Chapter 12

(© Lars Johansson/Fotolia)

Although each of these boats is rather large, from a distance their motion can be analyzed as if each were a particle.

Kinematics of a Particle

CHAPTER OBJECTIVES

- To introduce the concepts of position, displacement, velocity, and acceleration.
- To study particle motion along a straight line and represent this motion graphically.
- To investigate particle motion along a curved path using different coordinate systems.
- To present an analysis of dependent motion of two particles.
- To examine the principles of relative motion of two particles using translating axes.

12.1 Introduction

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces. Engineering mechanics is divided into two areas of study, namely, statics and dynamics. ***Statics*** is concerned with the equilibrium of a body that is either at rest or moves with constant velocity. Here we will consider ***dynamics***, which deals with the accelerated motion of a body. The subject of dynamics will be presented in two parts: *kinematics*, which treats only the geometric aspects of the motion, and *kinetics*, which is the analysis of the forces causing the motion. To develop these principles, the dynamics of a particle will be discussed first, followed by topics in rigid-body dynamics in two and then three dimensions.

Historically, the principles of dynamics developed when it was possible to make an accurate measurement of time. Galileo Galilei (1564–1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642–1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D'Alembert, Lagrange, and others.

There are many problems in engineering whose solutions require application of the principles of dynamics. Typically the structural design of any vehicle, such as an automobile or airplane, requires consideration of the motion to which it is subjected. This is also true for many mechanical devices, such as motors, pumps, movable tools, industrial manipulators, and machinery. Furthermore, predictions of the motions of artificial satellites, projectiles, and spacecraft are based on the theory of dynamics. With further advances in technology, there will be an even greater need for knowing how to apply the principles of this subject.

Problem Solving. Dynamics is considered to be more involved than statics since both the forces applied to a body and its motion must be taken into account. Also, many applications require using calculus, rather than just algebra and trigonometry. In any case, the most effective way of learning the principles of dynamics is ***to solve problems***. To be successful at this, it is necessary to present the work in a logical and orderly manner as suggested by the following sequence of steps:

1. Read the problem carefully and try to correlate the actual physical situation with the theory you have studied.
2. Draw any necessary diagrams and tabulate the problem data.
3. Establish a coordinate system and apply the relevant principles, generally in mathematical form.
4. Solve the necessary equations algebraically as far as practical; then, use a consistent set of units and complete the solution numerically. Report the answer with no more significant figures than the accuracy of the given data.
5. Study the answer using technical judgment and common sense to determine whether or not it seems reasonable.
6. Once the solution has been completed, review the problem. Try to think of other ways of obtaining the same solution.

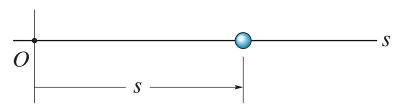
In applying this general procedure, do the work as neatly as possible. Being neat generally stimulates clear and orderly thinking, and vice versa.

12.2 Rectilinear Kinematics: Continuous Motion

We will begin our study of dynamics by discussing the kinematics of a particle that moves along a rectilinear or straight-line path. Recall that a *particle* has a mass but negligible size and shape. Therefore we must limit application to those objects that have dimensions that are of no consequence in the analysis of the motion. In most problems, we will be interested in bodies of finite size, such as rockets, projectiles, or vehicles. Each of these objects can be considered as a particle, as long as the motion is characterized by the motion of its mass center and any rotation of the body is neglected.

Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

Position. The straight-line path of a particle will be defined using a single coordinate axis s, Fig. 12–1a. The origin O on the path is a fixed point, and from this point the ***position coordinate*** s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from O to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s. Although the choice is arbitrary, in this case s is positive since the coordinate axis is positive to the right of the origin. Likewise, it is negative if the particle is located to the left of O. Realize that *position is a vector quantity* since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s, rather than in boldface $\mathbf{s}$, since the direction always remains along the coordinate axis.

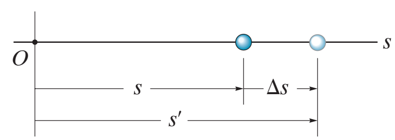


Position

(a)

Displacement. The ***displacement*** of the particle is defined as the *change in its position*. For example, if the particle moves from one point to another, Fig. 12–1b, the displacement is

$$\Delta s = s' - s$$



Displacement

(b)

Fig. 12–1

In this case Δs is *positive* since the particle's final position is to the *right* of its initial position, i.e., $s' > s$. Likewise, if the final position were to the *left* of its initial position, Δs would be *negative*.

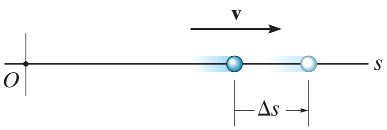
The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels. Specifically, the *distance traveled* is a *positive scalar* that represents the total length of path over which the particle travels.

Velocity. If the particle moves through a displacement Δs during the time interval Δt, the ***average velocity*** of the particle during this time interval is

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the ***instantaneous velocity*** is a vector defined as $v = \lim_{\Delta t \to 0} (\Delta s/\Delta t)$, or

$$(\overset{+}{\rightarrow}) \qquad \boxed{v = \frac{ds}{dt}} \qquad (12\text{–}1)$$



Velocity

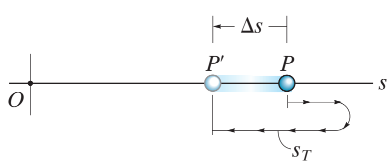
(c)

Since Δt or dt is always positive, the sign used to define the *sense* of the velocity is the same as that of Δs or ds. For example, if the particle is moving to the *right*, Fig. 12–1*c*, the velocity is *positive;* whereas if it is moving to the *left*, the velocity is *negative*. (This is emphasized here by the arrow written at the left of Eq. 12–1.) The *magnitude* of the velocity is known as the ***speed***, and it is generally expressed in units of m/s or ft/s.

Occasionally, the term "average speed" is used. The ***average speed*** is always a positive scalar and is defined as the total distance traveled by a particle, s_T, divided by the elapsed time Δt; i.e.,

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

For example, the particle in Fig. 12–1*d* travels along the path of length s_T in time Δt, so its average speed is $(v_{sp})_{avg} = s_T/\Delta t$, but its average velocity is $v_{avg} = -\Delta s/\Delta t$.



Average velocity and
Average speed

(d)

Fig. 12–1 (cont.)

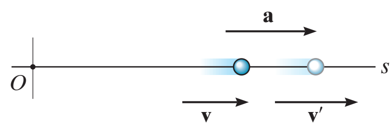
Acceleration. Provided the velocity of the particle is known at two points, the ***average acceleration*** of the particle during the time interval Δt is defined as

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v' - v$, Fig. 12–1*e*.

The ***instantaneous acceleration*** at time t is a *vector* that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$, or

$$(\overset{+}{\rightarrow}) \qquad \boxed{a = \frac{dv}{dt}} \qquad (12\text{–}2)$$



Acceleration

(e)

Substituting Eq. 12–1 into this result, we can also write

$$(\overset{+}{\rightarrow}) \qquad a = \frac{d^2 s}{dt^2}$$

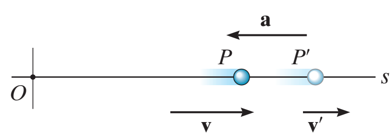
Both the average and instantaneous acceleration can be either positive or negative. In particular, when the particle is *slowing down*, or its speed is decreasing, the particle is said to be ***decelerating***. In this case, v' in Fig. 12–1*f* is *less* than v, and so $\Delta v = v' - v$ will be negative. Consequently, a will also be negative, and therefore it will act to the *left*, in the *opposite sense* to v. Also, notice that if the particle is originally at rest, then it can have an acceleration if a moment later it has a velocity v'; and, if the *velocity* is *constant*, then the *acceleration is zero* since $\Delta v = v - v = 0$. Units commonly used to express the magnitude of acceleration are m/s^2 or ft/s^2.

Finally, an important differential relation involving the displacement, velocity, and acceleration along the path may be obtained by eliminating the time differential dt between Eqs. 12–1 and 12–2. We have

$$dt = \frac{ds}{v} = \frac{dv}{a}$$

or

$$(\overset{+}{\rightarrow}) \qquad \boxed{a\,ds = v\,dv} \qquad (12\text{–}3)$$



Deceleration

(f)

Fig. 12–1 (cont.)

Although we have now produced three important kinematic equations, realize that the above equation is not independent of Eqs. 12–1 and 12–2.

When the ball is released, it has zero velocity but an acceleration of 9.81 m/s^2. (© R.C. Hibbeler)

Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\, ds = v\, dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\, dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} v = v_0 + a_c t \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}4)$$

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\, dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}5)$$

Velocity as a Function of Position. Either solve for t in Eq. 12–4 and substitute into Eq. 12–5, or integrate $v\, dv = a_c\, ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^{v} v\, dv = \int_{s_0}^{s} a_c\, ds$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} v^2 = v_0^2 + 2a_c(s - s_0) \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}6)$$

The algebraic signs of s_0, v_0, and a_c, used in the above three equations, are determined from the positive direction of the s axis as indicated by the arrow written at the left of each equation. Remember that these equations are useful *only when the acceleration is constant and when* $t = 0$, $s = s_0$, $v = v_0$. A typical example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the *downward* acceleration of the body when it is close to the earth is constant and approximately 9.81 m/s^2 or 32.2 ft/s^2. The proof of this is given in Example 13.2.

Important Points

- Dynamics is concerned with bodies that have accelerated motion.
- Kinematics is a study of the geometry of the motion.
- Kinetics is a study of the forces that cause the motion.
- Rectilinear kinematics refers to straight-line motion.
- Speed refers to the magnitude of velocity.
- Average speed is the total distance traveled divided by the total time. This is different from the average velocity, which is the displacement divided by the time.
- A particle that is slowing down is decelerating.
- A particle can have an acceleration and yet have zero velocity.
- The relationship $a\,ds = v\,dv$ is derived from $a = dv/dt$ and $v = ds/dt$, by eliminating dt.

During the time this vvvvket undergoes rectilinear motion, its altitude as a function of time can be measured and expressed as $s = s(t)$. Its velocity can then be found using $v = ds/dt$, and its acceleration can be determined from $a = dv/dt$. (© NASA)

Procedure for Analysis

Coordinate System.

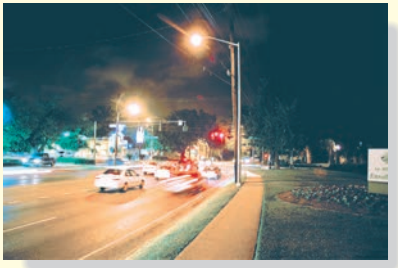
- Establish a position coordinate s along the path and specify its *fixed origin* and positive direction.
- Since motion is along a straight line, the vector quantities position, velocity, and acceleration can be represented as algebraic scalars. For analytical work the sense of s, v, and a is then defined by their *algebraic signs*.
- The positive sense for each of these scalars can be indicated by an arrow shown alongside each kinematic equation as it is applied.

Kinematic Equations.

- If a relation is known between any *two* of the four variables a, v, s, and t, then a third variable can be obtained by using one of the kinematic equations, $a = dv/dt$, $v = ds/dt$ or $a\,ds = v\,dv$, since each equation relates all three variables.*
- Whenever integration is performed, it is important that the position and velocity be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limits of integration if a definite integral is used.
- Remember that Eqs. 12–4 through 12–6 have only limited use. These equations apply *only* when the *acceleration is constant* and the initial conditions are $s = s_0$ and $v = v_0$ when $t = 0$.

*Some standard differentiation and integration formulas are given in Appendix A.

EXAMPLE 12.1



(© R.C. Hibbeler)

The car on the left in the photo and in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

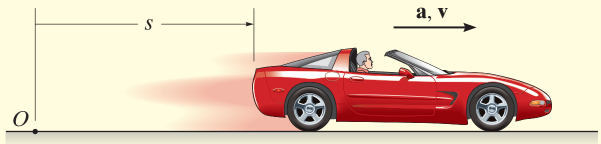


Fig. 12–2

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

*The *same result* can be obtained by evaluating a constant of integration C rather than using definite limits on the integral. For example, integrating $ds = (3t^2 + 2t)dt$ yields $s = t^3 + t^2 + C$. Using the condition that at $t = 0$, $s = 0$, then $C = 0$.

EXAMPLE 12.2

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)\ \text{m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

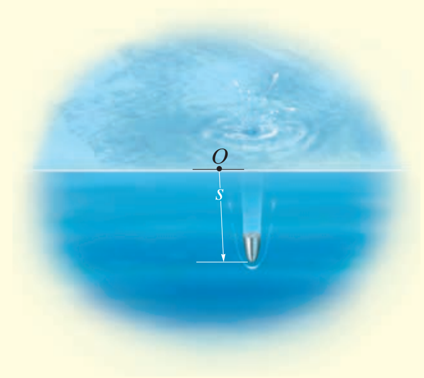


Fig. 12–3

SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O, Fig. 12–3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v, a, and t. (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

$$(+\downarrow) \qquad a = \frac{dv}{dt} = -0.4v^3$$

$$\int_{60\text{ m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\bigg|_{60}^{v} = t - 0$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\}\ \text{m/s}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559\ \text{m/s}\downarrow \qquad \textit{Ans.}$$

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s, v, and t. Using the initial condition $s = 0$, when $t = 0$, we have

$$(+\downarrow) \qquad v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\bigg|_0^t$$

$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\}\ \text{m}$$

When $t = 4$ s,

$$s = 4.43\ \text{m} \qquad \textit{Ans.}$$

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EXAMPLE 12.3

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

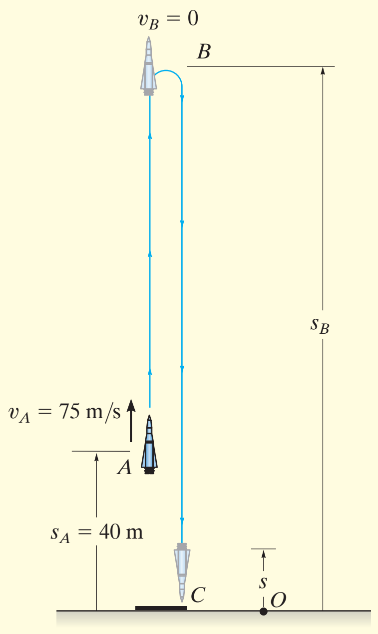

Fig. 12–4

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$ m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s^2 (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$

$$s_B = 327 \text{ m} \qquad \textit{Ans.}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward.

Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of 9.81 m/s^2, and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still 9.81 m/s^2 downward!

EXAMPLE 12.4

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate A to plate B, Fig. 12–5. If the particle is released from rest at the midpoint C, $s = 100$ mm, and the acceleration is $a = (4s)\ \text{m/s}^2$, where s is in meters, determine the velocity of the particle when it reaches plate B, $s = 200$ mm, and the time it takes to travel from C to B.

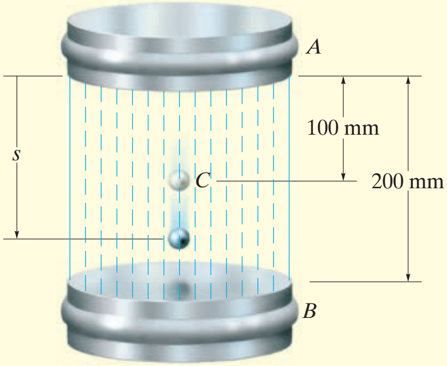

Fig. 12–5

SOLUTION

Coordinate System. As shown in Fig. 12–5, s is positive downward, measured from plate A.

Velocity. Since $a = f(s)$, the velocity as a function of position can be obtained by using $v\,dv = a\,ds$. Realizing that $v = 0$ at $s = 0.1$ m, we have

$(+\downarrow)$

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_{0.1\text{ m}}^s 4s\,ds$$

$$\frac{1}{2}v^2\Big|_0^v = \frac{4}{2}s^2\Big|_{0.1\text{ m}}^s$$

$$v = 2(s^2 - 0.01)^{1/2}\ \text{m/s} \qquad (1)$$

At $s = 200$ mm $= 0.2$ m,

$$v_B = 0.346\ \text{m/s} = 346\ \text{mm/s}\downarrow \qquad \textit{Ans.}$$

The positive root is chosen since the particle is traveling downward, i.e., in the $+s$ direction.

Time. The time for the particle to travel from C to B can be obtained using $v = ds/dt$ and Eq. 1, where $s = 0.1$ m when $t = 0$. From Appendix A,

$(+\downarrow)$

$$ds = v\,dt$$

$$= 2(s^2 - 0.01)^{1/2}dt$$

$$\int_{0.1}^s \frac{ds}{(s^2 - 0.01)^{1/2}} = \int_0^t 2\,dt$$

$$\ln\left(\sqrt{s^2 - 0.01} + s\right)\Big|_{0.1}^s = 2t\Big|_0^t$$

$$\ln\left(\sqrt{s^2 - 0.01} + s\right) + 2.303 = 2t$$

At $s = 0.2$ m,

$$t = \frac{\ln\left(\sqrt{(0.2)^2 - 0.01} + 0.2\right) + 2.303}{2} = 0.658\ \text{s} \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration cannot be used here because the acceleration changes with position, i.e., $a = 4s$.

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EXAMPLE 12.5

A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.

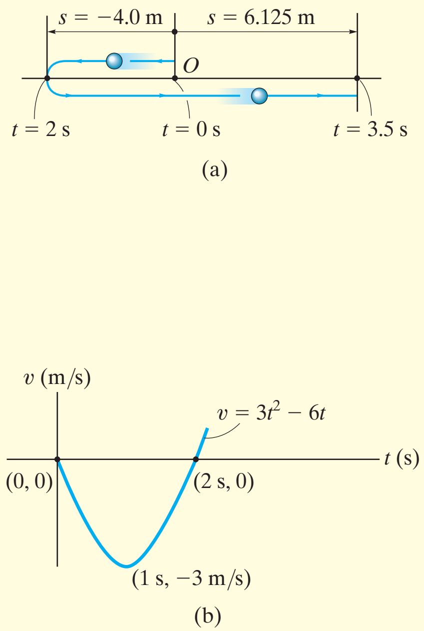


Fig. 12–6

SOLUTION

Coordinate System. Here positive motion is to the right, measured from the origin O, Fig. 12–6*a*.

Distance Traveled. Since $v = f(t)$, the position as a function of time may be found by integrating $v = ds/dt$ with $t = 0$, $s = 0$.

$(\overset{+}{\rightarrow})$

$$ds = v\,dt$$

$$= (3t^2 - 6t)\,dt$$

$$\int_0^s ds = \int_0^t (3t^2 - 6t)\,dt$$

$$s = (t^3 - 3t^2)\text{ m} \qquad (1)$$

In order to determine the distance traveled in 3.5 s, it is necessary to investigate the path of motion. If we consider a graph of the velocity function, Fig. 12–6*b*, then it reveals that for $0 < t < 2$ s the velocity is *negative*, which means the particle is traveling to the *left*, and for $t > 2$ s the velocity is *positive*, and hence the particle is traveling to the *right*. Also, note that $v = 0$ at $t = 2$ s. The particle's position when $t = 0$, $t = 2$ s, and $t = 3.5$ s can be determined from Eq. 1. This yields

$$s|_{t=0} = 0 \quad s|_{t=2\text{ s}} = -4.0\text{ m} \quad s|_{t=3.5\text{ s}} = 6.125\text{ m}$$

The path is shown in Fig. 12–6*a*. Hence, the distance traveled in 3.5 s is

$$s_T = 4.0 + 4.0 + 6.125 = 14.125\text{ m} = 14.1\text{ m} \qquad \textit{Ans.}$$

Velocity. The *displacement* from $t = 0$ to $t = 3.5$ s is

$$\Delta s = s|_{t=3.5\text{ s}} - s|_{t=0} = 6.125\text{ m} - 0 = 6.125\text{ m}$$

and so the average velocity is

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{6.125\text{ m}}{3.5\text{ s} - 0} = 1.75\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

The average speed is defined in terms of the *distance traveled* s_T. This positive scalar is

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{14.125\text{ m}}{3.5\text{ s} - 0} = 4.04\text{ m/s} \qquad \textit{Ans.}$$

NOTE: In this problem, the acceleration is $a = dv/dt = (6t - 6)$ m/s^2, which is not constant.

It is highly suggested that you test yourself on the solutions to these examples, by covering them over and then trying to think about which equations of kinematics must be used and how they are applied in order to determine the unknowns. Then before solving any of the problems, try and solve some of the Preliminary and Fundamental Problems which follow. The solutions and answers to all these problems are given in the back of the book. **Doing this throughout the book will help immensely in understanding how to apply the theory, and thereby develop your problem-solving skills.**

PRELIMINARY PROBLEM

P12–1.

a) If $s = (2t^3)$ m, where t is in seconds, determine v when $t = 2$ s.

b) If $v = (5s)$ m/s, where s is in meters, determine a at $s = 1$ m.

c) If $v = (4t + 5)$ m/s, where t is in seconds, determine a when $t = 2$ s.

d) If $a = 2$ m/s^2, determine v when $t = 2$ s if $v = 0$ when $t = 0$.

e) If $a = 2$ m/s^2, determine v at $s = 4$ m if $v = 3$ m/s at $s = 0$.

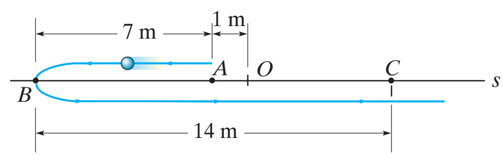
f) If $a = (s)$ m/s^2, where s is in meters, determine v when $s = 5$ m if $v = 0$ at $s = 4$ m.

g) If $a = 4$ m/s^2, determine s when $t = 3$ s if $v = 2$ m/s and $s = 2$ m when $t = 0$.

h) If $a = (8t^2)$ m/s^2, determine v when $t = 1$ s if $v = 0$ at $t = 0$.

i) If $s = (3t^2 + 2)$ m, determine v when $t = 2$ s.

j) When $t = 0$ the particle is at A. In four seconds it travels to B, then in another six seconds it travels to C. Determine the average velocity and the average speed. The origin of the coordinate is at O.



Prob. P12–1

12

FUNDAMENTAL PROBLEMS

F12–1. Initially, the car travels along a straight road with a speed of 35 m/s. If the brakes are applied and the speed of the car is reduced to 10 m/s in 15 s, determine the constant deceleration of the car.

Prob. F12–1

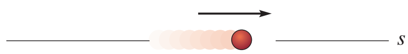
F12–2. A ball is thrown vertically upward with a speed of 15 m/s. Determine the time of flight when it returns to its original position.

Prob. F12–2

F12–3. A particle travels along a straight line with a velocity of $v = (4t - 3t^2)$ m/s, where t is in seconds. Determine the position of the particle when $t = 4$ s. $s = 0$ when $t = 0$.

F12–4. A particle travels along a straight line with a speed $v = (0.5t^3 - 8t)$ m/s, where t is in seconds. Determine the acceleration of the particle when $t = 2$ s.

F12–5. The position of the particle is given by $s = (2t^2 - 8t + 6)$ m, where t is in seconds. Determine the time when the velocity of the particle is zero, and the total distance traveled by the particle when $t = 3$ s.



Prob. F12–5

F12–6. A particle travels along a straight line with an acceleration of $a = (10 - 0.2s)$ m/s^2, where s is measured in meters. Determine the velocity of the particle when $s = 10$ m if $v = 5$ m/s at $s = 0$.

Prob. F12–6

F12–7. A particle moves along a straight line such that its acceleration is $a = (4t^2 - 2)$ m/s^2, where t is in seconds. When $t = 0$, the particle is located 2 m to the left of the origin, and when $t = 2$ s, it is 20 m to the left of the origin. Determine the position of the particle when $t = 4$ s.

F12–8. A particle travels along a straight line with a velocity of $v = (20 - 0.05s^2)$ m/s, where s is in meters. Determine the acceleration of the particle at $s = 15$ m.

PROBLEMS

12–1. Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)$ m/s^2, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

12–2. If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s^2 to the left.

12–3. A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

***12–4.** A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

12–5. The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

12–6. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint*: Plot the path to determine the total distance traveled.

12–7. A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

***12–8.** A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

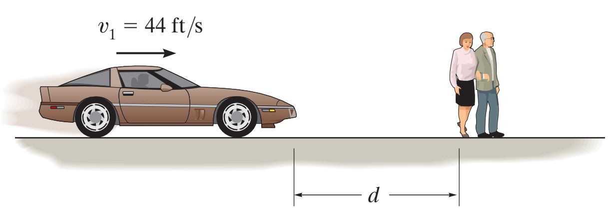
12–9. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)$ m/s^2, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

12–10. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

12–11. A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

***12–12.** Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

12–13. Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



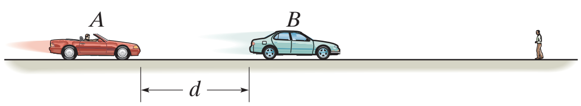
Prob. 12–13

12–14. The position of a particle along a straight-line path is defined by $s = (t^3 - 6t^2 - 15t + 7)$ ft, where t is in seconds. Determine the total distance traveled when $t = 10$ s. What are the particle's average velocity, average speed, and the instantaneous velocity and acceleration at this time?

12–15. A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

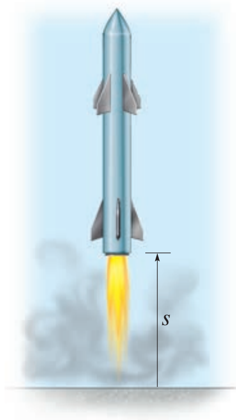
***12–16.** A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

12–17. Car B is traveling a distance d ahead of car A. Both cars are traveling at 60 ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at 12 ft/s^2. It takes the driver of car A 0.75 s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at 15 ft/s^2. Determine the minimum distance d be tween the cars so as to avoid a collision.



Prob. 12–17

12–18. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.

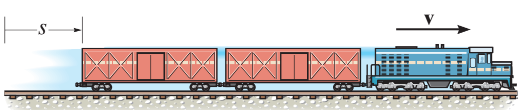


Prob. 12–18

12–19. A train starts from rest at station A and accelerates at 0.5 m/s^2 for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at 1 m/s^2 until it is brought to rest at station B. Determine the distance between the stations.

***12–20.** The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

12–21. A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.



Prob. 12–21

12–22. A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

12–23. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)$ m/s^2, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

***12–24.** The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

12–25. If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

12–26. The acceleration of a particle along a straight line is defined by $a = (2t - 9)$ m/s^2, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

12–27. When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2_f)(v^2_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

***12–28.** Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)$ ft/s^2 and $a_B = (12t^2 - 8)$ ft/s^2, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

12–29. A ball A is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

12–30. A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)$ m/s^2, where t is in seconds, determine the distance traveled before it stops.

12–31. The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

***12–32.** Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

12–33. As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81$ m/s^2 and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

12–34. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–36), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–33.

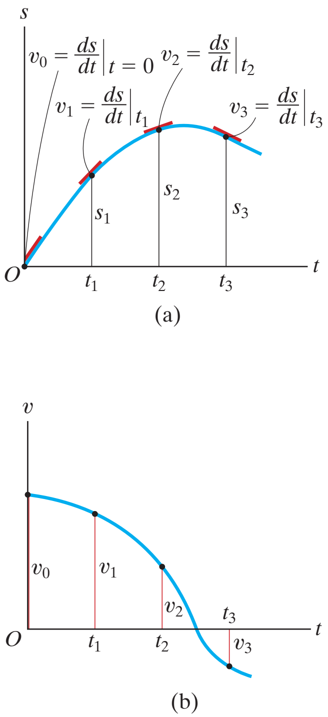


Fig. 12–7

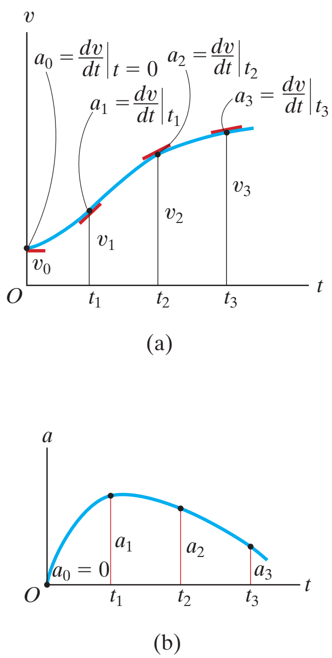


Fig. 12–8

12.3 Rectilinear Kinematics: Erratic Motion

When a particle has erratic or changing motion then its position, velocity, and acceleration *cannot* be described by a single continuous mathematical function along the entire path. Instead, a series of functions will be required to specify the motion at different intervals. For this reason, it is convenient to represent the motion as a graph. If a graph of the motion that relates any two of the variables s, v, a, t can be drawn, then this graph can be used to construct subsequent graphs relating two other variables since the variables are related by the differential relationships $v = ds/dt$, $a = dv/dt$, or $a\,ds = v\,dv$. Several situations occur frequently.

The *s–t*, *v–t*, and *a–t* Graphs. To construct the v–t graph given the s–t graph, Fig. 12–7*a*, the equation $v = ds/dt$ should be used, since it relates the variables s and t to v. This equation states that

$$\frac{ds}{dt} = v$$

$$\text{slope of } s\text{–}t \text{ graph} = \text{velocity}$$

For example, by measuring the slope on the s–t graph when $t = t_1$, the velocity is v_1, which is plotted in Fig. 12–7*b*. The v–t graph can be constructed by plotting this and other values at each instant.

The a–t graph can be constructed from the v–t graph in a similar manner, Fig. 12–8, since

$$\frac{dv}{dt} = a$$

$$\text{slope of } v\text{–}t \text{ graph} = \text{acceleration}$$

Examples of various measurements are shown in Fig. 12–8*a* and plotted in Fig. 12–8*b*.

