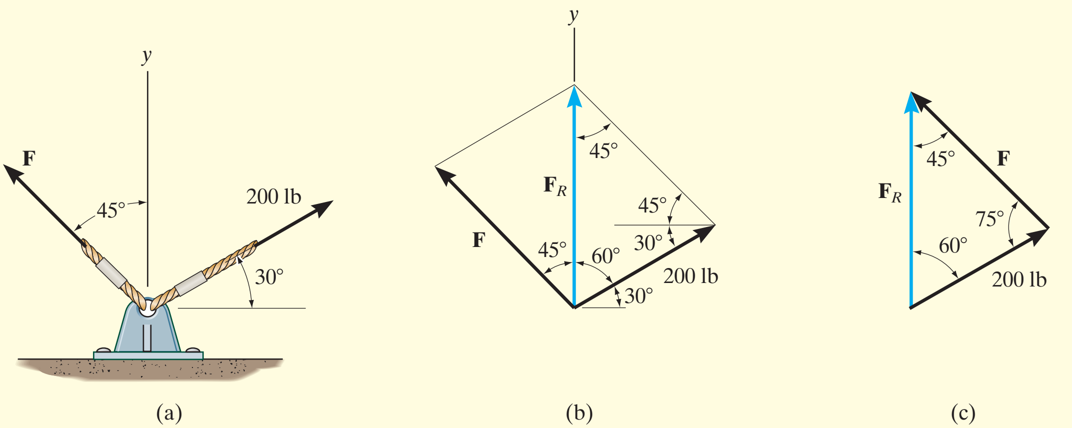


Fig. 2–13

SOLUTION

The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of $\mathbf{F}_R$ and **F** are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200\text{ lb}}{\sin 45^\circ}$$

$$F = 245\text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200\text{ lb}}{\sin 45^\circ}$$

$$F_R = 273\text{ lb} \qquad \textit{Ans.}$$

2

EXAMPLE 2.4

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive x axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

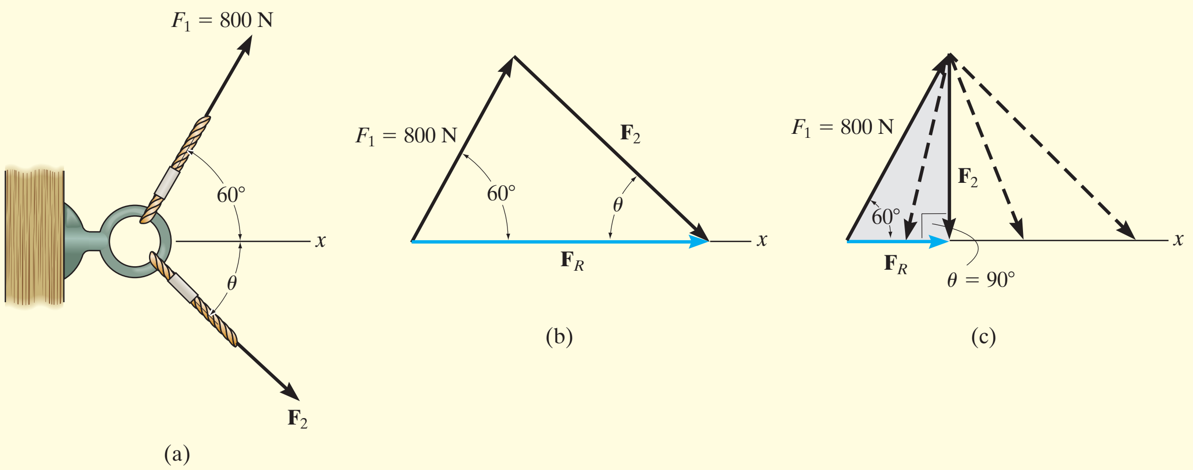


Fig. 2–14

SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

$$\theta = 90° \qquad \textit{Ans.}$$

Since the vector addition now forms the shaded right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800\text{ N})\cos 60° = 400\text{ N} \qquad \textit{Ans.}$$

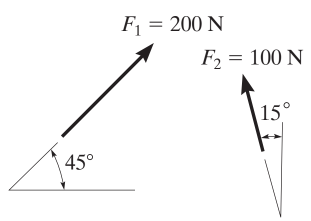
$$F_2 = (800\text{ N})\sin 60° = 693\text{ N} \qquad \textit{Ans.}$$

It is strongly suggested that you test yourself on the solutions to these examples, by covering them over and then trying to draw the parallelogram law, and thinking about how the sine and cosine laws are used to determine the unknowns. Then before solving any of the problems, try to solve the Preliminary Problems and some of the Fundamental Problems given on the next pages. The solutions and answers to these are given in the back of the book. Doing this throughout the book will help immensely in developing your problem-solving skills.

PRELIMINARY PROBLEMS

Partial solutions and answers to all Preliminary Problems are given in the back of the book.

P2–1. In each case, construct the parallelogram law to show $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$. Then establish the triangle rule, where $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$. Label all known and unknown sides and internal angles.



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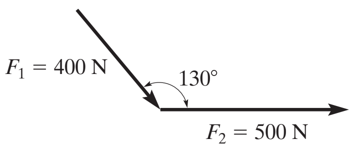

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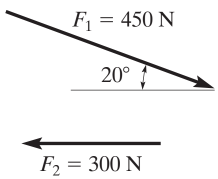
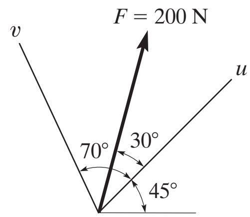


(c)

Prob. P2–1

P2–2. In each case, show how to resolve the force **F** into components acting along the u and v axes using the parallelogram law. Then establish the triangle rule to show $\mathbf{F}_R = \mathbf{F}_u + \mathbf{F}_v$. Label all known and unknown sides and interior angles.



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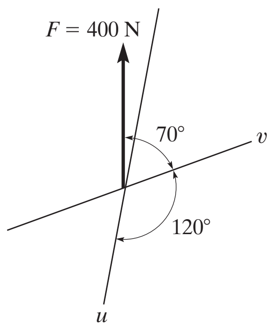

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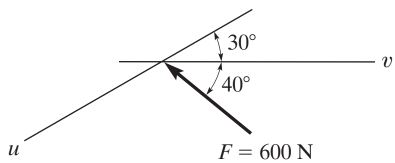

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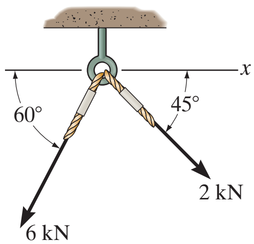
Prob. P2–2

FUNDAMENTAL PROBLEMS

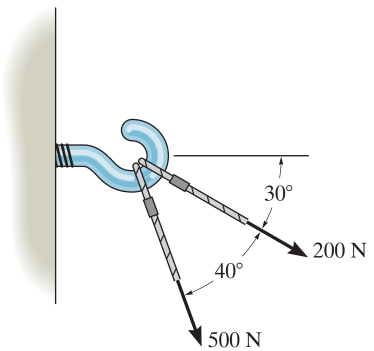
Partial solutions and answers to all Fundamental Problems are given in the back of the book.

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



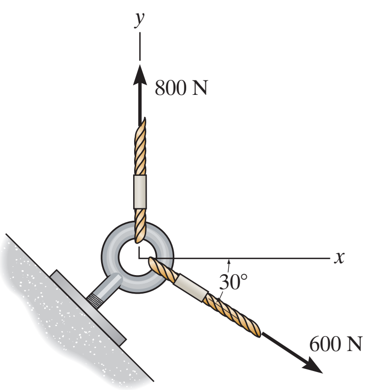
Prob. F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



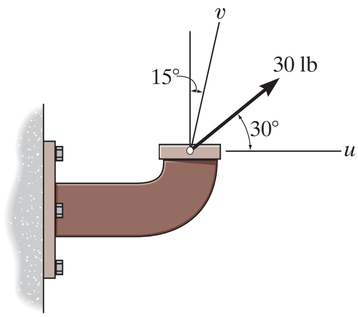
Prob. F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



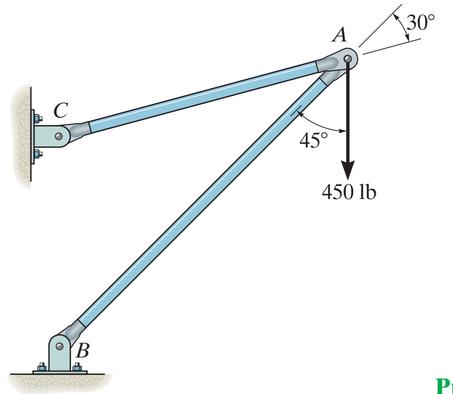
Prob. F2–3

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



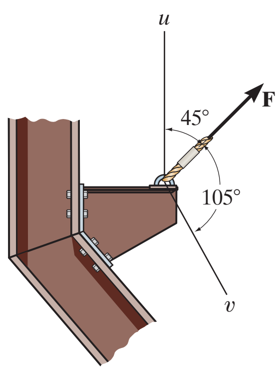
Prob. F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



Prob. F2–5

F2–6. If force $\mathbf{F}$ is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of $\mathbf{F}$ and the magnitude of its component $\mathbf{F}_v$ along the v axis.

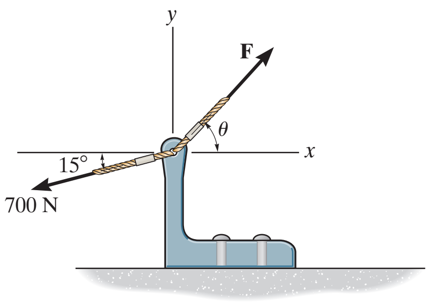


Prob. F2–6

PROBLEMS

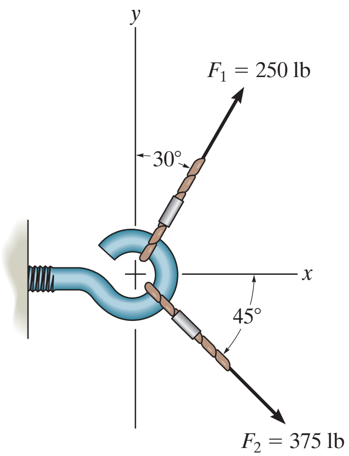
2–1. If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

2–2. If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



Probs. 2–1/2

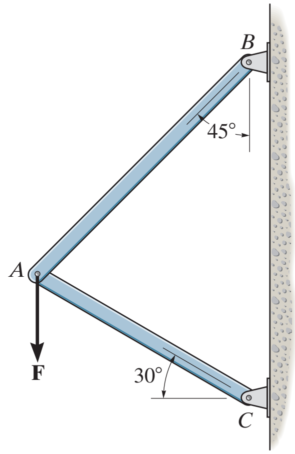
2–3. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



Prob. 2–3

***2–4.** The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.

2–5. Solve Prob. 2–4 with $F = 350$ lb.

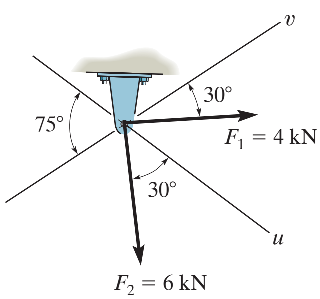


Probs. 2–4/5

2–6. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.

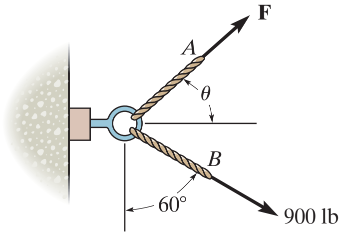
2–7. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

***2–8.** Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



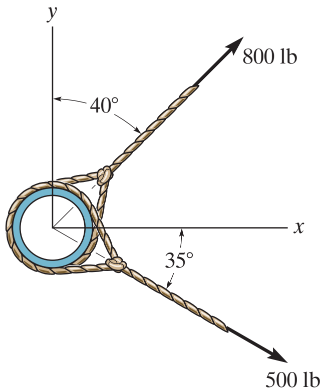
Probs. 2–6/7/8

2–9. If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope A and the corresponding angle θ.



Prob. 2–9

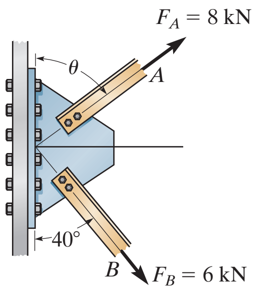
2–10. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Prob. 2–10

2–11. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

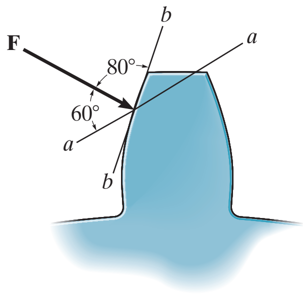
***2–12.** Determine the angle θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



Probs. 2–11/12

2–13. The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines aa and bb.

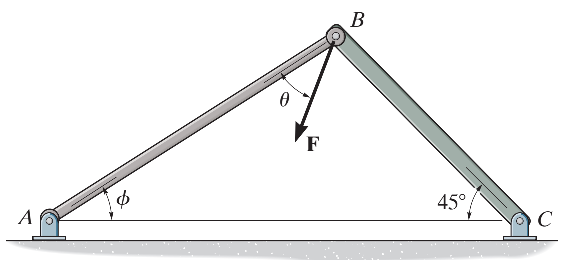
2–14. The component of force **F** acting along line aa is required to be 30 lb. Determine the magnitude of **F** and its component along line bb.



Probs. 2–13/14

2–15. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.

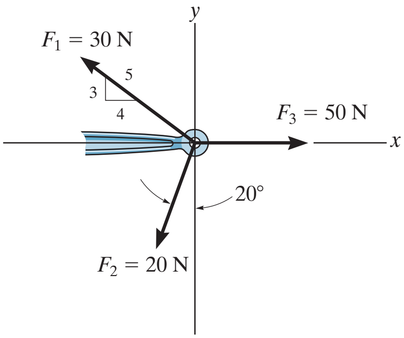
***2–16.** Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ $(0° \le \phi \le 45°)$ and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.



Probs. 2–15/16

2–17. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.

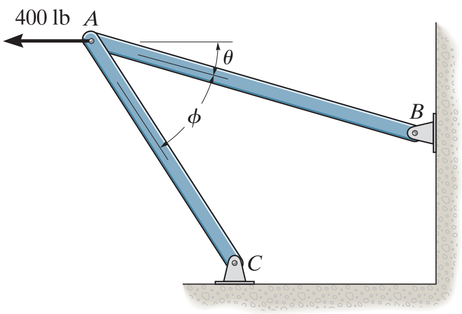
2–18. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



Probs. 2–17/18

2–19. Determine the design angle θ ($0° \le \theta \le 90°$) for strut AB so that the 400-lb horizontal force has a component of 500 lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.

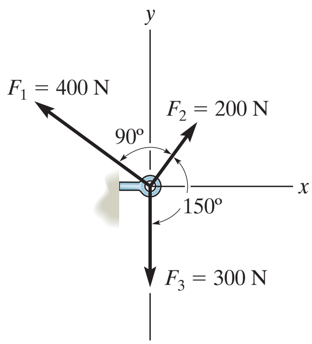
***2–20.** Determine the design angle ϕ ($0° \le \phi \le 90°$) between struts AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30°$.



Probs. 2–19/20

2–21. Determine the magnitude and direction of the resultant force, $\mathbf{F}_R$ measured counterclockwise from the positive x axis. Solve the problem by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.

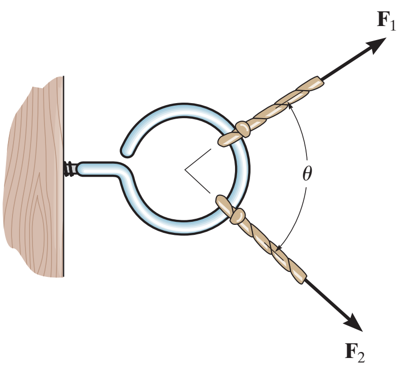
2–22. Determine the magnitude and direction of the resultant force, measured counterclockwise from the positive x axis. Solve l by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



Probs. 2–21/22

2–23. Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle θ ($0° \le \theta \le 180°$) between them, so that the resultant force has a magnitude of $F_R = 800$ N.

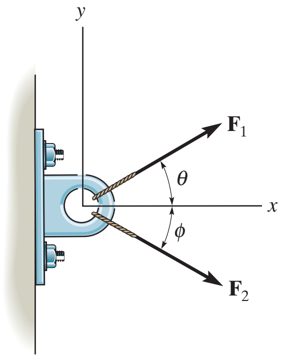
***2–24.** Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



Probs. 2–23/24

2

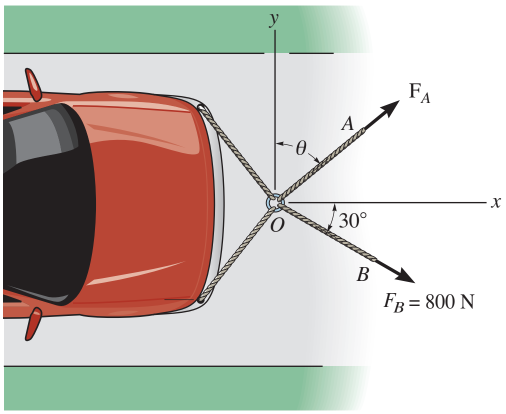
2–25. If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.



Prob. 2–25

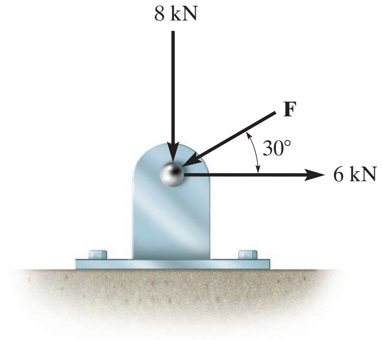
2–26. Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.

2–27. Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.



Probs. 2–26/27

***2–28.** Determine the magnitude of force $\mathbf{F}$ so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?

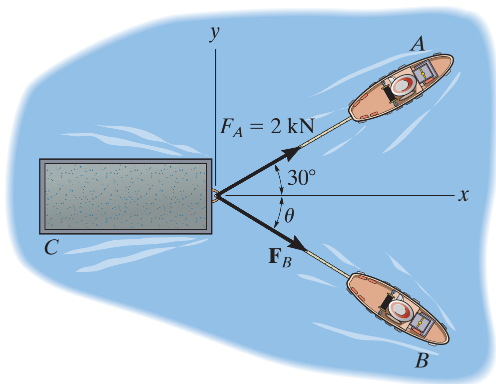


Prob. 2–28

2–29. If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.

2–30. If $\mathbf{F}_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise form the positive x axis.

2–31. If the resultant force of the two tugboats is required to be directed towards the positive x axis, and $\mathbf{F}_B$ is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



Probs. 2–29/30/31

2.4 Addition of a System of Coplanar Forces

When a force is resolved into two components along the x and y axes, the components are then called ***rectangular components***. For analytical work we can represent these components in one of two ways, using either scalar or Cartesian vector notation.

Scalar Notation. The rectangular components of force $\mathbf{F}$ shown in Fig. 2–15*a* are found using the parallelogram law, so that $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Because these components form a right triangle, they can be determined from

$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta$$

Instead of using the angle θ, however, the direction of $\mathbf{F}$ can also be defined using a small "slope" triangle, as in the example shown in Fig. 2–15*b*. Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives

$$\frac{F_x}{F} = \frac{a}{c}$$

or

$$F_x = F\left(\frac{a}{c}\right)$$

and

$$\frac{F_y}{F} = \frac{b}{c}$$

or

$$F_y = -F\left(\frac{b}{c}\right)$$

Here the y component is a *negative scalar* since $\mathbf{F}_y$ is directed along the negative y axis.

It is important to keep in mind that this positive and negative scalar notation is to be used only for computational purposes, not for graphical representations in figures. Throughout the book, the *head of a vector arrow* in *any figure* indicates the sense of the vector *graphically*; algebraic signs are not used for this purpose. Thus, the vectors in Figs. 2–15*a* and 2–15*b* are designated by using boldface (vector) notation.* Whenever italic symbols are written near vector arrows in figures, they indicate the *magnitude* of the vector, which is *always* a *positive* quantity.

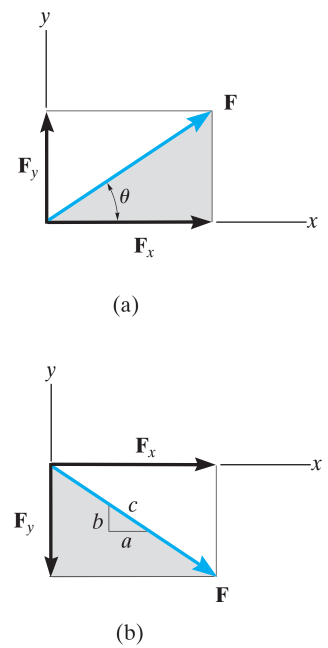


Fig. 2–15

*Negative signs are used only in figures with boldface notation when showing equal but opposite pairs of vectors, as in Fig. 2–2.

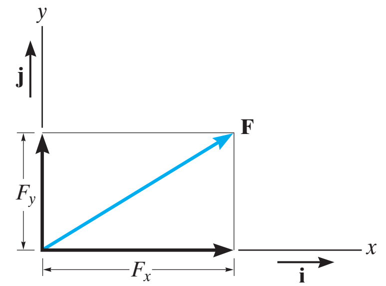


Fig. 2–16

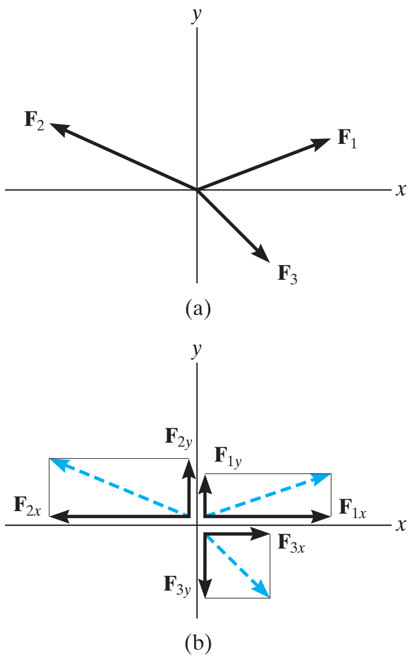
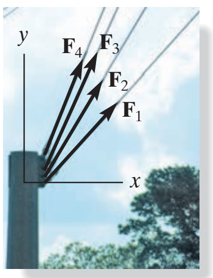


Fig. 2–17



The resultant force of the four cable forces acting on the post can be determined by adding algebraically the separate x and y components of each cable force. This resultant $\mathbf{F}_R$ produces the *same pulling effect* on the post as all four cables. (© Russell C. Hibbeler)

Cartesian Vector Notation. It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$. They are called unit vectors because they have a dimensionless magnitude of 1, and so they can be used to designate the *directions* of the x and y axes, respectively, Fig. 2–16.*

Since the *magnitude* of each component of $\mathbf{F}$ is *always a positive quantity*, which is represented by the (positive) scalars F_x and F_y, then we can express $\mathbf{F}$ as a ***Cartesian vector***,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

Coplanar Force Resultants. We can use either of the two methods just described to determine the resultant of several ***coplanar forces***, i.e., forces that all lie in the same plane. To do this, each force is first resolved into its x and y components, and then the respective components are added using *scalar algebra* since they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law. For example, consider the three concurrent forces in Fig. 2–17*a*, which have x and y components shown in Fig. 2–17*b*. Using Cartesian vector notation, each force is first represented as a Cartesian vector, i.e.,

$$\begin{aligned}
\mathbf{F}_1 &= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} \\
\mathbf{F}_2 &= -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} \\
\mathbf{F}_3 &= F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}
\end{aligned}$$

The vector resultant is therefore

$$\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\
&= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j} \\
&= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\
&= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}
\end{aligned}$$

If *scalar notation* is used, then indicating the positive directions of components along the x and y axes with symbolic arrows, we have

$$\begin{aligned}
&\overset{+}{\rightarrow} \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x} \\
&+\uparrow \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}
\end{aligned}$$

These are the *same* results as the $\mathbf{i}$ and $\mathbf{j}$ components of $\mathbf{F}_R$ determined above.