If the s–t curve for each interval of motion can be expressed by a mathematical function $s = s(t)$, then the equation of the v–t graph for the same interval can be obtained by differentiating this function with respect to time since $v = ds/dt$. Likewise, the equation of the a–t graph for the same interval can be determined by differentiating $v = v(t)$ since $a = dv/dt$. Since differentiation reduces a polynomial of degree n to that of degree $n - 1$, then if the s–t graph is parabolic (a second-degree curve), the v–t graph will be a sloping line (a first-degree curve), and the a–t graph will be a constant or a horizontal line (a zero-degree curve).

If the a–t graph is given, Fig. 12–9a, the v–t graph may be constructed using $a = dv/dt$, written as

$$\Delta v = \int a\, dt$$

$$\begin{array}{ccc} \text{change in velocity} & = & \text{area under } a\text{–}t \text{ graph} \end{array}$$

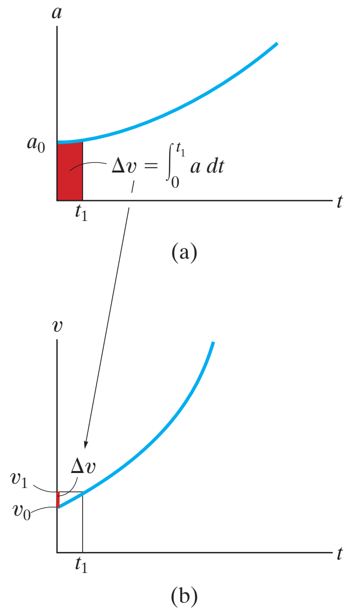


Fig. 12–9

Hence, to construct the v–t graph, we begin with the particle's initial velocity v_0 and then add to this small increments of area (Δv) determined from the a–t graph. In this manner successive points, $v_1 = v_0 + \Delta v$, etc., for the v–t graph are determined, Fig. 12–9b. Notice that an algebraic addition of the area increments of the a–t graph is necessary, since areas lying above the t axis correspond to an increase in v ("positive" area), whereas those lying below the axis indicate a decrease in v ("negative" area).

Similarly, if the v–t graph is given, Fig. 12–10a, it is possible to determine the s–t graph using $v = ds/dt$, written as

$$\Delta s = \int v\, dt$$

$$\begin{array}{ccc} \text{displacement} & = & \text{area under } v\text{–}t \text{ graph} \end{array}$$

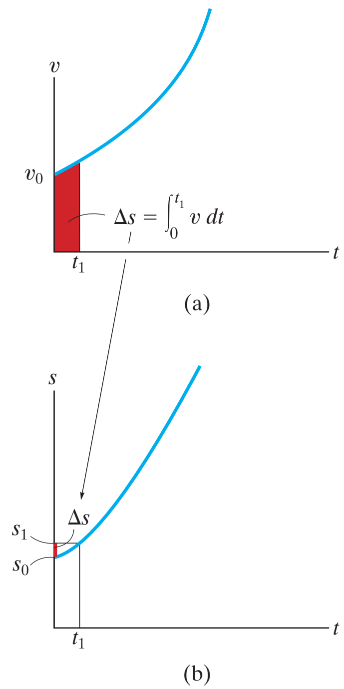


Fig. 12–10

In the same manner as stated above, we begin with the particle's initial position s_0 and add (algebraically) to this small area increments Δs determined from the v–t graph, Fig. 12–10b.

If segments of the a–t graph can be described by a series of equations, then each of these equations can be *integrated* to yield equations describing the corresponding segments of the v–t graph. In a similar manner, the s–t graph can be obtained by integrating the equations which describe the segments of the v–t graph. As a result, if the a–t graph is linear (a first-degree curve), integration will yield a v–t graph that is parabolic (a second-degree curve) and an s–t graph that is cubic (third-degree curve).

12

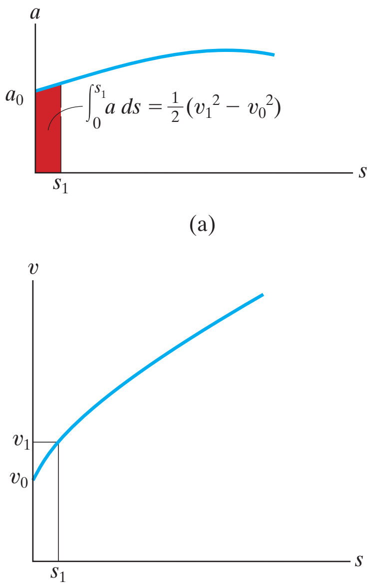


Fig. 12–11

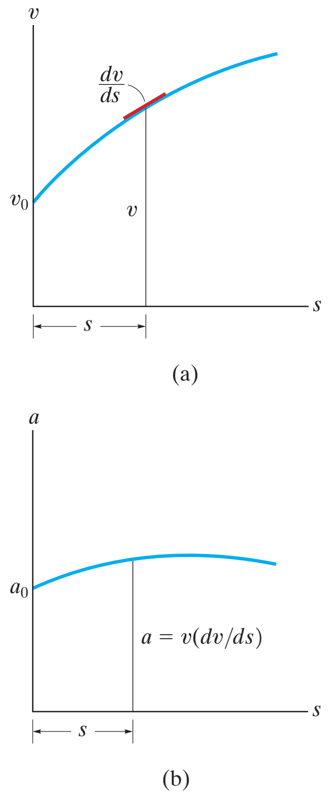


Fig. 12–12

The *v–s* and *a–s* Graphs.

If the a–s graph can be constructed, then points on the v–s graph can be determined by using $v\,dv = a\,ds$. Integrating this equation between the limits $v = v_0$ at $s = s_0$ and $v = v_1$ at $s = s_1$, we have,

$$\frac{1}{2}(v_1^2 - v_0^2) = \int_{s_0}^{s_1} a\,ds$$

area under
a–s graph

Therefore, if the red area in Fig. 12–11a is determined, and the initial velocity v_0 at $s_0 = 0$ is known, then $v_1 = \left(2\int_0^{s_1} a\,ds + v_0^2\right)^{1/2}$, Fig. 12–11$b$. Successive points on the v–s graph can be constructed in this manner.

If the v–s graph is known, the acceleration a at any position s can be determined using $a\,ds = v\,dv$, written as

$$a = v\left(\frac{dv}{ds}\right)$$

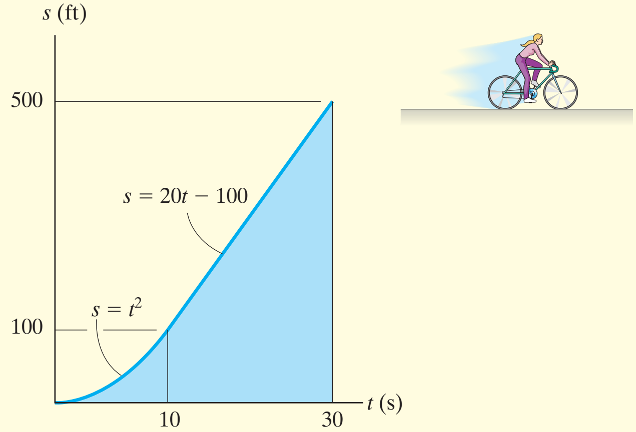
acceleration = velocity times slope of v–s graph

Thus, at any point (s, v) in Fig. 12–12a, the slope dv/ds of the v–s graph is measured. Then with v and dv/ds known, the value of a can be calculated, Fig. 12–12b.

The v–s graph can also be constructed from the a–s graph, or vice versa, by approximating the known graph in various intervals with mathematical functions, $v = f(s)$ or $a = g(s)$, and then using $a\,ds = v\,dv$ to obtain the other graph.

EXAMPLE 12.6

A bicycle moves along a straight road such that its position is described by the graph shown in Fig. 12–13*a*. Construct the v–t and a–t graphs for $0 \leq t \leq 30$ s.



(a)

SOLUTION

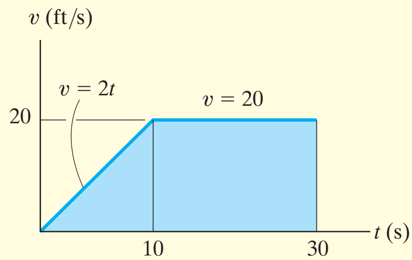
v–t Graph. Since $v = ds/dt$, the v–t graph can be determined by differentiating the equations defining the s–t graph, Fig. 12–13*a*. We have

$$0 \leq t < 10 \text{ s}; \qquad s = (t^2) \text{ ft} \qquad v = \frac{ds}{dt} = (2t) \text{ ft/s}$$

$$10 \text{ s} < t \leq 30 \text{ s}; \qquad s = (20t - 100) \text{ ft} \qquad v = \frac{ds}{dt} = 20 \text{ ft/s}$$

The results are plotted in Fig. 12–13*b*. We can also obtain specific values of v by measuring the *slope* of the s–t graph at a given instant. For example, at $t = 20$ s, the slope of the s–t graph is determined from the straight line from 10 s to 30 s, i.e.,

$$t = 20 \text{ s}; \qquad v = \frac{\Delta s}{\Delta t} = \frac{500 \text{ ft} - 100 \text{ ft}}{30 \text{ s} - 10 \text{ s}} = 20 \text{ ft/s}$$



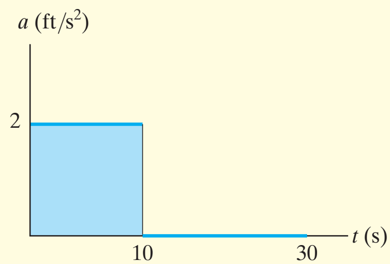
(b)

a–t Graph. Since $a = dv/dt$, the a–t graph can be determined by differentiating the equations defining the lines of the v–t graph. This yields

$$0 \leq t < 10 \text{ s}; \qquad v = (2t) \text{ ft/s} \qquad a = \frac{dv}{dt} = 2 \text{ ft/s}^2$$

$$10 < t \leq 30 \text{ s}; \qquad v = 20 \text{ ft/s} \qquad a = \frac{dv}{dt} = 0$$

The results are plotted in Fig. 12–13*c*.



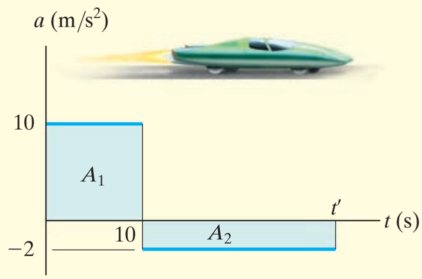
(c)

Fig. 12–13

NOTE: Show that $a = 2 \text{ ft/s}^2$ when $t = 5$ s by measuring the slope of the v–t graph.

12

EXAMPLE 12.7



(a)

The car in Fig. 12–14*a* starts from rest and travels along a straight track such that it accelerates at 10 m/s^2 for 10 s, and then decelerates at 2 m/s^2. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?

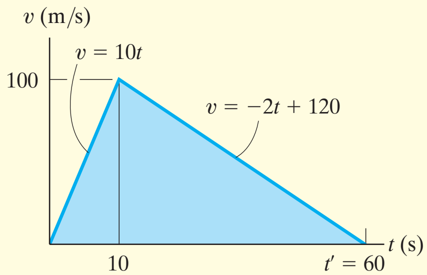
SOLUTION

***v–t* Graph.** Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

$$0 \le t < 10 \text{ s}; \qquad a = (10) \text{ m/s}^2; \qquad \int_0^v dv = \int_0^t 10\,dt, \qquad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

$$10 \text{ s} < t \le t';\ a = (-2) \text{ m/s}^2; \int_{100 \text{ m/s}}^{v} dv = \int_{10 \text{ s}}^{t} -2\,dt,\ v = (-2t + 120) \text{ m/s}$$



(b)

When $t = t'$ we require $v = 0$. This yields, Fig. 12–14*b*,

$$t' = 60 \text{ s} \qquad \textit{Ans.}$$

A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–14*a*. Thus

$$0 = 10 \text{ m/s}^2(10 \text{ s}) + (-2 \text{ m/s}^2)(t' - 10 \text{ s})$$

$$t' = 60 \text{ s} \qquad \textit{Ans.}$$

***s–t* Graph.** Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$$0 \le t \le 10 \text{ s}; \qquad v = (10t) \text{ m/s}; \qquad \int_0^s ds = \int_0^t 10t\,dt, \qquad s = (5t^2) \text{ m}$$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

$$10 \text{ s} \le t \le 60 \text{ s};\ v = (-2t + 120) \text{ m/s}; \int_{500 \text{ m}}^{s} ds = \int_{10 \text{ s}}^{t} (-2t + 120)\,dt$$

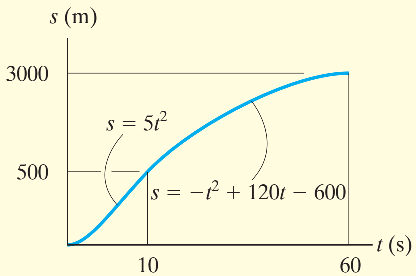
$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

$$s = (-t^2 + 120t - 600) \text{ m}$$

When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000 \text{ m} \qquad \textit{Ans.}$$

The s–t graph is shown in Fig. 12–14*c*.



(c)

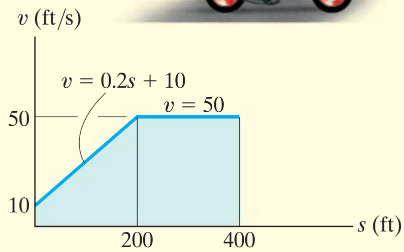
Fig. 12–14

NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60 \text{ s})(100 \text{ m/s}) = 3000 \text{ m} \qquad \textit{Ans.}$$

EXAMPLE 12.8

The v–s graph describing the motion of a motorcycle is shown in Fig. 12–15a. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 400$ ft.



(a)

SOLUTION

a–s Graph. Since the equations for segments of the v–s graph are given, the a–s graph can be determined using $a\,ds = v\,dv$.

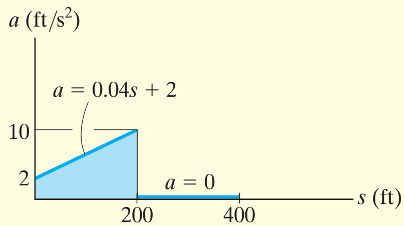
$0 \le s < 200$ ft; $\qquad v = (0.2s + 10)$ ft/s

$$a = v\frac{dv}{ds} = (0.2s + 10)\frac{d}{ds}(0.2s + 10) = 0.04s + 2$$

$200 \text{ ft} < s \le 400$ ft; $\qquad v = 50$ ft/s

$$a = v\frac{dv}{ds} = (50)\frac{d}{ds}(50) = 0$$

The results are plotted in Fig. 12–15b.



(b)

Fig. 12–15

Time. The time can be obtained using the v–s graph and $v = ds/dt$, because this equation relates v, s, and t. For the first segment of motion, $s = 0$ when $t = 0$, so

$0 \le s < 200$ ft; $\qquad v = (0.2s + 10)$ ft/s; $\qquad dt = \dfrac{ds}{v} = \dfrac{ds}{0.2s + 10}$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s + 10}$$

$$t = (5\ln(0.2s + 10) - 5\ln 10)\text{ s}$$

At $s = 200$ ft, $t = 5\ln[0.2(200) + 10] - 5\ln 10 = 8.05$ s. Therefore, using these initial conditions for the second segment of motion,

$200 \text{ ft} < s \le 400$ ft; $\qquad v = 50$ ft/s; $\qquad dt = \dfrac{ds}{v} = \dfrac{ds}{50}$

$$\int_{8.05\text{ s}}^t dt = \int_{200\text{ m}}^s \frac{ds}{50};$$

$$t - 8.05 = \frac{s}{50} - 4; \quad t = \left(\frac{s}{50} + 4.05\right)\text{s}$$

Therefore, at $s = 400$ ft,

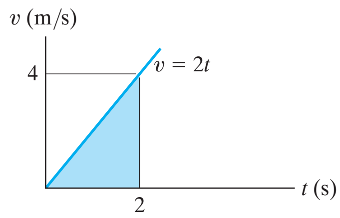
$$t = \frac{400}{50} + 4.05 = 12.0\text{ s} \qquad \textit{Ans.}$$

NOTE: The graphical results can be checked in part by calculating slopes. For example, at $s = 0$, $a = v(dv/ds) = 10(50 - 10)/200 = 2\text{ m/s}^2$. Also, the results can be checked in part by inspection. The v–s graph indicates the initial increase in velocity (acceleration) followed by constant velocity ($a = 0$).

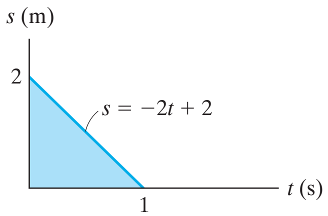
PRELIMINARY PROBLEM

P12–2.

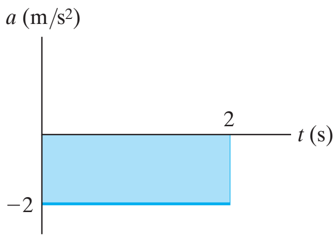
a) Draw the s–t and a–t graphs if $s = 0$ when $t = 0$.



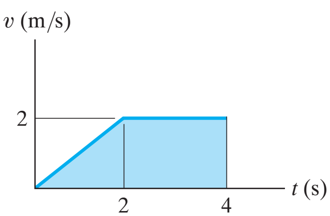
b) Draw the a–t and v–t graphs.



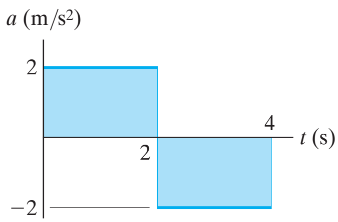
c) Draw the v–t and s–t graphs if $v = 0$, $s = 0$ when $t = 0$.



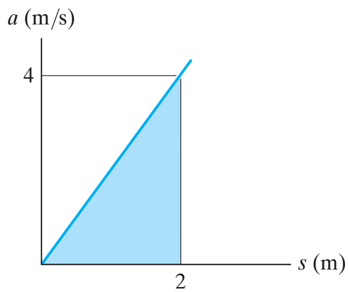
d) Determine s and a when $t = 3$ s if $s = 0$ when $t = 0$.



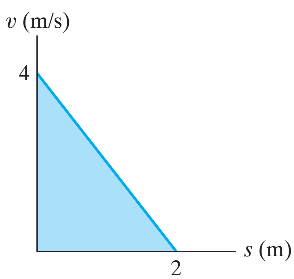
e) Draw the v–t graph if $v = 0$ when $t = 0$. Find the equation $v = f(t)$ for each segment.



f) Determine v at $s = 2$ m if $v = 1$ m/s at $s = 0$.



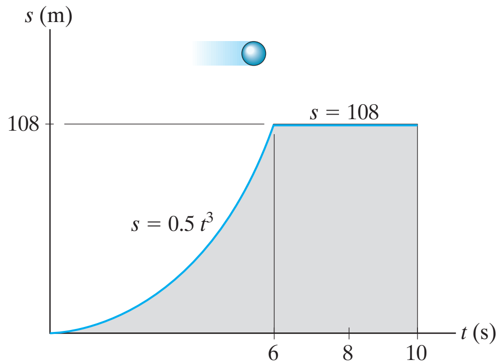
g) Determine a at $s = 1$ m.



Prob. P12–2

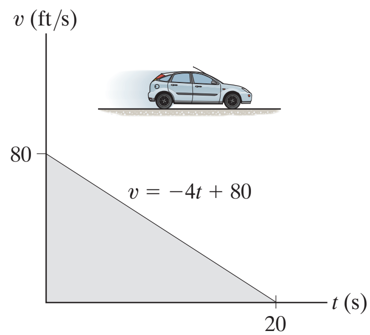
FUNDAMENTAL PROBLEMS

F12–9. The particle travels along a straight track such that its position is described by the s–t graph. Construct the v–t graph for the same time interval.



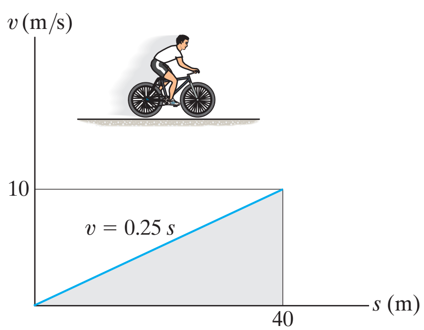
Prob. F12–9

F12–10. A van travels along a straight road with a velocity described by the graph. Construct the s–t and a–t graphs during the same period. Take $s = 0$ when $t = 0$.



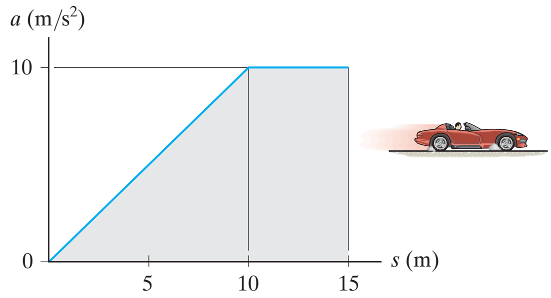
Prob. F12–10

F12–11. A bicycle travels along a straight road where its velocity is described by the v–s graph. Construct the a–s graph for the same interval.



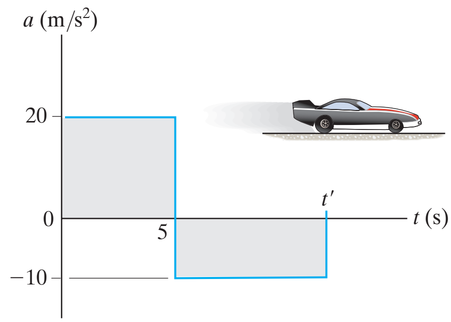
Prob. F12–11

F12–12. The sports car travels along a straight road such that its acceleration is described by the graph. Construct the v–s graph for the same interval and specify the velocity of the car when $s = 10$ m and $s = 15$ m.



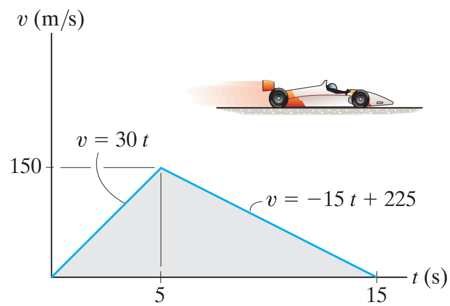
Prob. F12–12

F12–13. The dragster starts from rest and has an acceleration described by the graph. Construct the v–t graph for the time interval $0 \leq t \leq t'$, where t' is the time for the car to come to rest.



Prob. F12–13

F12–14. The dragster starts from rest and has a velocity described by the graph. Construct the s–t graph during the time interval $0 \leq t \leq 15$ s. Also, determine the total distance traveled during this time interval.

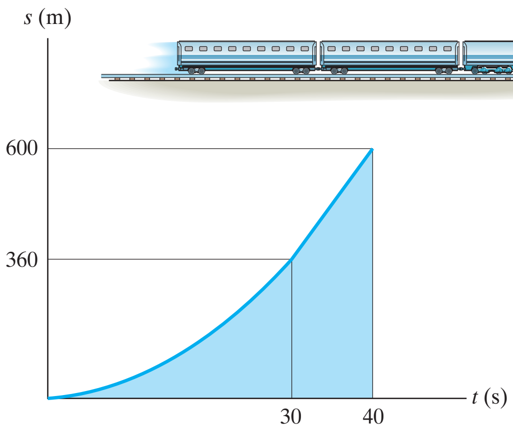


Prob. F12–14

PROBLEMS

12–35. A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

***12–36.** The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \leq t \leq 40$ s. For $0 \leq t \leq 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \geq 30$ s.



Prob. 12–36

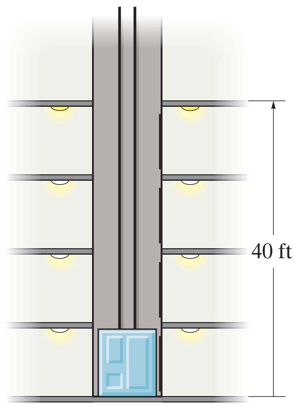
12–37. Two rockets start from rest at the same elevation. Rocket A accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket B accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s. Construct the a–t, v–t, and s–t graphs for each rocket until $t = 20$ s. What is the distance between the rockets when $t = 20$ s?

12–38. A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the v–t and a–t graphs for the time interval $0 \leq t \leq 4$ s.

12–39. If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 10$ s.

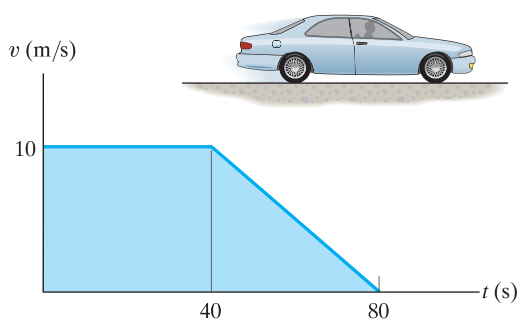
***12–40.** An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the s–t, v–t, and a–t graphs that describe the motion.

12–41. The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



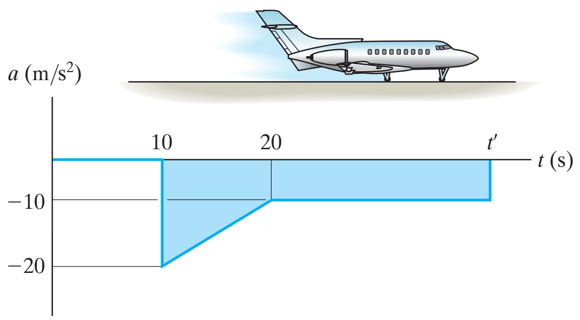
Prob. 12–41

12–42. The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



Prob. 12–42

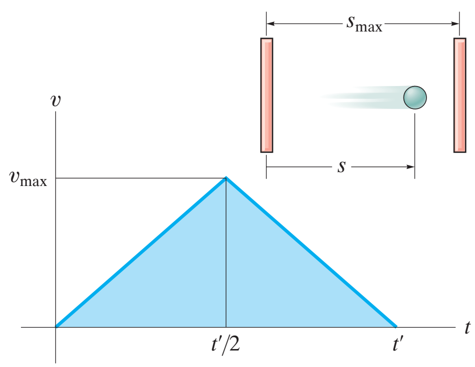
12–43. The motion of a jet plane just after landing on a runway is described by the a–t graph. Determine the time t' when the jet plane stops. Construct the v–t and s–t graphs for the motion. Here $s = 0$, and $v = 300$ ft/s when $t = 0$.



Prob. 12–43

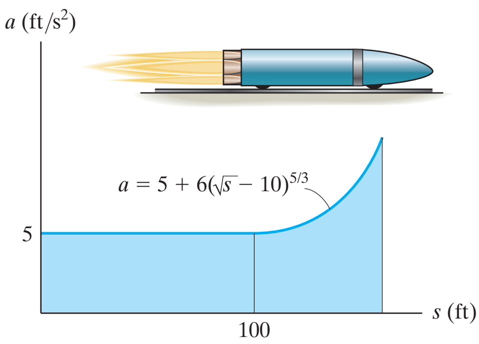
***12–44.** The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of $4\ \text{m/s}^2$. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.

12–45. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



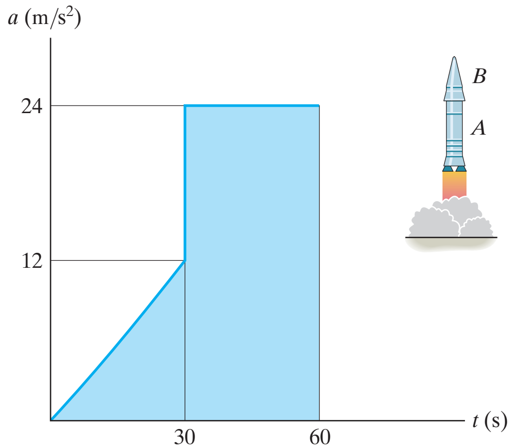
Probs. 12–44/45

12–46. The a–s graph for a rocket moving along a straight track has been experimentally determined. If the rocket starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



Prob. 12–46

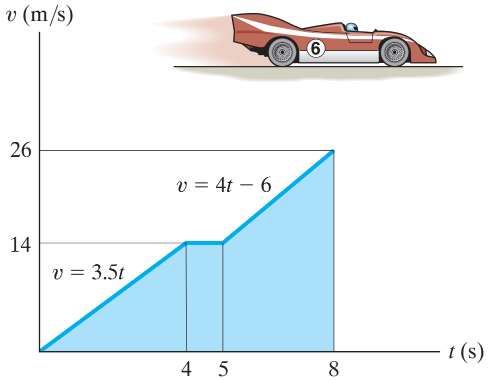
12–47. A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A, burns out and the second stage, B, ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



Prob. 12–47

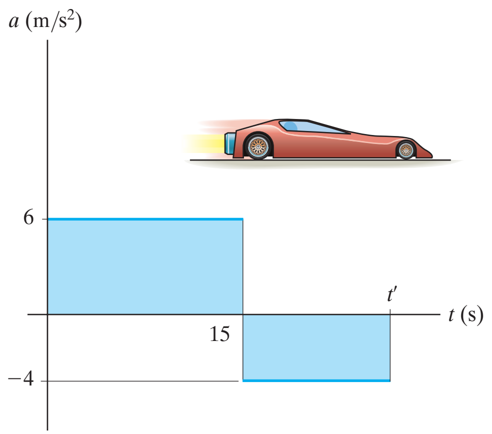
12

***12–48.** The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v–t graph. The flat part of the graph is caused by shifting gears. Draw the a–t graph and determine the maximum acceleration of the car.



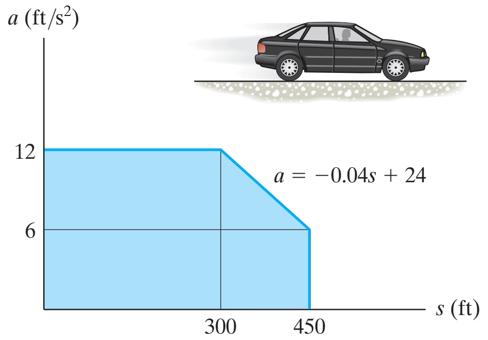
Prob. 12–48

12–49. The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0$, $s = 0$.



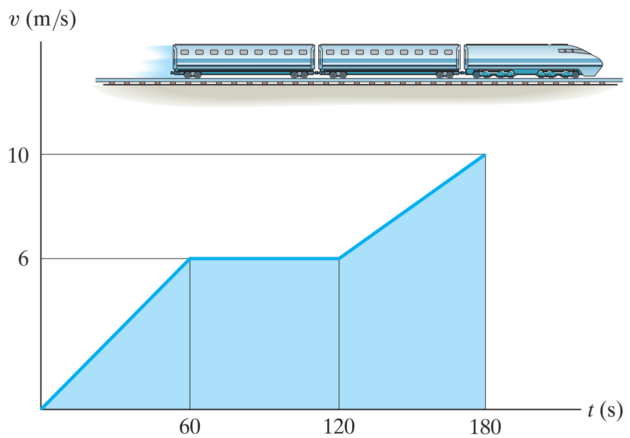
Prob. 12–49

12–50. The car starts from rest at $s = 0$ and is subjected to an acceleration shown by the a–s graph. Draw the v–s graph and determine the time needed to travel 200 ft.



Prob. 12–50

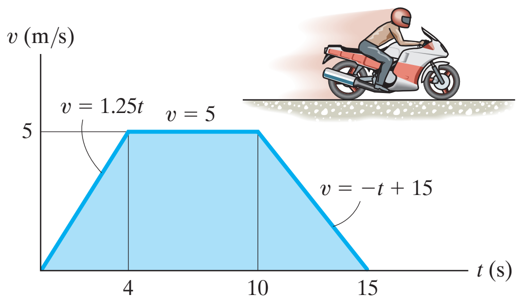
12–51. The v–t graph for a train has been experimentally determined. From the data, construct the s–t and a–t graphs for the motion for $0 \le t \le 180$ s. When $t = 0$, $s = 0$.



Prob. 12–51

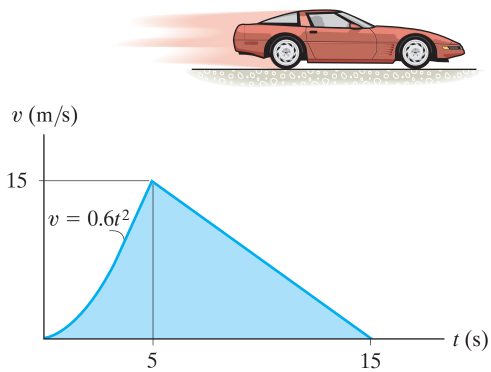
***12–52.** A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.

12–53. A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



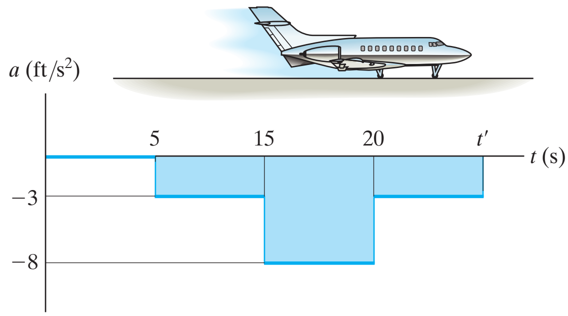
Probs. 12–52/53

12–54. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t and a–t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0$, $s = 0$.



Prob. 12–54

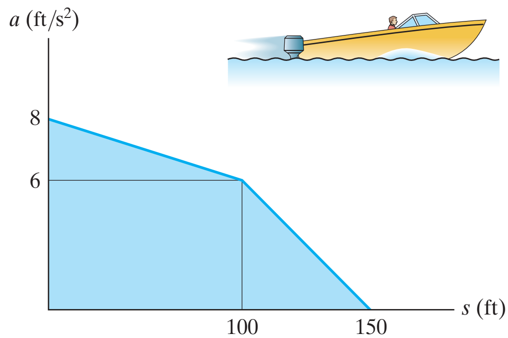
12–55. An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



Prob. 12–55

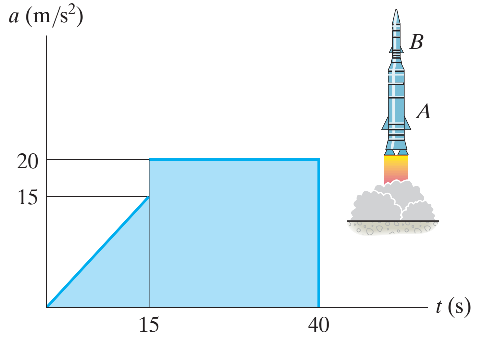
***12–56.** Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Determine the boat's speed when $s = 50$ ft, 100 ft, and 150 ft.

12–57. Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Construct the v–s graph.



Probs. 12–56/57

12–58. A two-stage rocket is fired vertically from rest with the acceleration shown. After 15 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 40$ s.



Prob. 12–58

12–59. The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

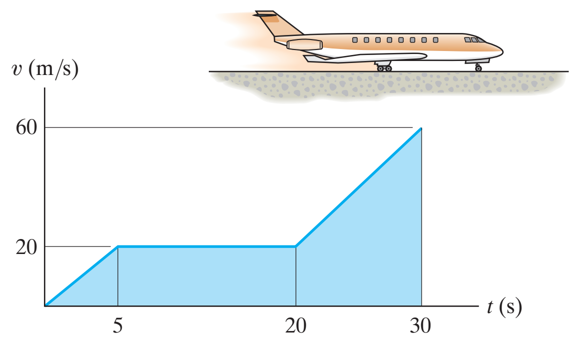
Plot the v–t graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

***12–60.** A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

12–61. Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

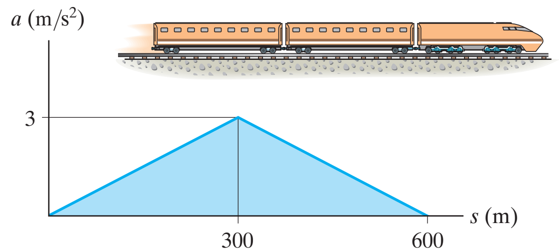
12–62. If the position of a particle is defined as $s = (5t - 3t^2)$ ft, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 10$ s.

12–63. From experimental data, the motion of a jet plane while traveling along a runway is defined by the v–t graph. Construct the s–t and a–t graphs for the motion. When $t = 0, s = 0$.



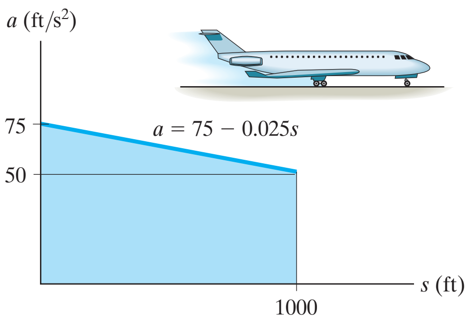
Prob. 12–63

***12–64.** The motion of a train is described by the a–s graph shown. Draw the v–s graph if $v = 0$ at $s = 0$.



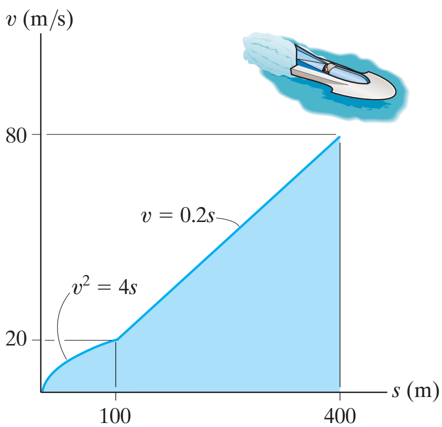
Prob. 12–64

12–65. The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 1000 ft. Also, how much time is required for it to travel 1000 ft?



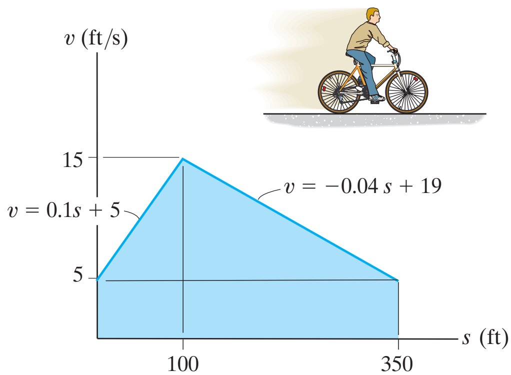
Prob. 12–65

12–66. The boat travels along a straight line with the speed described by the graph. Construct the s–t and a–s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



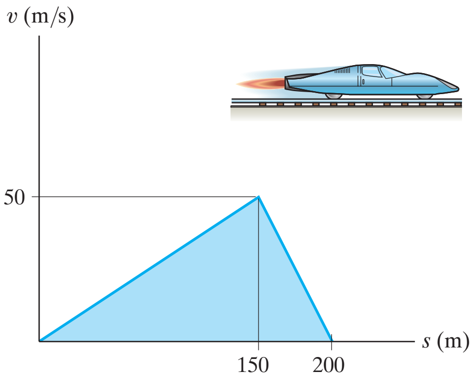
Prob. 12–66

12–67. The v–s graph of a cyclist traveling along a straight road is shown. Construct the a–s graph.



Prob. 12–67

***12–68.** The v–s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



Prob. 12–68

12.4 General Curvilinear Motion

Curvilinear motion occurs when a particle moves along a curved path. Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration.* In this section the general aspects of curvilinear motion are discussed, and in subsequent sections we will consider three types of coordinate systems often used to analyze this motion.

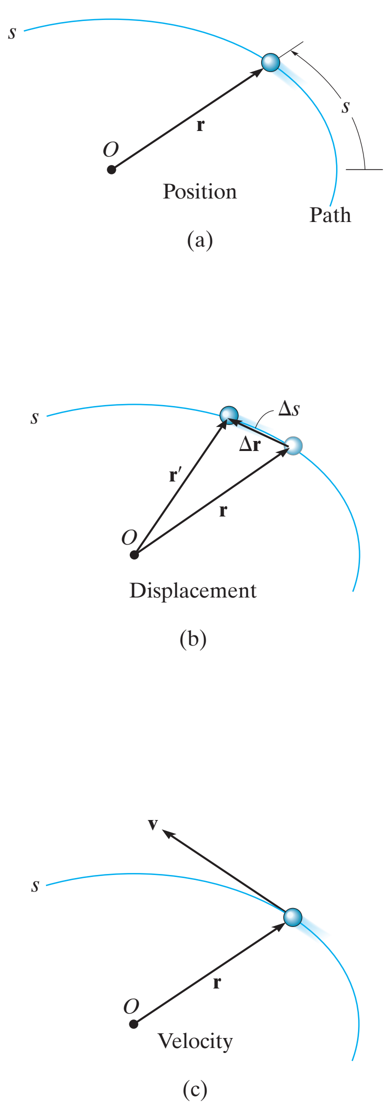


Fig. 12–16

Position. Consider a particle located at a point on a space curve defined by the path function $s(t)$, Fig. 12–16*a*. The position of the particle, measured from a fixed point O, will be designated by the *position vector* $\mathbf{r} = \mathbf{r}(t)$. Notice that both the magnitude and direction of this vector will change as the particle moves along the curve.

Displacement. Suppose that during a small time interval Δt the particle moves a distance Δs along the curve to a new position, defined by $\mathbf{r}' = \mathbf{r} + \Delta\mathbf{r}$, Fig. 12–16*b*. The *displacement* $\Delta\mathbf{r}$ represents the change in the particle's position and is determined by vector subtraction; i.e., $\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}$.

Velocity. During the time Δt, the *average velocity* of the particle is

$$\mathbf{v}_{\text{avg}} = \frac{\Delta\mathbf{r}}{\Delta t}$$

The *instantaneous velocity* is determined from this equation by letting $\Delta t \to 0$, and consequently the direction of $\Delta\mathbf{r}$ *approaches* the *tangent* to the curve. Hence, $\mathbf{v} = \lim_{\Delta t\to 0}(\Delta\mathbf{r}/\Delta t)$ or

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt}} \tag{12–7}$$

Since $d\mathbf{r}$ will be tangent to the curve, the *direction* of $\mathbf{v}$ is also *tangent to the curve*, Fig. 12–16*c*. The *magnitude* of $\mathbf{v}$, which is called the *speed*, is obtained by realizing that the length of the straight line segment $\Delta\mathbf{r}$ in Fig. 12–16*b* approaches the arc length Δs as $\Delta t \to 0$, we have $v = \lim_{\Delta t\to 0}(\Delta r/\Delta t) = \lim_{\Delta t\to 0}(\Delta s/\Delta t)$, or

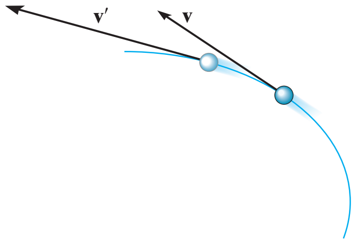
$$\boxed{v = \frac{ds}{dt}} \tag{12–8}$$

Thus, the *speed* can be obtained by differentiating the path function s with respect to time.

*A summary of some of the important concepts of vector analysis is given in Appendix B.

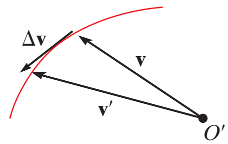
Acceleration. If the particle has a velocity $\mathbf{v}$ at time t and a velocity $\mathbf{v}' = \mathbf{v} + \Delta\mathbf{v}$ at $t + \Delta t$, Fig. 12–16*d*, then the *average acceleration* of the particle during the time interval Δt is

$$\mathbf{a}_{\text{avg}} = \frac{\Delta\mathbf{v}}{\Delta t}$$



(d)

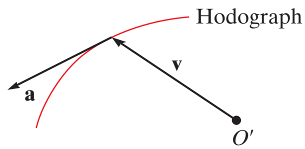
where $\Delta\mathbf{v} = \mathbf{v}' - \mathbf{v}$. To study this time rate of change, the two velocity vectors in Fig. 12–16*d* are plotted in Fig. 12–16*e* such that their tails are located at the fixed point O' and their arrowheads touch points on a curve. This curve is called a *hodograph*, and when constructed, it describes the locus of points for the arrowhead of the velocity vector in the same manner as the *path s* describes the locus of points for the arrowhead of the position vector, Fig. 12–16*a*.



(e)

To obtain the *instantaneous acceleration*, let $\Delta t \to 0$ in the above equation. In the limit $\Delta\mathbf{v}$ will approach the *tangent to the hodograph*, and so $\mathbf{a} = \lim_{\Delta t \to 0} (\Delta\mathbf{v}/\Delta t)$, or

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt}} \qquad (12\text{–}9)$$

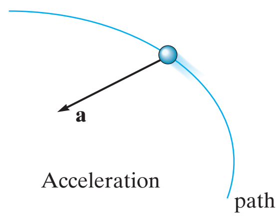


(f)

Substituting Eq. 12–7 into this result, we can also write

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$

By definition of the derivative, $\mathbf{a}$ acts *tangent to the hodograph*, Fig. 12–16*f*, and, *in general it is not tangent to the path of motion*, Fig. 12–16*g*. To clarify this point, realize that $\Delta\mathbf{v}$ and consequently $\mathbf{a}$ must account for the change made in *both* the magnitude *and* direction of the velocity $\mathbf{v}$ as the particle moves from one point to the next along the path, Fig. 12–16*d*. However, in order for the particle to follow any curved path, the directional change always "swings" the velocity vector toward the "inside" or "concave side" of the path, and therefore $\mathbf{a}$ *cannot* remain tangent to the path. In summary, $\mathbf{v}$ is always tangent to the *path* and $\mathbf{a}$ is always tangent to the *hodograph*.



(g)

Fig. 12–16

12.5 Curvilinear Motion: Rectangular Components

Occasionally the motion of a particle can best be described along a path that can be expressed in terms of its x, y, z coordinates.

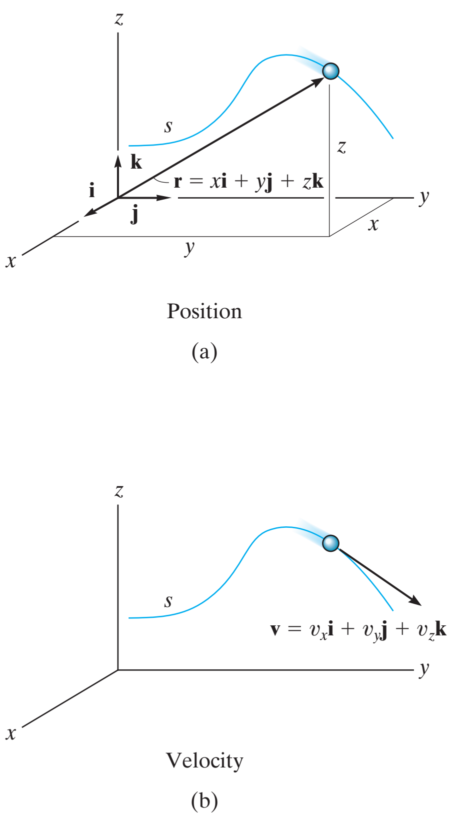


Fig. 12–17

Position. If the particle is at point (x, y, z) on the curved path s shown in Fig. 12–17*a*, then its location is defined by the *position vector*

$$\boxed{\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}} \qquad (12\text{–}10)$$

When the particle moves, the x, y, z components of $\mathbf{r}$ will be functions of time; i.e., $x = x(t)$, $y = y(t)$, $z = z(t)$, so that $\mathbf{r} = \mathbf{r}(t)$.

At any instant the *magnitude* of $\mathbf{r}$ is defined from Eq. B–3 in Appendix B as

$$r = \sqrt{x^2 + y^2 + z^2}$$

And the *direction* of $\mathbf{r}$ is specified by the unit vector $\mathbf{u}_r = \mathbf{r}/r$.

Velocity. The first time derivative of $\mathbf{r}$ yields the velocity of the particle. Hence,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$

When taking this derivative, it is necessary to account for changes in *both* the magnitude and direction of each of the vector's components. For example, the derivative of the $\mathbf{i}$ component of $\mathbf{r}$ is

$$\frac{d}{dt}(x\mathbf{i}) = \frac{dx}{dt}\mathbf{i} + x\frac{d\mathbf{i}}{dt}$$

The second term on the right side is zero, provided the x, y, z reference frame is *fixed*, and therefore the *direction* (and the *magnitude*) of $\mathbf{i}$ does not change with time. Differentiation of the $\mathbf{j}$ and $\mathbf{k}$ components may be carried out in a similar manner, which yields the final result,

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}} \qquad (12\text{–}11)$$

where

$$\boxed{v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}} \qquad (12\text{–}12)$$

The "dot" notation $\dot{x}$, $\dot{y}$, $\dot{z}$ represents the first time derivatives of $x = x(t)$, $y = y(t)$, $z = z(t)$, respectively.

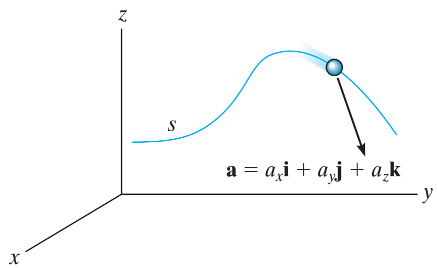
The velocity has a *magnitude* that is found from

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

and a *direction* that is specified by the unit vector $\mathbf{u}_v = \mathbf{v}/v$. As discussed in Sec. 12.4, this direction is *always tangent to the path*, as shown in Fig. 12–17*b*.

Acceleration. The acceleration of the particle is obtained by taking the first time derivative of Eq. 12–11 (or the second time derivative of Eq. 12–10). We have

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}} \qquad (12\text{–}13)$$



Acceleration

(c)

where

$$\boxed{\begin{aligned} a_x &= \dot{v}_x = \ddot{x} \\ a_y &= \dot{v}_y = \ddot{y} \\ a_z &= \dot{v}_z = \ddot{z} \end{aligned}} \qquad (12\text{–}14)$$

Here a_x, a_y, a_z represent, respectively, the first time derivatives of $v_x = v_x(t)$, $v_y = v_y(t)$, $v_z = v_z(t)$, or the second time derivatives of the functions $x = x(t)$, $y = y(t)$, $z = z(t)$.

The acceleration has a *magnitude*

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

and a *direction* specified by the unit vector $\mathbf{u}_a = \mathbf{a}/a$. Since $\mathbf{a}$ represents the time rate of *change* in both the magnitude and direction of the velocity, in general $\mathbf{a}$ will *not* be tangent to the path, Fig. 12–17*c*.

Important Points

- Curvilinear motion can cause changes in *both* the magnitude and direction of the position, velocity, and acceleration vectors.
- The velocity vector is always directed *tangent* to the path.
- In general, the acceleration vector is *not* tangent to the path, but rather, it is tangent to the hodograph.
- If the motion is described using rectangular coordinates, then the components along each of the axes do not change direction, only their magnitude and sense (algebraic sign) will change.
- By considering the component motions, the change in magnitude and direction of the particle's position and velocity are automatically taken into account.

Procedure for Analysis

Coordinate System.

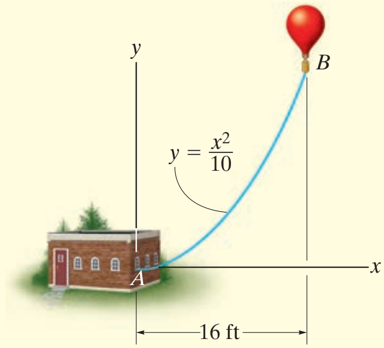
- A rectangular coordinate system can be used to solve problems for which the motion can conveniently be expressed in terms of its x, y, z components.

Kinematic Quantities.

- Since *rectilinear motion* occurs along *each coordinate axis*, the motion along each axis is found using $v = ds/dt$ and $a = dv/dt$; or in cases where the motion is not expressed as a function of time, the equation $a\,ds = v\,dv$ can be used.
- In two dimensions, the equation of the path $y = f(x)$ can be used to relate the x and y components of velocity and acceleration by applying the chain rule of calculus. A review of this concept is given in Appendix C.
- Once the x, y, z components of $\mathbf{v}$ and $\mathbf{a}$ have been determined, the magnitudes of these vectors are found from the Pythagorean theorem, Eq. B-3, and their coordinate direction angles from the components of their unit vectors, Eqs. B-4 and B-5.

EXAMPLE 12.9

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



(a)

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. When $t = 2$ s, $x = 8(2) = 16$ ft, Fig. 12–18*a*, and so

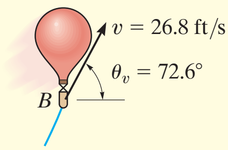
$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18*b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6^\circ \qquad \textit{Ans.}$$



(b)

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

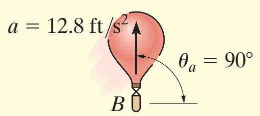
$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90^\circ \qquad \textit{Ans.}$$



(c)

Fig. 12–18

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (8t)^2/10 = 6.4t^2$ and then taking successive time derivatives.

EXAMPLE 12.10

(© R.C. Hibbeler)

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant upward velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of $y = 100$ m.

SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, due to constant velocity $v_y = 10$ m/s, so

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$y = 0.001x^2$$

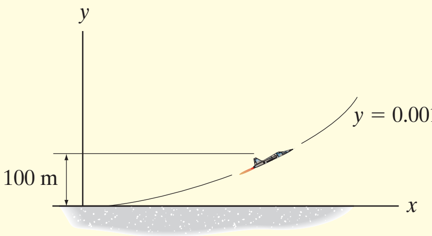
$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002x v_x \qquad (1)$$

Thus

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \qquad \textit{Ans.}$$



(a)

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = (0.002\dot{x})\dot{x} + 0.002x(\ddot{x}) = 0.002(v_x^2 + xa_x)$$

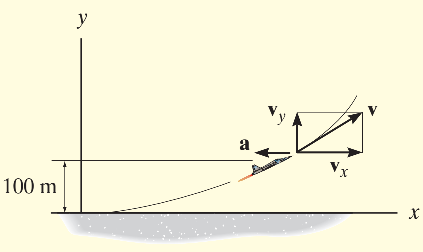
When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002\left[(15.81\text{ m/s})^2 + 316.2\text{ m}(a_x)\right]$$
$$a_x = -0.791\text{ m/s}^2$$

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2 \qquad \textit{Ans.}$$

These results are shown in Fig. 12–19*b*.



(b)

Fig. 12–19

12.6 Motion of a Projectile

The free-flight motion of a projectile is often studied in terms of its rectangular components. To illustrate the kinematic analysis, consider a projectile launched at point (x_0, y_0), with an initial velocity of $\mathbf{v}_0$, having components $(\mathbf{v}_0)_x$ and $(\mathbf{v}_0)_y$, Fig. 12–20. When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a *constant downward acceleration* of approximately $a_c = g = 9.81 \text{ m/s}^2$ or $g = 32.2 \text{ ft/s}^2$.*

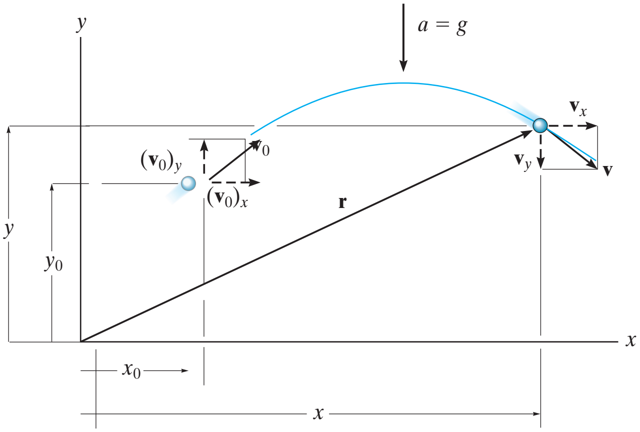
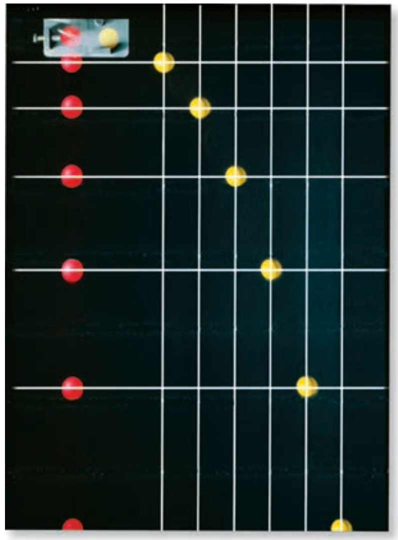


Fig. 12–20



Each picture in this sequence is taken after the same time interval. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity when released. Both balls accelerate downward at the same rate, and so they remain at the same elevation at any instant. This acceleration causes the difference in elevation between the balls to increase between successive photos. Also, note the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction remains constant. (© R.C. Hibbeler)

Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t \qquad\qquad v_x = (v_0)_x$$

$$(\xrightarrow{+}) \qquad x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad\qquad x = x_0 + (v_0)_x t$$

$$(\xrightarrow{+}) \qquad v^2 = v_0^2 + 2a_c(x - x_0); \qquad\qquad v_x = (v_0)_x$$

The first and last equations indicate that *the horizontal component of velocity always remains constant during the motion*.

Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

$$(+\uparrow) \qquad v = v_0 + a_c t; \qquad\qquad v_y = (v_0)_y - gt$$

$$(+\uparrow) \qquad y = y_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad\qquad y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$

$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(y - y_0); \qquad\qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

Recall that the last equation can be formulated on the basis of eliminating the time t from the first two equations, and therefore *only two of the above three equations are independent of one another*.

*This assumes that the earth's gravitational field does not vary with altitude.

12

Once thrown, the basketball follows a parabolic trajectory. (© R.C. Hibbeler)

To summarize, problems involving the motion of a projectile can have at most three unknowns since only three independent equations can be written; that is, *one* equation in the *horizontal direction* and *two* in the *vertical direction*. Once $\mathbf{v}_x$ and $\mathbf{v}_y$ are obtained, the resultant velocity $\mathbf{v}$, which is *always tangent* to the path, can be determined by the *vector sum* as shown in Fig. 12–20.

Procedure for Analysis

Coordinate System.

- Establish the fixed x, y coordinate axes and sketch the trajectory of the particle. Between any *two points* on the path specify the given problem data and identify the *three unknowns*. In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2 or 32.2 ft/s^2. The particle's initial and final velocities should be represented in terms of their x and y components.

- Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions.

Kinematic Equations.

- Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem.

Horizontal Motion.

- The *velocity* in the horizontal or x direction is *constant*, i.e., $v_x = (v_0)_x$, and

$$x = x_0 + (v_0)_x t$$

Vertical Motion.

- In the vertical or y direction *only two* of the following three equations can be used for solution.

$$v_y = (v_0)_y + a_c t$$

$$y = y_0 + (v_0)_y t + \tfrac{1}{2} a_c t^2$$

$$v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$

For example, if the particle's final velocity v_y is not needed, then the first and third of these equations will not be useful.