*For handwritten work, unit vectors are usually indicated using a circumflex, e.g., $\hat{\imath}$ and $\hat{\jmath}$. Also, realize that F_x and F_y in Fig. 2–16 represent the *magnitudes* of the components, which are *always positive scalars*. The directions are defined by $\mathbf{i}$ and $\mathbf{j}$. If instead we used scalar notation, then F_x and F_y could be positive or negative scalars, since they would account for *both* the magnitude and direction of the components.

We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e.,

$$\boxed{\begin{aligned}(F_R)_x &= \Sigma F_x \\ (F_R)_y &= \Sigma F_y\end{aligned}} \qquad (2\text{–}1)$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition, as shown in Fig. 2–17c. From this sketch, the magnitude of $\mathbf{F}_R$ is then found from the Pythagorean theorem; that is,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

The above concepts are illustrated numerically in the examples which follow.

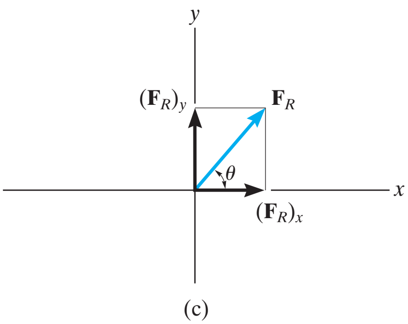


Fig. 2–17 (cont.)

Important Points

- The resultant of several coplanar forces can easily be determined if an x, y coordinate system is established and the forces are resolved along the axes.
- The direction of each force is specified by the angle its line of action makes with one of the axes, or by a slope triangle.
- The orientation of the x and y axes is arbitrary, and their positive direction can be specified by the Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$.
- The x and y components of the *resultant force* are simply the algebraic addition of the components of all the coplanar forces.
- The magnitude of the resultant force is determined from the Pythagorean theorem, and when the resultant components are sketched on the x and y axes, Fig. 2–17c, the direction θ can be determined from trigonometry.

EXAMPLE 2.5

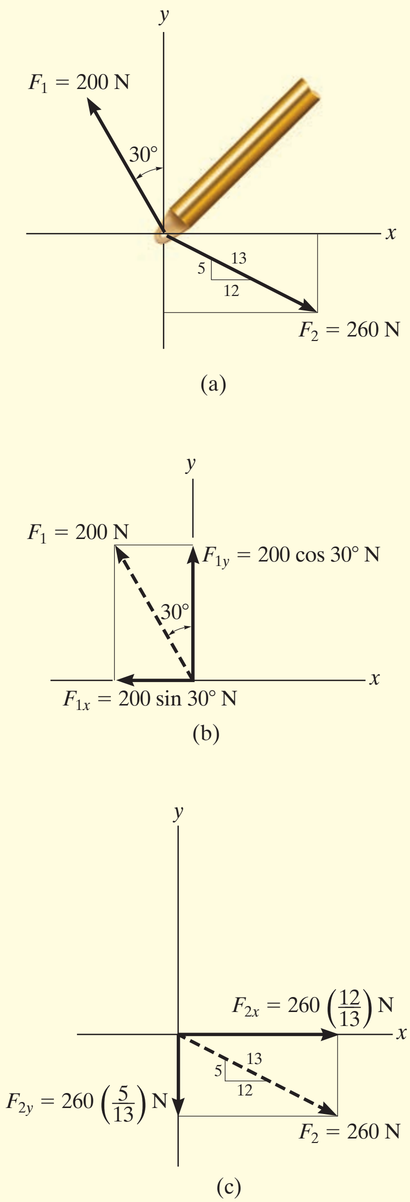


Fig. 2–18

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig. 2–18a. Express each force as a Cartesian vector.

SOLUTION

Scalar Notation. By the parallelogram law, $\mathbf{F}_1$ is resolved into x and y components, Fig. 2–18b. Since $\mathbf{F}_{1x}$ acts in the $-x$ direction, and $\mathbf{F}_{1y}$ acts in the $+y$ direction, we have

$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow \qquad \textit{Ans.}$$

$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow \qquad \textit{Ans.}$$

The force $\mathbf{F}_2$ is resolved into its x and y components, as shown in Fig. 2–18c. Here the *slope* of the line of action for the force is indicated. From this "slope triangle" we could obtain the angle θ, e.g., $\theta = \tan^{-1}\left(\frac{5}{12}\right)$, and then proceed to determine the magnitudes of the components in the same manner as for $\mathbf{F}_1$. The easier method, however, consists of using proportional parts of similar triangles, i.e.,

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

Similarly,

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$

Notice how the magnitude of the *horizontal component*, $\mathbf{F}_{2x}$, was obtained by multiplying the force magnitude by the ratio of the *horizontal leg* of the slope triangle divided by the hypotenuse; whereas the magnitude of the *vertical component*, F_{2y}, was obtained by multiplying the force magnitude by the ratio of the *vertical leg* divided by the hypotenuse. Hence, using scalar notation to represent these components, we have

$$F_{2x} = 240 \text{ N} = 240 \text{ N} \rightarrow \qquad \textit{Ans.}$$

$$F_{2y} = -100 \text{ N} = 100 \text{ N} \downarrow \qquad \textit{Ans.}$$

Cartesian Vector Notation. Having determined the magnitudes and directions of the components of each force, we can express each force as a Cartesian vector.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$

EXAMPLE 2.6

The link in Fig. 2–19*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.

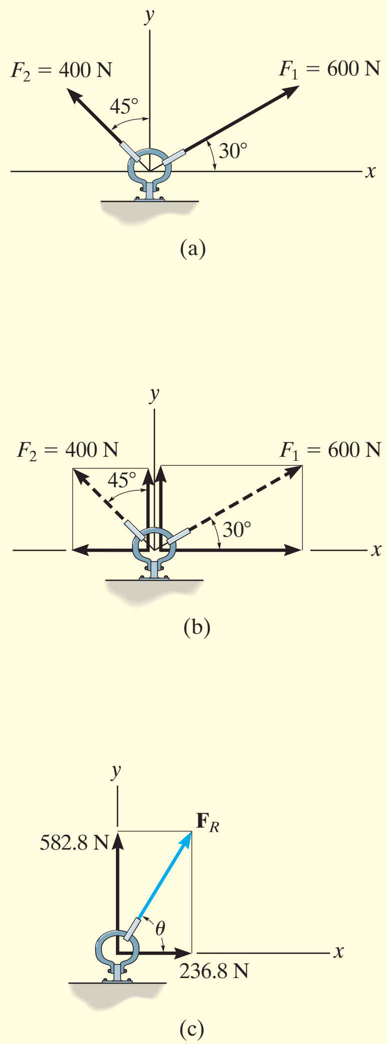


Fig. 2–19

SOLUTION I

Scalar Notation. First we resolve each force into its x and y components, Fig. 2–19*b*, then we sum these components algebraically.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N}$$
$$= 236.8 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N}$$
$$= 582.8 \text{ N}\uparrow$$

The resultant force, shown in Fig. 2–19*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2}$$
$$= 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition,

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9^\circ \qquad \textit{Ans.}$$

SOLUTION II

Cartesian Vector Notation. From Fig. 2–19*b*, each force is first expressed as a Cartesian vector.

$$\mathbf{F}_1 = \{600 \cos 30^\circ \mathbf{i} + 600 \sin 30^\circ \mathbf{j}\} \text{ N}$$
$$\mathbf{F}_2 = \{-400 \sin 45^\circ \mathbf{i} + 400 \cos 45^\circ \mathbf{j}\} \text{ N}$$

Then,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N})\mathbf{i}$$
$$+ (600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N})\mathbf{j}$$
$$= \{236.8\mathbf{i} + 582.8\mathbf{j}\} \text{ N}$$

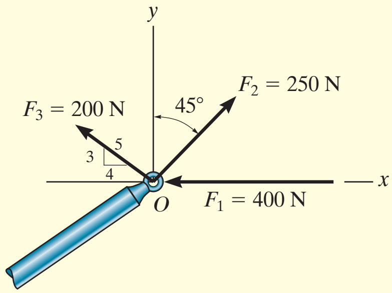
The magnitude and direction of $\mathbf{F}_R$ are determined in the same manner as before.

NOTE: Comparing the two methods of solution, notice that the use of scalar notation is more efficient since the components can be found *directly*, without first having to express each force as a Cartesian vector before adding the components. Later, however, we will show that Cartesian vector analysis is very beneficial for solving three-dimensional problems.

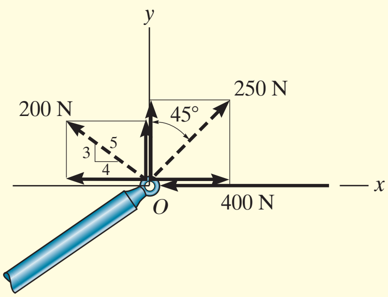
2

EXAMPLE 2.7

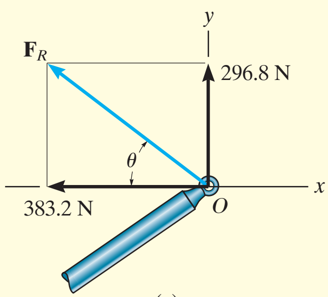
The end of the boom O in Fig. 2–20*a* is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



(a)



(b)



(c)

Fig. 2–20

SOLUTION

Each force is resolved into its x and y components, Fig. 2–20*b*. Summing the x components, we have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400 \text{ N} + 250 \sin 45° \text{ N} - 200\left(\tfrac{4}{5}\right) \text{ N}$$
$$= -383.2 \text{ N} = 383.2 \text{ N} \leftarrow$$

The negative sign indicates that F_{Rx} acts to the left, i.e., in the negative x direction, as noted by the small arrow. Obviously, this occurs because F_1 and F_3 in Fig. 2–20*b* contribute a greater pull to the left than F_2 which pulls to the right. Summing the y components yields

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250 \cos 45° \text{ N} + 200\left(\tfrac{3}{5}\right) \text{ N}$$
$$= 296.8 \text{ N}\uparrow$$

The resultant force, shown in Fig. 2–20*c*, has a *magnitude* of

$$F_R = \sqrt{(-383.2 \text{ N})^2 + (296.8 \text{ N})^2}$$
$$= 485 \text{ N} \qquad \textit{Ans.}$$

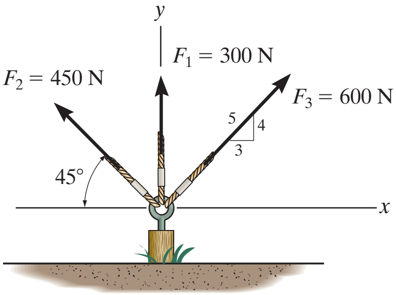
From the vector addition in Fig. 2–20*c*, the direction angle θ is

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8° \qquad \textit{Ans.}$$

NOTE: Application of this method is more convenient, compared to using two applications of the parallelogram law, first to add $\mathbf{F}_1$ and $\mathbf{F}_2$ then adding $\mathbf{F}_3$ to this resultant.

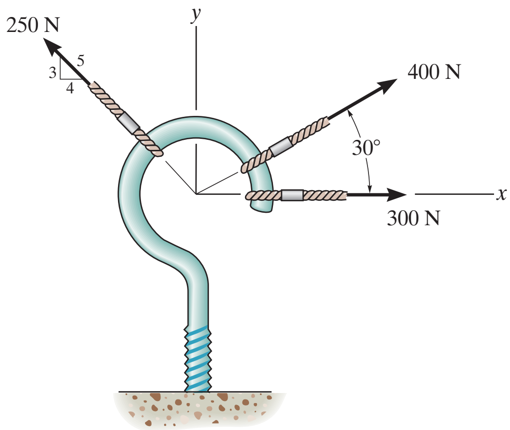
FUNDAMENTAL PROBLEMS

F2–7. Resolve each force acting on the post into its x and y components.



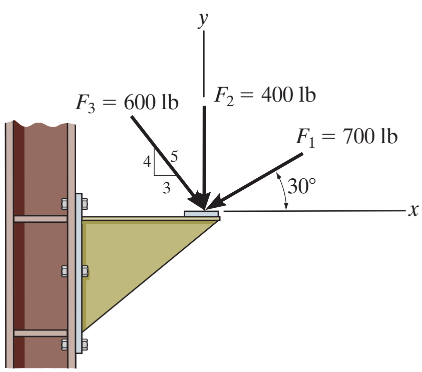
Prob. F2–7

F2–8. Determine the magnitude and direction of the resultant force.



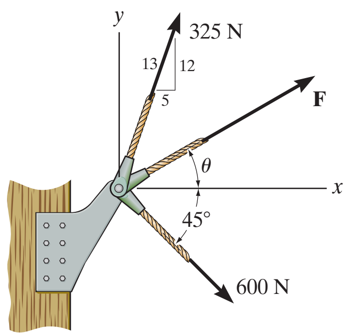
Prob. F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



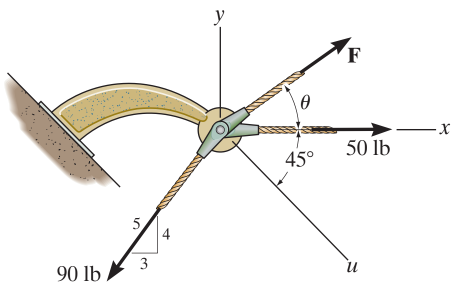
Prob. F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of **F** and its direction θ.



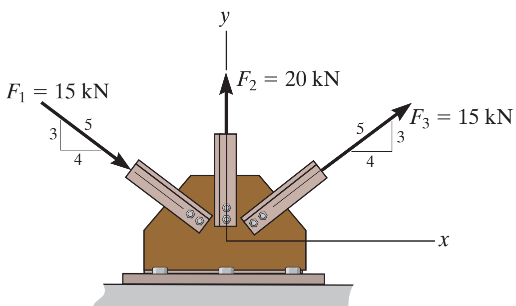
Prob. F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of **F** and its direction θ.



Prob. F2–11

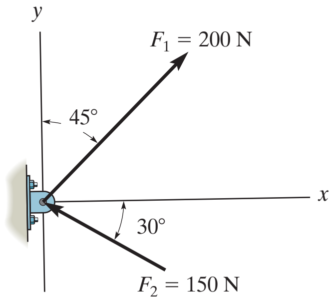
F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



Prob. F2–12

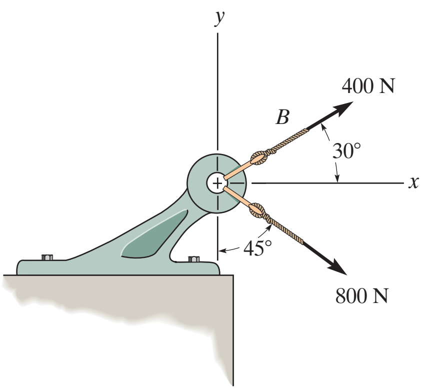
PROBLEMS

*2–32. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Prob. 2–32

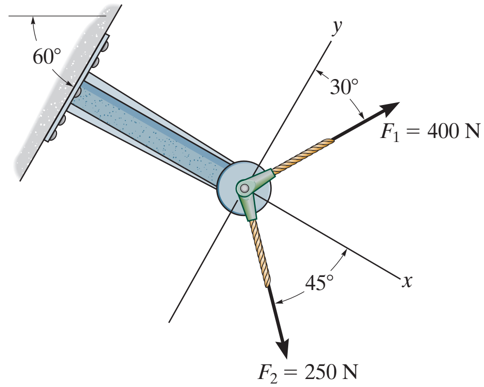
2–33. Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



Prob. 2–33

2–34. Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.

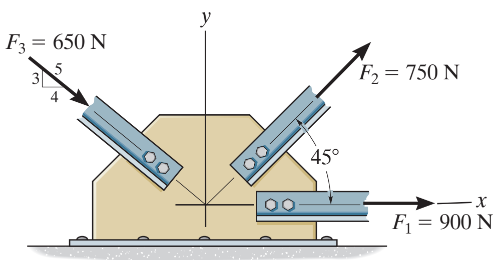
2–35. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Probs. 2–34/35

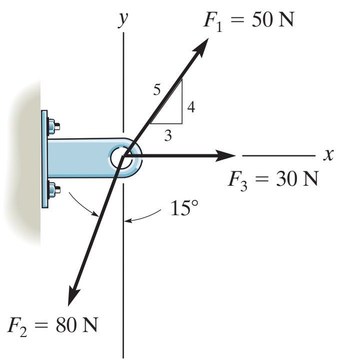
*2–36. Resolve each force acting on the *gusset plate* into its x and y components, and express each force as a Cartesian vector.

2–37. Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



Probs. 2–36/37

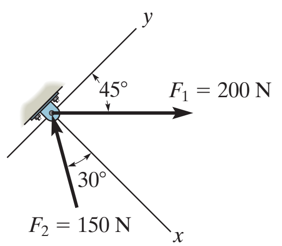
2–38. Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



Prob. 2–38

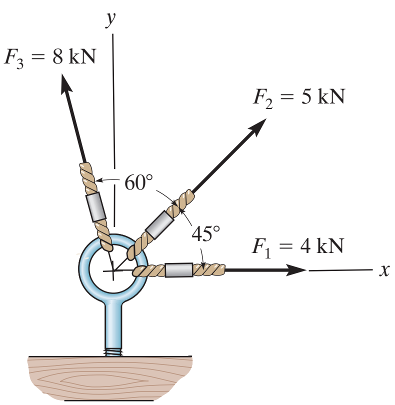
2–39. Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.

***2–40.** Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Probs. 2–39/40

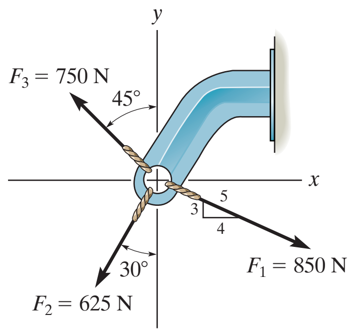
2–41. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Prob. 2–41

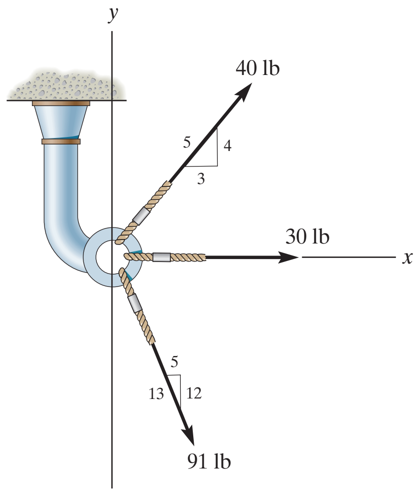
2–42. Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.

2–43. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



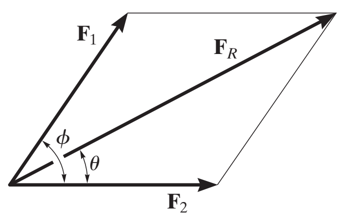
Probs. 2–42/43

***2–44.** Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



Prob. 2–44

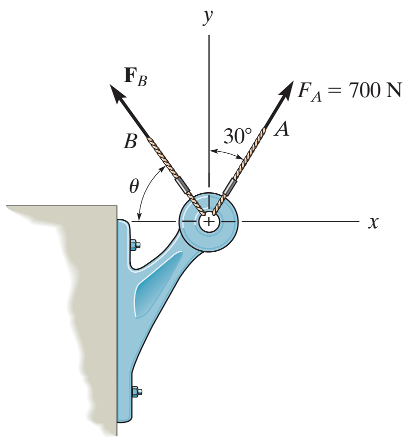
2–45. Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.



Prob. 2–45

2–46. Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.

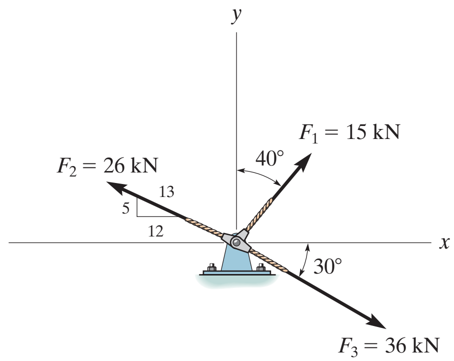
2–47. Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



Probs. 2–46/47

2–50. Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.

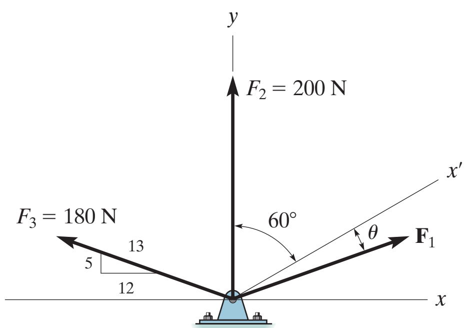
2–51. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Probs. 2–50/51

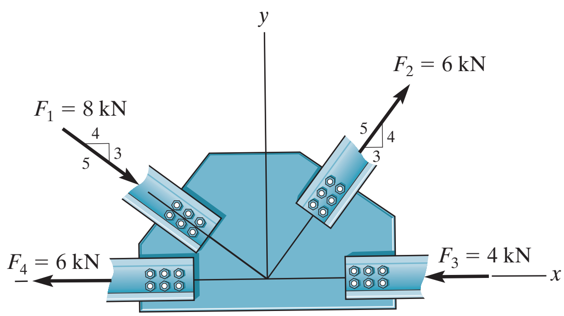
***2–48.** Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 800 N.