Gravel falling off the end of this conveyor belt follows a path that can be predicted using the equations of constant acceleration. In this way the location of the accumulated pile can be determined. Rectangular coordinates are used for the analysis since the acceleration is only in the vertical direction. (© R.C. Hibbeler)

EXAMPLE 12.11

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range *R* where sacks begin to pile up.

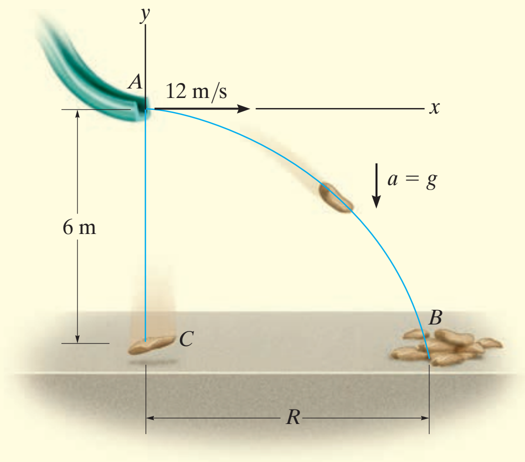


Fig. 12–21

SOLUTION

Coordinate System. The origin of coordinates is established at the beginning of the path, point *A*, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12\text{ m/s}$ and $(v_A)_y = 0$. Also, between points *A* and *B* the acceleration is $a_y = -9.81\text{ m/s}^2$. Since $(v_B)_x = (v_A)_x = 12\text{ m/s}$, the three unknowns are $(v_B)_y$, *R*, and the time of flight t_{AB}. Here we do not need to determine $(v_B)_y$.

Vertical Motion. The vertical distance from *A* to *B* is known, and therefore we can obtain a direct solution for t_{AB} by using the equation

$(+\uparrow)$
$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$
$$-6\text{ m} = 0 + 0 + \tfrac{1}{2}(-9.81\text{ m/s}^2)t_{AB}^2$$
$$t_{AB} = 1.11\text{ s} \qquad \textit{Ans.}$$

Horizontal Motion. Since t_{AB} has been calculated, *R* is determined as follows:

$(\xrightarrow{+})$
$$x_B = x_A + (v_A)_x t_{AB}$$
$$R = 0 + 12\text{ m/s}\,(1.11\text{ s})$$
$$R = 13.3\text{ m} \qquad \textit{Ans.}$$

NOTE: The calculation for t_{AB} also indicates that if a sack were released *from rest* at *A*, it would take the same amount of time to strike the floor at *C*, Fig. 12–21.

12

EXAMPLE 12.12

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.

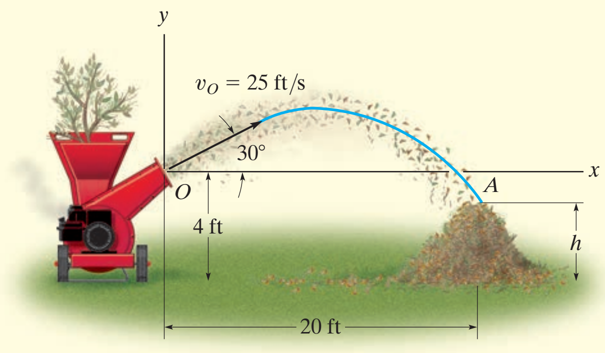


Fig. 12–22

SOLUTION

Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30°) \text{ ft/s} = 21.65 \text{ ft/s} \rightarrow$$

$$(v_O)_y = (25 \sin 30°) \text{ ft/s} = 12.5 \text{ ft/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65$ ft/s and $a_y = -32.2 \text{ ft/s}^2$. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$$(\overset{+}{\rightarrow}) \qquad x_A = x_O + (v_O)_x t_{OA}$$

$$20 \text{ ft} = 0 + (21.65 \text{ ft/s}) t_{OA}$$

$$t_{OA} = 0.9238 \text{ s}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \quad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2} a_c t_{OA}^2$$

$$(h - 4 \text{ ft}) = 0 + (12.5 \text{ ft/s})(0.9238 \text{ s}) + \tfrac{1}{2}(-32.2 \text{ ft/s}^2)(0.9238 \text{ s})^2$$

$$h = 1.81 \text{ ft} \qquad \textit{Ans.}$$

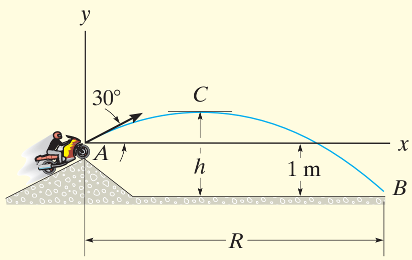
NOTE: We can determine $(v_A)_y$ by using $(v_A)_y = (v_O)_y + a_c t_{OA}$.

EXAMPLE 12.13

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Fig. 12–23*a* remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.

(© R.C. Hibbeler)

(a)



(b)

Fig. 12–23

SOLUTION

Coordinate System. As shown in Fig. 12–23*b*, the origin of the coordinates is established at A. Between the end points of the path AB the three unknowns are the initial speed v_A, range R, and the vertical component of velocity $(v_B)_y$.

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A.

$(+\uparrow)$
$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2}a_c t_{AB}^2$$
$$-1\text{ m} = 0 + v_A \sin 30°(1.5\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(1.5\text{ s})^2$$
$$v_A = 13.38\text{ m/s} = 13.4\text{ m/s} \qquad \textit{Ans.}$$

Horizontal Motion. The range R can now be determined.

$(\overset{+}{\rightarrow})$
$$x_B = x_A + (v_A)_x t_{AB}$$
$$R = 0 + 13.38 \cos 30°\text{ m/s}(1.5\text{ s})$$
$$= 17.4\text{ m} \qquad \textit{Ans.}$$

In order to find the maximum height h we will consider the path AC, Fig. 12–23*b*. Here the three unknowns are the time of flight t_{AC}, the horizontal distance from A to C, and the height h. At the maximum height $(v_C)_y = 0$, and since v_A is known, we can determine h *directly* without considering t_{AC} using the following equation.

$$(v_C)_y^2 = (v_A)_y^2 + 2a_c[y_C - y_A]$$
$$0^2 = (13.38 \sin 30°\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)[(h - 1\text{ m}) - 0]$$
$$h = 3.28\text{ m} \qquad \textit{Ans.}$$

NOTE: Show that the bike will strike the ground at B with a velocity having components of

$$(v_B)_x = 11.6\text{ m/s} \rightarrow, \quad (v_B)_y = 8.02\text{ m/s}\downarrow$$

12

PRELIMINARY PROBLEMS

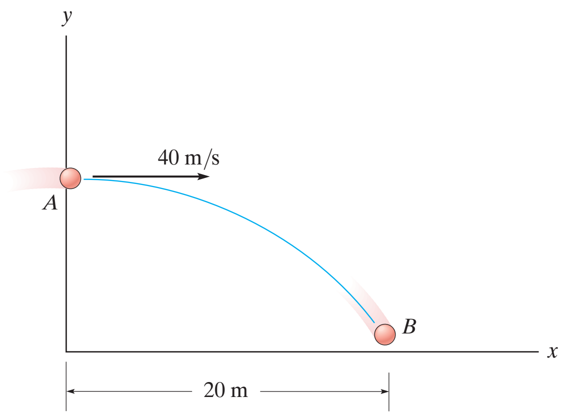
P12–3. Use the chain-rule and find $\dot{y}$ and $\ddot{y}$ in terms of $x, \dot{x}$ and $\ddot{x}$ if

a) $y = 4x^2$

b) $y = 3e^x$

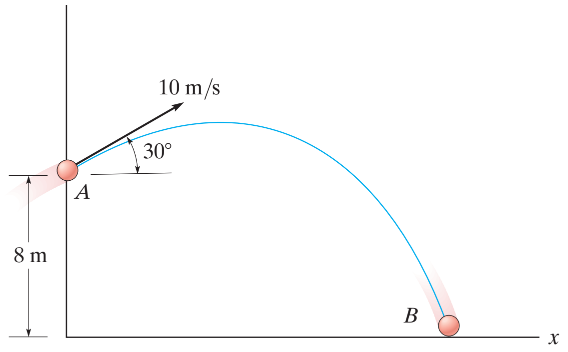
c) $y = 6 \sin x$

P12–4. The particle travels from A to B. Identify the three unknowns, and write the three equations needed to solve for them.



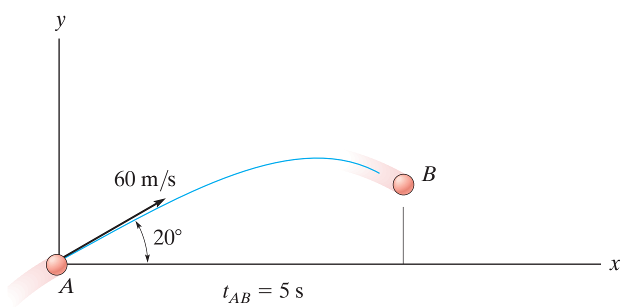
Prob. P12–4

P12–5. The particle travels from A to B. Identify the three unknowns, and write the three equations needed to solve for them.



Prob. P12–5

P12–6. The particle travels from A to B. Identify the three unknowns, and write the three equations needed to solve for them.

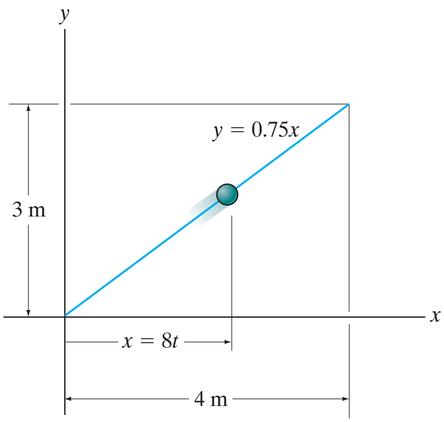


Prob. P12–6

FUNDAMENTAL PROBLEMS

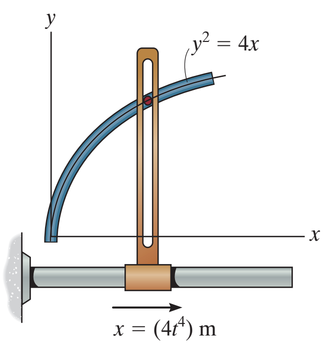
F12–15. If the x and y components of a particle's velocity are $v_x = (32t)$ m/s and $v_y = 8$ m/s, determine the equation of the path $y = f(x)$, if $x = 0$ and $y = 0$ when $t = 0$.

F12–16. A particle is traveling along the straight path. If its position along the x axis is $x = (8t)$ m, where t is in seconds, determine its speed when $t = 2$ s.



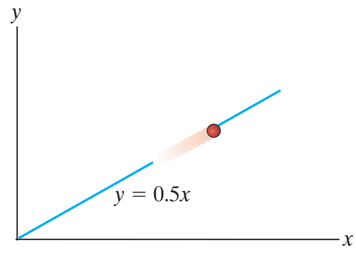
Prob. F12–16

F12–17. A particle is constrained to travel along the path. If $x = (4t^4)$ m, where t is in seconds, determine the magnitude of the particle's velocity and acceleration when $t = 0.5$ s.



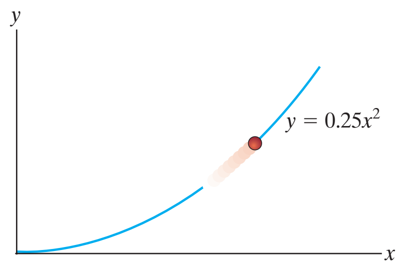
Prob. F12–17

F12–18. A particle travels along a straight-line path $y = 0.5x$. If the x component of the particle's velocity is $v_x = (2t^2)$ m/s, where t is in seconds, determine the magnitude of the particle's velocity and acceleration when $t = 4$ s.



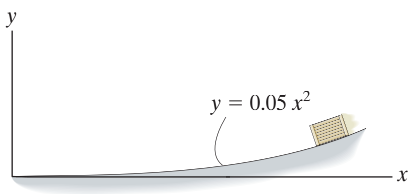
Prob. F12–18

F12–19. A particle is traveling along the parabolic path $y = 0.25x^2$. If $x = 8$ m, $v_x = 8$ m/s, and $a_x = 4$ m/s^2 when $t = 2$ s, determine the magnitude of the particle's velocity and acceleration at this instant.



Prob. F12–19

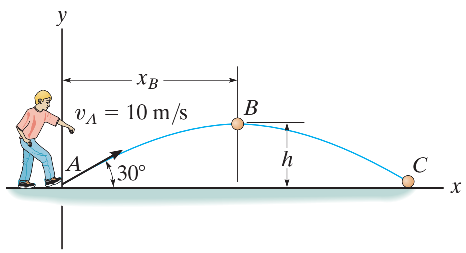
F12–20. The box slides down the slope described by the equation $y = (0.05x^2)$ m, where x is in meters. If the box has x components of velocity and acceleration of $v_x = -3$ m/s and $a_x = -1.5$ m/s^2 at $x = 5$ m, determine the y components of the velocity and the acceleration of the box at this instant.



Prob. F12–20

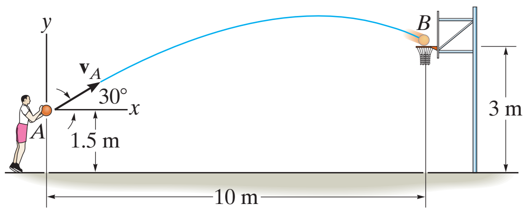
F12–21. The ball is kicked from point A with the initial velocity $v_A = 10\ \text{m/s}$. Determine the maximum height h it reaches.

F12–22. The ball is kicked from point A with the initial velocity $v_A = 10\ \text{m/s}$. Determine the range R, and the speed when the ball strikes the ground.



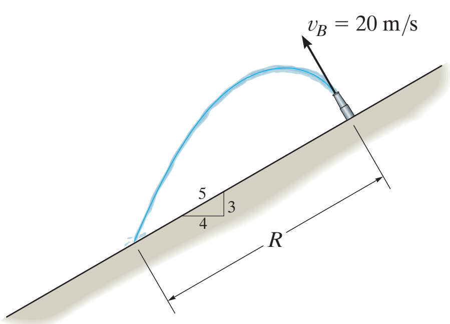
Prob. F12–21/22

F12–23. Determine the speed at which the basketball at A must be thrown at the angle of 30° so that it makes it to the basket at B.



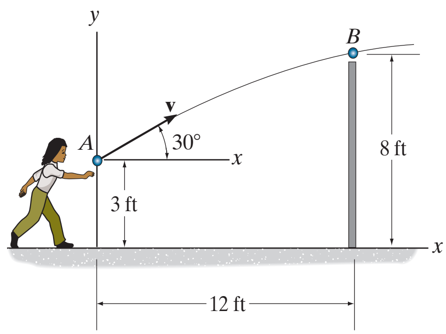
Prob. F12–23

F12–24. Water is sprayed at an angle of 90° from the slope at 20 m/s. Determine the range R.



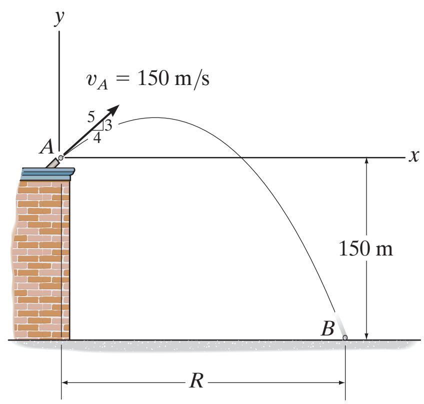
Prob. F12–24

F12–25. A ball is thrown from A. If it is required to clear the wall at B, determine the minimum magnitude of its initial velocity $\mathbf{v}_A$.



Prob. F12–25

F12–26. A projectile is fired with an initial velocity of $v_A = 150\ \text{m/s}$ off the roof of the building. Determine the range R where it strikes the ground at B.



Prob. F12–26

PROBLEMS

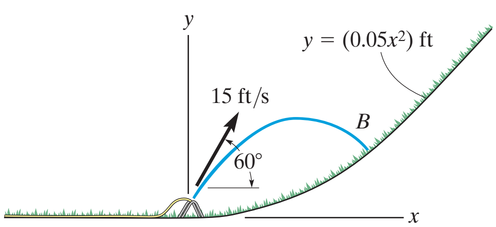
12–69. If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

12–70. The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

12–71. A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s². Determine the particle's position (x, y, z) at $t = 1$ s.

***12–72.** The velocity of a particle is given by $v = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

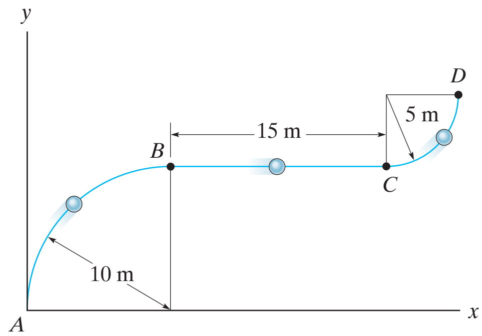
12–73. The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity of 15 ft/s as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill. Assume that the hill is defined by the equation $y = (0.05x^2)$ ft and neglect the size of the sprinkler.



Prob. 12–73

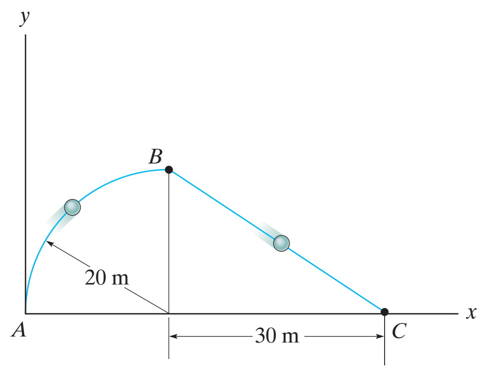
12–74. A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s². Determine the particle's position (x, y, z) when $t = 2$ s.

12–75. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



Prob. 12–75

***12–76.** A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A. Determine its average speed when it goes around the closed path.

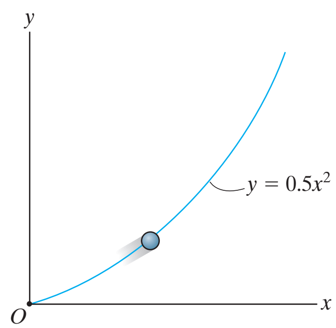


Prob. 12–76

12–77. The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

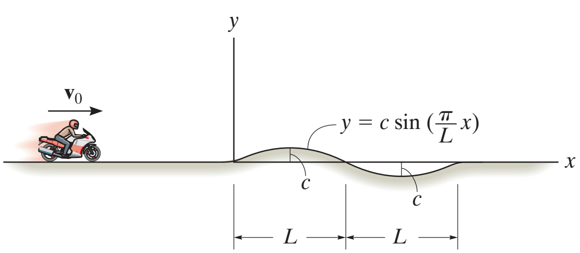
12–78. A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

12–79. The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



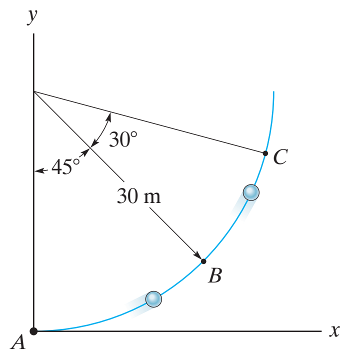
Prob. 12–79

***12–80.** The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



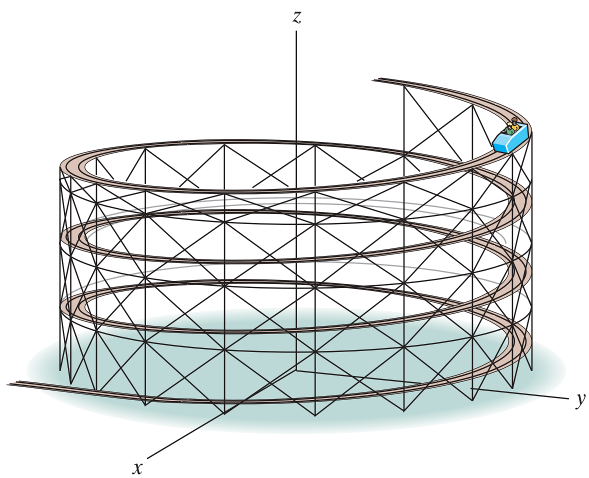
Prob. 12–80

12–81. A particle travels along the curve from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



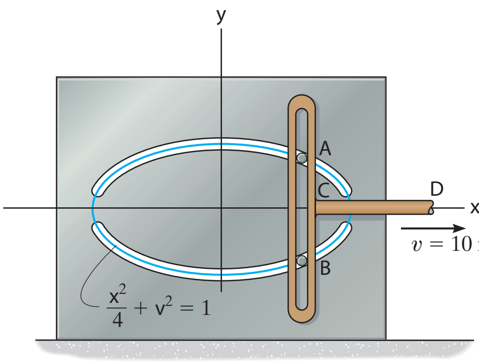
Prob. 12–81

12–82. The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



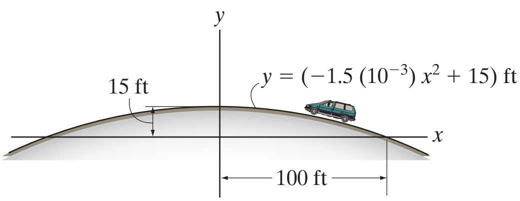
Prob. 12–82

12–83. Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



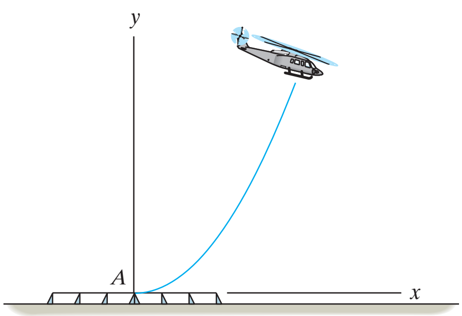
Prob. 12–83

***12–84.** The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



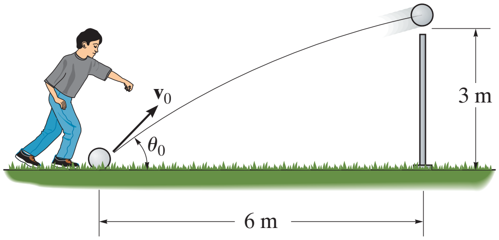
Prob. 12–84

12–85. The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



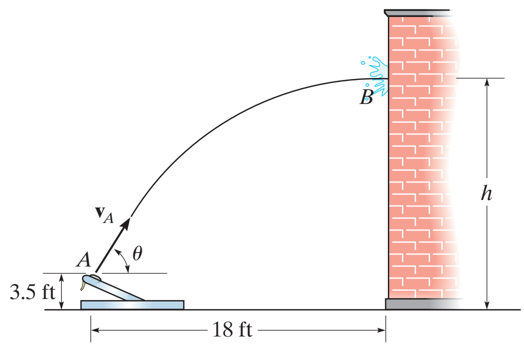
Prob. 12–85

12–86. Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



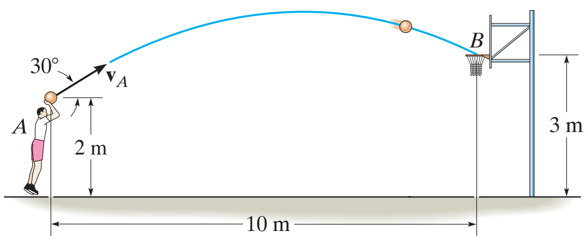
Prob. 12–86

12–87. The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.



Prob. 12–87

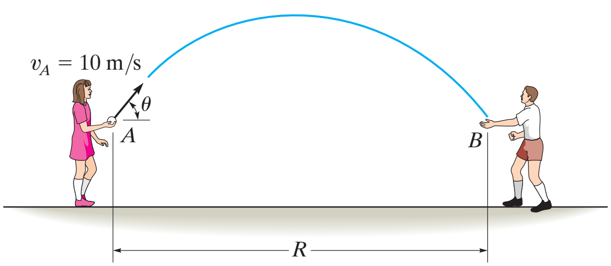
***12–88.** Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



Prob. 12–88

12–89. The girl at A can throw a ball at $v_A = 10$ m/s. Calculate the maximum possible range $R = R_{max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.

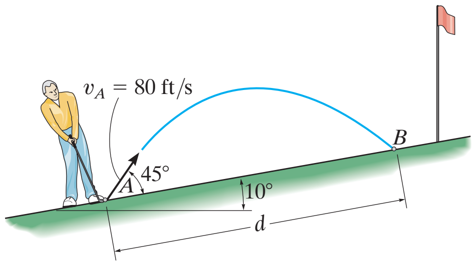
12–90. Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15°, i.e., $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. Assume the ball is caught at the same elevation from which it is thrown.



Probs. 12–89/90

12–93. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.

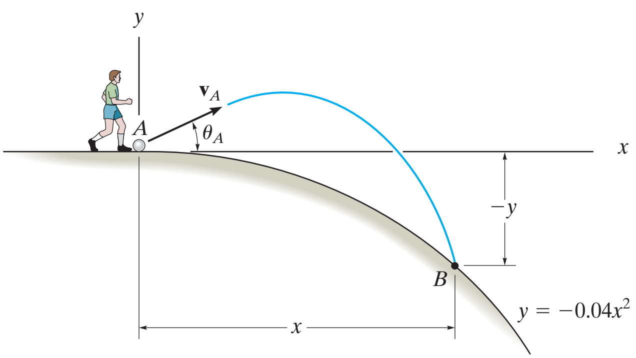
12–94. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the speed at which it strikes the ground at B and the time of flight from A to B.



Probs. 12–93/94

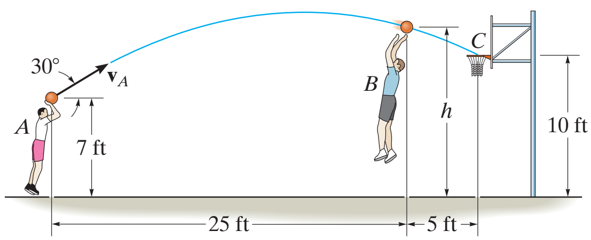
12–91. The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.

***12–92.** The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



Probs. 12–91/92

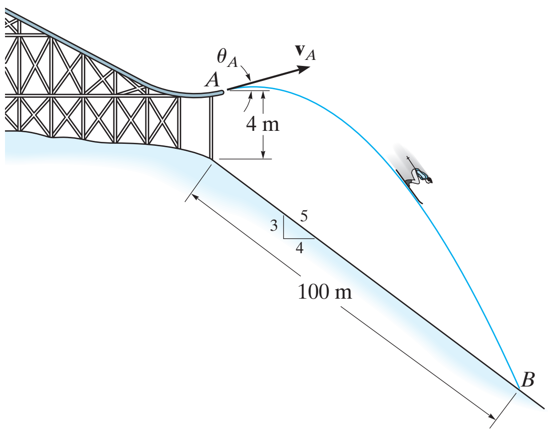
12–95. The basketball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



Prob. 12–95

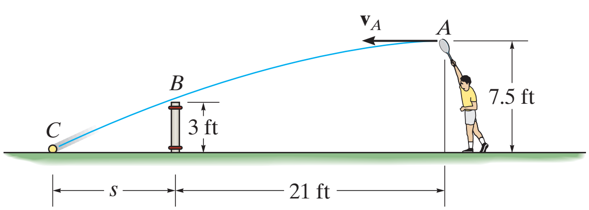
***12–96.** It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.

12–97. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



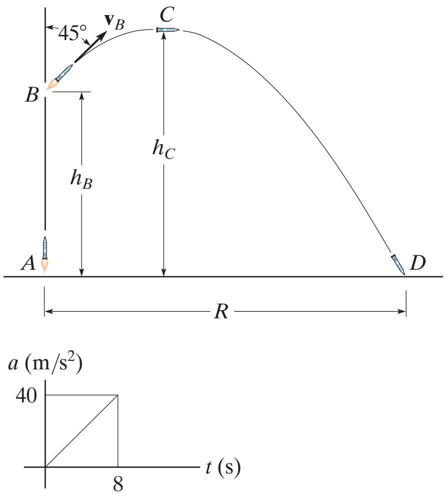
Probs. 12–96/97

12–98. Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



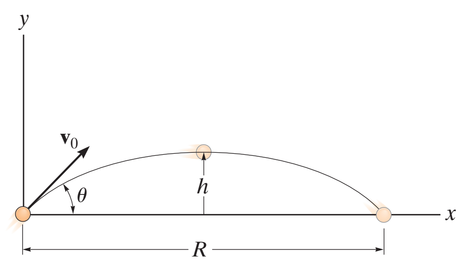
Prob. 12–98

12–99. The missile at A takes off from rest and rises vertically to B, where its fuel runs out in 8 s. If the acceleration varies with time as shown, determine the missile's height h_B and speed v_B. If by internal controls the missile is then suddenly pointed 45° as shown, and allowed to travel in free flight, determine the maximum height attained, h_C, and the range R to where it crashes at D.