2–49. If $F_1 = 300$ N and $\theta = 10°$, determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force acting on the bracket.



Probs. 2–48/49

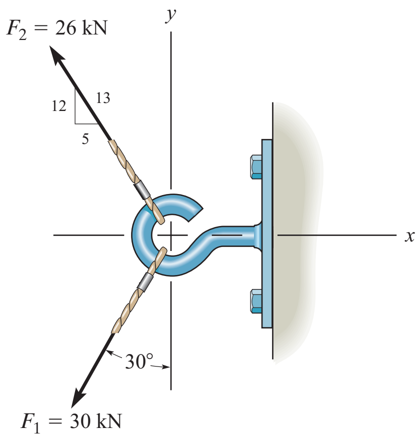
***2–52.** Determine the x and y components of each force acting on the *gusset plate* of a bridge truss. Show that the resultant force is zero.



Prob. 2–52

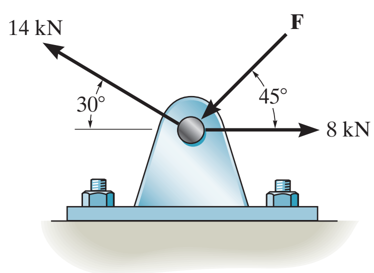
2–53. Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.

2–54. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Probs. 2–53/54

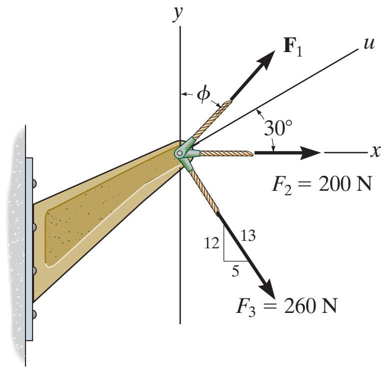
2–55. Determine the magnitude of force $\mathbf{F}$ so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



Prob. 2–55

***2–56.** If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.

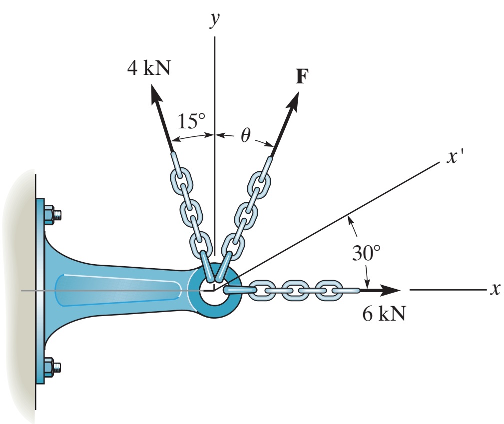
2–57. If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.



Probs. 2–56/57

2–58. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}$ so that the resultant force is directed along the positive x' axis and has a magnitude of 8 kN.

2–59. If $F = 5$ kN and $\theta = 30°$, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Probs. 2–58/59

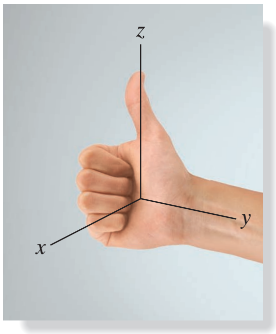


Fig. 2–21 (© Russell C. Hibbeler)

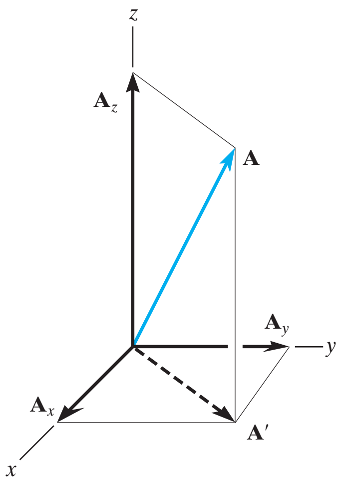


Fig. 2–22

2.5 Cartesian Vectors

The operations of vector algebra, when applied to solving problems in *three dimensions*, are greatly simplified if the vectors are first represented in Cartesian vector form. In this section we will present a general method for doing this; then in the next section we will use this method for finding the resultant force of a system of concurrent forces.

Right-Handed Coordinate System. We will use a right-handed coordinate system to develop the theory of vector algebra that follows. A rectangular coordinate system is said to be ***right-handed*** if the thumb of the right hand points in the direction of the positive z axis when the right-hand fingers are curled about this axis and directed from the positive x towards the positive y axis, Fig. 2–21.

Rectangular Components of a Vector. A vector $\mathbf{A}$ may have one, two, or three rectangular components along the x, y, z coordinate axes, depending on how the vector is oriented relative to the axes. In general, though, when $\mathbf{A}$ is directed within an octant of the x, y, z frame, Fig. 2–22, then by two successive applications of the parallelogram law, we may resolve the vector into components as $\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$ and then $\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$. Combining these equations, to eliminate $\mathbf{A}'$, $\mathbf{A}$ is represented by the vector sum of its *three* rectangular components,

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z \qquad (2\text{–}2)$$

Cartesian Unit Vectors. In three dimensions, the set of Cartesian unit vectors, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, is used to designate the directions of the x, y, z axes, respectively. As stated in Sec. 2–4, the *sense* (or arrowhead) of these vectors will be represented analytically by a plus or minus sign, depending on whether they are directed along the positive or negative $x, y,$ or z axes. The positive Cartesian unit vectors are shown in Fig. 2–23.

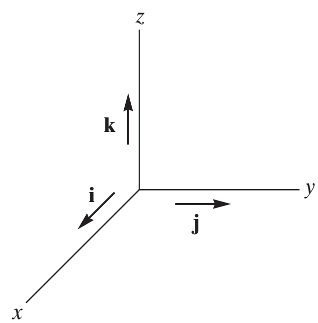


Fig. 2–23

Cartesian Vector Representation. Since the three components of $\mathbf{A}$ in Eq. 2–2 act in the positive $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ directions, Fig. 2–24, we can write $\mathbf{A}$ in Cartesian vector form as

$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}} \tag{2–3}$$

There is a distinct advantage to writing vectors in this manner. Separating the *magnitude* and *direction* of each *component vector* will simplify the operations of vector algebra, particularly in three dimensions.

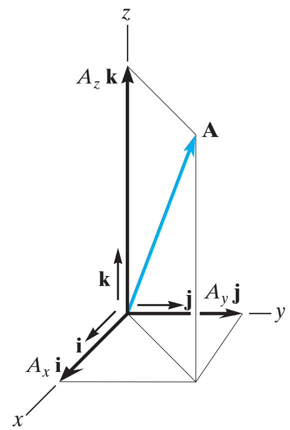


Fig. 2–24

Magnitude of a Cartesian Vector. It is always possible to obtain the magnitude of $\mathbf{A}$ provided it is expressed in Cartesian vector form. As shown in Fig. 2–25, from the blue right triangle, $A = \sqrt{A'^2 + A_z^2}$, and from the gray right triangle, $A' = \sqrt{A_x^2 + A_y^2}$. Combining these equations to eliminate A' yields

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{2–4}$$

Hence, the magnitude of $\mathbf{A}$ *is equal to the positive square root of the sum of the squares of its components.*

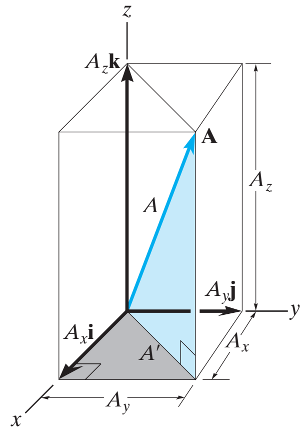


Fig. 2–25

Coordinate Direction Angles. We will define the *direction* of $\mathbf{A}$ by the ***coordinate direction angles*** α (alpha), β (beta), and γ (gamma), measured between the *tail* of $\mathbf{A}$ and the *positive x, y, z* axes provided they are located at the tail of $\mathbf{A}$, Fig. 2–26. Note that regardless of where $\mathbf{A}$ is directed, each of these angles will be between 0° and 180°.

To determine α, β, and γ, consider the projection of $\mathbf{A}$ onto the x, y, z axes, Fig. 2–27. Referring to the colored right triangles shown in the figure, we have

$$\boxed{\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}} \tag{2–5}$$

These numbers are known as the ***direction cosines*** of $\mathbf{A}$. Once they have been obtained, the coordinate direction angles α, β, γ can then be determined from the inverse cosines.

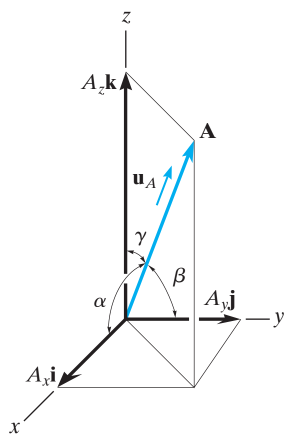


Fig. 2–26

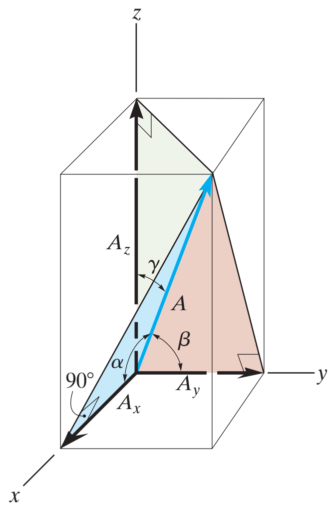


Fig. 2–27

An easy way of obtaining these direction cosines is to form a unit vector $\mathbf{u}_A$ in the direction of A, Fig. 2–26. If $\mathbf{A}$ is expressed in Cartesian vector form, $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$, then $\mathbf{u}_A$ will have a magnitude of one and be dimensionless provided $\mathbf{A}$ is divided by its magnitude, i.e.,

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}} \tag{2–6}$$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. By comparison with Eqs. 2–5, it is seen that *the* $\mathbf{i}, \mathbf{j}, \mathbf{k}$ *components* of $\mathbf{u}_A$ *represent the direction cosines of* $\mathbf{A}$, i.e.,

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k} \tag{2–7}$$

Since the magnitude of a vector is equal to the positive square root of the sum of the squares of the magnitudes of its components, and $\mathbf{u}_A$ has a magnitude of one, then from the above equation an important relation among the direction cosines can be formulated as

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1} \tag{2–8}$$

Here we can see that if only *two* of the coordinate angles are known, the third angle can be found using this equation.

Finally, if the magnitude and coordinate direction angles of $\mathbf{A}$ are known, then $\mathbf{A}$ may be expressed in Cartesian vector form as

$$\boxed{\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}} \tag{2–9}$$

Transverse and Azmuth Angles.

Sometimes, the direction of $\mathbf{A}$ can be specified using two angles, namely, a ***transverse angle*** θ and an ***azmuth angle*** ϕ (phi), such as shown in Fig. 2–28. The components of $\mathbf{A}$ can then be determined by applying trigonometry first to the light blue right triangle, which yields

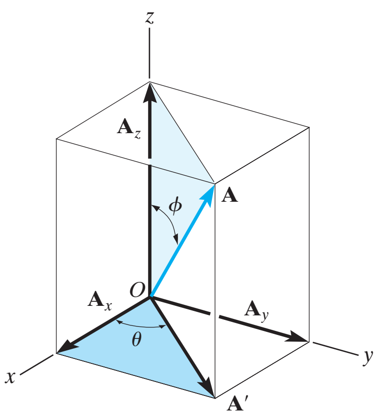


Fig. 2–28

$$A_z = A\cos\phi$$

and

$$A' = A\sin\phi$$

Now applying trigonometry to the dark blue right triangle,

$$A_x = A'\cos\theta = A\sin\phi\cos\theta$$

$$A_y = A'\sin\theta = A\sin\phi\sin\theta$$

Therefore **A** written in Cartesian vector form becomes

$$\mathbf{A} = A \sin \phi \cos \theta\, \mathbf{i} + A \sin \phi \sin \theta\, \mathbf{j} + A \cos \phi\, \mathbf{k}$$

You should not memorize this equation, rather it is important to understand how the components were determined using trigonometry.

2.6 Addition of Cartesian Vectors

The addition (or subtraction) of two or more vectors is greatly simplified if the vectors are expressed in terms of their Cartesian components. For example, if $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ and $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$, Fig. 2–29, then the resultant vector, **R**, has components which are the scalar sums of the **i**, **j**, **k** components of **A** and **B**, i.e.,

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$$

If this is generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as

$$\boxed{\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}} \qquad (2\text{–}10)$$

Here ΣF_x, ΣF_y, and ΣF_z represent the algebraic sums of the respective x, y, z or **i**, **j**, **k** components of each force in the system.

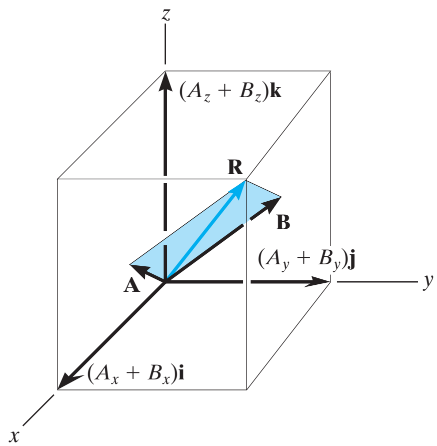


Fig. 2–29

Cartesian vector analysis provides a convenient method for finding both the resultant force and its components in three dimensions. (© Russell C. Hibbeler)

Important Points

- A Cartesian vector **A** has **i**, **j**, **k** components along the x, y, z axes. If **A** is known, its magnitude is defined by $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.
- The direction of a Cartesian vector can be defined by the three angles α, β, γ, measured from the *positive* x, y, z axes to the *tail* of the vector. To find these angles formulate a unit vector in the direction of **A**, i.e., $\mathbf{u}_A = \mathbf{A}/A$, and determine the inverse cosines of its components. Only two of these angles are independent of one another; the third angle is found from $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.
- The direction of a Cartesian vector can also be specified using a transverse angle θ and azimuth angle ϕ.

EXAMPLE 2.8

Express the force **F** shown in Fig. 2–30*a* as a Cartesian vector.

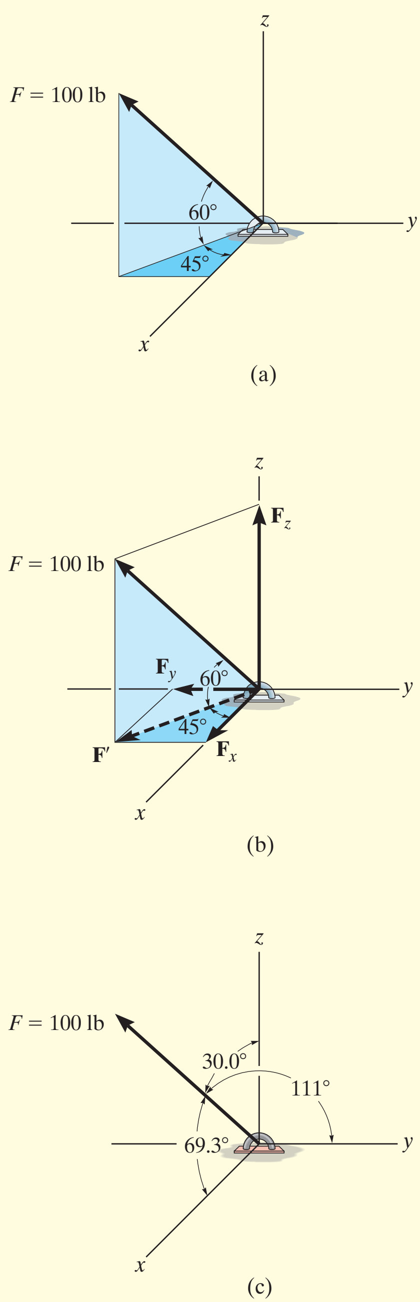


Fig. 2–30

SOLUTION

The angles of 60° and 45° defining the direction of **F** are *not* coordinate direction angles. Two successive applications of the parallelogram law are needed to resolve **F** into its *x, y, z* components. First $\mathbf{F} = \mathbf{F}' + \mathbf{F}_z$, then $\mathbf{F}' = \mathbf{F}_x + \mathbf{F}_y$, Fig. 2–30*b*. By trigonometry, the magnitudes of the components are

$$F_z = 100 \sin 60° \text{ lb} = 86.6 \text{ lb}$$

$$F' = 100 \cos 60° \text{ lb} = 50 \text{ lb}$$

$$F_x = F' \cos 45° = 50 \cos 45° \text{ lb} = 35.4 \text{ lb}$$

$$F_y = F' \sin 45° = 50 \sin 45° \text{ lb} = 35.4 \text{ lb}$$

Realizing that $\mathbf{F}_y$ has a direction defined by $-\mathbf{j}$, we have

$$\mathbf{F} = \{35.4\mathbf{i} - 35.4\mathbf{j} + 86.6\mathbf{k}\} \text{ lb} \qquad \textit{Ans.}$$

To show that the magnitude of this vector is indeed 100 lb, apply Eq. 2–4,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2} = 100 \text{ lb}$$

If needed, the coordinate direction angles of **F** can be determined from the components of the unit vector acting in the direction of **F**. Hence,

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$= \frac{35.4}{100}\mathbf{i} - \frac{35.4}{100}\mathbf{j} + \frac{86.6}{100}\mathbf{k}$$

$$= 0.354\mathbf{i} - 0.354\mathbf{j} + 0.866\mathbf{k}$$

so that

$$\alpha = \cos^{-1}(0.354) = 69.3°$$

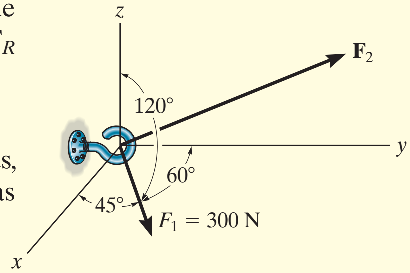
$$\beta = \cos^{-1}(-0.354) = 111°$$

$$\gamma = \cos^{-1}(0.866) = 30.0°$$

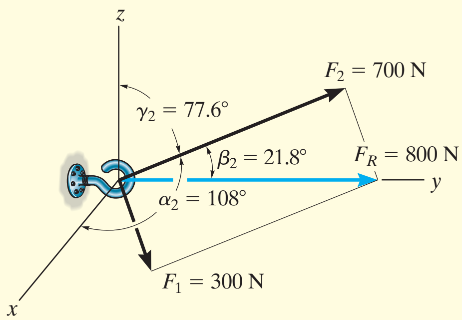
These results are shown in Fig. 2–30*c*.

EXAMPLE 2.9

Two forces act on the hook shown in Fig. 2–31*a*. Specify the magnitude of $\mathbf{F}_2$ and its coordinate direction angles so that the resultant force $\mathbf{F}_R$ acts along the positive y axis and has a magnitude of 800 N.



(a)



(b)

Fig. 2–31

SOLUTION

To solve this problem, the resultant force $\mathbf{F}_R$ and its two components, $\mathbf{F}_1$ and $\mathbf{F}_2$, will each be expressed in Cartesian vector form. Then, as shown in Fig. 2–31*b*, it is necessary that $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$.

Applying Eq. 2–9,

$$\begin{aligned}\mathbf{F}_1 &= F_1 \cos \alpha_1 \mathbf{i} + F_1 \cos \beta_1 \mathbf{j} + F_1 \cos \gamma_1 \mathbf{k} \\ &= 300 \cos 45° \,\mathbf{i} + 300 \cos 60° \,\mathbf{j} + 300 \cos 120° \,\mathbf{k} \\ &= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\} \text{ N} \\ \mathbf{F}_2 &= F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\end{aligned}$$

Since $\mathbf{F}_R$ has a magnitude of 800 N and acts in the $+\mathbf{j}$ direction,

$$\mathbf{F}_R = (800 \text{ N})(+\mathbf{j}) = \{800\mathbf{j}\} \text{ N}$$

We require

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ 800\mathbf{j} &= 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k} \\ 800\mathbf{j} &= (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}\end{aligned}$$

To satisfy this equation the $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components of $\mathbf{F}_R$ must be equal to the corresponding $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components of $(\mathbf{F}_1 + \mathbf{F}_2)$. Hence,

$$\begin{aligned}0 &= 212.1 + F_{2x} & F_{2x} &= -212.1 \text{ N} \\ 800 &= 150 + F_{2y} & F_{2y} &= 650 \text{ N} \\ 0 &= -150 + F_{2z} & F_{2z} &= 150 \text{ N}\end{aligned}$$

The magnitude of $\mathbf{F}_2$ is thus

$$\begin{aligned}F_2 &= \sqrt{(-212.1 \text{ N})^2 + (650 \text{ N})^2 + (150 \text{ N})^2} \\ &= 700 \text{ N}\end{aligned}$$ *Ans.*

We can use Eq. 2–9 to determine $\alpha_2, \beta_2, \gamma_2$.

$$\cos \alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108°$$ *Ans.*

$$\cos \beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8°$$ *Ans.*

$$\cos \gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6°$$ *Ans.*

These results are shown in Fig. 2–31*b*.