Prob. 12–99

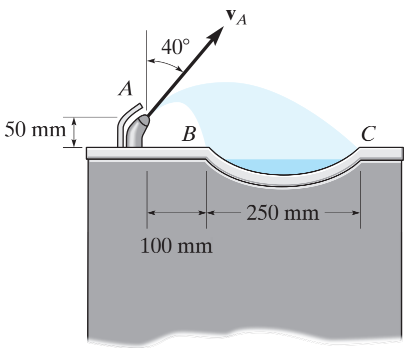
***12–100.** The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



Prob. 12–100

12

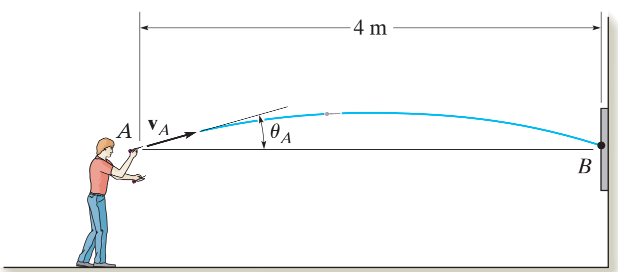
12–101. The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



Prob. 12–101

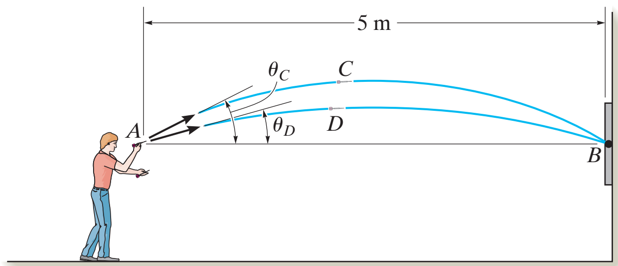
12–102. If the dart is thrown with a speed of 10 m/s, determine the shortest possible time before it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?

12–103. If the dart is thrown with a speed of 10 m/s, determine the longest possible time when it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



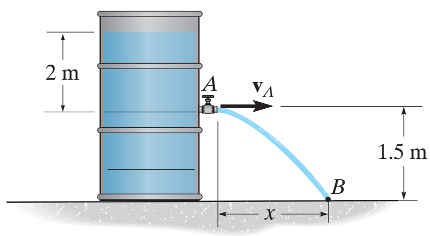
Probs. 12–102/103

***12–104.** The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C ($>\theta_D$), then the second dart is thrown at θ_D.



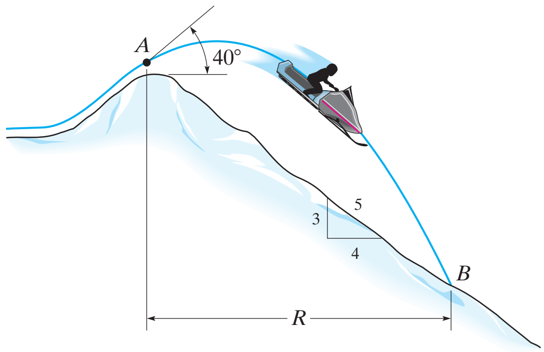
Prob. 12–104

12–105. The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2\,gh}$, where $h = 2$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



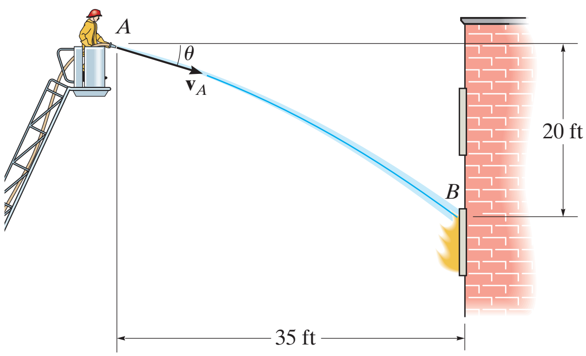
Prob. 12–105

12–106. The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



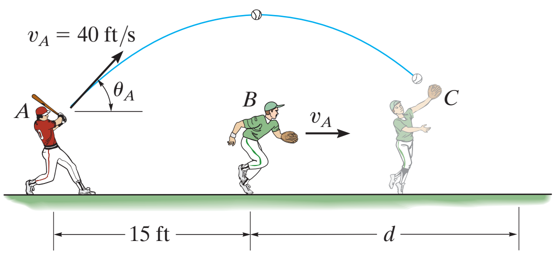
Prob. 12–106

12–107. The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



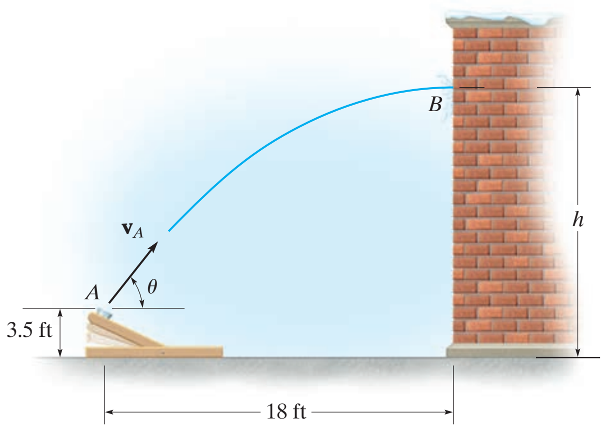
Prob. 12–107

***12–108.** The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60°$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.

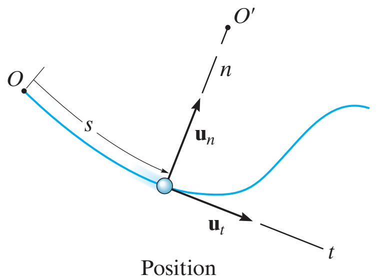


Prob. 12–108

12–109. The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.

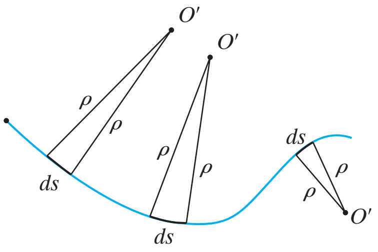


Prob. 12–109



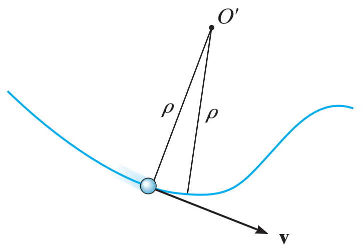
Position

(a)



Radius of curvature

(b)



Velocity

(c)

Fig. 12–24

12.7 Curvilinear Motion: Normal and Tangential Components

When the path along which a particle travels is *known*, then it is often convenient to describe the motion using n and t coordinate axes which act normal and tangent to the path, respectively, and at the instant considered have their *origin located at the particle*.

Planar Motion. Consider the particle shown in Fig. 12–24*a*, which moves in a plane along a fixed curve, such that at a given instant it is at position s, measured from point O. We will now consider a coordinate system that has its origin on the curve, and at the instant considered this origin happens to *coincide* with the location of the particle. The t axis is *tangent* to the curve at the point and is positive in the direction of *increasing* s. We will designate this positive direction with the unit vector $\mathbf{u}_t$. A unique choice for the *normal axis* can be made by noting that geometrically the curve is constructed from a series of differential arc segments ds, Fig. 12–24*b*. Each segment ds is formed from the arc of an associated circle having a *radius of curvature* ρ (rho) and *center of curvature* O'. The normal axis n is perpendicular to the t axis with its positive sense directed *toward* the center of curvature O', Fig. 12–24*a*. This positive direction, which is *always* on the concave side of the curve, will be designated by the unit vector $\mathbf{u}_n$. The plane which contains the n and t axes is referred to as the embracing or *osculating plane*, and in this case it is fixed in the plane of motion.*

Velocity. Since the particle moves, s is a function of time. As indicated in Sec. 12.4, the particle's velocity $\mathbf{v}$ has a *direction* that is *always tangent to the path*, Fig. 12–24*c*, and a *magnitude* that is determined by taking the time derivative of the path function $s = s(t)$, i.e., $v = ds/dt$ (Eq. 12–8). Hence

$$\boxed{\mathbf{v} = v\mathbf{u}_t} \tag{12–15}$$

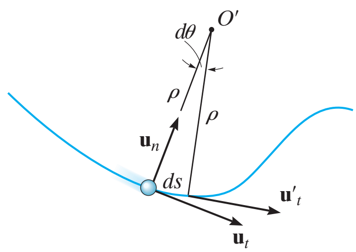
where

$$\boxed{v = \dot{s}} \tag{12–16}$$

*The osculating plane may also be defined as the plane which has the greatest contact with the curve at a point. It is the limiting position of a plane contacting both the point and the arc segment ds. As noted above, the osculating plane is always coincident with a plane curve; however, each point on a three-dimensional curve has a unique osculating plane.

Acceleration. The acceleration of the particle is the time rate of change of the velocity. Thus,

$$\mathbf{a} = \dot{\mathbf{v}} = \dot{v}\mathbf{u}_t + v\dot{\mathbf{u}}_t \tag{12–17}$$

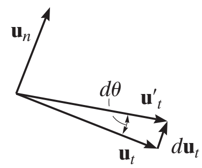


(d)

In order to determine the time derivative $\dot{\mathbf{u}}_t$, note that as the particle moves along the arc ds in time dt, $\mathbf{u}_t$ preserves its magnitude of unity; however, its *direction* changes, and becomes $\mathbf{u}'_t$, Fig. 12–24*d*. As shown in Fig. 12–24*e*, we require $\mathbf{u}'_t = \mathbf{u}_t + d\mathbf{u}_t$. Here $d\mathbf{u}_t$ stretches between the arrowheads of $\mathbf{u}_t$ and $\mathbf{u}'_t$, which lie on an infinitesimal arc of radius $u_t = 1$. Hence, $d\mathbf{u}_t$ has a *magnitude* of $du_t = (1)\,d\theta$, and its *direction* is defined by $\mathbf{u}_n$. Consequently, $d\mathbf{u}_t = d\theta\mathbf{u}_n$, and therefore the time derivative becomes $\dot{\mathbf{u}}_t = \dot{\theta}\mathbf{u}_n$. Since $ds = \rho d\theta$, Fig. 12–24*d*, then $\dot{\theta} = \dot{s}/\rho$, and therefore

$$\dot{\mathbf{u}}_t = \dot{\theta}\mathbf{u}_n = \frac{\dot{s}}{\rho}\mathbf{u}_n = \frac{v}{\rho}\mathbf{u}_n$$

Substituting into Eq. 12–17, **a** can be written as the sum of its two components,



(e)

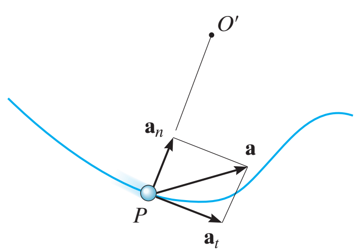
$$\boxed{\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n} \tag{12–18}$$

where

$$\boxed{a_t = \dot{v}} \quad \text{or} \quad \boxed{a_t ds = v\,dv} \tag{12–19}$$

and

$$\boxed{a_n = \frac{v^2}{\rho}} \tag{12–20}$$



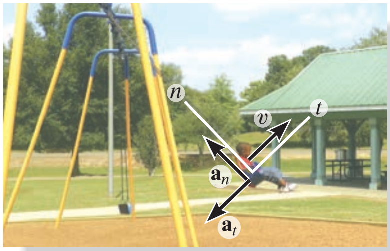
Acceleration

(f)

Fig. 12–24 (cont.)

These two mutually perpendicular components are shown in Fig. 12–24*f*. Therefore, the *magnitude* of acceleration is the positive value of

$$a = \sqrt{a_t^2 + a_n^2} \tag{12–21}$$



As the boy swings upward with a velocity $\mathbf{v}$, his motion can be analyzed using n–t coordinates. As he rises, the magnitude of his velocity (speed) is decreasing, and so a_t will be negative. The rate at which the direction of his velocity changes is a_n, which is always positive, that is, towards the center of rotation. (© R.C. Hibbeler)

To better understand these results, consider the following two special cases of motion.

1. If the particle moves along a straight line, then $\rho \to \infty$ and from Eq. 12–20, $a_n = 0$. Thus $a = a_t = \dot{v}$, and we can conclude that the *tangential component of acceleration represents the time rate of change in the magnitude of the velocity.*
2. If the particle moves along a curve with a constant speed, then $a_t = \dot{v} = 0$ and $a = a_n = v^2/\rho$. Therefore, the *normal component of acceleration represents the time rate of change in the direction of the velocity*. Since $\mathbf{a}_n$ *always* acts towards the center of curvature, this component is sometimes referred to as the *centripetal* (or center seeking) *acceleration*.

As a result of these interpretations, a particle moving along the curved path in Fig. 12–25 will have accelerations directed as shown.

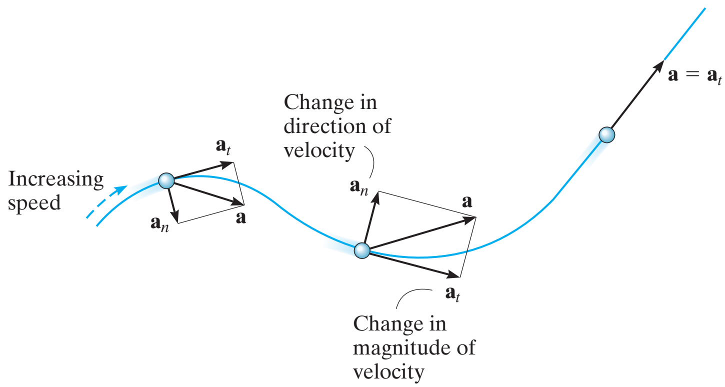


Fig. 12–25

Three-Dimensional Motion.

If the particle moves along a space curve, Fig. 12–26, then at a given instant the t axis is uniquely specified; however, an infinite number of straight lines can be constructed normal to the tangent axis. As in the case of planar motion, we will choose the positive n axis directed toward the path's center of curvature O'. This axis is referred to as the *principal normal* to the curve. With the n and t axes so defined, Eqs. 12–15 through 12–21 can be used to determine $\mathbf{v}$ and $\mathbf{a}$. Since $\mathbf{u}_t$ and $\mathbf{u}_n$ are always perpendicular to one another and lie in the osculating plane, for spatial motion a third unit vector, $\mathbf{u}_b$, defines the *binormal axis b* which is perpendicular to $\mathbf{u}_t$ and $\mathbf{u}_n$, Fig. 12–26.

Since the three unit vectors are related to one another by the vector cross product, e.g., $\mathbf{u}_b = \mathbf{u}_t \times \mathbf{u}_n$, Fig. 12–26, it may be possible to use this relation to establish the direction of one of the axes, if the directions of the other two are known. For example, no motion occurs in the $\mathbf{u}_b$ direction, and if this direction and $\mathbf{u}_t$ are known, then $\mathbf{u}_n$ can be determined, where in this case $\mathbf{u}_n = \mathbf{u}_b \times \mathbf{u}_t$, Fig. 12–26. Remember, though, that $\mathbf{u}_n$ is always on the concave side of the curve.

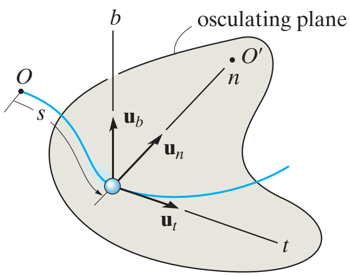


Fig. 12–26

Procedure for Analysis

Coordinate System.

- Provided the *path* of the particle is *known*, we can establish a set of n and t coordinates having a *fixed origin*, which is coincident with the particle at the instant considered.
- The positive tangent axis acts in the direction of motion and the positive normal axis is directed toward the path's center of curvature.

Velocity.

- The particle's *velocity* is always tangent to the path.
- The magnitude of velocity is found from the time derivative of the path function.

$$v = \dot{s}$$

Tangential Acceleration.

- The tangential component of acceleration is the result of the time rate of change in the *magnitude* of velocity. This component acts in the positive s direction if the particle's speed is increasing or in the opposite direction if the speed is decreasing.
- The relations between a_t, v, t, and s are the same as for rectilinear motion, namely,

$$a_t = \dot{v} \quad a_t\,ds = v\,dv$$

- If a_t is constant, $a_t = (a_t)_c$, the above equations, when integrated, yield

$$s = s_0 + v_0 t + \tfrac{1}{2}(a_t)_c t^2$$
$$v = v_0 + (a_t)_c t$$
$$v^2 = v_0^2 + 2(a_t)_c(s - s_0)$$

Normal Acceleration.

- The normal component of acceleration is the result of the time rate of change in the *direction* of the velocity. This component is *always* directed toward the center of curvature of the path, i.e., along the positive n axis.
- The magnitude of this component is determined from

$$a_n = \frac{v^2}{\rho}$$

- If the path is expressed as $y = f(x)$, the radius of curvature ρ at any point on the path is determined from the equation

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

The derivation of this result is given in any standard calculus text.

Once the rotation is constant, the riders will then have only a normal component of acceleration. (© R.C. Hibbeler)

Motorists traveling along this cloverleaf interchange experience a normal acceleration due to the change in direction of their velocity. A tangential component of acceleration occurs when the cars' speed is increased or decreased. (© R.C. Hibbeler)

EXAMPLE 12.14

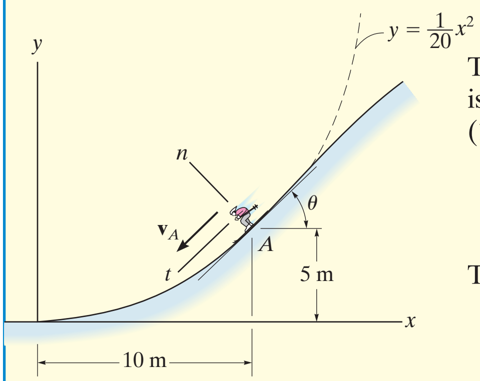
When the skier reaches point A along the parabolic path in Fig. 12–27a, he has a speed of 6 m/s which is increasing at 2 m/s^2. Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

SOLUTION

Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of $\mathbf{v}$ and $\mathbf{a}$ along these axes, Fig. 12–27a.

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10$ m, $dy/dx = 1$. Hence, at A, $\mathbf{v}$ makes an angle of $\theta = \tan^{-1}1 = 45°$ with the x axis, Fig. 12–27b. Therefore,

$$v_A = 6 \text{ m/s} \quad 45° \swarrow \qquad \textit{Ans.}$$



(a)

The acceleration is determined from $\mathbf{a} = \dot{v}\mathbf{u}_t + (v^2/\rho)\mathbf{u}_n$. However, it is first necessary to determine the radius of curvature of the path at A (10 m, 5 m). Since $d^2y/dx^2 = \frac{1}{10}$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{\left|\frac{1}{10}\right|}\right|_{x=10\text{ m}} = 28.28 \text{ m}$$

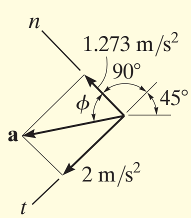
The acceleration becomes

$$\mathbf{a}_A = \dot{v}\mathbf{u}_t + \frac{v^2}{\rho}\mathbf{u}_n$$
$$= 2\mathbf{u}_t + \frac{(6 \text{ m/s})^2}{28.28 \text{ m}}\mathbf{u}_n$$
$$= \{2\mathbf{u}_t + 1.273\mathbf{u}_n\} \text{ m/s}^2$$

As shown in Fig. 12–27b,

$$a = \sqrt{(2 \text{ m/s}^2)^2 + (1.273 \text{ m/s}^2)^2} = 2.37 \text{ m/s}^2$$

$$\phi = \tan^{-1}\frac{2}{1.273} = 57.5°$$



(b)

Fig. 12–27

Thus, $45° + 90° + 57.5° - 180° = 12.5°$ so that,

$$a = 2.37 \text{ m/s}^2 \quad 12.5° \swarrow \qquad \textit{Ans.}$$

NOTE: By using n, t coordinates, we were able to readily solve this problem through the use of Eq. 12–18, since it accounts for the separate changes in the magnitude and direction of $\mathbf{v}$.

EXAMPLE 12.15

A race car C travels around the horizontal circular track that has a radius of 300 ft, Fig. 12–28. If the car increases its speed at a constant rate of 7 ft/s^2, starting from rest, determine the time needed for it to reach an acceleration of 8 ft/s^2. What is its speed at this instant?

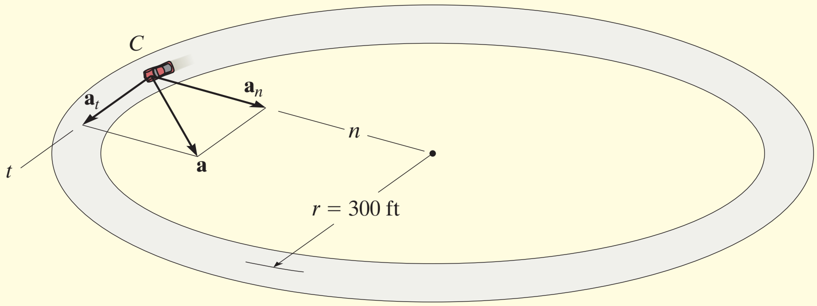


Fig. 12–28

SOLUTION

Coordinate System. The origin of the n and t axes is coincident with the car at the instant considered. The t axis is in the direction of motion, and the positive n axis is directed toward the center of the circle. This coordinate system is selected since the path is known.

Acceleration. The magnitude of acceleration can be related to its components using $a = \sqrt{a_t^2 + a_n^2}$. Here $a_t = 7$ ft/s^2. Since $a_n = v^2/\rho$, the velocity as a function of time must be determined first.

$$v = v_0 + (a_t)_c t$$
$$v = 0 + 7t$$

Thus

$$a_n = \frac{v^2}{\rho} = \frac{(7t)^2}{300} = 0.163t^2 \text{ ft/s}^2$$

The time needed for the acceleration to reach 8 ft/s^2 is therefore

$$a = \sqrt{a_t^2 + a_n^2}$$
$$8 \text{ ft/s}^2 = \sqrt{(7 \text{ ft/s}^2)^2 + (0.163t^2)^2}$$

Solving for the positive value of t yields

$$0.163t^2 = \sqrt{(8 \text{ ft/s}^2)^2 - (7 \text{ ft/s}^2)^2}$$
$$t = 4.87 \text{ s} \qquad \textit{Ans.}$$

Velocity. The speed at time $t = 4.87$ s is

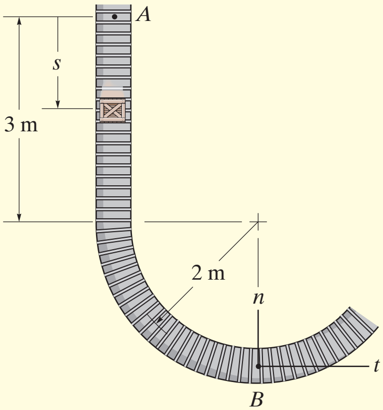
$$v = 7t = 7(4.87) = 34.1 \text{ ft/s} \qquad \textit{Ans.}$$

NOTE: Remember the velocity will always be tangent to the path, whereas the acceleration will be directed within the curvature of the path.

12

EXAMPLE 12.16

(a)



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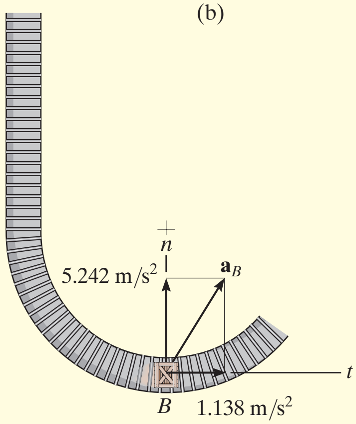


(c)

Fig. 12–29

The boxes in Fig. 12–29*a* travel along the industrial conveyor. If a box as in Fig. 12–29*b* starts from rest at A and increases its speed such that $a_t = (0.2t)\ \text{m/s}^2$, where t is in seconds, determine the magnitude of its acceleration when it arrives at point B.

SOLUTION

Coordinate System. The position of the box at any instant is defined from the fixed point A using the position or path coordinate s, Fig. 12–29*b*. The acceleration is to be determined at B, so the origin of the n, t axes is at this point.

Acceleration. To determine the acceleration components $a_t = \dot{v}$ and $a_n = v^2/\rho$, it is first necessary to formulate v and $\dot{v}$ so that they may be evaluated at B. Since $v_A = 0$ when $t = 0$, then

$$a_t = \dot{v} = 0.2t \qquad (1)$$

$$\int_0^v dv = \int_0^t 0.2t\, dt$$

$$v = 0.1t^2 \qquad (2)$$

The time needed for the box to reach point B can be determined by realizing that the position of B is $s_B = 3 + 2\pi(2)/4 = 6.142$ m, Fig. 12–29*b*, and since $s_A = 0$ when $t = 0$ we have

$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^{6.142\text{ m}} ds = \int_0^{t_B} 0.1t^2 dt$$

$$6.142\text{ m} = 0.0333t_B^3$$

$$t_B = 5.690\text{s}$$

Substituting into Eqs. 1 and 2 yields

$$(a_B)_t = \dot{v}_B = 0.2(5.690) = 1.138\ \text{m/s}^2$$

$$v_B = 0.1(5.69)^2 = 3.238\ \text{m/s}$$

At B, $\rho_B = 2$ m, so that

$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(3.238\ \text{m/s})^2}{2\text{ m}} = 5.242\ \text{m/s}^2$$

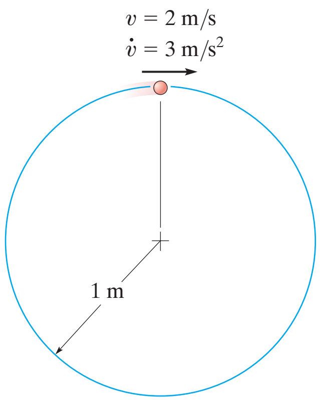
The magnitude of $\mathbf{a}_B$, Fig. 12–29*c*, is therefore

$$a_B = \sqrt{(1.138\ \text{m/s}^2)^2 + (5.242\ \text{m/s}^2)^2} = 5.36\ \text{m/s}^2 \qquad \textit{Ans.}$$

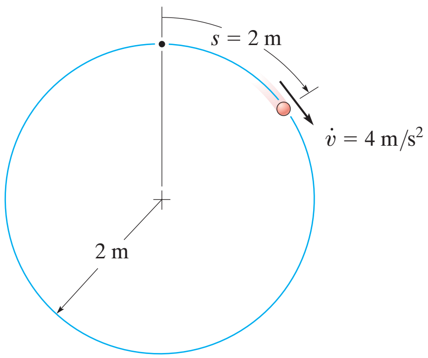
PRELIMINARY PROBLEM

P12–7.

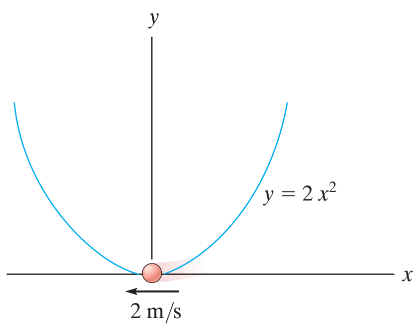
a) Determine the acceleration at the instant shown.



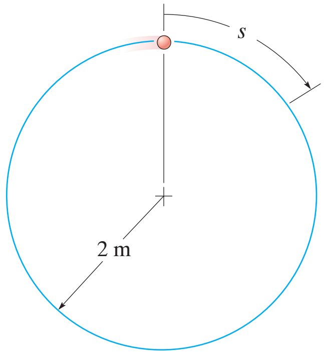
b) Determine the increase in speed and the normal component of acceleration at $s = 2$ m. At $s = 0$, $v = 0$.



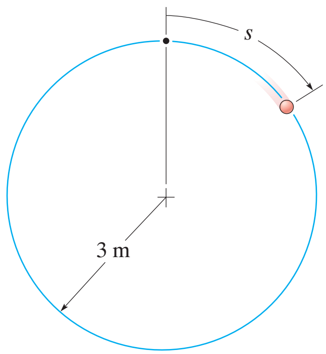
c) Determine the acceleration at the instant shown. The particle has a constant speed of 2 m/s.



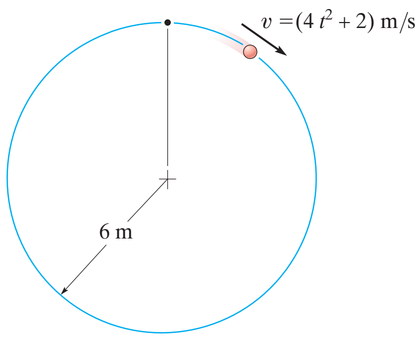
d) Determine the normal and tangential components of acceleration at $s = 0$ if $v = (4s + 1)$ m/s, where s is in meters.



e) Determine the acceleration at $s = 2$ m if $\dot{v} = (2\,s)$ m/s^2, where s is in meters. At $s = 0$, $v = 1$ m/s.



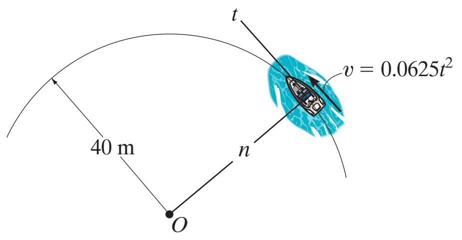
f. Determine the acceleration when $t = 1$ s if $v = (4t^2 + 2)$ m/s, where t is in seconds.



Prob. P12–7

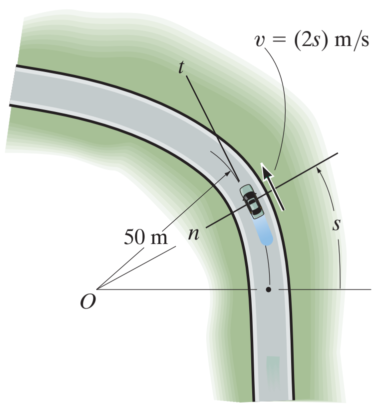
FUNDAMENTAL PROBLEMS

F12–27. The boat is traveling along the circular path with a speed of $v = (0.0625t^2)$ m/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 10$ s.