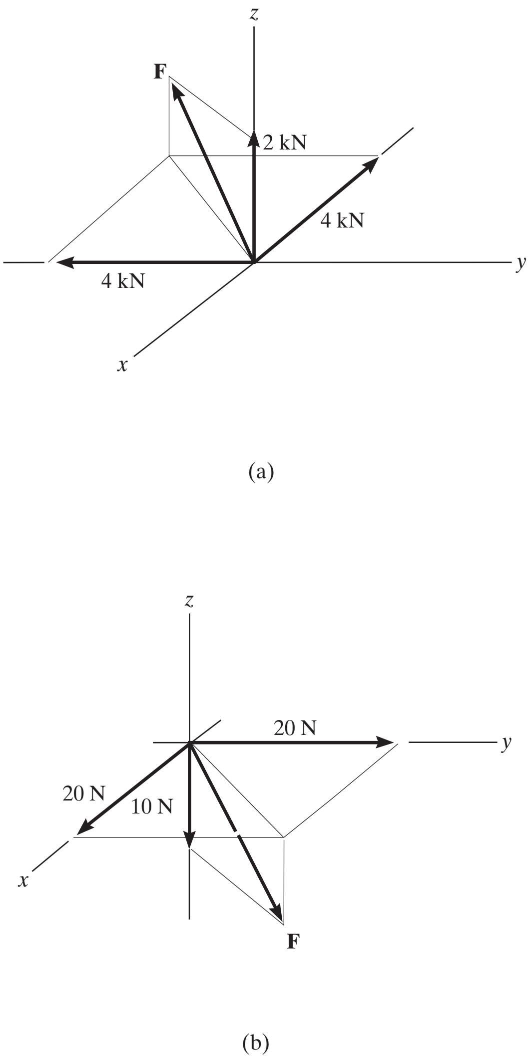
PRELIMINARY PROBLEMS

P2–3. Sketch the following forces on the x, y, z coordinate axes. Show α, β, γ.

a) $\mathbf{F} = \{50\mathbf{i} + 60\mathbf{j} - 10\mathbf{k}\}$ kN

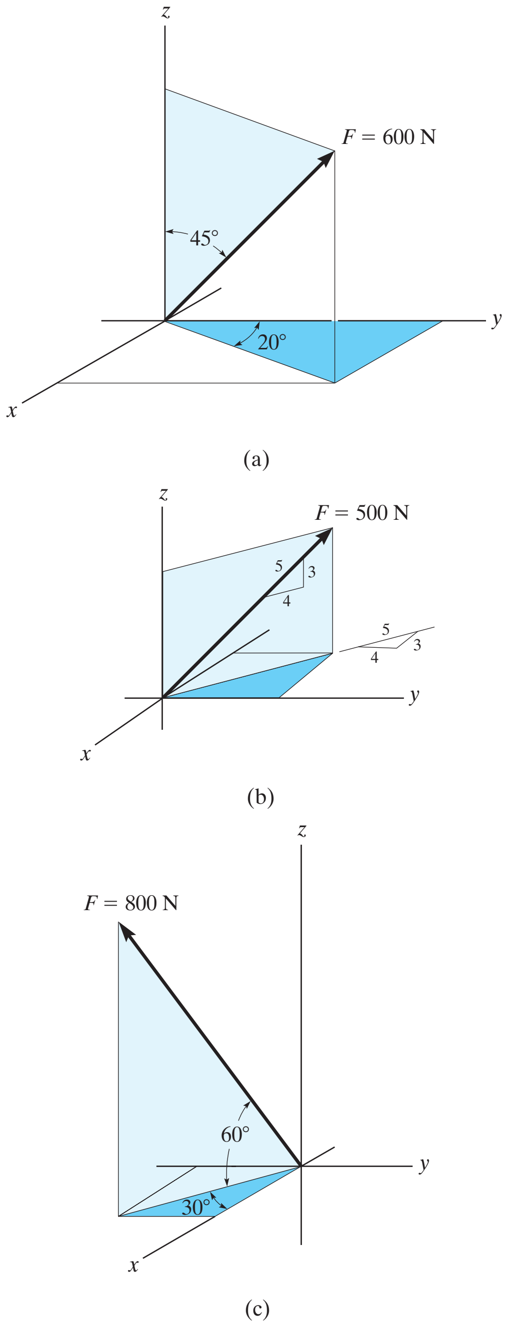
b) $\mathbf{F} = \{-40\mathbf{i} - 80\mathbf{j} + 60\mathbf{k}\}$ kN

P2–4. In each case, establish **F** as a Cartesian vector, and find the magnitude of **F** and the direction cosine of β.



Prob. P2–4

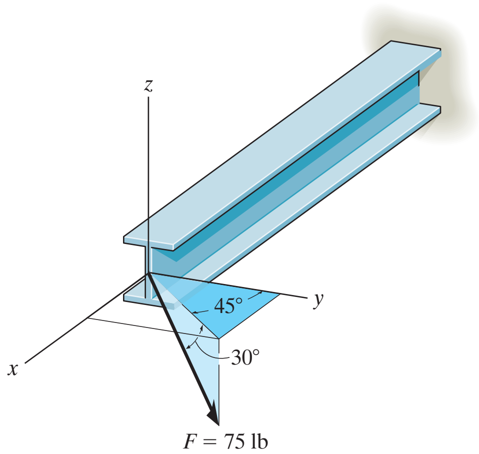
P2–5. Show how to resolve each force into its x, y, z components. Set up the calculation used to find the magnitude of each component.



Prob. P2–5

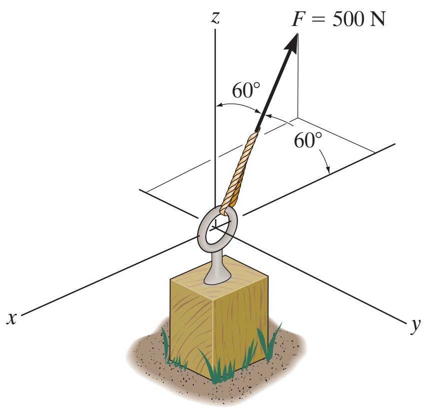
FUNDAMENTAL PROBLEMS

F2–13. Determine the coordinate direction angles of the force.



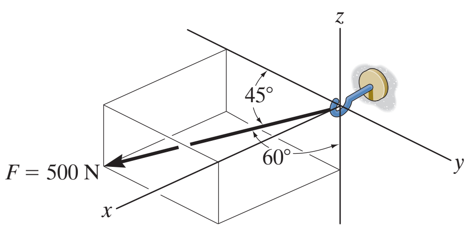
Prob. F2–13

F2–14. Express the force as a Cartesian vector.



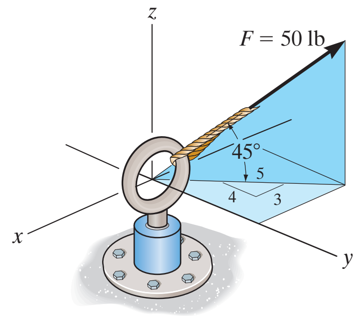
Prob. F2–14

F2–15. Express the force as a Cartesian vector.



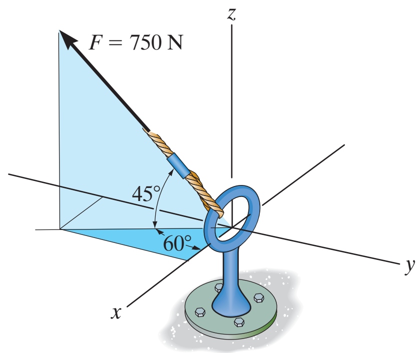
Prob. F2–15

F2–16. Express the force as a Cartesian vector.



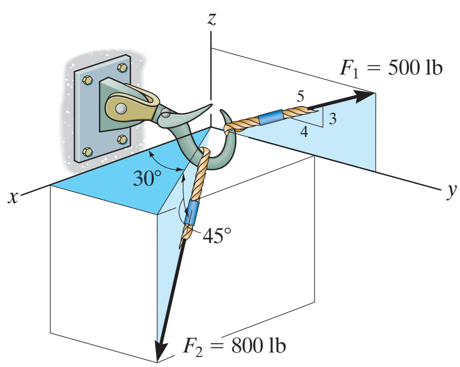
Prob. F2–16

F2–17. Express the force as a Cartesian vector.



Prob. F2–17

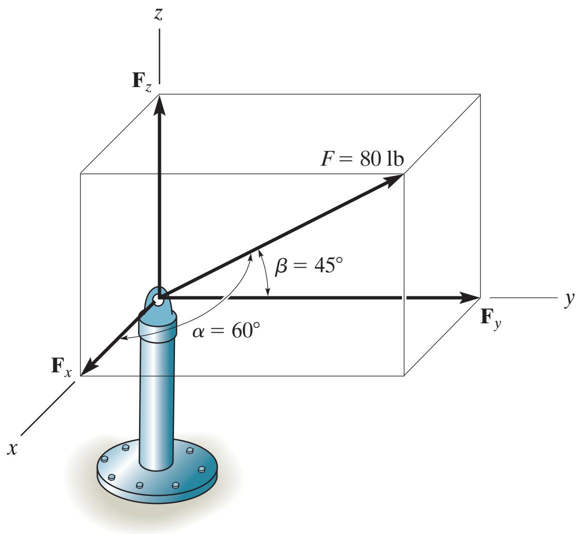
F2–18. Determine the resultant force acting on the hook.



Prob. F2–18

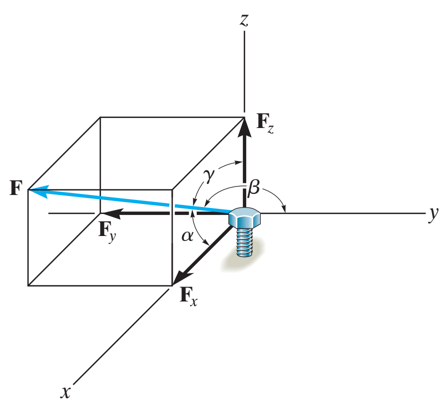
PROBLEMS

***2–60.** The force **F** has a magnitude of 80 lb and acts within the octant shown. Determine the magnitudes of the *x, y, z* components of **F**.



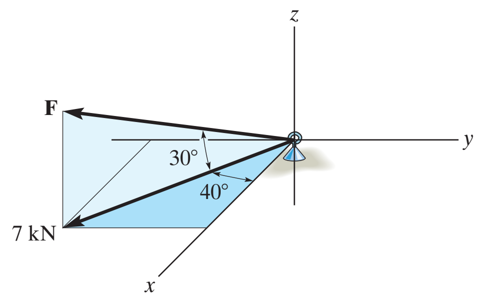
Prob. 2–60

2–61. The bolt is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



Prob. 2–61

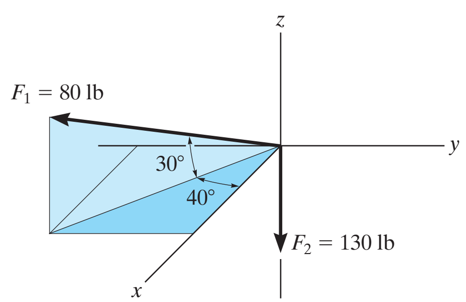
2–62. Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the *x–y* plane is 7 kN.



Prob. 2–62

2–63. Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.

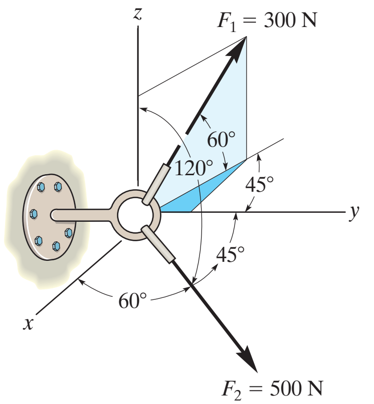
***2–64.** Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



Probs. 2–63/64

2–65. The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.

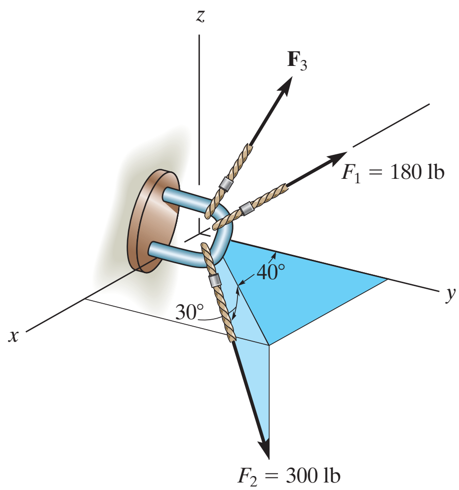
2–66. Determine the coordinate direction angles of $\mathbf{F}_1$.



Probs. 2–65/66

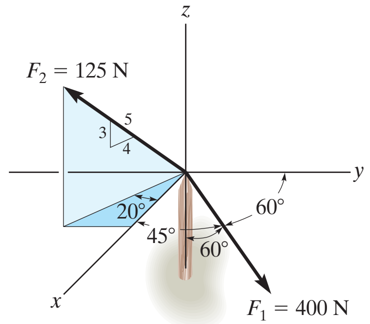
2–67. Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.

***2–68.** Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



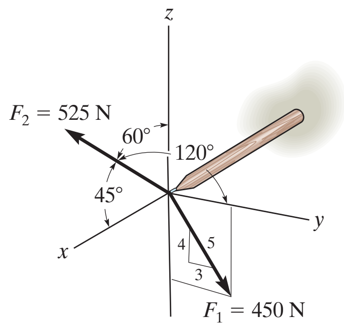
Probs. 2–67/68

2–69. Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



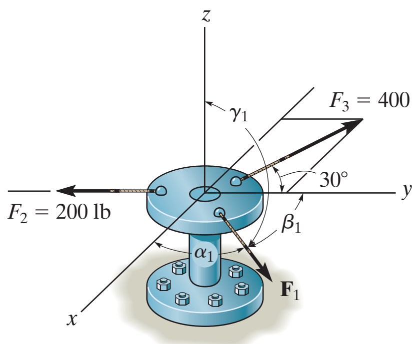
Prob. 2–69

2–70. Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



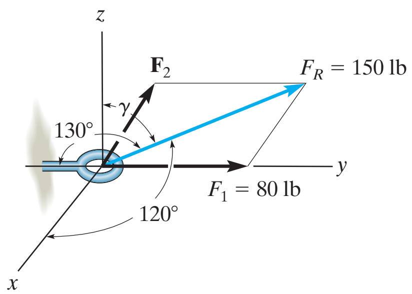
Prob. 2–70

2–71. Specify the magnitude and coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = \{-350\mathbf{k}\}$ lb. Note that $\mathbf{F}_3$ lies in the x–y plane.



Prob. 2–71

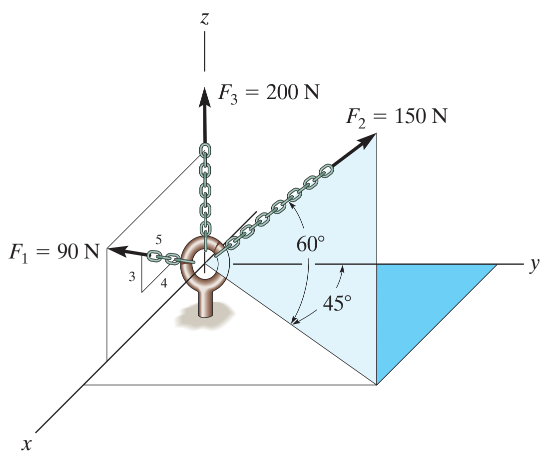
***2–72.** Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If the resultant force $\mathbf{F}_R$ has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



Prob. 2–72

2–73. Express each force in Cartesian vector form.

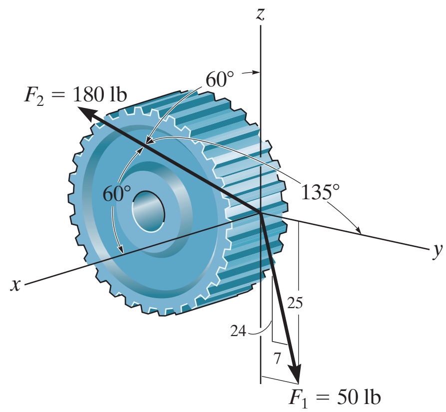
2–74. Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



Probs. 2–73/74

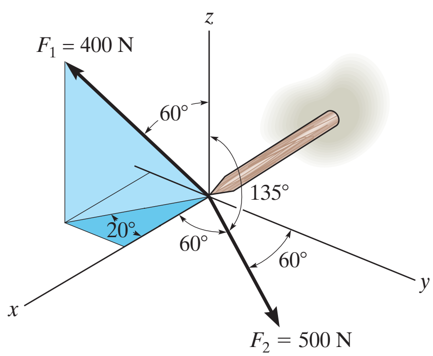
2–75. The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.

***2–76.** The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



Probs. 2–75/76

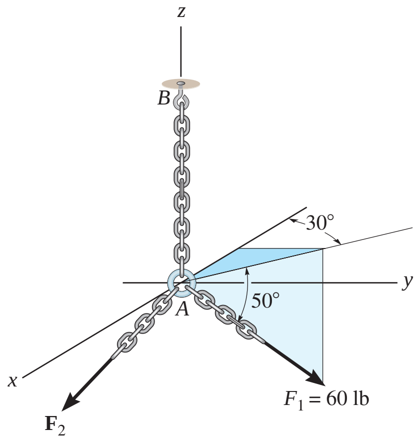
2–77. Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



Prob. 2–77

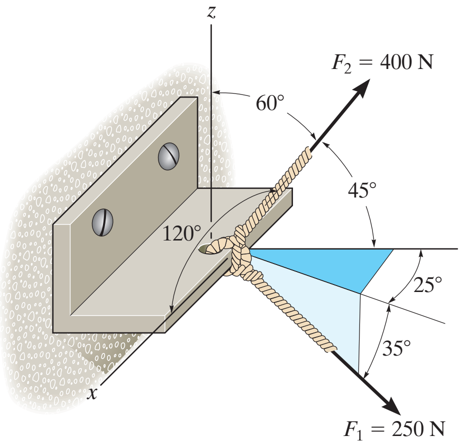
2–78. The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.

2–79. Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.



Probs. 2–78/79

***2–80.** The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force $\mathbf{F}_R$. Find the magnitude and coordinate direction angles of the resultant force.

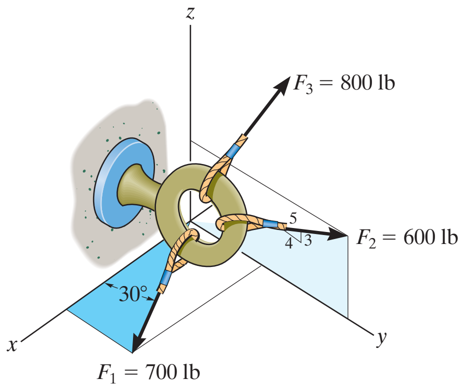


Prob. 2–80

2–81. If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 60°$ and $\gamma_3 = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.

2–82. If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$, and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.

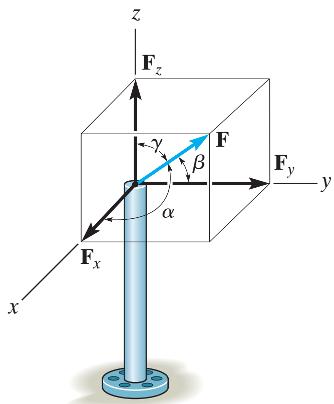
2–83. If the direction of the resultant force acting on the eyebolt is defined by the unit vector $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, determine the coordinate direction angles of $\mathbf{F}_3$ and the magnitude of $\mathbf{F}_R$.



Probs. 2–81/82/83

***2–84.** The pole is subjected to the force $\mathbf{F}$, which has components acting along the x, y, z axes as shown. If the magnitude of $\mathbf{F}$ is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.

2–85. The pole is subjected to the force $\mathbf{F}$ which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of $\mathbf{F}$ and $\mathbf{F}_y$.



Probs. 2–84/85

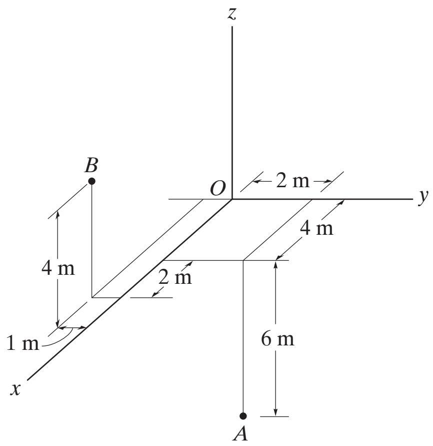


Fig. 2–32

2.7 Position Vectors

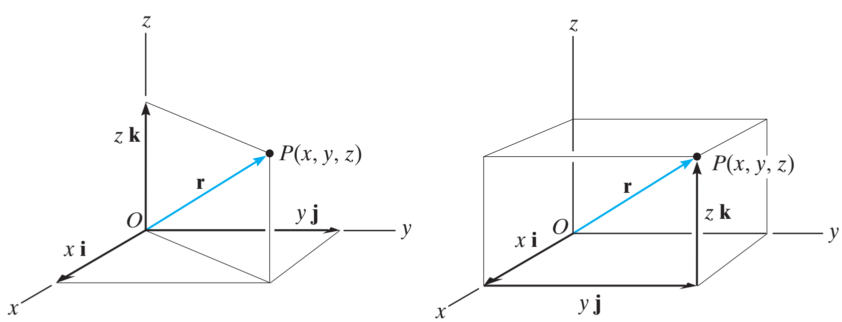
In this section we will introduce the concept of a position vector. It will be shown that this vector is of importance in formulating a Cartesian force vector directed between two points in space.

x, y, z Coordinates. Throughout the book we will use a *right-handed* coordinate system to reference the location of points in space. We will also use the convention followed in many technical books, which requires the positive z axis to be directed *upward* (the zenith direction) so that it measures the height of an object or the altitude of a point. The x, y axes then lie in the horizontal plane, Fig. 2–32. Points in space are located relative to the origin of coordinates, O, by successive measurements along the x, y, z axes. For example, the coordinates of point A are obtained by starting at O and measuring $x_A = +4$ m along the x axis, then $y_A = +2$ m along the y axis, and finally $z_A = -6$ m along the z axis, so that $A(4\text{ m}, 2\text{ m}, -6\text{ m})$. In a similar manner, measurements along the x, y, z axes from O to B yield the coordinates of B, that is, $B(6\text{ m}, -1\text{ m}, 4\text{ m})$.

Position Vector. A ***position vector*** **r** is defined as a fixed vector which locates a point in space relative to another point. For example, if **r** extends from the origin of coordinates, O, to point $P(x, y, z)$, Fig. 2–33*a*, then **r** can be expressed in Cartesian vector form as

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

Note how the head-to-tail vector addition of the three components yields vector **r**, Fig. 2–33*b*. Starting at the origin O, one "travels" x in the $+\mathbf{i}$ direction, then y in the $+\mathbf{j}$ direction, and finally z in the $+\mathbf{k}$ direction to arrive at point $P(x, y, z)$.



(a) (b)

Fig. 2–33

In the more general case, the position vector may be directed from point A to point B in space, Fig. 2–34*a*. This vector is also designated by the symbol $\mathbf{r}$. As a matter of convention, we will *sometimes* refer to this vector with *two subscripts* to indicate from and to the point where it is directed. Thus, $\mathbf{r}$ can also be designated as $\mathbf{r}_{AB}$. Also, note that $\mathbf{r}_A$ and $\mathbf{r}_B$ in Fig. 2–34*a* are referenced with only one subscript since they extend from the origin of coordinates.