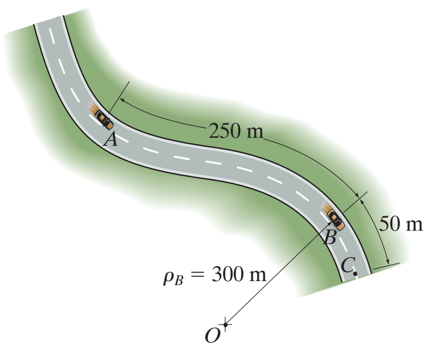
Prob. F12–27

F12–28. The car is traveling along the road with a speed of $v = (2\,s)$ m/s, where s is in meters. Determine the magnitude of its acceleration when $s = 10$ m.



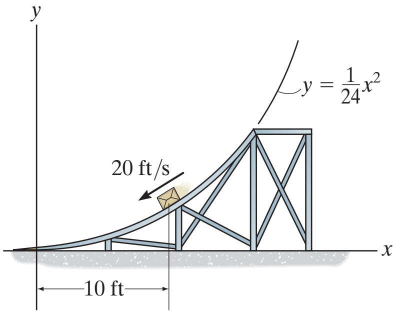
Prob. F12–28

F12–29. If the car decelerates uniformly along the curved road from 25 m/s at A to 15 m/s at C, determine the acceleration of the car at B.



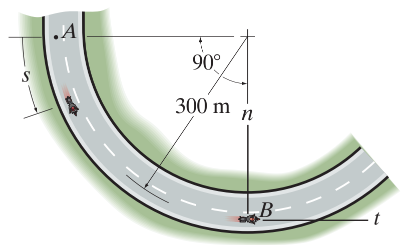
Prob. F12–29

F12–30. When $x = 10$ ft, the crate has a speed of 20 ft/s which is increasing at 6 ft/s^2. Determine the direction of the crate's velocity and the magnitude of the crate's acceleration at this instant.



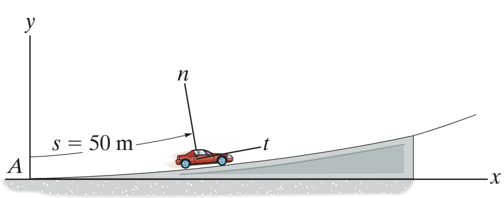
Prob. F12–30

F12–31. If the motorcycle has a deceleration of $a_t = -(0.001s)$ m/s^2 and its speed at position A is 25 m/s, determine the magnitude of its acceleration when it passes point B.



Prob. F12–31

F12–32. The car travels up the hill with a speed of $v = (0.2s)$ m/s, where s is in meters, measured from A. Determine the magnitude of its acceleration when it is at point $s = 50$ m, where $\rho = 500$ m.



Prob. F12–32

PROBLEMS

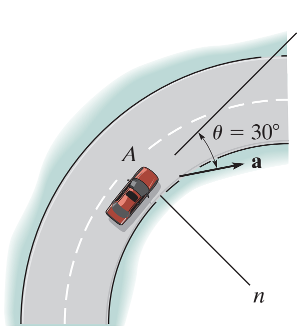
12–110. An automobile is traveling on a curve having a radius of 800 ft. If the acceleration of the automobile is 5 ft/s^2, determine the constant speed at which the automobile is traveling.

12–111. Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

***12–112.** A boat has an initial speed of 16 ft/s. If it then increases its speed along a circular path of radius $\rho = 80$ ft at the rate of $\dot{v} = (1.5s)$ ft/s, where s is in feet, determine the time needed for the boat to travel $s = 50$ ft.

12–113. The position of a particle is defined by $\mathbf{r} = \{4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 1$ s.

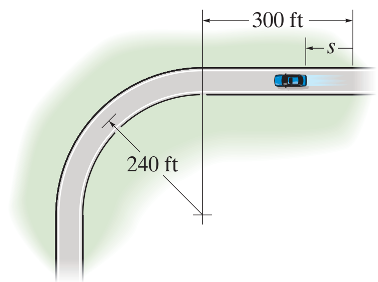
12–114. The automobile has a speed of 80 ft/s at point A and an acceleration having a magnitude of 10 ft/s^2, acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.



Prob. 12–114

12–115. The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.

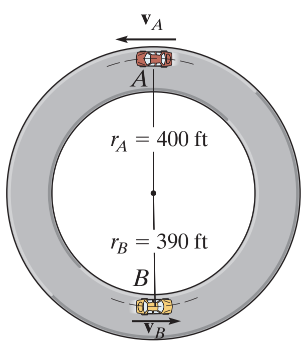
***12–116.** The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



Probs. 12–115/116

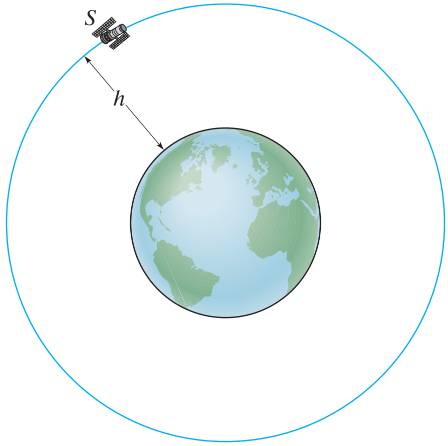
12–117. The two cars A and B travel along the circular path at constant speeds $v_A = 80$ ft/s and $v_B = 100$ ft/s, respectively. If they are at the positions shown when $t = 0$, determine the time when the cars are side by side, and the time when they are 90° apart.

12–118. Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 60 ft/s and is increasing its speed at the rate of 15 ft/s^2 until it travels for a distance of 100π ft, after which it maintains a constant speed. Car B has a speed of 120 ft/s and is decreasing its speed at 15 ft/s^2 until it travels a distance of 65π ft, after which it maintains a constant speed. Determine the time when they come side by side.



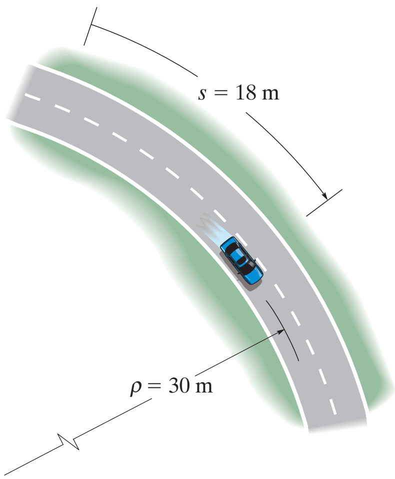
Probs. 12–117/118

12–119. The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2, determine the altitude h. Assume the earth's diameter to be 12 713 km.



Prob. 12–119

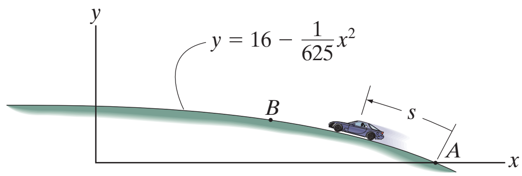
***12–120.** The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



Prob. 12–120

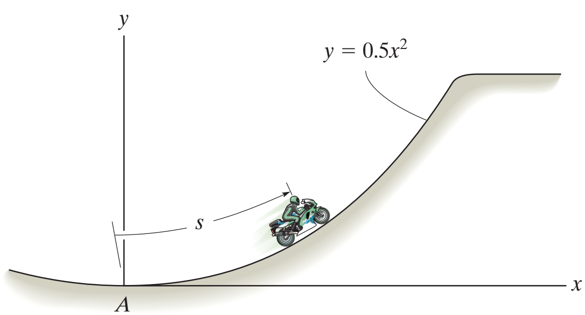
12–121. The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m and $x = 50$ m.

12–122. If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5$ m/s^2, determine the magnitude of the car's acceleration when $s = 101.68$ m and $x = 0$.



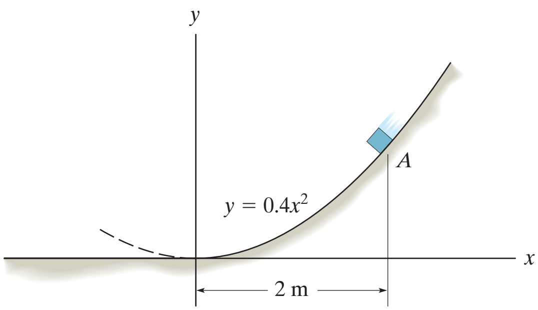
Probs. 12–121/122

12–123. The motorcycle is traveling at 1 m/s when it is at A. If the speed is then increased at $\dot{v} = 0.1$ m/s^2, determine its speed and acceleration at the instant $t = 5$ s.



Prob. 12–123

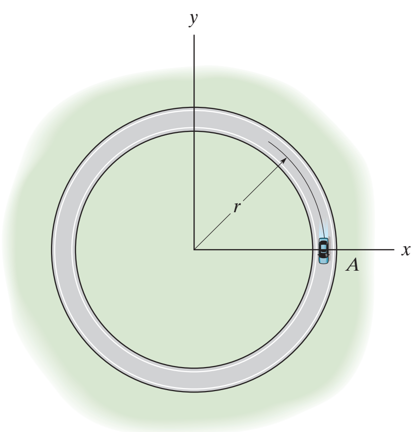
***12–124.** The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at $A(x_A = 2\text{ m}, y_A = 1.6\text{ m})$, the speed is $v = 8\text{ m/s}$ and the increase in speed is $dv/dt = 4\text{ m/s}^2$. Determine the magnitude of the acceleration of the box at this instant.



Prob. 12–124

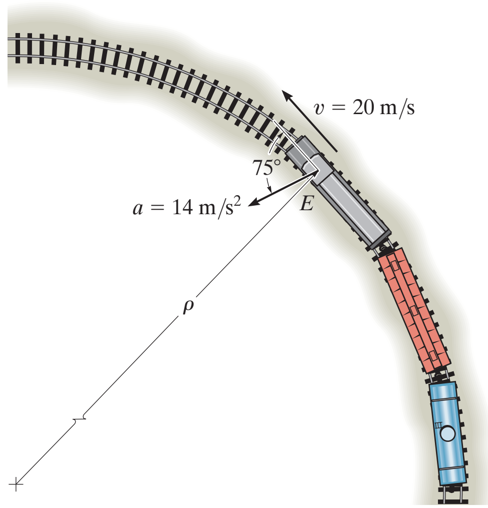
12–125. The car travels around the circular track having a radius of $r = 300\text{ m}$ such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $\dot{v} = (0.06t)\text{ m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.

12–126. The car travels around the portion of a circular track having a radius of $r = 500\text{ ft}$ such that when it is at point A it has a velocity of 2 ft/s, which is increasing at the rate of $\dot{v} = (0.002t)\text{ ft/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths the way around the track.



Probs. 12–125/126

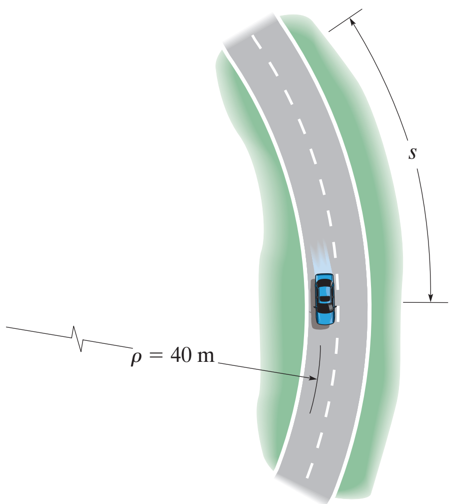
12–127. At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



Prob. 12–127

***12–128.** The car has an initial speed $v_0 = 20\text{ m/s}$. If it increases its speed along the circular track at $s = 0$, $a_t = (0.8s)\text{ m/s}^2$, where s is in meters, determine the time needed for the car to travel $s = 25\text{ m}$.

12–129. The car starts from rest at $s = 0$ and increases its speed at $a_t = 4\text{ m/s}^2$. Determine the time when the magnitude of acceleration becomes 20 m/s^2. At what position s does this occur?



Probs. 12–128/129

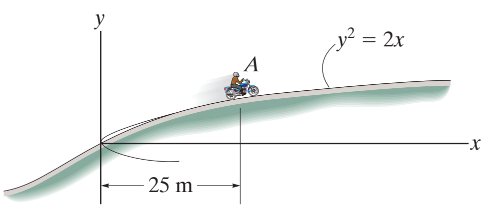
12–130. A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

12–131. A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s^2.

***12–132.** Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

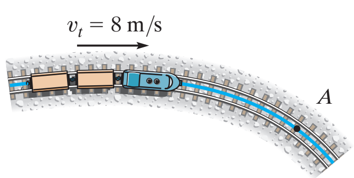
12–133. A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\ v)$ m/s^2, determine the magnitude of the particle's acceleration four seconds later.

12–134. The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



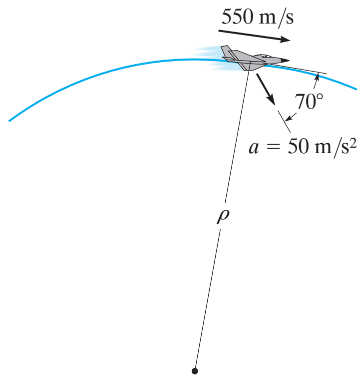
Prob. 12–134

12–135. When $t = 0$, the train has a speed of 8 m/s, which is increasing at 0.5 m/s^2. Determine the magnitude of the acceleration of the engine when it reaches point A, at $t = 20$ s. Here the radius of curvature of the tracks is $\rho_A = 400$ m.



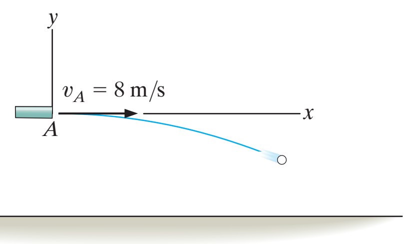
Prob. 12–135

***12–136.** At a given instant the jet plane has a speed of 550 m/s and an acceleration of 50 m/s^2 acting in the direction shown. Determine the rate of increase in the plane's speed, and also the radius of curvature ρ of the path.



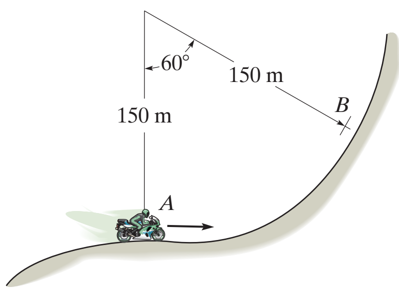
Prob. 12–136

12–137. The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



Prob. 12–137

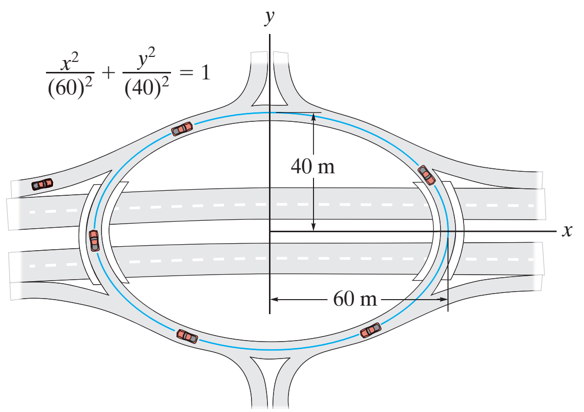
12–138. The motorcycle is traveling at 40 m/s when it is at A. If the speed is then decreased at $\dot{v} = -(0.05\ s)$ m/s^2, where s is in meters measured from A, determine its speed and acceleration when it reaches B.



Prob. 12–138

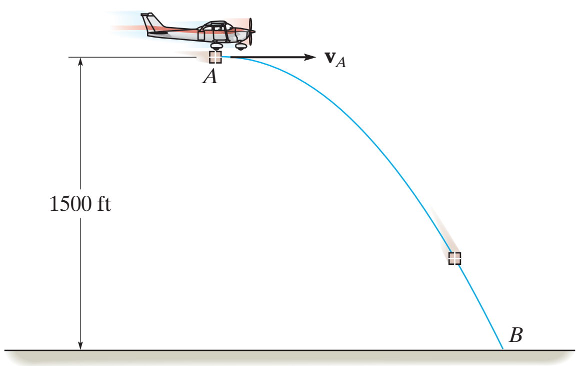
12–139. Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the minimum acceleration experienced by the passengers.

***12–140.** Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



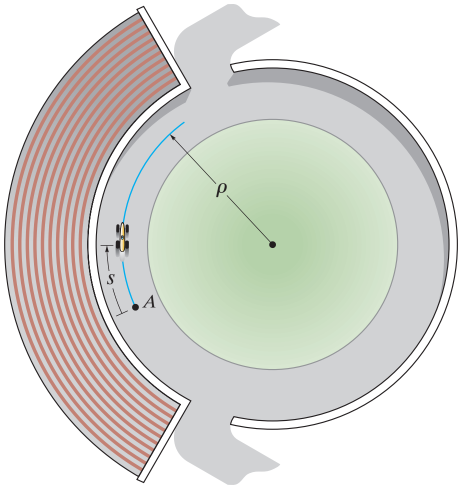
Probs. 12–139/140

12–141. A package is dropped from the plane which is flying with a constant horizontal velocity of $v_A = 150$ ft/s. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity of $v_A = 150$ ft/s, and (b) *just before* it strikes the ground at B.



Prob. 12–141

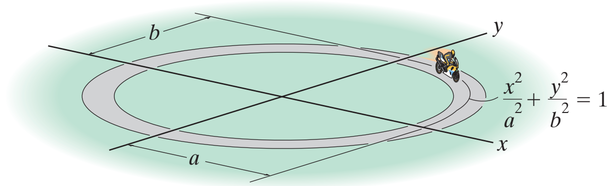
12–142. The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



Prob. 12–142

12–143. The motorcycle travels along the elliptical track at a constant speed v. Determine its greatest acceleration if $a > b$.

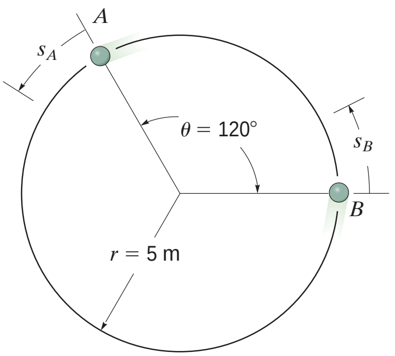
***12–144.** The motorcycle travels along the elliptical track at a constant speed v. Determine its smallest acceleration if $a > b$.



Probs. 12–143/144

12–145. Particles A and B are traveling counter-clockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A)\ \text{m/s}^2$, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?

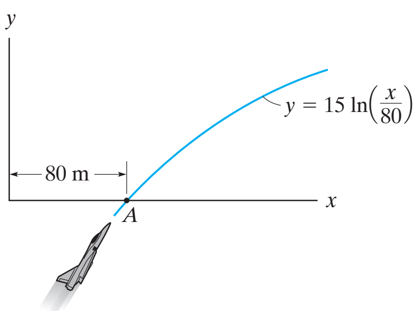
12–146. Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t\ \text{m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



Probs. 12–145/146

12–147. The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s² when it reaches point A. Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.

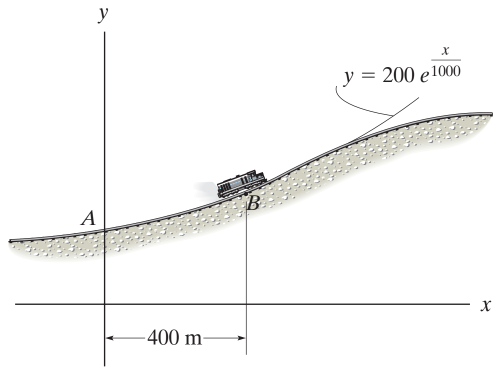
***12–148.** The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point $A(y = 0)$.



Probs. 12–147/148

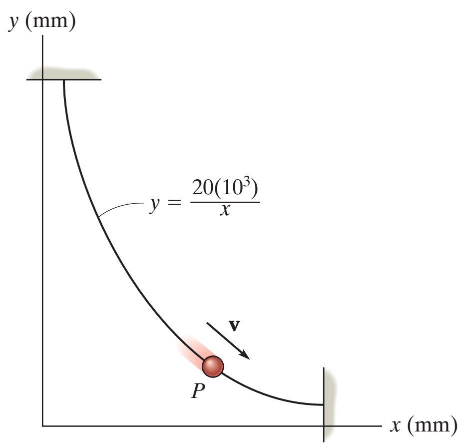
12–149. The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5\ \text{m/s}^2$. Determine the magnitude of acceleration of the train at this point.

12–150. The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25\ \text{m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B, where $s_{AB} = 412$ m.



Probs. 12–149/150

12–151. The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.

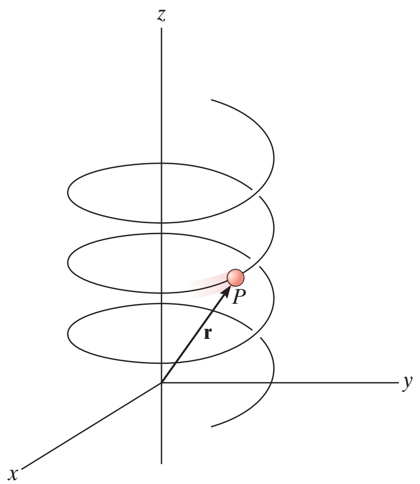


Prob. 12–151

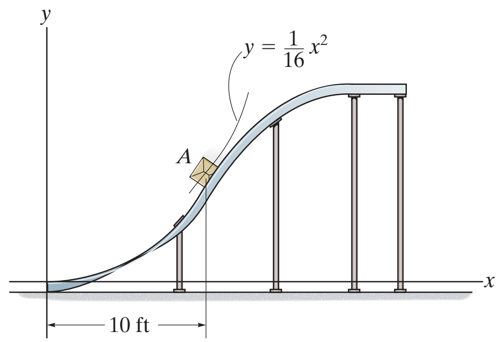
***12–152.** A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint*: Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?

12–153. The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

12–154. If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3\text{ ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



Prob. 12–152

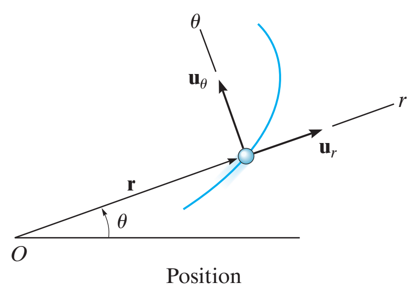


Prob. 12–154

12.8 Curvilinear Motion: Cylindrical Components

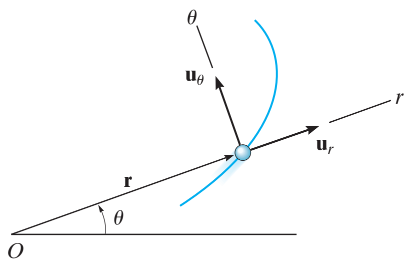
Sometimes the motion of the particle is constrained on a path that is best described using cylindrical coordinates. If motion is restricted to the plane, then polar coordinates are used.

Polar Coordinates. We can specify the location of the particle shown in Fig. 12–30*a* using a *radial coordinate* r, which extends outward from the fixed origin O to the particle, and a *transverse coordinate* θ, which is the counterclockwise angle between a fixed reference line and the r axis. The angle is generally measured in degrees or radians, where 1 rad $= 180°/\pi$. The positive directions of the r and θ coordinates are defined by the unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$, respectively. Here $\mathbf{u}_r$ is in the direction of increasing r when θ is held fixed, and $\mathbf{u}_\theta$ is in a direction of increasing θ when r is held fixed. Note that these directions are perpendicular to one another.



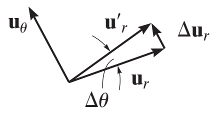
(a)

Fig. 12–30



Position

(a)



(b)

Position. At any instant the position of the particle, Fig. 12–30*a*, is defined by the position vector

$$\mathbf{r} = r\mathbf{u}_r \tag{12–22}$$

Velocity. The instantaneous velocity $\mathbf{v}$ is obtained by taking the time derivative of $\mathbf{r}$. Using a dot to represent the time derivative, we have

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{u}_r + r\dot{\mathbf{u}}_r$$

To evaluate $\dot{\mathbf{u}}_r$, notice that $\mathbf{u}_r$ only changes its direction with respect to time, since by definition the magnitude of this vector is always one unit. Hence, during the time Δt, a change Δr will not cause a change in the direction of $\mathbf{u}_r$; however, a change $\Delta\theta$ will cause $\mathbf{u}_r$ to become $\mathbf{u}'_r$, where $\mathbf{u}'_r = \mathbf{u}_r + \Delta\mathbf{u}_r$, Fig. 12–30*b*. The time change in $\mathbf{u}_r$ is then $\Delta\mathbf{u}_r$. For small angles $\Delta\theta$ this vector has a magnitude $\Delta u_r \approx 1(\Delta\theta)$ and acts in the $\mathbf{u}_\theta$ direction. Therefore, $\Delta\mathbf{u}_r = \Delta\theta\mathbf{u}_\theta$, and so

$$\dot{\mathbf{u}}_r = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u}_r}{\Delta t} = \left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \right) \mathbf{u}_\theta$$

$$\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta \tag{12–23}$$

Substituting into the above equation, the velocity can be written in component form as

$$\boxed{\mathbf{v} = v_r\mathbf{u}_r + v_\theta\mathbf{u}_\theta} \tag{12–24}$$

where

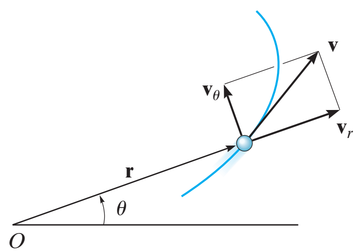
$$\boxed{\begin{aligned} v_r &= \dot{r} \\ v_\theta &= r\dot{\theta} \end{aligned}} \tag{12–25}$$

These components are shown graphically in Fig. 12–30*c*. The *radial component* $\mathbf{v}_r$ is a measure of the rate of increase or decrease in the length of the radial coordinate, i.e., $\dot{r}$; whereas the *transverse component* $\mathbf{v}_\theta$ can be interpreted as the rate of motion along the circumference of a circle having a radius r. In particular, the term $\dot{\theta} = d\theta/dt$ is called the *angular velocity*, since it indicates the time rate of change of the angle θ. Common units used for this measurement are rad/s.

Since $\mathbf{v}_r$ and $\mathbf{v}_\theta$ are mutually perpendicular, the *magnitude* of velocity or speed is simply the positive value of

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} \tag{12–26}$$

and the *direction* of $\mathbf{v}$ is, of course, tangent to the path, Fig. 12–30*c*.



Velocity

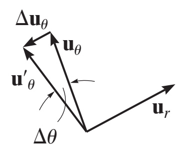
(c)

Fig. 12–30 (cont.)

Acceleration. Taking the time derivatives of Eq. 12–24, using Eqs. 12–25, we obtain the particle's instantaneous acceleration,

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{r}\mathbf{u}_r + \dot{r}\dot{\mathbf{u}}_r + \dot{r}\dot{\theta}\mathbf{u}_\theta + r\ddot{\theta}\mathbf{u}_\theta + r\dot{\theta}\dot{\mathbf{u}}_\theta$$

To evaluate $\dot{\mathbf{u}}_\theta$, it is necessary only to find the change in the direction of $\mathbf{u}_\theta$ since its magnitude is always unity. During the time Δt, a change Δr will not change the direction of $\mathbf{u}_\theta$, however, a change $\Delta\theta$ will cause $\mathbf{u}_\theta$ to become $\mathbf{u}'_\theta$, where $\mathbf{u}'_\theta = \mathbf{u}_\theta + \Delta\mathbf{u}_\theta$, Fig. 12–30*d*. The time change in $\mathbf{u}_\theta$ is thus $\Delta\mathbf{u}_\theta$. For small angles this vector has a magnitude $\Delta u_\theta \approx 1(\Delta\theta)$ and acts in the $-\mathbf{u}_r$ direction; i.e., $\Delta\mathbf{u}_\theta = -\Delta\theta\mathbf{u}_r$. Thus,



(d)

$$\dot{\mathbf{u}}_\theta = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{u}_\theta}{\Delta t} = -\left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\right)\mathbf{u}_r$$

$$\dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \qquad (12–27)$$

Substituting this result and Eq. 12–23 into the above equation for **a**, we can write the acceleration in component form as

$$\boxed{\mathbf{a} = a_r\mathbf{u}_r + a_\theta\mathbf{u}_\theta} \qquad (12–28)$$

where

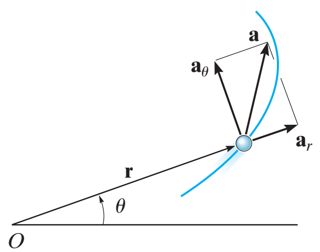
$$\boxed{\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{aligned}} \qquad (12–29)$$

The term $\ddot{\theta} = d^2\theta/dt^2 = d/dt(d\theta/dt)$ is called the *angular acceleration* since it measures the change made in the angular velocity during an instant of time. Units for this measurement are rad/s^2.

Since $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are always perpendicular, the *magnitude* of acceleration is simply the positive value of

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} \qquad (12–30)$$

The *direction* is determined from the vector addition of its two components. In general, **a** will *not* be tangent to the path, Fig. 12–30*e*.