From Fig. 2–34*a*, by the head-to-tail vector addition, using the triangle rule, we require

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

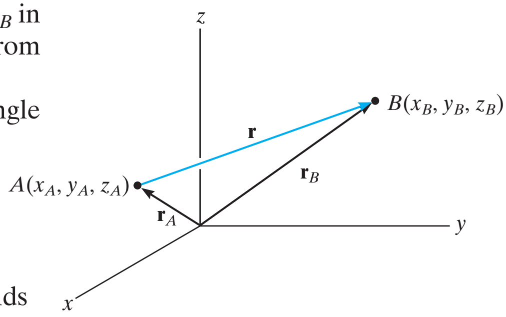
Solving for $\mathbf{r}$ and expressing $\mathbf{r}_A$ and $\mathbf{r}_B$ in Cartesian vector form yields

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$

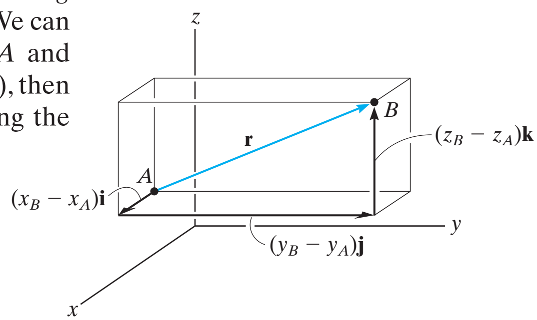
or

$$\boxed{\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}} \qquad (2\text{–}11)$$

Thus, the $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ *components of the position vector* $\mathbf{r}$ *may be formed by taking the coordinates of the tail of the vector* $A(x_A, y_A, z_A)$ *and subtracting them from the corresponding coordinates of the head* $B(x_B, y_B, z_B)$. We can also form these components *directly*, Fig. 2–34*b*, by starting at A and moving through a distance of $(x_B - x_A)$ along the positive x axis ($+\mathbf{i}$), then $(y_B - y_A)$ along the positive y axis ($+\mathbf{j}$), and finally $(z_B - z_A)$ along the positive z axis ($+\mathbf{k}$) to get to B.

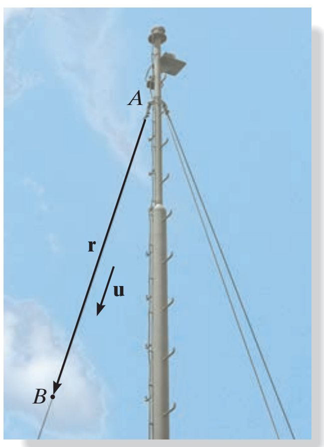


(a)



(b)

Fig. 2–34



If an x, y, z coordinate system is established, then the coordinates of two points A and B on the cable can be determined. From this the position vector $\mathbf{r}$ acting along the cable can be formulated. Its magnitude represents the distance from A to B, and its unit vector, $\mathbf{u} = \mathbf{r}/r$, gives the direction defined by α, β, γ. (© Russell C. Hibbeler)

EXAMPLE 2.10

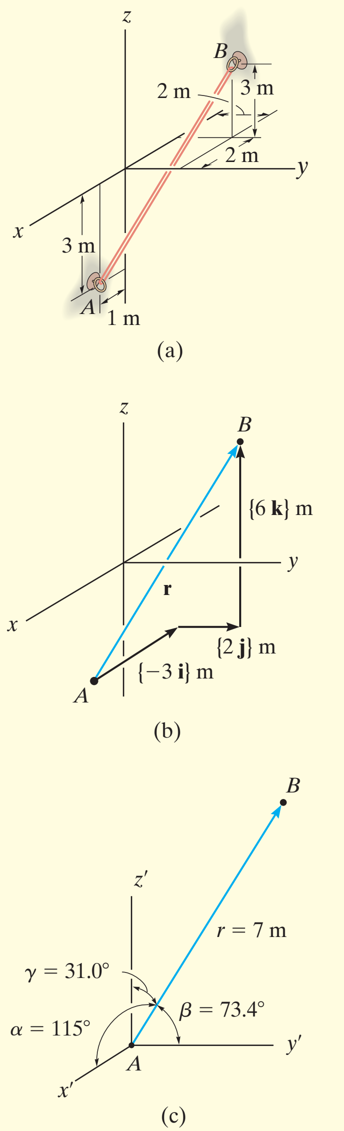


Fig. 2–35

An elastic rubber band is attached to points A and B as shown in Fig. 2–35a. Determine its length and its direction measured from A toward B.

SOLUTION

We first establish a position vector from A to B, Fig. 2–35b. In accordance with Eq. 2–11, the coordinates of the tail $A(1\text{ m}, 0, -3\text{ m})$ are subtracted from the coordinates of the head $B(-2\text{ m}, 2\text{ m}, 3\text{ m})$, which yields

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

These components of $\mathbf{r}$ can also be determined *directly* by realizing that they represent the direction and distance one must travel along each axis in order to move from A to B, i.e., along the x axis $\{-3\mathbf{i}\}$ m, along the y axis $\{2\mathbf{j}\}$ m, and finally along the z axis $\{6\mathbf{k}\}$ m.

The length of the rubber band is therefore

$$r = \sqrt{(-3\text{ m})^2 + (2\text{ m})^2 + (6\text{ m})^2} = 7\text{ m} \qquad \textit{Ans.}$$

Formulating a unit vector in the direction of $\mathbf{r}$, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

The components of this unit vector give the coordinate direction angles

$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115° \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4° \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0° \qquad \textit{Ans.}$$

NOTE: These angles are measured from the *positive axes* of a localized coordinate system placed at the tail of $\mathbf{r}$, as shown in Fig. 2–35c.

2.8 Force Vector Directed Along a Line

Quite often in three-dimensional statics problems, the direction of a force is specified by two points through which its line of action passes. Such a situation is shown in Fig. 2–36, where the force **F** is directed along the cord AB. We can formulate **F** as a Cartesian vector by realizing that it has the *same direction* and *sense* as the position vector **r** directed from point A to point B on the cord. This common direction is specified by the ***unit vector*** $\mathbf{u} = \mathbf{r}/r$. Hence,

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

Although we have represented **F** symbolically in Fig. 2–36, note that it has *units of force*, unlike **r**, which has units of length.

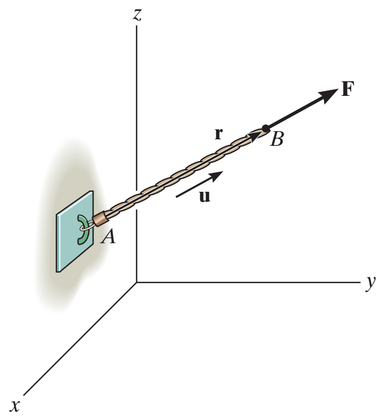


Fig. 2–36

The force **F** acting along the rope can be represented as a Cartesian vector by establishing x, y, z axes and first forming a position vector **r** along the length of the rope. Then the corresponding unit vector $\mathbf{u} = \mathbf{r}/r$ that defines the direction of both the rope and the force can be determined. Finally, the magnitude of the force is combined with its direction, $\mathbf{F} = F\mathbf{u}$. (© Russell C. Hibbeler)

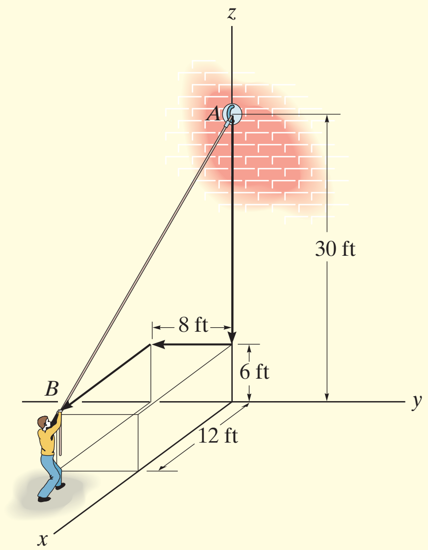
Important Points

- A position vector locates one point in space relative to another point.
- The easiest way to formulate the components of a position vector is to determine the distance and direction that must be traveled along the x, y, z directions—going from the tail to the head of the vector.
- A force **F** acting in the direction of a position vector **r** can be represented in Cartesian form if the unit vector **u** of the position vector is determined and it is multiplied by the magnitude of the force, i.e., $\mathbf{F} = F\mathbf{u} = F(\mathbf{r}/r)$.

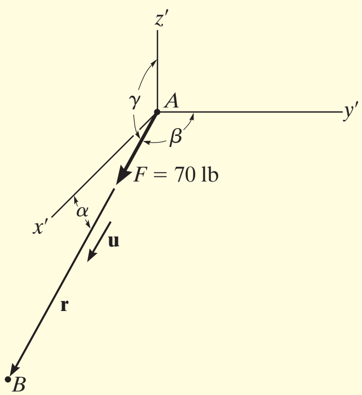
2

2

EXAMPLE 2.11



(a)



(b)

Fig. 2–37

The man shown in Fig. 2–37*a* pulls on the cord with a force of 70 lb. Represent this force acting on the support A as a Cartesian vector and determine its direction.

SOLUTION

Force **F** is shown in Fig. 2–37*b*. The *direction* of this vector, **u**, is determined from the position vector **r**, which extends from A to B. Rather than using the coordinates of the end points of the cord, **r** can be determined *directly* by noting in Fig. 2–37*a* that one must travel from A $\{-24\mathbf{k}\}$ ft, then $\{-8\mathbf{j}\}$ ft, and finally $\{12\mathbf{i}\}$ ft to get to B. Thus,

$$\mathbf{r} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ ft}$$

The magnitude of **r**, which represents the *length* of cord AB, is

$$r = \sqrt{(12\text{ ft})^2 + (-8\text{ ft})^2 + (-24\text{ ft})^2} = 28\text{ ft}$$

Forming the unit vector that defines the direction and sense of both **r** and **F**, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = \frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}$$

Since **F** has a *magnitude* of 70 lb and a *direction* specified by **u**, then

$$\mathbf{F} = F\mathbf{u} = 70\text{ lb}\left(\frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right)$$

$$= \{30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}\}\text{ lb} \qquad \textit{Ans.}$$

The coordinate direction angles are measured between **r** (or **F**) and the *positive axes* of a localized coordinate system with origin placed at A, Fig. 2–37*b*. From the components of the unit vector:

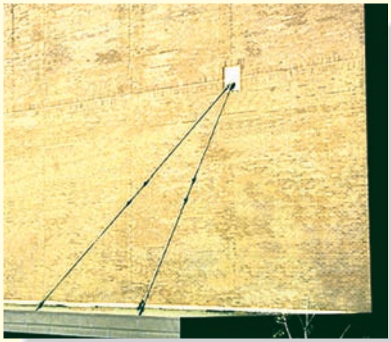
$$\alpha = \cos^{-1}\left(\frac{12}{28}\right) = 64.6^\circ \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{-8}{28}\right) = 107^\circ \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-24}{28}\right) = 149^\circ \qquad \textit{Ans.}$$

NOTE: These results make sense when compared with the angles identified in Fig. 2–37*b*.

EXAMPLE 2.12



(© Russell C. Hibbeler)

The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the wall hook at A as shown in Fig. 2–38*a*, determine the resultant force acting at A. Express the result as a Cartesian vector.

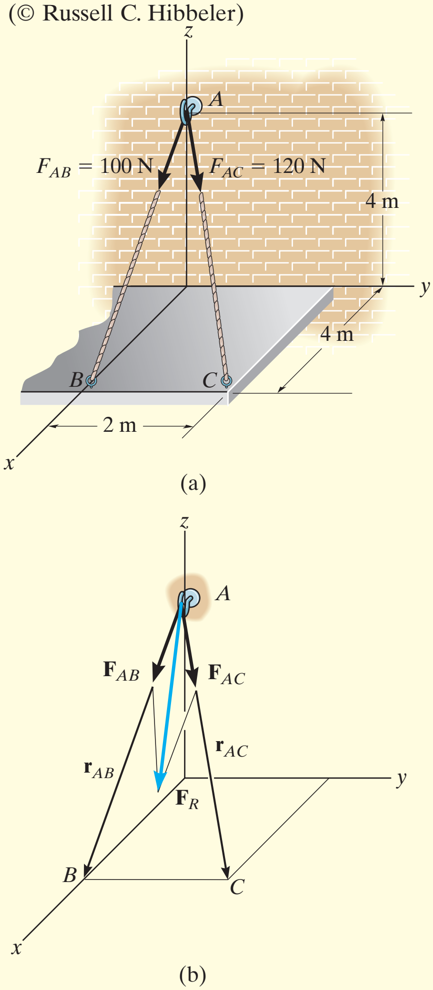


Fig. 2–38

SOLUTION

The resultant force $\mathbf{F}_R$ is shown graphically in Fig. 2–38*b*. We can express this force as a Cartesian vector by first formulating $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ as Cartesian vectors and then adding their components. The directions of $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ are specified by forming unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ along the cables. These unit vectors are obtained from the associated position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$. With reference to Fig. 2–38*a*, to go from A to B, we must travel $\{-4\mathbf{k}\}$ m, and then $\{4\mathbf{i}\}$ m. Thus,

$$\mathbf{r}_{AB} = \{4\mathbf{i} - 4\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(4\text{ m})^2 + (-4\text{ m})^2} = 5.66\text{ m}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = (100\text{ N})\left(\frac{4}{5.66}\mathbf{i} - \frac{4}{5.66}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\text{ N}$$

To go from A to C, we must travel $\{-4\mathbf{k}\}$ m, then $\{2\mathbf{j}\}$ m, and finally $\{4\mathbf{i}\}$. Thus,

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(4\text{ m})^2 + (2\text{ m})^2 + (-4\text{ m})^2} = 6\text{ m}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = (120\text{ N})\left(\frac{4}{6}\mathbf{i} + \frac{2}{6}\mathbf{j} - \frac{4}{6}\mathbf{k}\right)$$

$$= \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\text{ N}$$

The resultant force is therefore

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\text{ N} + \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\text{ N}$$

$$= \{151\mathbf{i} + 40\mathbf{j} - 151\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

EXAMPLE 2.13

The force in Fig. 2–39*a* acts on the hook. Express it as a Cartesian vector.

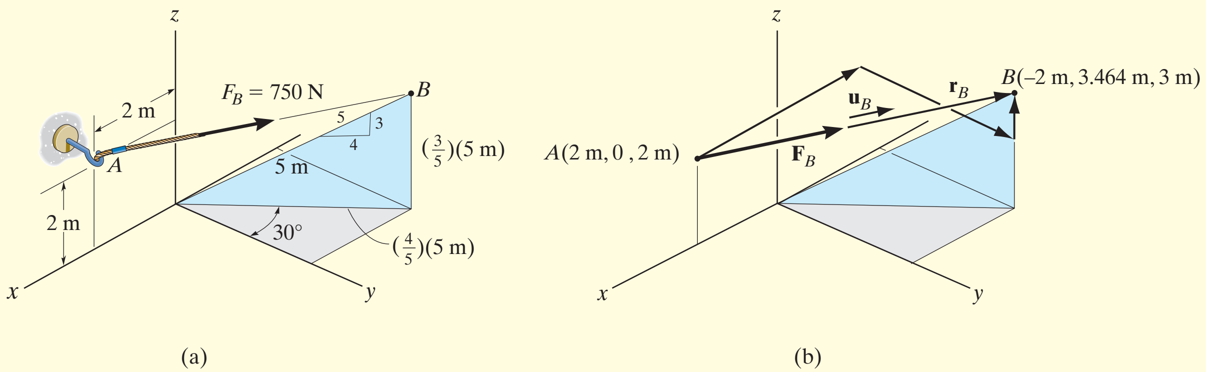


Fig. 2–39

SOLUTION

As shown in Fig. 2–39*b*, the coordinates for points A and B are

$$A(2\text{ m},\ 0,\ 2\text{ m})$$

and

$$B\left[-\left(\frac{4}{5}\right)5\sin 30^\circ\text{ m},\ \left(\frac{4}{5}\right)5\cos 30^\circ\text{ m},\ \left(\frac{3}{5}\right)5\text{ m}\right]$$

or

$$B(-2\text{ m},\ 3.464\text{ m},\ 3\text{ m})$$

Therefore, to go from A to B, one must travel $\{-4\mathbf{i}\}$ m, then $\{3.464\mathbf{j}\}$ m, and finally $\{1\mathbf{k}\}$ m. Thus,

$$\mathbf{u}_B = \left(\frac{\mathbf{r}_B}{r_B}\right) = \frac{\{-4\mathbf{i} + 3.464\mathbf{j} + 1\mathbf{k}\}\text{ m}}{\sqrt{(-4\text{ m})^2 + (3.464\text{ m})^2 + (1\text{ m})^2}}$$

$$= -0.7428\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k}$$

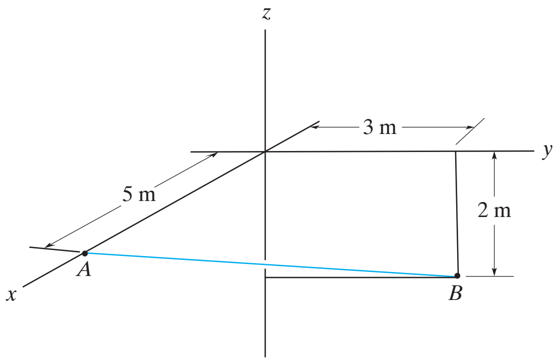
Force $\mathbf{F}_B$ expressed as a Cartesian vector becomes

$$\mathbf{F}_B = F_B\,\mathbf{u}_B = (750\text{ N})(-0.74281\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k})$$

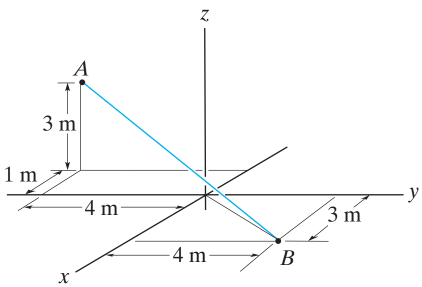
$$= \{-557\mathbf{i} + 482\mathbf{j} + 139\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

PRELIMINARY PROBLEMS

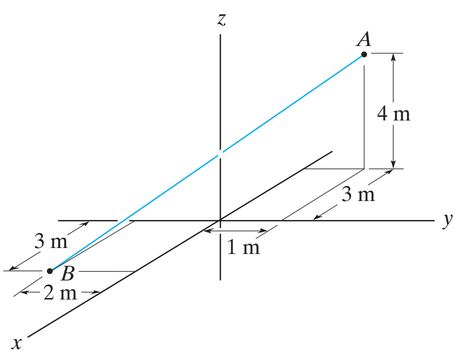
P2–6. In each case, establish a position vector from point A to point B.



(a)



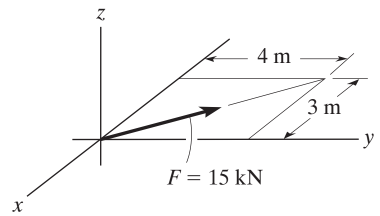
(b)



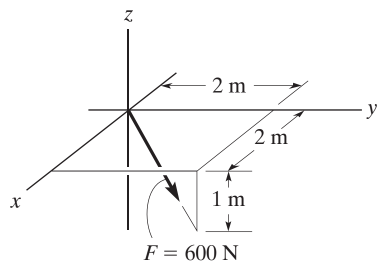
(c)

Prob. P2–6

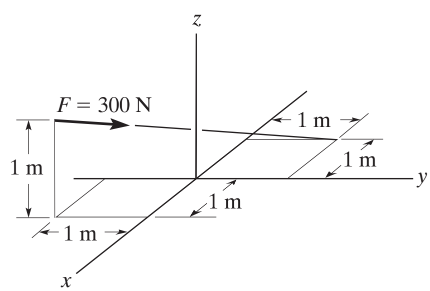
P2–7. In each case, express **F** as a Cartesian vector.



(a)



(b)



(c)

Prob. P2–7

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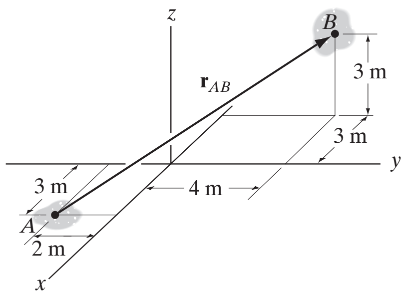
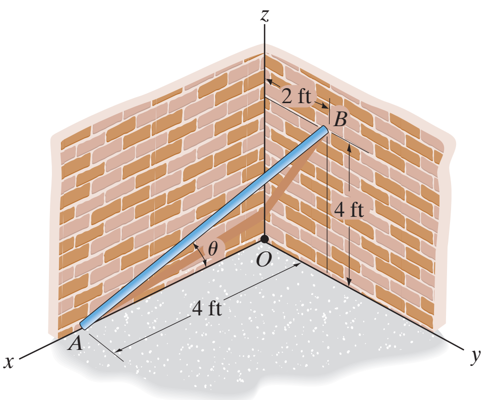
FUNDAMENTAL PROBLEMS

F2–19. Express the position vector $\mathbf{r}_{AB}$ in Cartesian vector form, then determine its magnitude and coordinate direction angles.



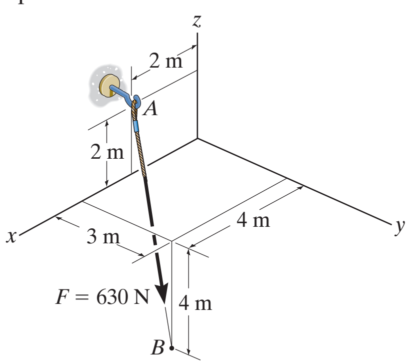
Prob. F2–19

F2–20. Determine the length of the rod and the position vector directed from A to B. What is the angle θ?



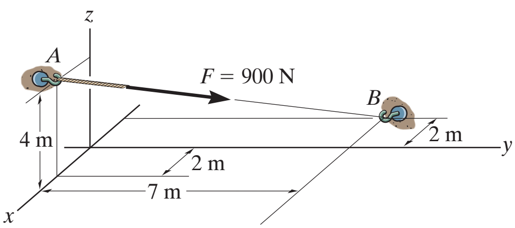
Prob. F2–20

F2–21. Express the force as a Cartesian vector.



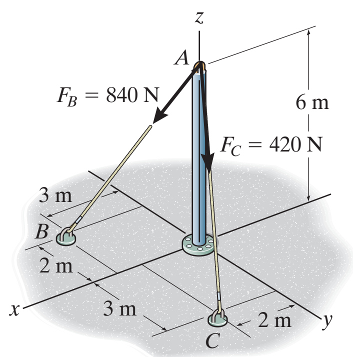
Prob. F2–21

F2–22. Express the force as a Cartesian vector.



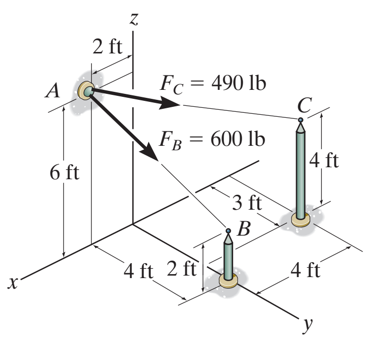
Prob. F2–22

F2–23. Determine the magnitude of the resultant force at A.



Prob. F2–23

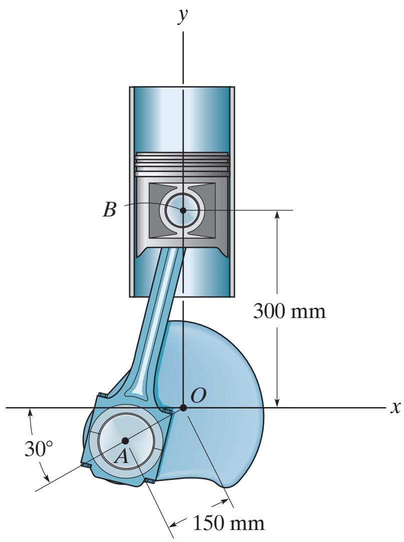
F2–24. Determine the resultant force at A.



Prob. F2–24

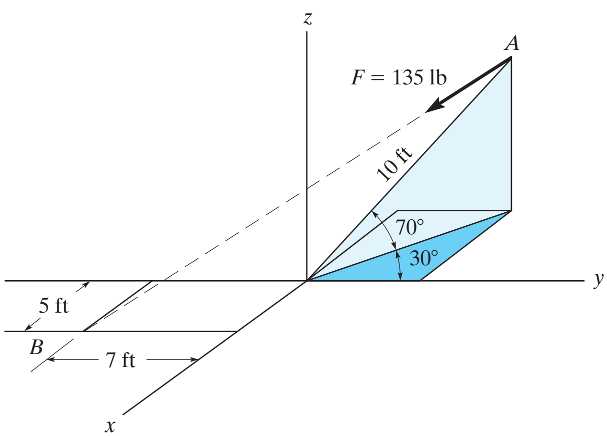
PROBLEMS

2–86. Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



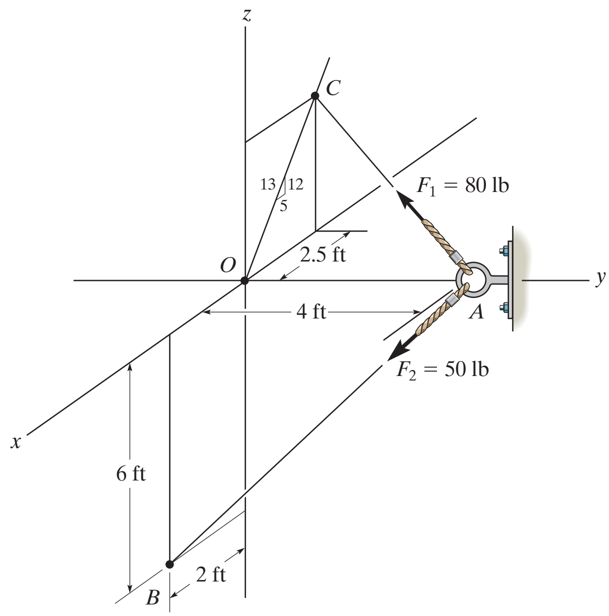
Prob. 2–86

2–87. Express force $\mathbf{F}$ as a Cartesian vector; then determine its coordinate direction angles.



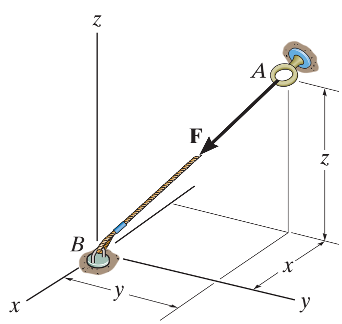
Prob. 2–87

***2–88.** Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–88

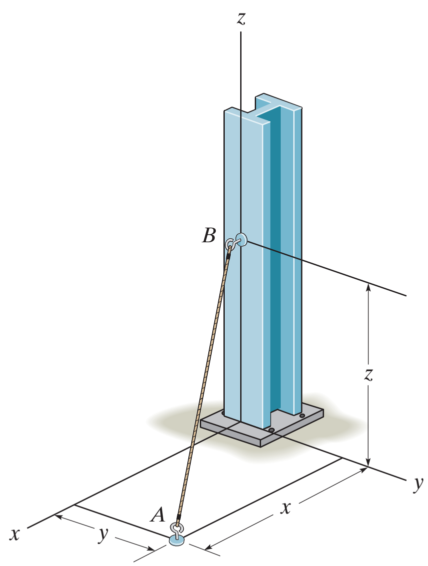
2–89. If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



Prob. 2–89

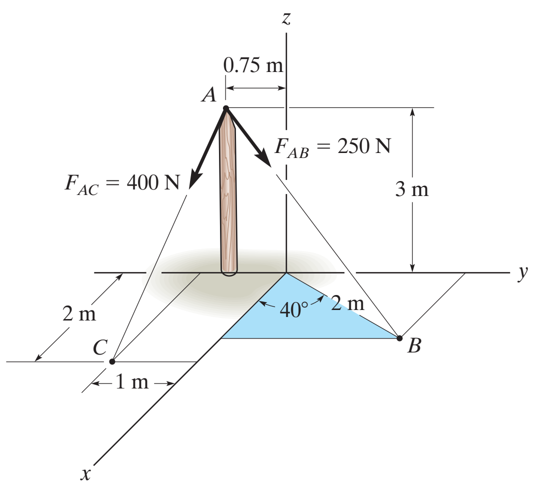
2–90. The 8-m-long cable is anchored to the ground at A. If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.

2–91. The 8-m-long cable is anchored to the ground at A. If $z = 5$ m, determine the location $+x, +y$ of point A. Choose a value such that $x = y$.



Probs. 2–90/91

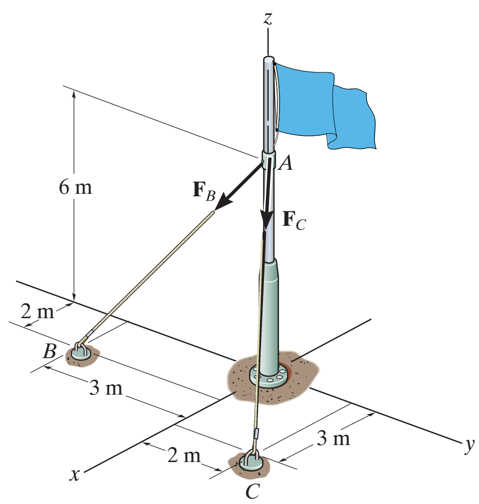
***2–92.** Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–92

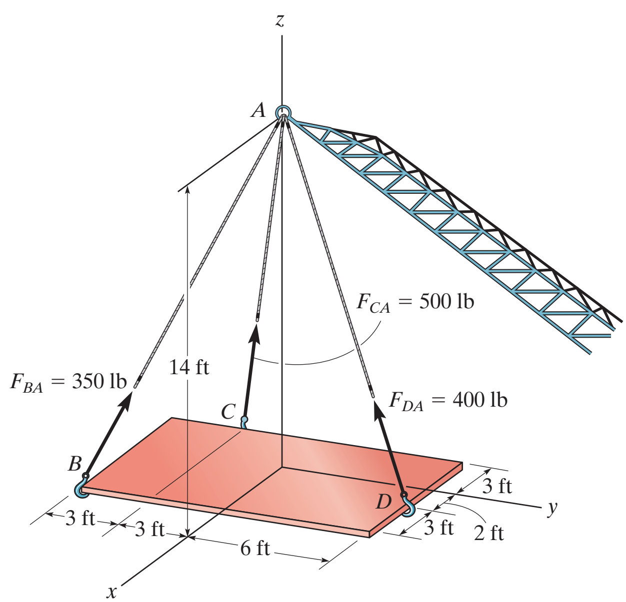
2–93. If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

2–94. If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



Probs. 2–93/94

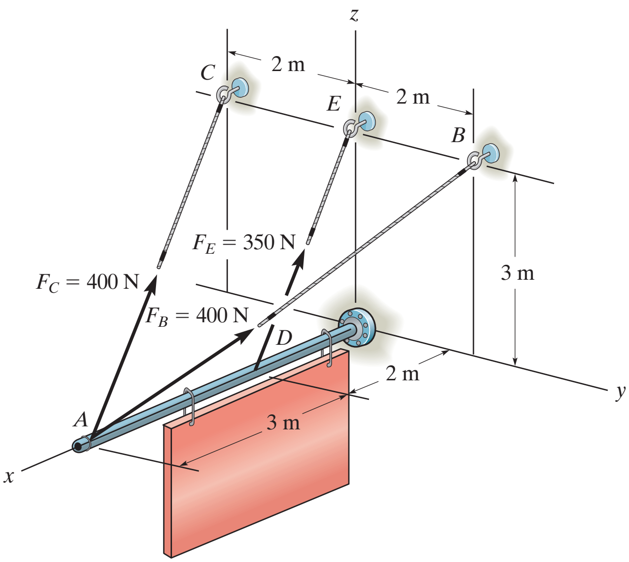
2–95. The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



Prob. 2–95

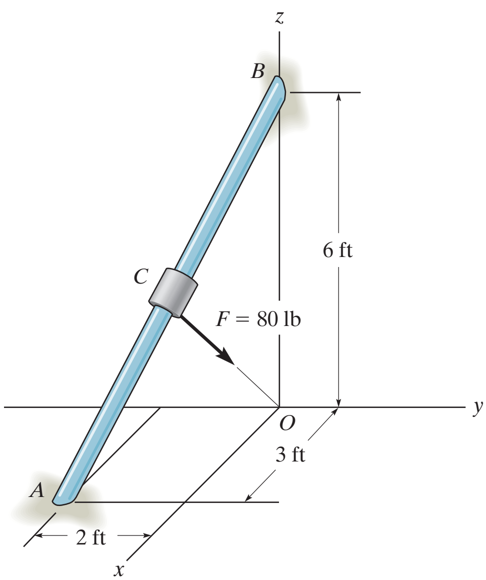
***2–96.** The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.

2–97. Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting on the sign at point A.



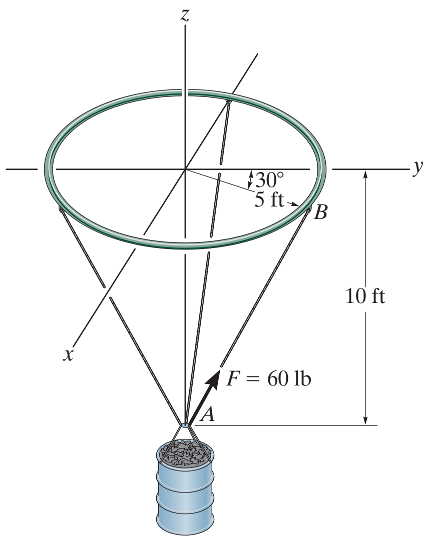
Probs. 2–96/97

2–98. The force **F** has a magnitude of 80 lb and acts at the midpoint C of the thin rod. Express the force as a Cartesian vector.



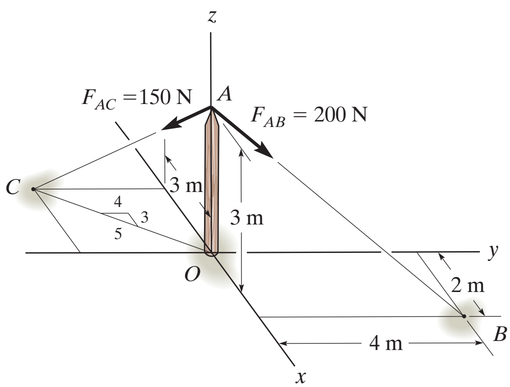
Prob. 2–98

2–99. The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



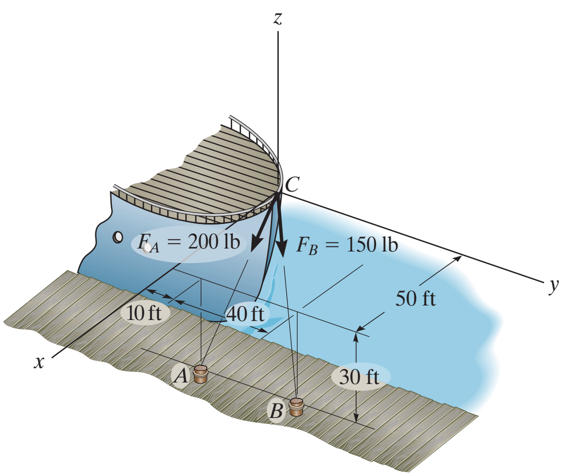
Prob. 2–99

***2–100.** Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



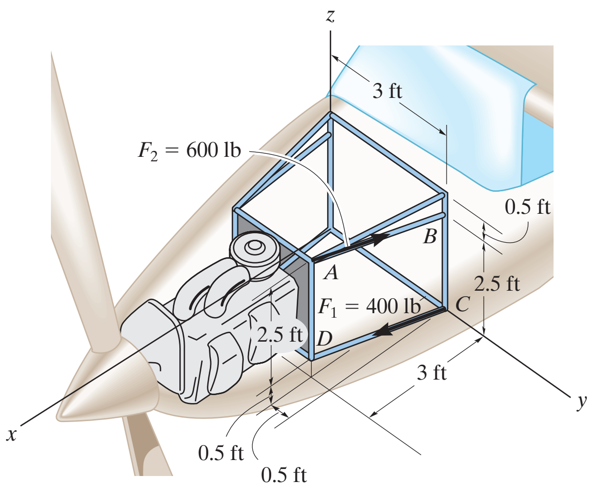
Prob. 2–100

2–101. The two mooring cables exert forces on the stern of a ship as shown. Represent each force as as Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.



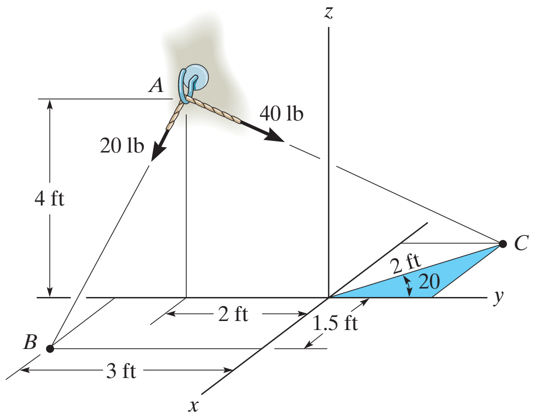
Prob. 2–101

2–102. The engine of the lightweight plane is supported by struts that are connected to the space truss that makes up the structure of the plane. The anticipated loading in two of the struts is shown. Express each of those forces as Cartesian vector.



Prob. 2–102

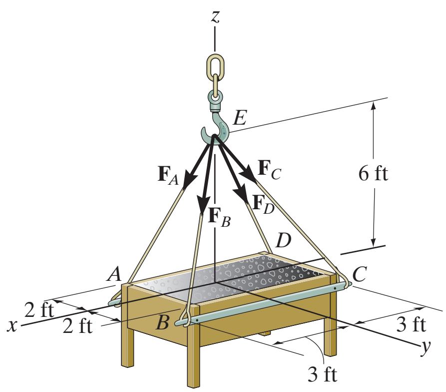
2–103. Determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–103

***2–104.** If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.

2–105. If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



Probs. 2–104/105

2.9 Dot Product

Occasionally in statics one has to find the angle between two lines or the components of a force parallel and perpendicular to a line. In two dimensions, these problems can readily be solved by trigonometry since the geometry is easy to visualize. In three dimensions, however, this is often difficult, and consequently vector methods should be employed for the solution. The dot product, which defines a particular method for "multiplying" two vectors, can be used to solve the above-mentioned problems.

The ***dot product*** of vectors **A** and **B**, written $\mathbf{A}\cdot\mathbf{B}$ and read "**A** dot **B**," is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle θ between their tails, Fig. 2–40. Expressed in equation form,

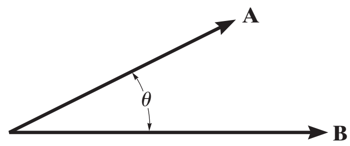


Fig. 2–40

$$\boxed{\mathbf{A}\cdot\mathbf{B} = AB\cos\theta} \qquad (2\text{–}12)$$

where $0^\circ \leq \theta \leq 180^\circ$. The dot product is often referred to as the *scalar product* of vectors since the result is a *scalar* and not a vector.

Laws of Operation.

1. Commutative law: $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
2. Multiplication by a scalar: $a(\mathbf{A}\cdot\mathbf{B}) = (a\mathbf{A})\cdot\mathbf{B} = \mathbf{A}\cdot(a\mathbf{B})$
3. Distributive law: $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$

It is easy to prove the first and second laws by using Eq. 2–12. The proof of the distributive law is left as an exercise (see Prob. 2–112).

Cartesian Vector Formulation.

Equation 2–12 must be used to find the dot product for any two Cartesian unit vectors. For example, $\mathbf{i}\cdot\mathbf{i} = (1)(1)\cos 0^\circ = 1$ and $\mathbf{i}\cdot\mathbf{j} = (1)(1)\cos 90^\circ = 0$. If we want to find the dot product of two general vectors **A** and **B** that are expressed in Cartesian vector form, then we have

$$\begin{aligned}
\mathbf{A}\cdot\mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\
&= A_xB_x(\mathbf{i}\cdot\mathbf{i}) + A_xB_y(\mathbf{i}\cdot\mathbf{j}) + A_xB_z(\mathbf{i}\cdot\mathbf{k}) \\
&\quad + A_yB_x(\mathbf{j}\cdot\mathbf{i}) + A_yB_y(\mathbf{j}\cdot\mathbf{j}) + A_yB_z(\mathbf{j}\cdot\mathbf{k}) \\
&\quad + A_zB_x(\mathbf{k}\cdot\mathbf{i}) + A_zB_y(\mathbf{k}\cdot\mathbf{j}) + A_zB_z(\mathbf{k}\cdot\mathbf{k})
\end{aligned}$$

Carrying out the dot-product operations, the final result becomes

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z} \qquad (2\text{–}13)$$

Thus, to determine the dot product of two Cartesian vectors, multiply their corresponding x, y, z components and sum these products algebraically. Note that the result will be either a positive or negative *scalar*, or it could be zero.

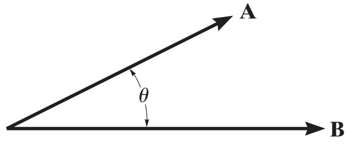
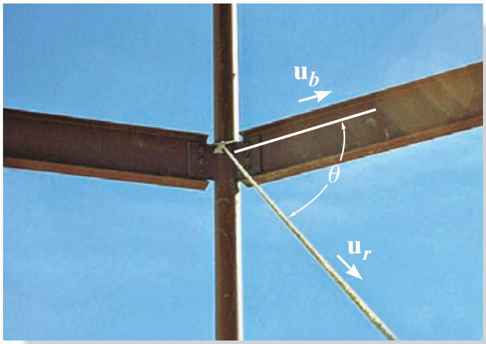
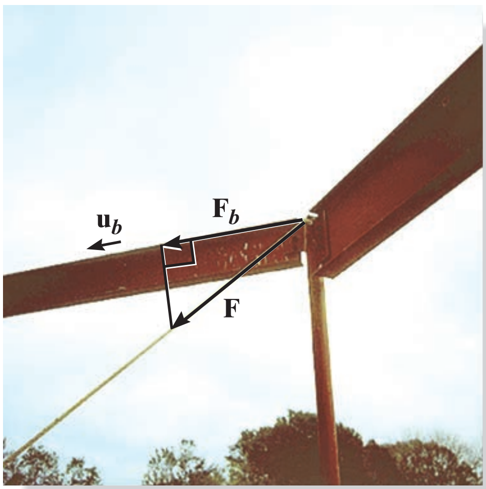


Fig. 2–40 (Repeated)



The angle θ between the rope and the beam can be determined by formulating unit vectors along the beam and rope and then using the dot product $\mathbf{u}_b \cdot \mathbf{u}_r = (1)(1)\cos\theta$. (© Russell C. Hibbeler)



The projection of the cable force $\mathbf{F}$ along the beam can be determined by first finding the unit vector $\mathbf{u}_b$ that defines this direction. Then apply the dot product, $F_b = \mathbf{F} \cdot \mathbf{u}_b$. (© Russell C. Hibbeler)

Applications. The dot product has two important applications in mechanics.

- ***The angle formed between two vectors or intersecting lines.*** The angle θ between the tails of vectors $\mathbf{A}$ and $\mathbf{B}$ in Fig. 2–40 can be determined from Eq. 2–12 and written as

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right) \qquad 0^\circ \le \theta \le 180^\circ$$

Here $\mathbf{A} \cdot \mathbf{B}$ is found from Eq. 2–13. In particular, notice that if $\mathbf{A} \cdot \mathbf{B} = 0, \theta = \cos^{-1} 0 = 90^\circ$ so that $\mathbf{A}$ will be *perpendicular* to $\mathbf{B}$.

- ***The components of a vector parallel and perpendicular to a line.*** The component of vector $\mathbf{A}$ parallel to or collinear with the line aa in Fig. 2–40 is defined by A_a where $A_a = A \cos\theta$. This component is sometimes referred to as the ***projection*** of $\mathbf{A}$ onto the line, since a *right angle* is formed in the construction. If the *direction* of the line is specified by the unit vector $\mathbf{u}_a$, then since $u_a = 1$, we can determine the magnitude of A_a directly from the dot product (Eq. 2–12); i.e.,

$$A_a = A\cos\theta = \mathbf{A} \cdot \mathbf{u}_a$$

Hence, the scalar projection of $\mathbf{A}$ *along a line is determined from the dot product of* $\mathbf{A}$ *and the unit vector* $\mathbf{u}_a$ *which defines the direction of the line*. Notice that if this result is positive, then $\mathbf{A}_a$ has a directional sense which is the same as $\mathbf{u}_a$, whereas if A_a is a negative scalar, then $\mathbf{A}_a$ has the opposite sense of direction to $\mathbf{u}_a$.

The component $\mathbf{A}_a$ represented as a *vector* is therefore

$$\mathbf{A}_a = A_a \mathbf{u}_a$$

The component of $\mathbf{A}$ that is *perpendicular* to line aa can also be obtained, Fig. 2–41. Since $\mathbf{A} = \mathbf{A}_a + \mathbf{A}_\perp$, then $\mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_a$. There are two possible ways of obtaining $A_\perp$. One way would be to determine θ from the dot product, $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{u}_A / A)$, then $A_\perp = A\sin\theta$. Alternatively, if A_a is known, then by Pythagorean's theorem we can also write $A_\perp = \sqrt{A^2 - A_a{}^2}$.

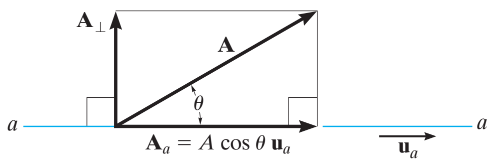


Fig. 2–41

Important Points

- The dot product is used to determine the angle between two vectors or the projection of a vector in a specified direction.
- If vectors **A** and **B** are expressed in Cartesian vector form, the dot product is determined by multiplying the respective x, y, z scalar components and algebraically adding the results, i.e., $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.
- From the definition of the dot product, the angle formed between the tails of vectors **A** and **B** is $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{B}/AB)$.
- The magnitude of the projection of vector **A** along a line aa whose direction is specified by $\mathbf{u}_a$ is determined from the dot product $A_a = \mathbf{A} \cdot \mathbf{u}_a$.