Acceleration

(e)

12

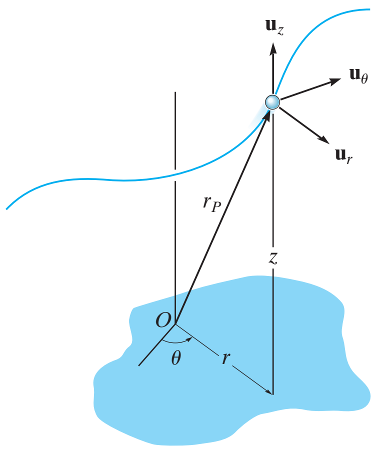


Fig. 12–31

The spiral motion of this girl can be followed by using cylindrical components. Here the radial coordinate r is constant, the transverse coordinate θ will increase with time as the girl rotates about the vertical, and her altitude z will decrease with time. (© R.C. Hibbeler)

Cylindrical Coordinates. If the particle moves along a space curve as shown in Fig. 12–31, then its location may be specified by the three *cylindrical coordinates*, r, θ, z. The z coordinate is identical to that used for rectangular coordinates. Since the unit vector defining its direction, $\mathbf{u}_z$, is constant, the time derivatives of this vector are zero, and therefore the position, velocity, and acceleration of the particle can be written in terms of its cylindrical coordinates as follows:

$$\mathbf{r}_P = r\mathbf{u}_r + z\mathbf{u}_z$$

$$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta + \dot{z}\mathbf{u}_z \tag{12–31}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta + \ddot{z}\mathbf{u}_z \tag{12–32}$$

Time Derivatives. The above equations require that we obtain the time derivatives $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ in order to evaluate the r and θ components of $\mathbf{v}$ and $\mathbf{a}$. Two types of problems generally occur:

1. If the polar coordinates are specified as time parametric equations, $r = r(t)$ and $\theta = \theta(t)$, then the time derivatives can be found directly.
2. If the time-parametric equations are not given, then the path $r = f(\theta)$ must be known. Using the chain rule of calculus we can then find the relation between $\dot{r}$ and $\dot{\theta}$, and between $\ddot{r}$ and $\ddot{\theta}$. Application of the chain rule, along with some examples, is explained in Appendix C.

Procedure for Analysis

Coordinate System.

- Polar coordinates are a suitable choice for solving problems when data regarding the angular motion of the radial coordinate r is given to describe the particle's motion. Also, some paths of motion can conveniently be described in terms of these coordinates.
- To use polar coordinates, the origin is established at a fixed point, and the radial line r is directed to the particle.
- The transverse coordinate θ is measured from a fixed reference line to the radial line.

Velocity and Acceleration.

- Once r and the four time derivatives $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ have been evaluated at the instant considered, their values can be substituted into Eqs. 12–25 and 12–29 to obtain the radial and transverse components of $\mathbf{v}$ and $\mathbf{a}$.
- If it is necessary to take the time derivatives of $r = f(\theta)$, then the chain rule of calculus must be used. See Appendix C.
- Motion in three dimensions requires a simple extension of the above procedure to include $\dot{z}$ and $\ddot{z}$.

EXAMPLE 12.17

The amusement park ride shown in Fig. 12–32*a* consists of a chair that is rotating in a horizontal circular path of radius r such that the arm OB has an angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$. Determine the radial and transverse components of velocity and acceleration of the passenger. Neglect his size in the calculation.

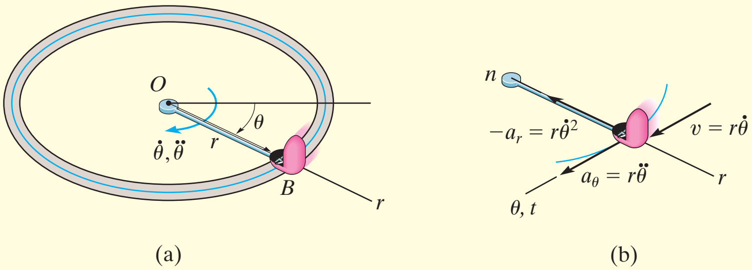


Fig. 12–32

SOLUTION

Coordinate System. Since the angular motion of the arm is reported, polar coordinates are chosen for the solution, Fig. 12–32*a*. Here θ is not related to r, since the radius is constant for all θ.

Velocity and Acceleration. It is first necessary to specify the first and second time derivatives of r and θ. Since r is *constant*, we have

$$r = r \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

Thus,

$$v_r = \dot{r} = 0 \qquad \textit{Ans.}$$

$$v_\theta = r\dot{\theta} \qquad \textit{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2 \qquad \textit{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r\ddot{\theta} \qquad \textit{Ans.}$$

These results are shown in Fig. 12–32*b*.

NOTE: The n, t axes are also shown in Fig. 12–32*b*, which in this special case of circular motion happen to be *collinear* with the r and θ axes, respectively. Since $v = v_\theta = v_t = r\dot{\theta}$, then by comparison,

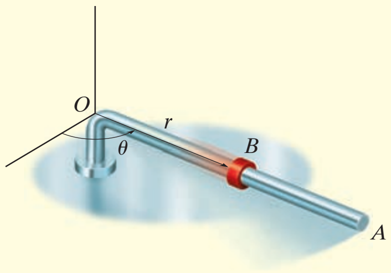
$$-a_r = a_n = \frac{v^2}{\rho} = \frac{(r\dot{\theta})^2}{r} = r\dot{\theta}^2$$

$$a_\theta = a_t = \frac{dv}{dt} = \frac{d}{dt}(r\dot{\theta}) = \frac{dr}{dt}\dot{\theta} + r\frac{d\dot{\theta}}{dt} = 0 + r\ddot{\theta}$$

12

EXAMPLE 12.18

The rod OA in Fig. 12–33a rotates in the horizontal plane such that $\theta = (t^3)$ rad. At the same time, the collar B is sliding outward along OA so that $r = (100t^2)$ mm. If in both cases t is in seconds, determine the velocity and acceleration of the collar when $t = 1$ s.



(a)

SOLUTION

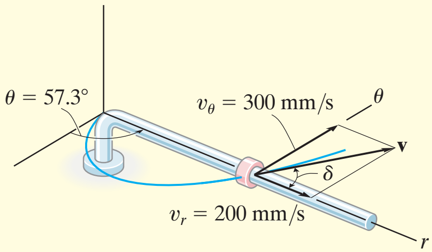
Coordinate System. Since time-parametric equations of the path are given, it is not necessary to relate r to θ.

Velocity and Acceleration. Determining the time derivatives and evaluating them when $t = 1$ s, we have

$$r = 100t^2\Big|_{t=1\text{ s}} = 100\text{ mm} \qquad \theta = t^3\Big|_{t=1\text{ s}} = 1\text{ rad} = 57.3^\circ$$

$$\dot{r} = 200t\Big|_{t=1\text{ s}} = 200\text{ mm/s} \qquad \dot{\theta} = 3t^2\Big|_{t=1\text{ s}} = 3\text{ rad/s}$$

$$\ddot{r} = 200\Big|_{t=1\text{ s}} = 200\text{ mm/s}^2 \qquad \ddot{\theta} = 6t\Big|_{t=1\text{ s}} = 6\text{ rad/s}^2.$$



(b)

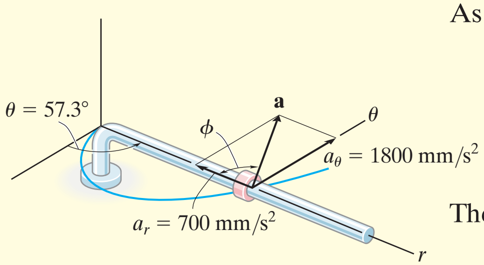
As shown in Fig. 12–33b,

$$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta$$
$$= 200\mathbf{u}_r + 100(3)\mathbf{u}_\theta = \{200\mathbf{u}_r + 300\mathbf{u}_\theta\}\text{ mm/s}$$

The magnitude of $\mathbf{v}$ is

$$v = \sqrt{(200)^2 + (300)^2} = 361\text{ mm/s} \qquad \textit{Ans.}$$

$$\delta = \tan^{-1}\left(\frac{300}{200}\right) = 56.3^\circ \qquad \delta + 57.3^\circ = 114^\circ \qquad \textit{Ans.}$$



(c)

Fig. 12–33

As shown in Fig. 12–33c,

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$
$$= [200 - 100(3)^2]\mathbf{u}_r + [100(6) + 2(200)3]\mathbf{u}_\theta$$
$$= \{-700\mathbf{u}_r + 1800\mathbf{u}_\theta\}\text{ mm/s}^2$$

The magnitude of $\mathbf{a}$ is

$$a = \sqrt{(-700)^2 + (1800)^2} = 1930\text{ mm/s}^2 \qquad \textit{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{1800}{700}\right) = 68.7^\circ \qquad (180^\circ - \phi) + 57.3^\circ = 169^\circ \qquad \textit{Ans.}$$

NOTE: The velocity is tangent to the path; however, the acceleration is directed within the curvature of the path, as expected.

EXAMPLE 12.19

The searchlight in Fig. 12–34*a* casts a spot of light along the face of a wall that is located 100 m from the searchlight. Determine the magnitudes of the velocity and acceleration at which the spot appears to travel across the wall at the instant $\theta = 45^\circ$. The searchlight rotates at a constant rate of $\dot{\theta} = 4$ rad/s.

SOLUTION

Coordinate System. Polar coordinates will be used to solve this problem since the angular rate of the searchlight is given. To find the necessary time derivatives it is first necessary to relate r to θ. From Fig. 12–34*a*,

$$r = 100/\cos\theta = 100\sec\theta$$

Velocity and Acceleration. Using the chain rule of calculus, noting that $d(\sec\theta) = \sec\theta\tan\theta\,d\theta$, and $d(\tan\theta) = \sec^2\theta\,d\theta$, we have

$$\begin{aligned}
\dot{r} &= 100(\sec\theta\tan\theta)\dot{\theta} \\
\ddot{r} &= 100(\sec\theta\tan\theta)\dot{\theta}(\tan\theta)\dot{\theta} + 100\sec\theta(\sec^2\theta)\dot{\theta}(\dot{\theta}) \\
&\quad + 100\sec\theta\tan\theta(\ddot{\theta}) \\
&= 100\sec\theta\tan^2\theta\,(\dot{\theta})^2 + 100\sec^3\theta\,(\dot{\theta})^2 + 100(\sec\theta\tan\theta)\ddot{\theta}
\end{aligned}$$

Since $\dot{\theta} = 4$ rad/s $=$ constant, then $\ddot{\theta} = 0$, and the above equations, when $\theta = 45^\circ$, become

$$\begin{aligned}
r &= 100\sec 45^\circ = 141.4 \\
\dot{r} &= 400\sec 45^\circ\tan 45^\circ = 565.7 \\
\ddot{r} &= 1600\,(\sec 45^\circ\tan^2 45^\circ + \sec^3 45^\circ) = 6788.2
\end{aligned}$$

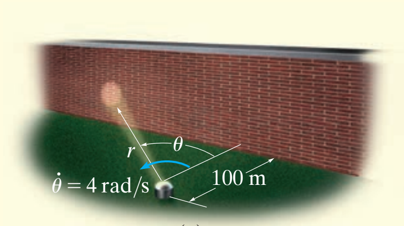
As shown in Fig. 12–34*b*,

$$\begin{aligned}
\mathbf{v} &= \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta \\
&= 565.7\mathbf{u}_r + 141.4(4)\mathbf{u}_\theta \\
&= \{565.7\mathbf{u}_r + 565.7\mathbf{u}_\theta\}\ \text{m/s} \\
v &= \sqrt{v_r^2 + v_\theta^2} = \sqrt{(565.7)^2 + (565.7)^2} \\
&= 800\ \text{m/s}
\end{aligned}$$ *Ans.*

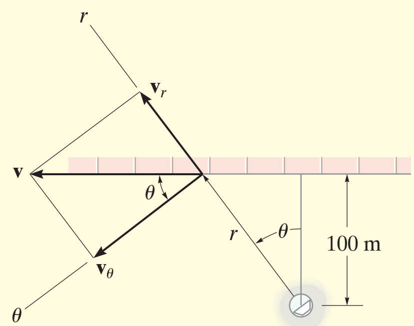
As shown in Fig. 12–34c,

$$\begin{aligned}
\mathbf{a} &= (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta \\
&= [6788.2 - 141.4(4)^2]\mathbf{u}_r + [141.4(0) + 2(565.7)4]\mathbf{u}_\theta \\
&= \{4525.5\mathbf{u}_r + 4525.5\mathbf{u}_\theta\}\ \text{m/s}^2 \\
a &= \sqrt{a_r^2 + a_\theta^2} = \sqrt{(4525.5)^2 + (4525.5)^2} \\
&= 6400\ \text{m/s}^2
\end{aligned}$$ *Ans.*

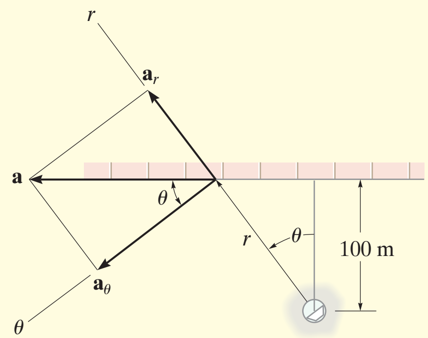
NOTE: It is also possible to find a without having to calculate $\ddot{r}$ (or a_r). As shown in Fig. 12–34*d*, since $a_\theta = 4525.5$ m/s^2, then by vector resolution, $a = 4525.5/\cos 45^\circ = 6400$ m/s^2.



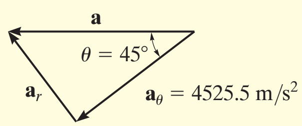
(a)



(b)



(c)

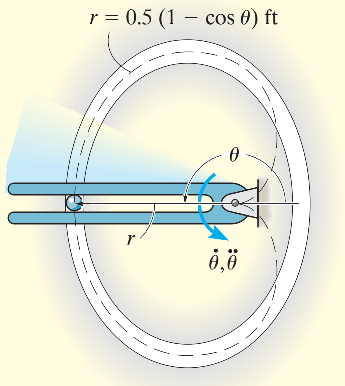


(d)

Fig. 12–34

12

EXAMPLE 12.20



(a)

Due to the rotation of the forked rod, the ball in Fig. 12–35*a* travels around the slotted path, a portion of which is in the shape of a cardioid, $r = 0.5(1 - \cos\theta)$ ft, where θ is in radians. If the ball's velocity is $v = 4$ ft/s and its acceleration is $a = 30$ ft/s^2 at the instant $\theta = 180°$, determine the angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$ of the fork.

SOLUTION

Coordinate System. This path is most unusual, and mathematically it is best expressed using polar coordinates, as done here, rather than rectangular coordinates. Also, since $\dot{\theta}$ and $\ddot{\theta}$ must be determined, then r, θ coordinates are an obvious choice.

Velocity and Acceleration. The time derivatives of r and θ can be determined using the chain rule.

$$r = 0.5(1 - \cos\theta)$$
$$\dot{r} = 0.5(\sin\theta)\dot{\theta}$$
$$\ddot{r} = 0.5(\cos\theta)\dot{\theta}(\dot{\theta}) + 0.5(\sin\theta)\ddot{\theta}$$

Evaluating these results at $\theta = 180°$, we have

$$r = 1 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = -0.5\dot{\theta}^2$$

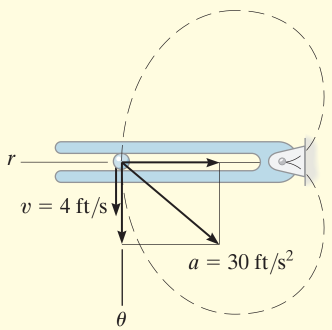
Since $v = 4$ ft/s, using Eq. 12–26 to determine $\dot{\theta}$ yields

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$$
$$4 = \sqrt{(0)^2 + (1\dot{\theta})^2}$$
$$\dot{\theta} = 4 \text{ rad/s} \qquad \textit{Ans.}$$

In a similar manner, $\ddot{\theta}$ can be found using Eq. 12–30.

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$$
$$30 = \sqrt{[-0.5(4)^2 - 1(4)^2]^2 + [1\ddot{\theta} + 2(0)(4)]^2}$$
$$(30)^2 = (-24)^2 + \ddot{\theta}^2$$
$$\ddot{\theta} = 18 \text{ rad/s}^2 \qquad \textit{Ans.}$$

Vectors **a** and **v** are shown in Fig. 12–35*b*.



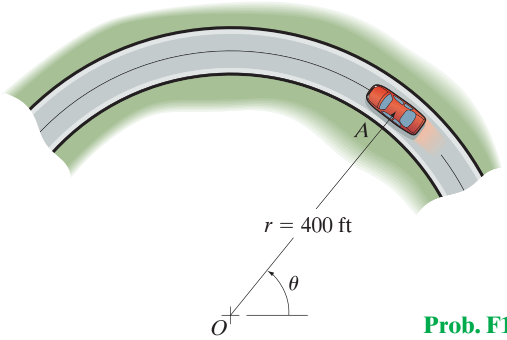
(b)

Fig. 12–35

NOTE: At this location, the θ and t (tangential) axes will coincide. The $+n$ (normal) axis is directed to the right, opposite to $+r$.

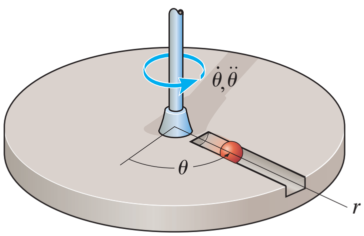
FUNDAMENTAL PROBLEMS

F12–33. The car has a speed of 55 ft/s. Determine the angular velocity $\dot{\theta}$ of the radial line OA at this instant.



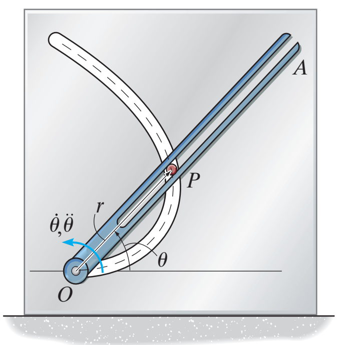
Prob. F12–33

F12–34. The platform is rotating about the vertical axis such that at any instant its angular position is $\theta = (4t^{3/2})$ rad, where t is in seconds. A ball rolls outward along the radial groove so that its position is $r = (0.1t^3)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the ball when $t = 1.5$ s.



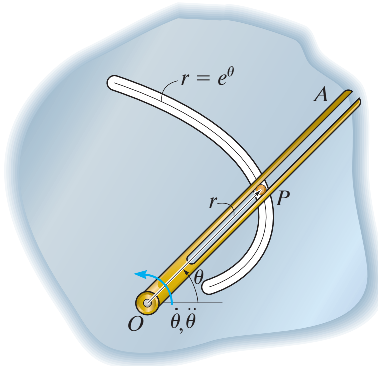
Prob. F12–34

F12–35. Peg P is driven by the fork link OA along the curved path described by $r = (2\theta)$ ft. At the instant $\theta = \pi/4$ rad, the angular velocity and angular acceleration of the link are $\dot{\theta} = 3$ rad/s and $\ddot{\theta} = 1$ rad/s^2. Determine the magnitude of the peg's acceleration at this instant.



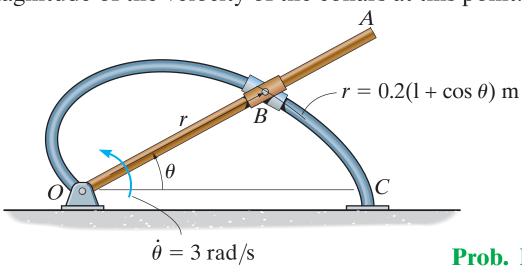
Prob. F12–35

F12–36. Peg P is driven by the forked link OA along the path described by $r = e^{\theta}$, where r is in meters. When $\theta = \frac{\pi}{4}$ rad, the link has an angular velocity and angular acceleration of $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 4$ rad/s^2. Determine the radial and transverse components of the peg's acceleration at this instant.



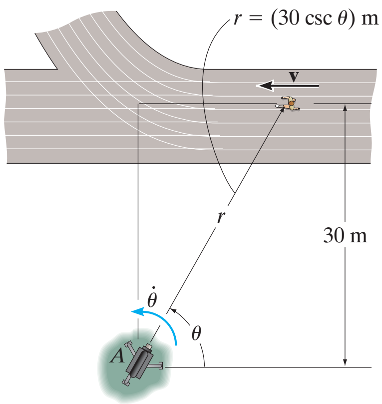
Prob. F12–36

F12–37. The collars are pin connected at B and are free to move along rod OA and the curved guide OC having the shape of a cardioid, $r = [0.2(1 + \cos\theta)]$ m. At $\theta = 30°$, the angular velocity of OA is $\dot{\theta} = 3$ rad/s. Determine the magnitude of the velocity of the collars at this point.



Prob. F12–37

F12–38. At the instant $\theta = 45°$, the athlete is running with a constant speed of 2 m/s. Determine the angular velocity at which the camera must turn in order to follow the motion.



Prob. F12–38

PROBLEMS

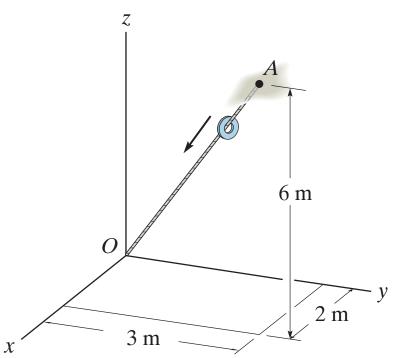
12–155. A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

***12–156.** For a short time a rocket travels up and to the right at a constant speed of 800 m/s along the parabolic path $y = 600 - 35x^2$. Determine the radial and transverse components of velocity of the rocket at the instant $\theta = 60°$, where θ is measured counterclockwise from the x axis.

12–157. A particle moves along a path defined by polar coordinates $r = (2e^t)$ ft and $\theta = (8t^2)$ rad, where t is in seconds. Determine the components of its velocity and acceleration when $t = 1$ s.

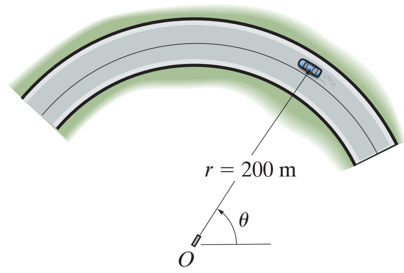
12–158. An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h^2. If the propeller has a diameter of 6 ft and is rotating at a constant angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

12–159. The small washer is sliding down the cord OA. When it is at the midpoint, its speed is 28 m/s and its acceleration is 7 m/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



Prob. 12–159

***12–160.** A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



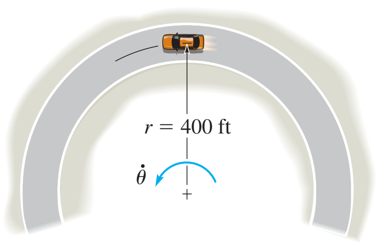
Prob. 12–160

12–161. If a particle moves along a path such that $r = (2 \cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

12–162. If a particle moves along a path such that $r = (e^{at})$ m and $\theta = t$, where t is in seconds, plot the path $r = f(\theta)$, and determine the particle's radial and transverse components of velocity and acceleration.

12–163. The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.

***12–164.** The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.

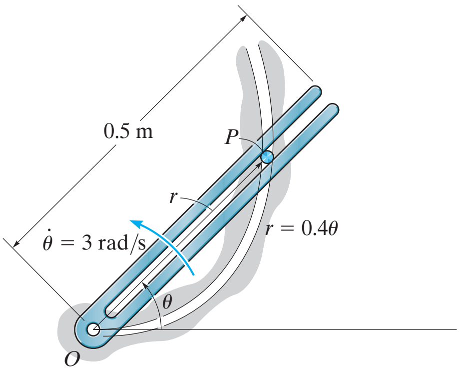


Probs. 12–163/164

12–165. The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

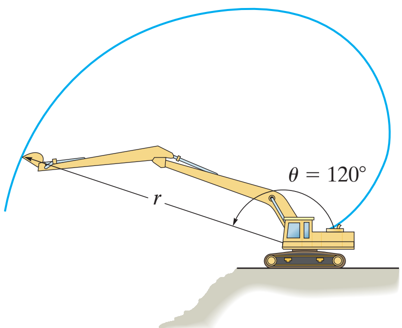
12–166. A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ and the argument for the sine are in radians, and t is in seconds. Determine the magnitude of the acceleration of the particle at $\theta = 30^\circ$. The particle starts from rest at $\theta = 0^\circ$.

12–167. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



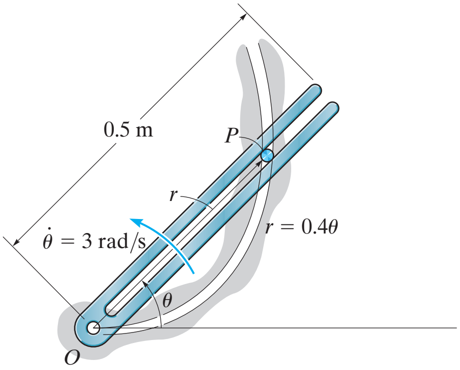
Prob. 12–167

***12–168.** For a short time the bucket of the backhoe traces the path of the cardioid $r = 25(1 - \cos\theta)$ ft. Determine the magnitudes of the velocity and acceleration of the bucket when $\theta = 120^\circ$ if the boom is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.2$ rad/s^2 at the instant shown.



Prob. 12–168

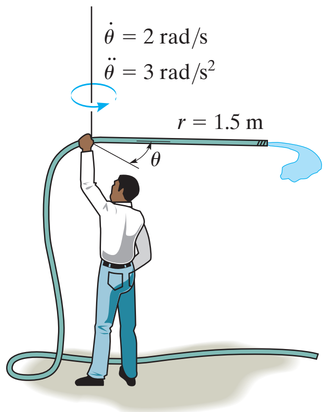
12–169. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



Prob. 12–169

12–170. A particle moves in the $x-y$ plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

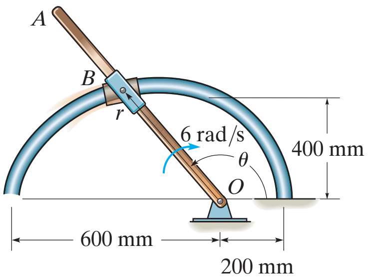
12–171. At the instant shown, the man is twirling a hose over his head with an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If it is assumed that the hose lies in a horizontal plane, and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 1.5$ m.



Prob. 12–171

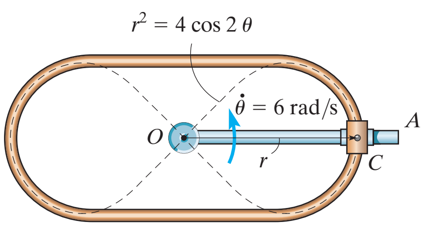
***12–172.** The rod OA rotates clockwise with a constant angular velocity of 6 rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 150°$.

12–173. Determine the magnitude of the acceleration of the slider blocks in Prob. 12–172 when $\theta = 150°$.



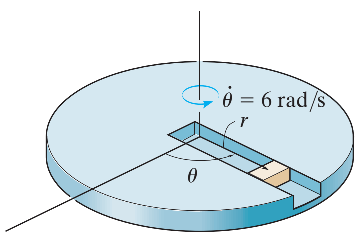
Probs. 12–172/173

12–174. A double collar C is pin connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = (4 \cos 2\theta)$ ft^2, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0°$ as shown. Rod OA is rotating at a constant rate of $\dot{\theta} = 6$ rad/s.



Prob. 12–174

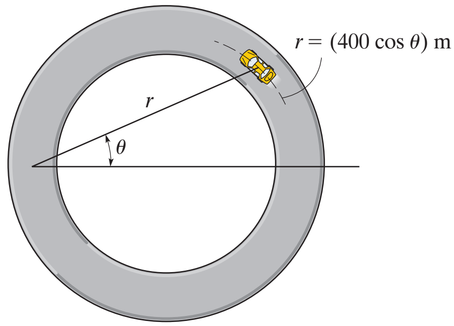
12–175. A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



Prob. 12–175

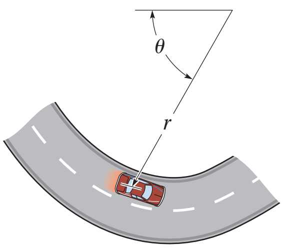
***12–176.** The car travels around the circular track with a constant speed of 20 m/s. Determine the car's radial and transverse components of velocity and acceleration at the instant $\theta = \pi/4$ rad.

12–177. The car travels around the circular track such that its transverse component is $\theta = (0.006t^2)$ rad, where t is in seconds. Determine the car's radial and transverse components of velocity and acceleration at the instant $t = 4$ s.



Probs. 12–176/177

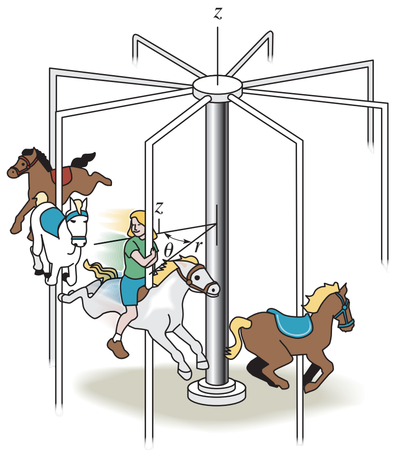
12–178. The car travels along a road which for a short distance is defined by $r = (200/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 35$ ft/s, determine the radial and transverse components of its velocity when $\theta = \pi/3$ rad.



Prob. 12–178

12–179. A horse on the merry-go-round moves according to the equations $r = 8$ ft, $\theta = (0.6t)$ rad, and $z = (1.5 \sin \theta)$ ft, where t is in seconds. Determine the cylindrical components of the velocity and acceleration of the horse when $t = 4$ s.