EXAMPLE 2.14

Determine the magnitudes of the projection of the force **F** in Fig. 2–42 onto the u and v axes.

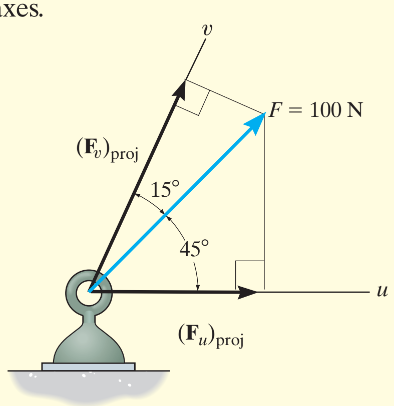


Fig. 2–42

SOLUTION

Projections of Force. The graphical representation of the *projections* is shown in Fig. 2–42. From this figure, the magnitudes of the projections of **F** onto the u and v axes can be obtained by trigonometry:

$$(F_u)_{\text{proj}} = (100\text{ N})\cos 45^\circ = 70.7\text{ N} \qquad \textit{Ans.}$$

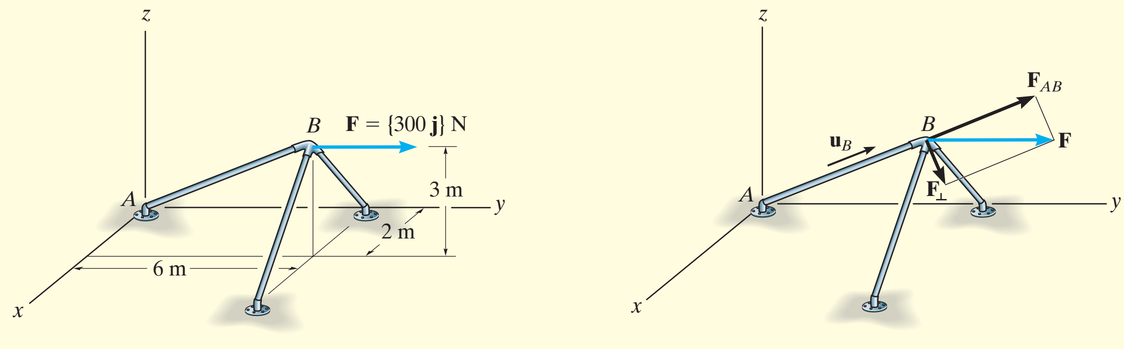
$$(F_v)_{\text{proj}} = (100\text{ N})\cos 15^\circ = 96.6\text{ N} \qquad \textit{Ans.}$$

NOTE: These projections are not equal to the magnitudes of the components of force **F** along the u and v axes found from the parallelogram law. They will only be equal if the u and v axes are *perpendicular* to one another.

EXAMPLE 2.15

2

The frame shown in Fig. 2–43*a* is subjected to a horizontal force $\mathbf{F} = \{300\mathbf{j}\}$ N. Determine the magnitudes of the components of this force parallel and perpendicular to member AB.



(a) (b)

Fig. 2–43

SOLUTION

The magnitude of the component of $\mathbf{F}$ along AB is equal to the dot product of $\mathbf{F}$ and the unit vector $\mathbf{u}_B$, which defines the direction of AB, Fig. 2–43*b*. Since

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.286\,\mathbf{i} + 0.857\,\mathbf{j} + 0.429\,\mathbf{k}$$

then

$$\begin{aligned} F_{AB} &= F\cos\theta = \mathbf{F}\cdot\mathbf{u}_B = (300\mathbf{j})\cdot(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}) \\ &= (0)(0.286) + (300)(0.857) + (0)(0.429) \\ &= 257.1\text{ N} \end{aligned}$$ *Ans.*

Since the result is a positive scalar, $\mathbf{F}_{AB}$ has the same sense of direction as $\mathbf{u}_B$, Fig. 2–43*b*.

Expressing $\mathbf{F}_{AB}$ in Cartesian vector form, we have

$$\begin{aligned} \mathbf{F}_{AB} = F_{AB}\mathbf{u}_B &= (257.1\text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}) \\ &= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\}\text{N} \end{aligned}$$ *Ans.*

The perpendicular component, Fig. 2–43*b*, is therefore

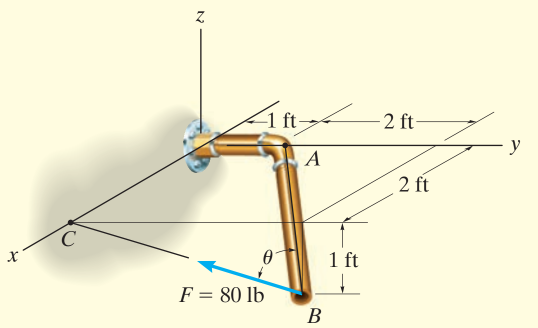
$$\begin{aligned} \mathbf{F}_\perp &= \mathbf{F} - \mathbf{F}_{AB} = 300\mathbf{j} - (73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}) \\ &= \{-73.5\mathbf{i} + 79.6\mathbf{j} - 110\mathbf{k}\}\text{N} \end{aligned}$$

Its magnitude can be determined either from this vector or by using the Pythagorean theorem, Fig. 2–43*b*:

$$\begin{aligned} F_\perp &= \sqrt{F^2 - F_{AB}^2} = \sqrt{(300\text{ N})^2 - (257.1\text{ N})^2} \\ &= 155\text{ N} \end{aligned}$$ *Ans.*

EXAMPLE 2.16

The pipe in Fig. 2–44*a* is subjected to the force of $F = 80$ lb. Determine the angle θ between **F** and the pipe segment BA and the projection of **F** along this segment.



(a)

SOLUTION

Angle θ. First we will establish position vectors from B to A and B to C; Fig. 2–44*b*. Then we will determine the angle θ between the tails of these two vectors.

$$\mathbf{r}_{BA} = \{-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\} \text{ ft}, \quad r_{BA} = 3 \text{ ft}$$
$$\mathbf{r}_{BC} = \{-3\mathbf{j} + 1\mathbf{k}\} \text{ ft}, \quad r_{BC} = \sqrt{10} \text{ ft}$$

Thus,

$$\cos\theta = \frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}} = \frac{(-2)(0) + (-2)(-3) + (1)(1)}{3\sqrt{10}} = 0.7379$$

$$\theta = 42.5^\circ \qquad \textit{Ans.}$$

Components of F. The component of **F** along BA is shown in Fig. 2–44*c*. We must first formulate the unit vector along BA and force **F** as Cartesian vectors.

$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{(-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k})}{3} = -\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$$

$$\mathbf{F} = 80 \text{ lb}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = 80\left(\frac{-3\mathbf{j} + 1\mathbf{k}}{\sqrt{10}}\right) = -75.89\mathbf{j} + 25.30\mathbf{k}$$

Thus,

$$F_{BA} = \mathbf{F} \cdot \mathbf{u}_{BA} = (-75.89\mathbf{j} + 25.30\mathbf{k}) \cdot \left(-\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right)$$
$$= 0\left(-\frac{2}{3}\right) + (-75.89)\left(-\frac{2}{3}\right) + (25.30)\left(\frac{1}{3}\right)$$
$$= 59.0 \text{ lb} \qquad \textit{Ans.}$$

NOTE: Since θ has been calculated, then also, $F_{BA} = F\cos\theta = 80 \text{ lb} \cos 42.5^\circ = 59.0$ lb.

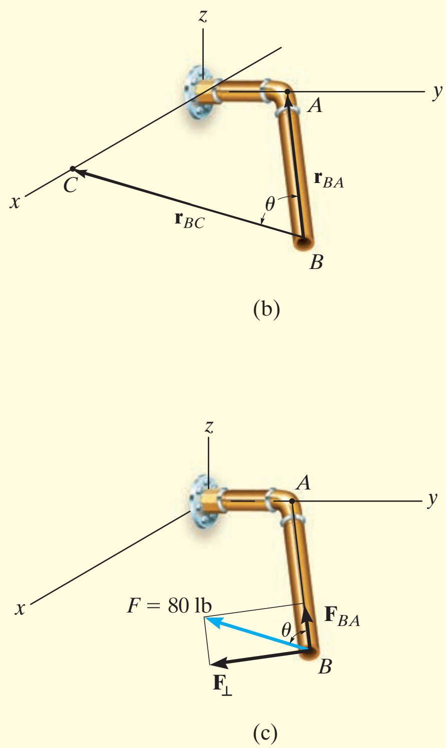
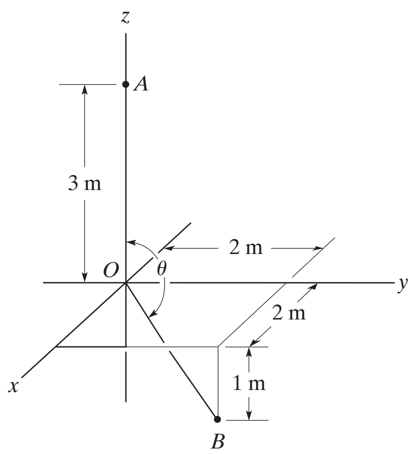


Fig. 2–44

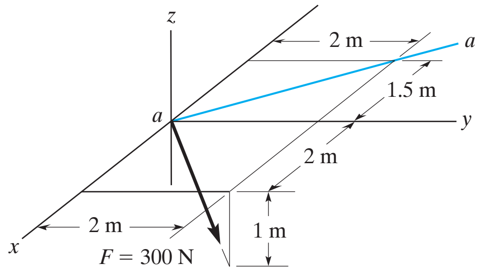
PRELIMINARY PROBLEMS

P2–8. In each case, set up the dot product to find the angle θ. Do not calculate the result.

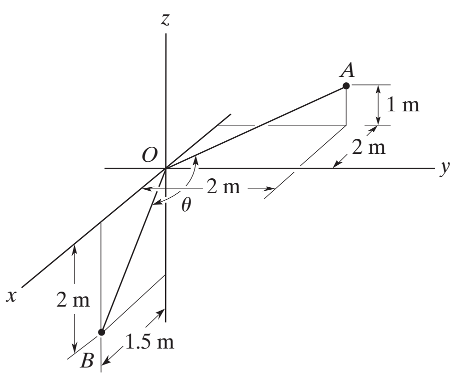
P2–9. In each case, set up the dot product to find the magnitude of the projection of the force **F** along *a*-*a* axes. Do not calculate the result.



(a)

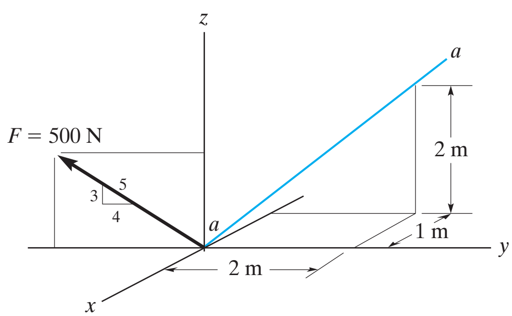


(a)



(b)

Prob. P2–8

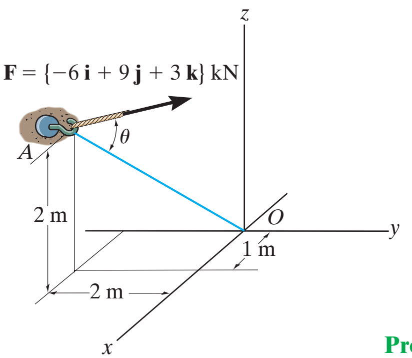


(b)

Prob. P2–9

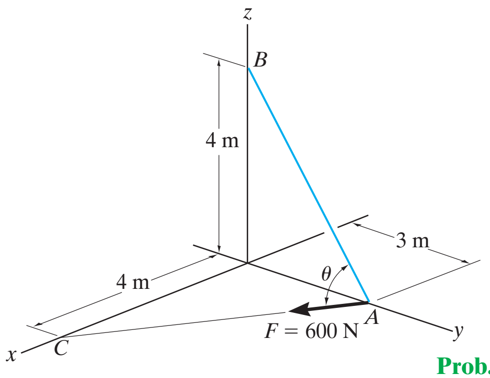
FUNDAMENTAL PROBLEMS

F2–25. Determine the angle θ between the force and the line AO.



Prob. F2–25

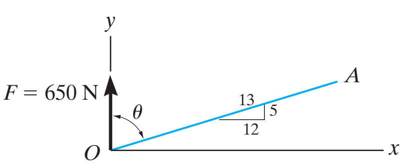
F2–26. Determine the angle θ between the force and the line AB.



Prob. F2–26

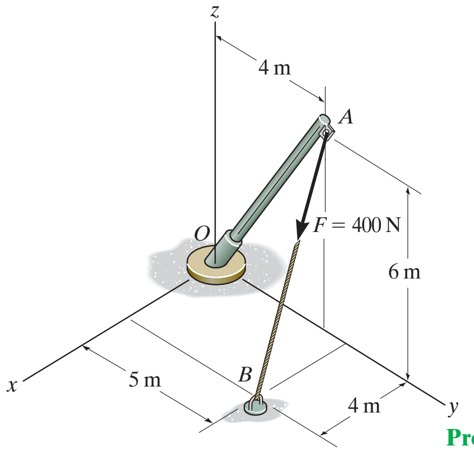
F2–27. Determine the angle θ between the force and the line OA.

F2–28. Determine the projected component of the force along the line OA.



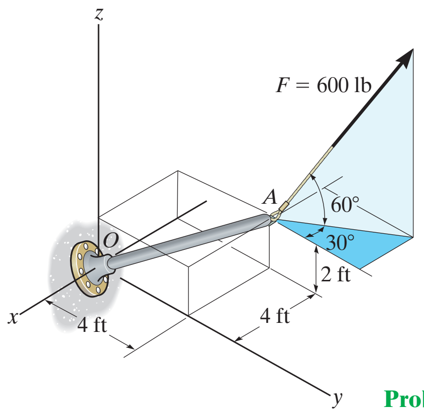
Probs. F2–27/28

F2–29. Find the magnitude of the projected component of the force along the pipe AO.



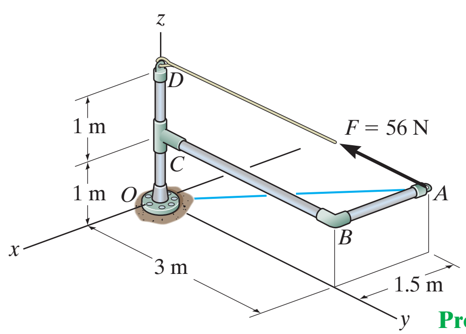
Prob. F2–29

F2–30. Determine the components of the force acting parallel and perpendicular to the axis of the pole.



Prob. F2–30

F2–31. Determine the magnitudes of the components of the force $F = 56$ N acting along and perpendicular to line AO.

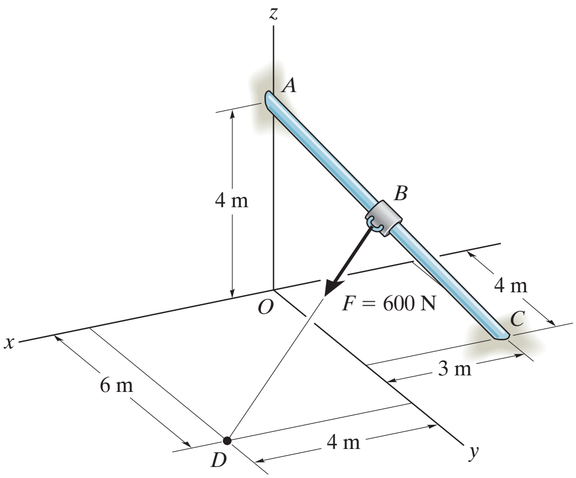


Prob. F2–31

2

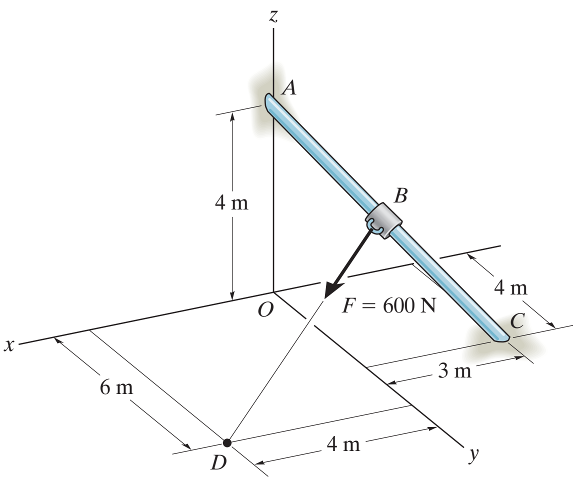
PROBLEMS

2–106. Express the force **F** in Cartesian vector form if it acts at the midpoint B of the rod.



Prob. 2–106

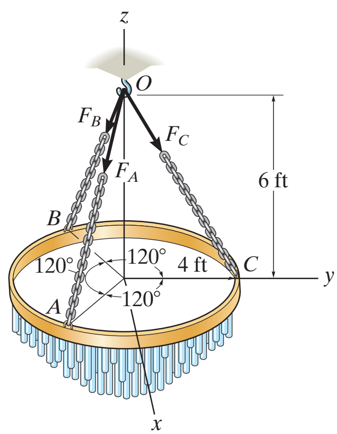
2–107. Express force **F** in Cartesian vector form if point B is located 3 m along the rod from end C.



Prob. 2–107

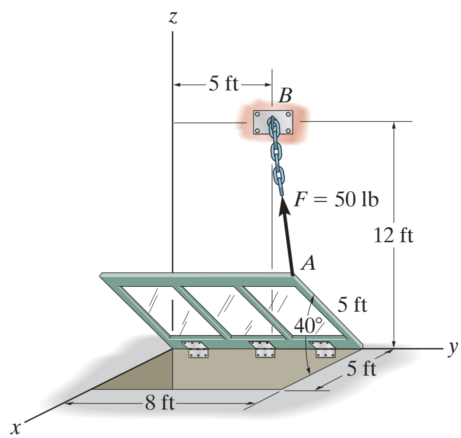
***2–108.** The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.

2–109. The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



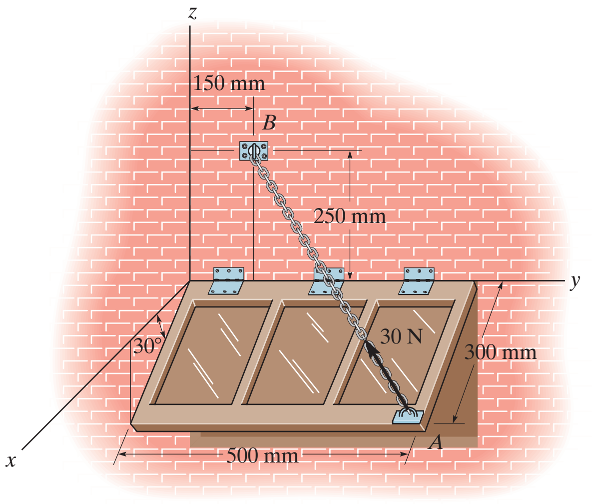
Probs. 2–108/109

2–110. The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



Prob. 2–110

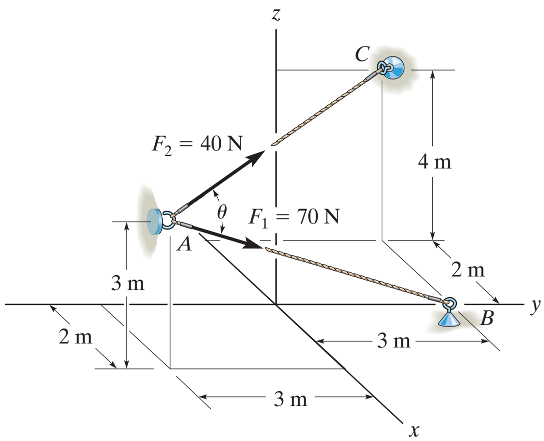
2–111. The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



Prob. 2–111

2–114. Determine the angle θ between the two cables.

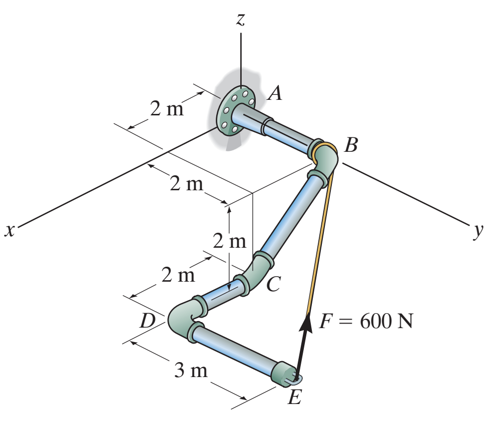
2–115. Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



Probs. 2–114/115

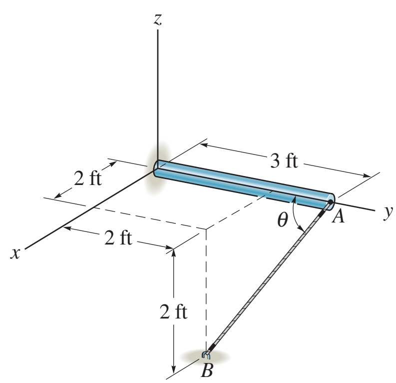
*__2–112.__ Given the three vectors **A, B,** and **D,** show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

2–113. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



Probs. 2–112/113

*__2–116.__ Determine the angle θ between the y axis of the pole and the wire AB.

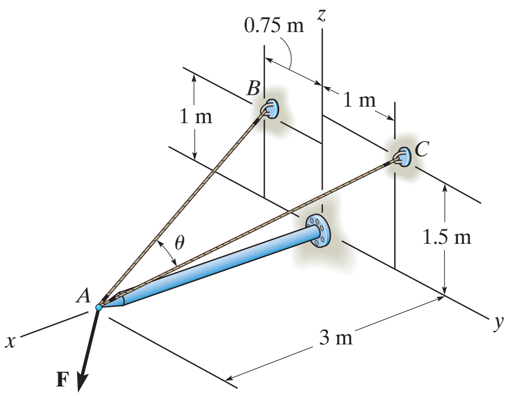


Prob. 2–116

2–117. Determine the magnitudes of the projected components of the force $\mathbf{F} = [60\mathbf{i} + 12\mathbf{j} - 40\mathbf{k}]$ N along the cables AB and AC.

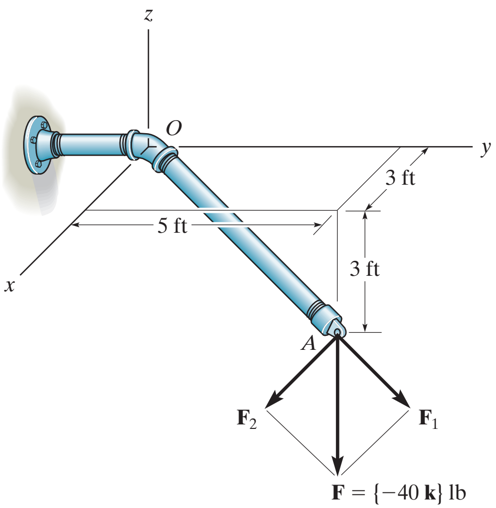
2–118. Determine the angle θ between cables AB and AC.

2



Probs. 2–117/118

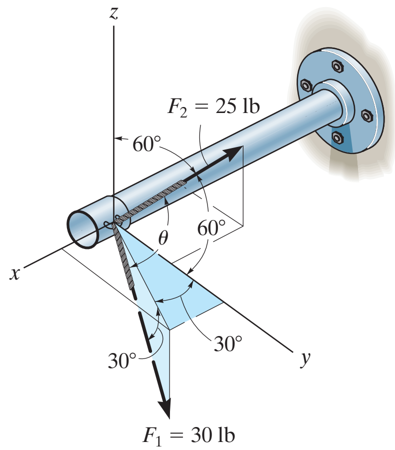
2–119. A force of $\mathbf{F} = \{-40\mathbf{k}\}$ lb acts at the end of the pipe. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which are directed along the pipe's axis and perpendicular to it.



Prob. 2–119

***2–120.** Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.

2–121. Determine the angle θ between the two cables attached to the pipe.

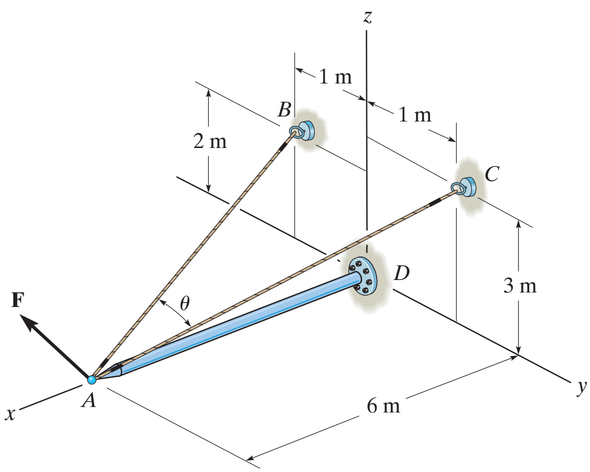


Probs. 2–120/121

2–122. Determine the angle θ between the cables AB and AC.

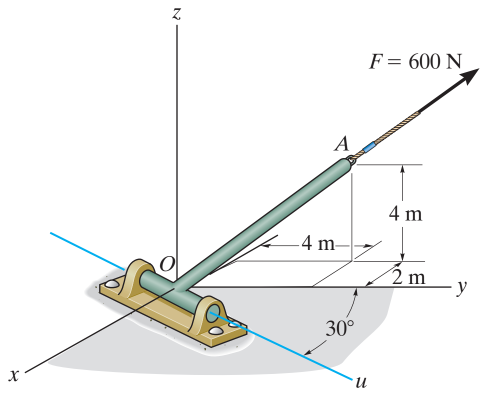
2–123. Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable BA.

***2–124.** Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable CA.



Probs. 2–122/123/124

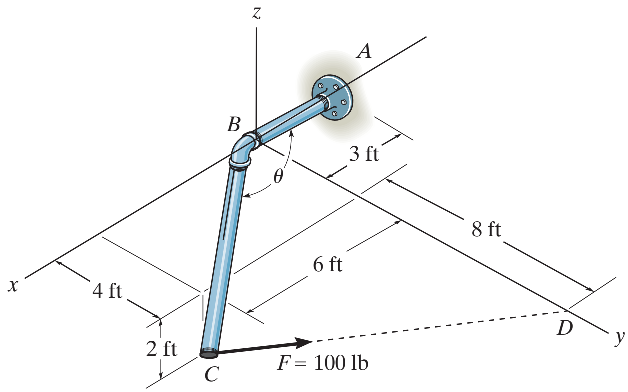
2–125. Determine the magnitude of the projection of force $F = 600$ N along the u axis.



Prob. 2–125

2–126. Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.

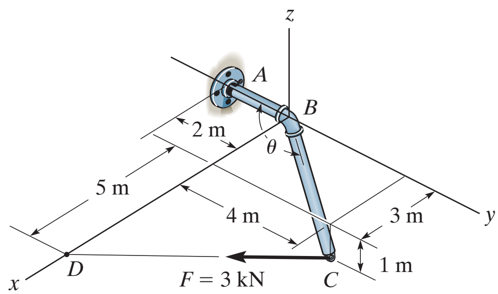
2–127. Determine the angle θ between pipe segments BA and BC.



Probs. 2–126/127

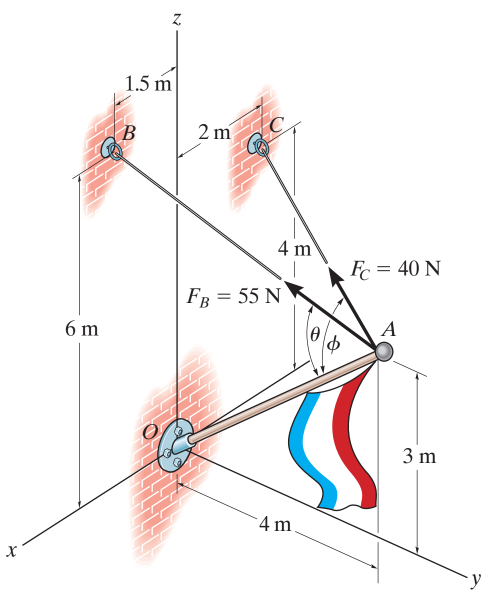
***2–128.** Determine the angle θ between BA and BC.

2–129. Determine the magnitude of the projected component of the 3 kN force acting along the axis BC of the pipe.



Probs. 2–128/129

2–130. Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.

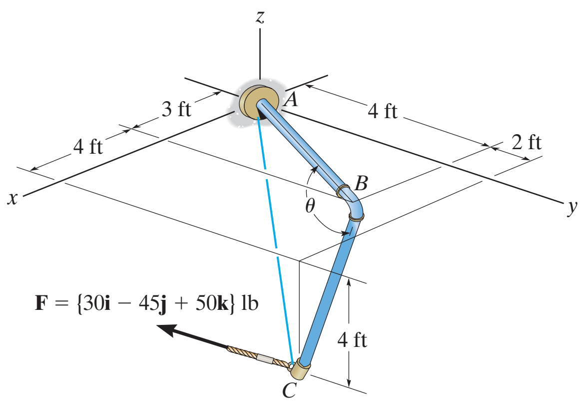


Prob. 2–130

2–131. Determine the magnitudes of the components of **F** acting along and perpendicular to segment BC of the pipe assembly.

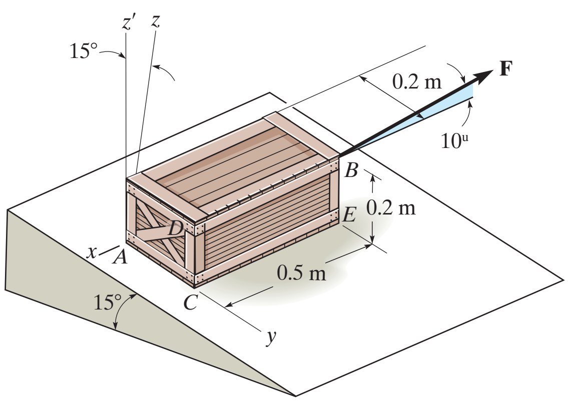
***2–132.** Determine the magnitude of the projected component of **F** along AC. Express this component as a Cartesian vector.

2–133. Determine the angle θ between the pipe segments BA and BC.



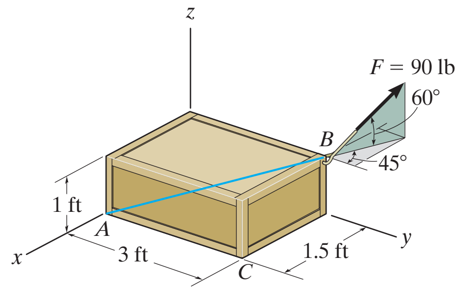
Probs. 2–131/132/133

2–134. If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x–z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that **F** makes with the diagonal AB of the crate.



Prob. 2–134

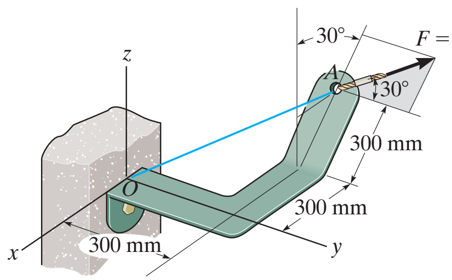
2–135. Determine the magnitudes of the components of the force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



Prob. 2–135

***2–136.** Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.

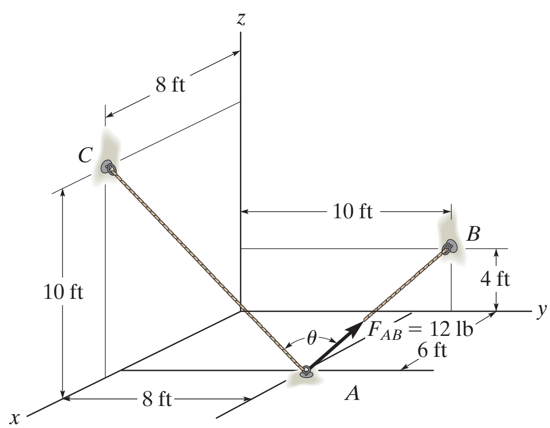
2–137. Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



Probs. 2–136/137

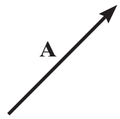
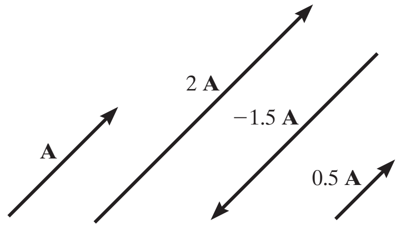
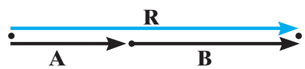
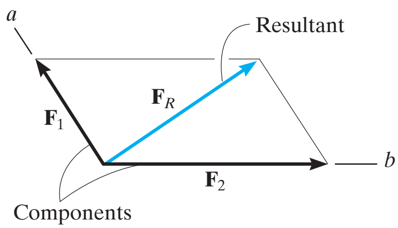
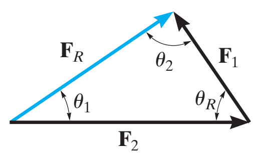
2–138. Determine the angle θ between the two cables.

2–139. Determine the projected component of the force $F = 12$ lb acting in the direction of cable AC. Express the result as a Cartesian vector.



Probs. 2–138/139

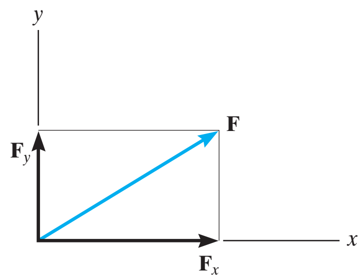
CHAPTER REVIEW

A scalar is a positive or negative number; e.g., mass and temperature. A vector has a magnitude and direction, where the arrowhead represents the sense of the vector.		
Multiplication or division of a vector by a scalar will change only the magnitude of the vector. If the scalar is negative, the sense of the vector will change so that it acts in the opposite sense.		
If vectors are collinear, the resultant is simply the algebraic or scalar addition.	$R = A + B$	
Parallelogram Law Two forces add according to the parallelogram law. The *components* form the sides of the parallelogram and the *resultant* is the diagonal. To find the components of a force along any two axes, extend lines from the head of the force, parallel to the axes, to form the components.		
To obtain the components of the resultant, show how the forces add by tip-to-tail using the triangle rule, and then use the law of cosines and the law of sines to calculate their values.	$F_R = \sqrt{F_1^2 + F_2^2 - 2\,F_1F_2 \cos\theta_R}$ $\dfrac{F_1}{\sin\theta_1} = \dfrac{F_2}{\sin\theta_2} = \dfrac{F_R}{\sin\theta_R}$	

2

Rectangular Components: Two Dimensions

Vectors $\mathbf{F}_x$ and $\mathbf{F}_y$ are rectangular components of $\mathbf{F}$.



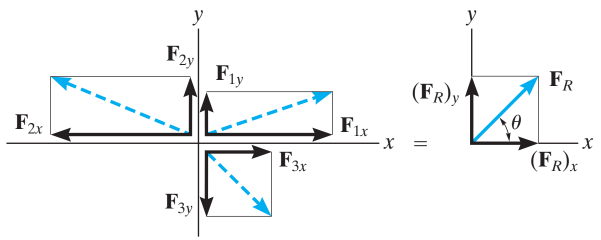
The resultant force is determined from the algebraic sum of its components.

$$(F_R)_x = \Sigma F_x$$

$$(F_R)_y = \Sigma F_y$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

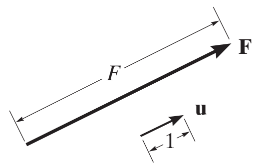
$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$



Cartesian Vectors

The unit vector $\mathbf{u}$ has a length of 1, no units, and it points in the direction of the vector $\mathbf{F}$.

$$\mathbf{u} = \frac{\mathbf{F}}{F}$$



A force can be resolved into its Cartesian components along the x, y, z axes so that $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$.

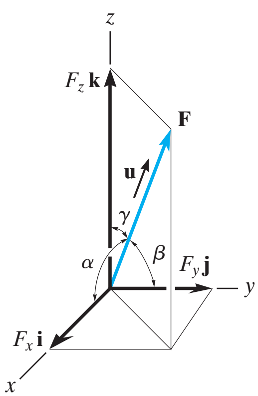
The magnitude of $\mathbf{F}$ is determined from the positive square root of the sum of the squares of its components.

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

The coordinate direction angles α, β, γ are determined by formulating a unit vector in the direction of $\mathbf{F}$. The x, y, z components of $\mathbf{u}$ represent $\cos\alpha, \cos\beta, \cos\gamma$.

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$\mathbf{u} = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$



The coordinate direction angles are related so that only two of the three angles are independent of one another.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

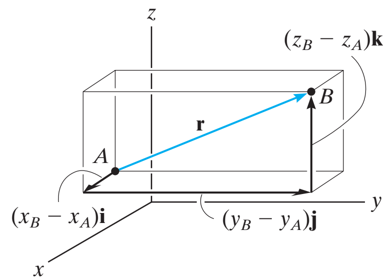
To find the resultant of a concurrent force system, express each force as a Cartesian vector and add the **i**, **j**, **k** components of all the forces in the system.

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

Position and Force Vectors

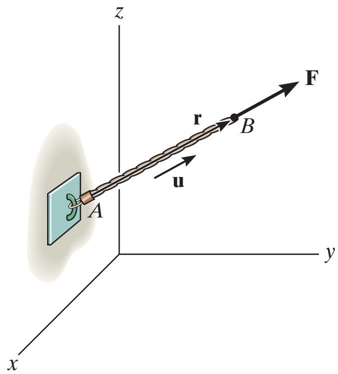
A position vector locates one point in space relative to another. The easiest way to formulate the components of a position vector is to determine the distance and direction that one must travel along the x, y, and z directions—going from the tail to the head of the vector.

$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$



If the line of action of a force passes through points A and B, then the force acts in the same direction as the position vector **r**, which is defined by the unit vector **u**. The force can then be expressed as a Cartesian vector.

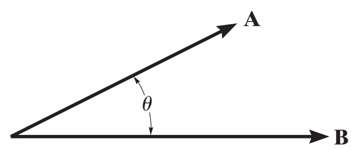
$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$



Dot Product

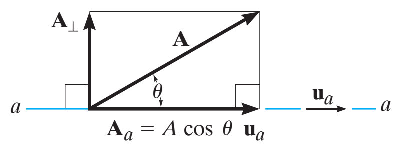
The dot product between two vectors **A** and **B** yields a scalar. If **A** and **B** are expressed in Cartesian vector form, then the dot product is the sum of the products of their x, y, and z components.

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$



The dot product can be used to determine the angle between **A** and **B**.

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$



The dot product is also used to determine the projected component of a vector **A** onto an axis aa defined by its unit vector $\mathbf{u}_a$.

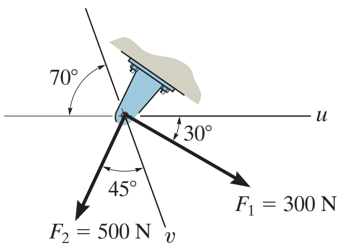
$$\mathbf{A}_a = A\cos\theta\,\mathbf{u}_a = (\mathbf{A}\cdot\mathbf{u}_a)\mathbf{u}_a$$

REVIEW PROBLEMS

Partial solutions and answers to all Review Problems are given in the back of the book.

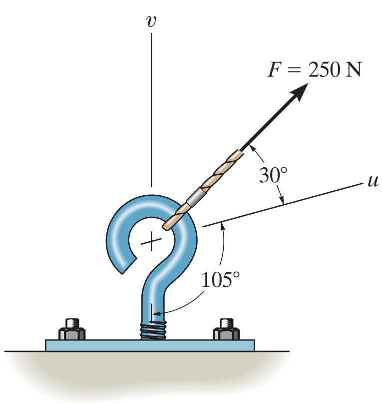
2

R2–1. Determine the magnitude of the resultant force $\mathbf{F}_R$ and its direction, measured clockwise from the positive u axis.



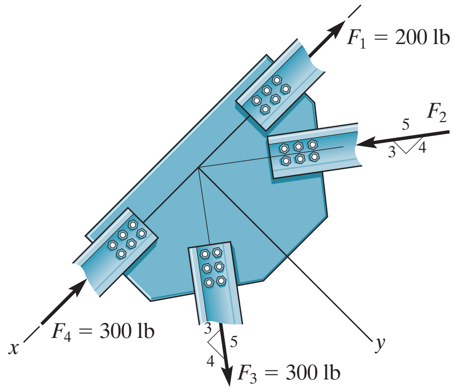
Prob. R2–1

R2–2. Resolve **F** into components along the u and v axes and determine the magnitudes of these components.



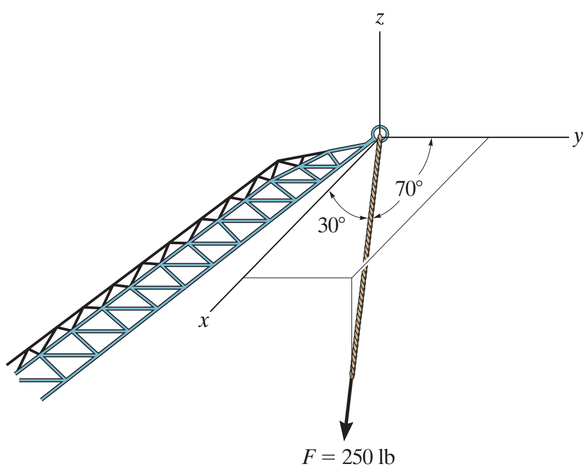
Prob. R2–2

R2–3. Determine the magnitude of the resultant force acting on the *gusset plate* of the bridge truss.



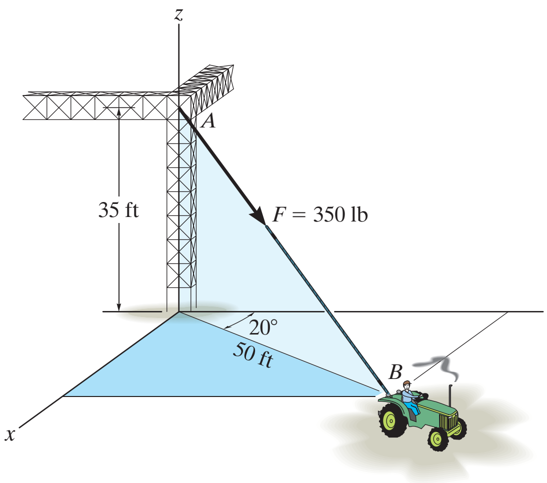
Prob. R2–3

R2–4. The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



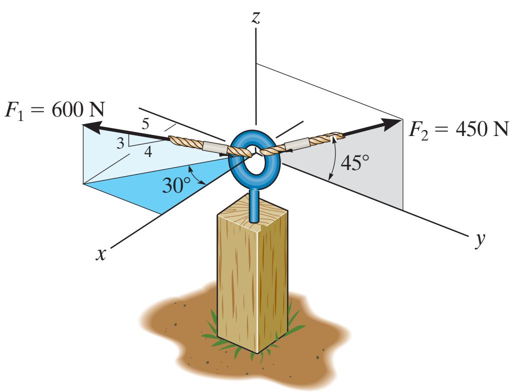
Prob. R2–4

R2–5. The cable attached to the tractor at B exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



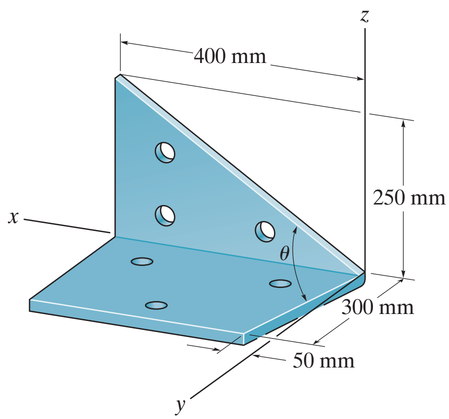
Prob. R2–5

R2–6. Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



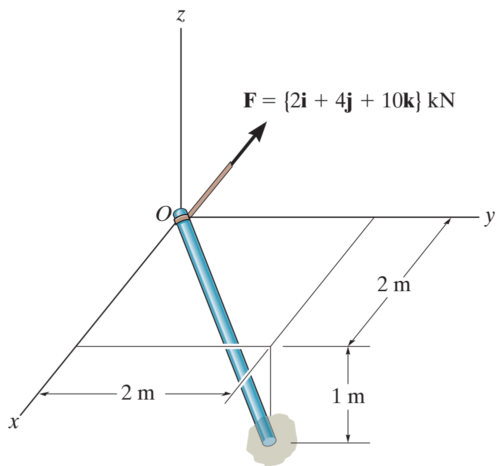
Prob. R2–6

R2–7. Determine the angle θ between the edges of the sheet-metal bracket.



Prob. R2–7

R2–8. Determine the projection of the force $\mathbf{F}$ along the pole.



Prob. R2–8