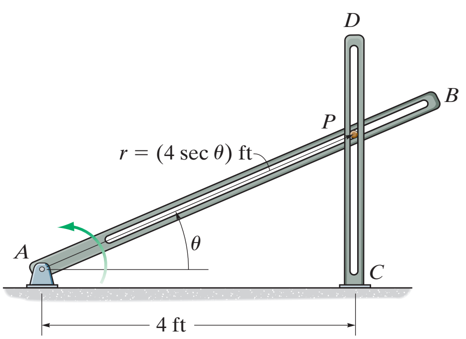
***12–180.** A horse on the merry-go-round moves according to the equations $r = 8$ ft, $\dot{\theta} = 2$ rad/s and $z = (1.5 \sin \theta)$ ft, where t is in seconds. Determine the maximum and minimum magnitudes of the velocity and acceleration of the horse during the motion.



Probs. 12–179/180

12–181. If the slotted arm AB rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB.

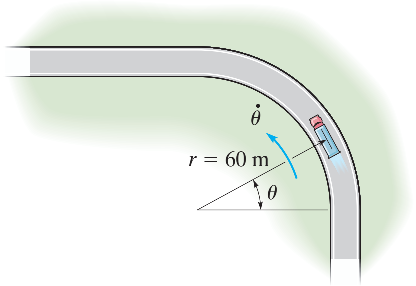
12–182. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB. When $\theta = 30°$, the angular velocity and angular acceleration of arm AB are $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the peg P at this instant.



Probs. 12–181/182

12–183. A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.

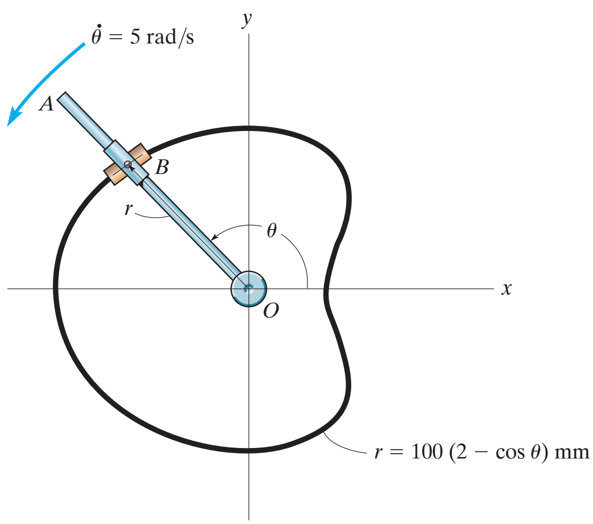
***12–184.** A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s^2, Determine the truck's radial and transverse components of acceleration.



Probs. 12–183/184

12–185. The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos \theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 120°$.

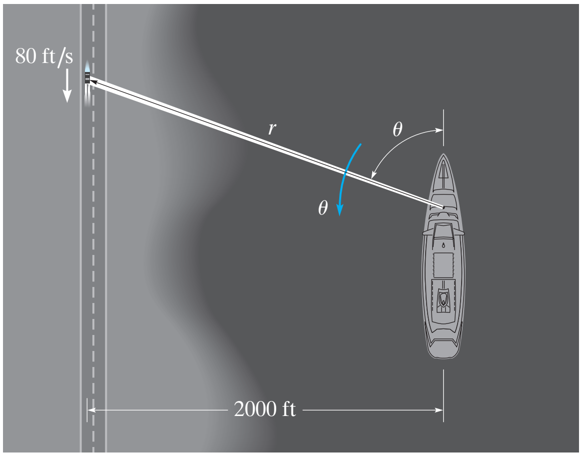
12–186. Determine the magnitude of the acceleration of the slider blocks in Prob. 12–185 when $\theta = 120°$.



Probs. 12–185/186

12–187. The searchlight on the boat anchored 2000 ft from shore is turned on the automobile, which is traveling along the straight road at a constant speed 80 ft/s, Determine the angular rate of rotation of the light when the automobile is $r = 3000$ ft from the boat.

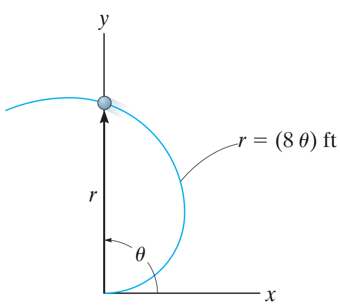
***12–188.** If the car in Prob. 12–187 is accelerating at 15 ft/s^2 at the instant $r = 3000$ ft determine the required angular acceleration $\ddot{\theta}$ of the light at this instant.



Probs. 12–187/188

12–189. A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.

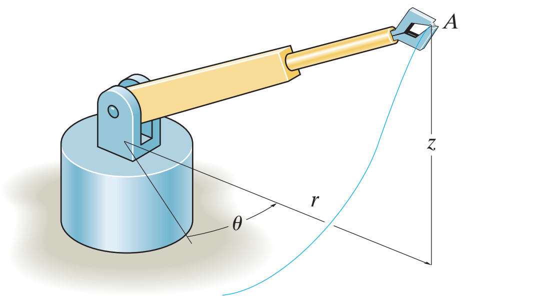
12–190. Solve Prob. 12–189 if the particle has an angular acceleration $\ddot{\theta} = 5$ rad/s^2 when $\dot{\theta} = 4$ rad/s at $\theta = \pi/2$ rad.



Probs. 12–189/190

12–191. The arm of the robot moves so that $r = 3$ ft is constant, and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.

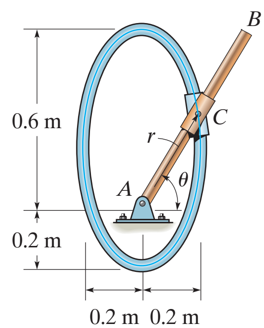
***12–192.** For a short time the arm of the robot is extending such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



Probs. 12–191/192

12–193. The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5\,t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |(0.4 \sin \theta + 0.2)|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0, \theta = 0$. Use Simpson's rule with $n = 50$ to determine θ at $t = 1$ s.

12–194. The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the mechanism is to be designed so that the largest speed given to the collar is 6 m/s, determine the required constant angular velocity $\dot{\theta}$ of rod AB. The path defined by the fixed rod is $r = (0.4 \sin \theta + 0.2)$ m.



Probs. 12–193/194

12.9 Absolute Dependent Motion Analysis of Two Particles

In some types of problems the motion of one particle will *depend* on the corresponding motion of another particle. This dependency commonly occurs if the particles, here represented by blocks, are interconnected by inextensible cords which are wrapped around pulleys. For example, the movement of block A downward along the inclined plane in Fig. 12–36 will cause a corresponding movement of block B up the other incline. We can show this mathematically by first specifying the location of the blocks using *position coordinates* s_A and s_B. Note that each of the coordinate axes is (1) measured from a *fixed* point (O) or *fixed* datum line, (2) measured along each inclined plane *in the direction of motion* of each block, and (3) has a positive sense from the fixed datums to A and to B. If the total cord length is l_T, the two position coordinates are related by the equation

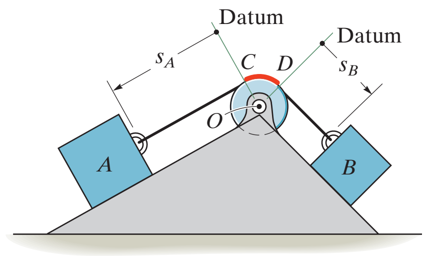


Fig. 12–36

$$s_A + l_{CD} + s_B = l_T$$

Here l_{CD} is the length of the cord passing over arc CD. Taking the time derivative of this expression, realizing that l_{CD} and l_T *remain constant*, while s_A and s_B measure the segments of the cord that change in length, we have

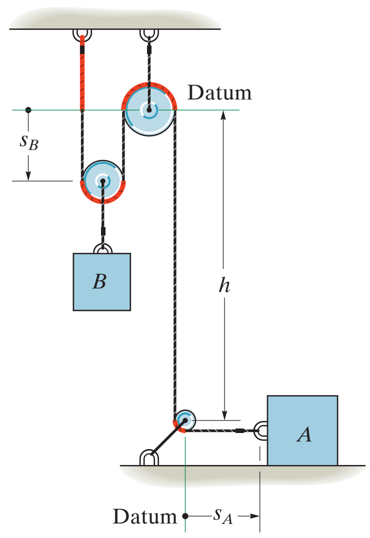
$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \qquad \text{or} \qquad v_B = -v_A$$

The negative sign indicates that when block A has a velocity downward, i.e., in the direction of positive s_A, it causes a corresponding upward velocity of block B; i.e., B moves in the negative s_B direction.

In a similar manner, time differentiation of the velocities yields the relation between the accelerations, i.e.,

$$a_B = -a_A$$

A more complicated example is shown in Fig. 12–37*a*. In this case, the position of block A is specified by s_A, and the position of the *end* of the cord from which block B is suspended is defined by s_B. As above, we have chosen position coordinates which (1) have their origin at fixed points or datums, (2) are measured in the direction of motion of each block, and (3) from the fixed datums are positive to the right for s_A and positive downward for s_B. During the motion, the length of the red colored segments of the cord in Fig. 12–37*a remains constant*. If l represents the total length of cord minus these segments, then the position coordinates can be related by the equation



(a)

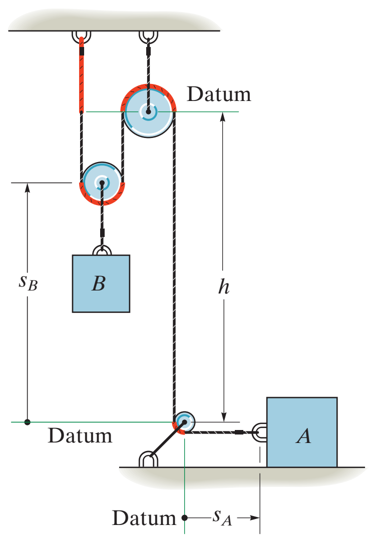
Fig. 12–37

$$2s_B + h + s_A = l$$

Since l and h are constant during the motion, the two time derivatives yield

$$2v_B = -v_A \qquad 2a_B = -a_A$$

Hence, when B moves downward ($+s_B$), A moves to the left ($-s_A$) with twice the motion.



(b)

Fig. 12–37 (cont.)

This example can also be worked by defining the position of block B from the center of the bottom pulley (a fixed point), Fig. 12–37*b*. In this case

$$2(h - s_B) + h + s_A = l$$

Time differentiation yields

$$2v_B = v_A \qquad 2a_B = a_A$$

Here the signs are the same. Why?

The cable is wrapped around the pulleys on this crane in order to reduce the required force needed to hoist a load. (© R.C. Hibbeler)

Procedure for Analysis

The above method of relating the dependent motion of one particle to that of another can be performed using algebraic scalars or position coordinates provided each particle moves along a rectilinear path. When this is the case, only the magnitudes of the velocity and acceleration of the particles will change, not their line of direction.

Position-Coordinate Equation.

- Establish each position coordinate with an origin located at a *fixed* point or datum.
- It is *not necessary* that the *origin* be the *same* for each of the coordinates; however, it is *important* that each coordinate axis selected be directed along the *path of motion* of the particle.
- Using geometry or trigonometry, relate the position coordinates to the total length of the cord, l_T, or to that portion of cord, l, which *excludes* the segments that do not change length as the particles move—such as arc segments wrapped over pulleys.
- If a problem involves a *system* of two or more cords wrapped around pulleys, then the position of a point on one cord must be related to the position of a point on another cord using the above procedure. Separate equations are written for a fixed length of each cord of the system and the positions of the two particles are then related by these equations (see Examples 12.22 and 12.23).

Time Derivatives.

- Two successive time derivatives of the position-coordinate equations yield the required velocity and acceleration equations which relate the motions of the particles.
- The signs of the terms in these equations will be consistent with those that specify the positive and negative sense of the position coordinates.

EXAMPLE 12.21

Determine the speed of block A in Fig. 12–38 if block B has an upward speed of 6 ft/s.

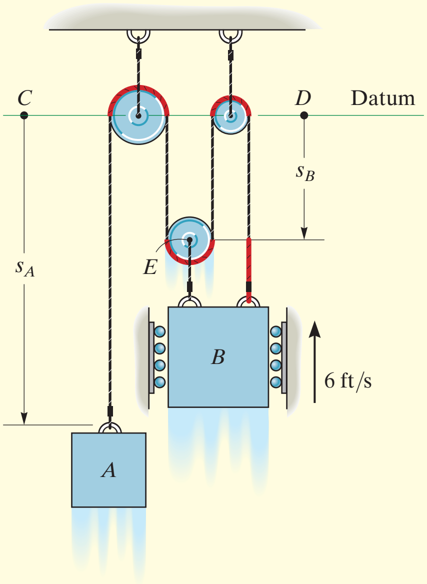


Fig. 12–38

SOLUTION

Position-Coordinate Equation. There is *one cord* in this system having segments which change length. Position coordinates s_A and s_B will be used since each is measured from a fixed point (C or D) and extends along each block's *path of motion*. In particular, s_B is directed to point E since motion of B and E is the *same*.

The red colored segments of the cord in Fig. 12–38 remain at a constant length and do not have to be considered as the blocks move. The remaining length of cord, l, is also constant and is related to the changing position coordinates s_A and s_B by the equation

$$s_A + 3s_B = l$$

Time Derivative. Taking the time derivative yields

$$v_A + 3v_B = 0$$

so that when $v_B = -6$ ft/s (upward),

$$v_A = 18 \text{ ft/s} \downarrow \qquad \textit{Ans.}$$

EXAMPLE 12.22

Determine the speed of A in Fig. 12–39 if B has an upward speed of 6 ft/s.

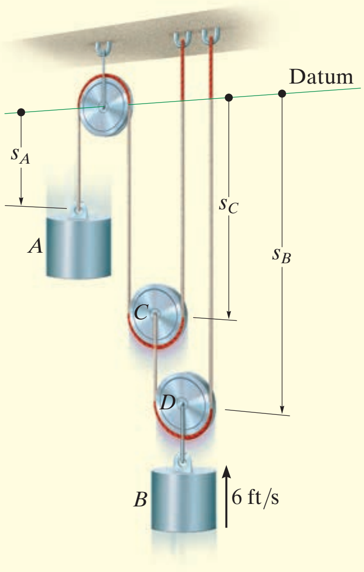


Fig. 12–39

SOLUTION

Position-Coordinate Equation. As shown, the positions of blocks A and B are defined using coordinates s_A and s_B. Since the system has *two cords* with segments that change length, it will be necessary to use a third coordinate, s_C, in order to relate s_A to s_B. In other words, the length of one of the cords can be expressed in terms of s_A and s_C, and the length of the other cord can be expressed in terms of s_B and s_C.

The red colored segments of the cords in Fig. 12–39 do not have to be considered in the analysis. Why? For the remaining cord lengths, say l_1 and l_2, we have

$$s_A + 2s_C = l_1 \qquad s_B + (s_B - s_C) = l_2$$

Time Derivative. Taking the time derivative of these equations yields

$$v_A + 2v_C = 0 \qquad 2v_B - v_C = 0$$

Eliminating v_C produces the relationship between the motions of each cylinder.

$$v_A + 4v_B = 0$$

so that when $v_B = -6$ ft/s (upward),

$$v_A = +24 \text{ ft/s} = 24 \text{ ft/s} \downarrow \qquad \textit{Ans.}$$

EXAMPLE 12.23

Determine the speed of block B in Fig. 12–40 if the end of the cord at A is pulled down with a speed of 2 m/s.

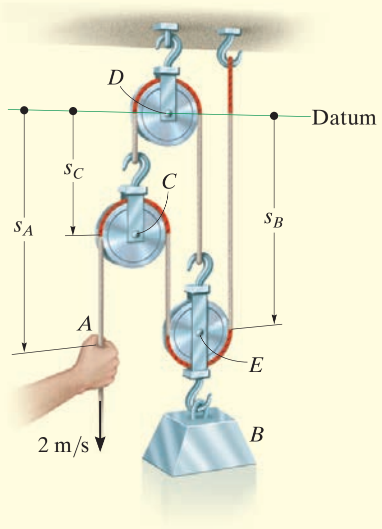


Fig. 12–40

SOLUTION

Position-Coordinate Equation. The position of point A is defined by s_A, and the position of block B is specified by s_B since point E on the pulley will have the *same motion* as the block. Both coordinates are measured from a horizontal datum passing through the *fixed* pin at pulley D. Since the system consists of *two* cords, the coordinates s_A and s_B cannot be related directly. Instead, by establishing a third position coordinate, s_C, we can now express the length of one of the cords in terms of s_B and s_C, and the length of the other cord in terms of s_A, s_B, and s_C.

Excluding the red colored segments of the cords in Fig. 12–40, the remaining constant cord lengths l_1 and l_2 (along with the hook and link dimensions) can be expressed as

$$s_C + s_B = l_1$$
$$(s_A - s_C) + (s_B - s_C) + s_B = l_2$$

Time Derivative. The time derivative of each equation gives

$$v_C + v_B = 0$$
$$v_A - 2v_C + 2v_B = 0$$

Eliminating v_C, we obtain

$$v_A + 4v_B = 0$$

so that when $v_A = 2$ m/s (downward),

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textit{Ans.}$$

12

EXAMPLE 12.24

A man at A is hoisting a safe S as shown in Fig. 12–41 by walking to the right with a constant velocity $v_A = 0.5$ m/s. Determine the velocity and acceleration of the safe when it reaches the elevation of 10 m. The rope is 30 m long and passes over a small pulley at D.

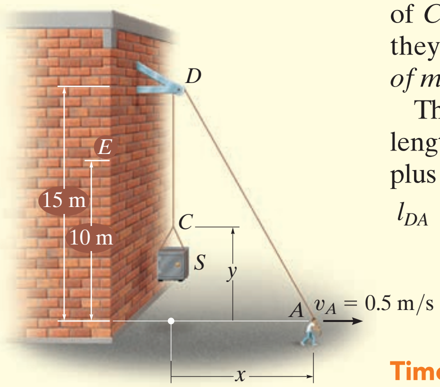


Fig. 12–41

SOLUTION

Position-Coordinate Equation. This problem is unlike the previous examples since rope segment DA changes *both direction and magnitude*. However, the ends of the rope, which define the positions of C and A, are specified by means of the x and y coordinates since they must be measured from a fixed point and *directed along the paths of motion* of the ends of the rope.

The x and y coordinates may be related since the rope has a fixed length $l = 30$ m, which at all times is equal to the length of segment DA plus CD. Using the Pythagorean theorem to determine l_{DA}, we have $l_{DA} = \sqrt{(15)^2 + x^2}$; also, $l_{CD} = 15 - y$. Hence,

$$l = l_{DA} + l_{CD}$$

$$30 = \sqrt{(15)^2 + x^2} + (15 - y)$$

$$y = \sqrt{225 + x^2} - 15 \qquad (1)$$

Time Derivatives. Taking the time derivative, using the chain rule (see Appendix C), where $v_S = dy/dt$ and $v_A = dx/dt$, yields

$$v_S = \frac{dy}{dt} = \left[\frac{1}{2}\frac{2x}{\sqrt{225 + x^2}}\right]\frac{dx}{dt}$$

$$= \frac{x}{\sqrt{225 + x^2}}v_A \qquad (2)$$

At $y = 10$ m, x is determined from Eq. 1, i.e., $x = 20$ m. Hence, from Eq. 2 with $v_A = 0.5$ m/s,

$$v_S = \frac{20}{\sqrt{225 + (20)^2}}(0.5) = 0.4 \text{ m/s} = 400 \text{ mm/s} \uparrow \qquad \textit{Ans.}$$

The acceleration is determined by taking the time derivative of Eq. 2. Since v_A is constant, then $a_A = dv_A/dt = 0$, and we have

$$a_S = \frac{d^2y}{dt^2} = \left[\frac{-x(dx/dt)}{(225 + x^2)^{3/2}}\right]xv_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]\left(\frac{dx}{dt}\right)v_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]x\frac{dv_A}{dt} = \frac{225v_A^2}{(225 + x^2)^{3/2}}$$

At $x = 20$ m, with $v_A = 0.5$ m/s, the acceleration becomes

$$a_S = \frac{225(0.5 \text{ m/s})^2}{[225 + (20 \text{ m})^2]^{3/2}} = 0.00360 \text{ m/s}^2 = 3.60 \text{ mm/s}^2 \uparrow \qquad \textit{Ans.}$$

NOTE: The constant velocity at A causes the other end C of the rope to have an acceleration since $\mathbf{v}_A$ causes segment DA to change its direction as well as its length.

12.10 Relative-Motion of Two Particles Using Translating Axes

Throughout this chapter the absolute motion of a particle has been determined using a single fixed reference frame. There are many cases, however, where the path of motion for a particle is complicated, so that it may be easier to analyze the motion in parts by using two or more frames of reference. For example, the motion of a particle located at the tip of an airplane propeller, while the plane is in flight, is more easily described if one observes first the motion of the airplane from a fixed reference and then superimposes (vectorially) the circular motion of the particle measured from a reference attached to the airplane.

In this section *translating frames of reference* will be considered for the analysis.

Position. Consider particles A and B, which move along the arbitrary paths shown in Fig. 12–42. The *absolute position* of each particle, $\mathbf{r}_A$ and $\mathbf{r}_B$, is measured from the common origin O of the *fixed* x, y, z reference frame. The origin of a second frame of reference x', y', z' is attached to and moves with particle A. The axes of this frame are *only permitted to translate* relative to the fixed frame. The position of B measured relative to A is denoted by the *relative-position vector* $\mathbf{r}_{B/A}$. Using vector addition, the three vectors shown in Fig. 12–42 can be related by the equation

$$\boxed{\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}} \qquad (12\text{–}33)$$

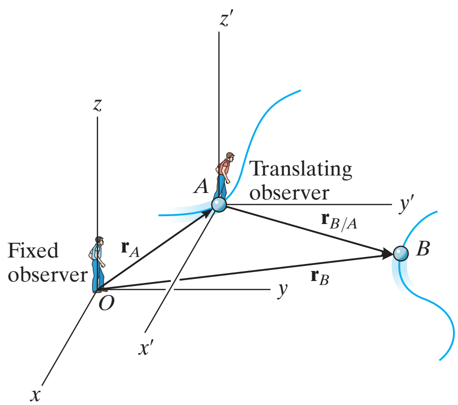


Fig. 12–42

Velocity. An equation that relates the velocities of the particles is determined by taking the time derivative of the above equation; i.e.,

$$\boxed{\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}} \qquad (12\text{–}34)$$

Here $\mathbf{v}_B = d\mathbf{r}_B/dt$ and $\mathbf{v}_A = d\mathbf{r}_A/dt$ refer to *absolute velocities*, since they are observed from the fixed frame; whereas the *relative velocity* $\mathbf{v}_{B/A} = d\mathbf{r}_{B/A}/dt$ is observed from the translating frame. It is important to note that since the x', y', z' axes translate, the *components* of $\mathbf{r}_{B/A}$ will *not* change direction and therefore the time derivative of these components will only have to account for the change in their magnitudes. Equation 12–34 therefore states that the velocity of B is equal to the velocity of A plus (vectorially) the velocity of "B with respect to A," as measured by the *translating observer* fixed in the x', y', z' reference frame.

Acceleration. The time derivative of Eq. 12–34 yields a similar vector relation between the *absolute* and *relative accelerations* of particles A and B.

$$\boxed{\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}} \tag{12–35}$$

Here $\mathbf{a}_{B/A}$ is the acceleration of B as seen by the observer located at A and translating with the x', y', z' reference frame.*

Procedure for Analysis

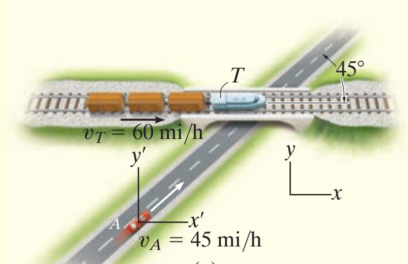
- When applying the relative velocity and acceleration equations, it is first necessary to specify the particle A that is the origin for the translating x', y', z' axes. Usually this point has a *known* velocity or acceleration.
- Since vector addition forms a triangle, there can be at most *two unknowns*, represented by the magnitudes and/or directions of the vector quantities.
- These unknowns can be solved for either graphically, using trigonometry (law of sines, law of cosines), or by resolving each of the three vectors into rectangular or Cartesian components, thereby generating a set of scalar equations.

The pilots of these close-flying planes must be aware of their relative positions and velocities at all times in order to avoid a collision. (© R.C. Hibbeler)

* An easy way to remember the setup of these equations is to note the "cancellation" of the subscript A between the two terms, e.g., $\mathbf{a}_B = \mathbf{a}_{\cancel{A}} + \mathbf{a}_{B/\cancel{A}}$.

EXAMPLE 12.25

A train travels at a constant speed of 60 mi/h and crosses over a road as shown in Fig. 12–43*a*. If the automobile A is traveling at 45 mi/h along the road, determine the magnitude and direction of the velocity of the train relative to the automobile.



(a)

SOLUTION I

Vector Analysis. The relative velocity $\mathbf{v}_{T/A}$ is measured from the translating x', y' axes attached to the automobile, Fig. 12–43*a*. It is determined from $\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$. Since $\mathbf{v}_T$ and $\mathbf{v}_A$ are known in *both* magnitude and direction, the unknowns become the x and y components of $\mathbf{v}_{T/A}$. Using the x, y axes in Fig. 12–43*a*, we have

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$60\mathbf{i} = (45 \cos 45°\mathbf{i} + 45 \sin 45°\mathbf{j}) + \mathbf{v}_{T/A}$$

$$\mathbf{v}_{T/A} = \{28.2\mathbf{i} - 31.8\mathbf{j}\} \text{ mi/h}$$

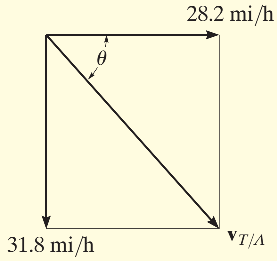
The magnitude of $\mathbf{v}_{T/A}$ is thus

$$v_{T/A} = \sqrt{(28.2)^2 + (-31.8)^2} = 42.5 \text{ mi/h} \qquad \textit{Ans.}$$

From the direction of each component, Fig. 12–43*b*, the direction of $\mathbf{v}_{T/A}$ is

$$\tan\theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{31.8}{28.2}$$

$$\theta = 48.5° \measuredangle \qquad \textit{Ans.}$$



(b)

Note that the vector addition shown in Fig. 12–43*b* indicates the correct sense for $\mathbf{v}_{T/A}$. This figure anticipates the answer and can be used to check it.

SOLUTION II

Scalar Analysis. The unknown components of $\mathbf{v}_{T/A}$ can also be determined by applying a scalar analysis. We will assume these components act in the *positive* x and y directions. Thus,

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$\begin{bmatrix} 60 \text{ mi/h} \\ \rightarrow \end{bmatrix} = \begin{bmatrix} 45 \text{ mi/h} \\ \measuredangle^{45°} \end{bmatrix} + \begin{bmatrix} (v_{T/A})_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (v_{T/A})_y \\ \uparrow \end{bmatrix}$$

Resolving each vector into its x and y components yields

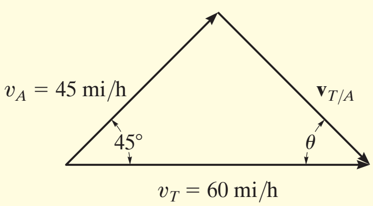
$$(\xrightarrow{+}) \qquad 60 = 45 \cos 45° + (v_{T/A})_x + 0$$

$$(+\uparrow) \qquad 0 = 45 \sin 45° + 0 + (v_{T/A})_y$$

Solving, we obtain the previous results,

$$(v_{T/A})_x = 28.2 \text{ mi/h} = 28.2 \text{ mi/h} \rightarrow$$

$$(v_{T/A})_y = -31.8 \text{ mi/h} = 31.8 \text{ mi/h} \downarrow$$



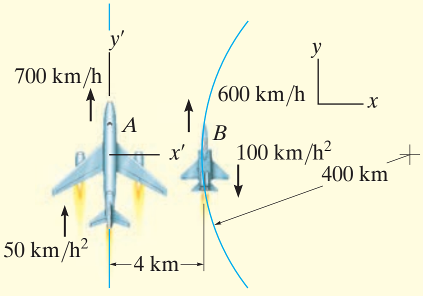
(c)

Fig. 12–43

12

EXAMPLE 12.26

Plane A in Fig. 12–44a is flying along a straight-line path, whereas plane B is flying along a circular path having a radius of curvature of $\rho_B = 400$ km. Determine the velocity and acceleration of B as measured by the pilot of A.



(a)

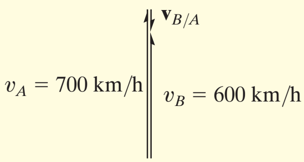
SOLUTION

Velocity. The origin of the x *and* y axes are located at an arbitrary fixed point. Since the motion relative to plane A is to be determined, the *translating frame of reference* x', y' is attached to it, Fig. 12–44a. Applying the relative-velocity equation in scalar form since the velocity vectors of both planes are parallel at the instant shown, we have

$$(+\uparrow) \qquad v_B = v_A + v_{B/A}$$

$$600\text{ km/h} = 700\text{ km/h} + v_{B/A}$$

$$v_{B/A} = -100\text{ km/h} = 100\text{ km/h}\downarrow \qquad \textit{Ans.}$$



(b)

The vector addition is shown in Fig. 12–44b.

Acceleration. Plane B has both tangential and normal components of acceleration since it is flying along a *curved path*. From Eq. 12–20, the magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600\text{ km/h})^2}{400\text{ km}} = 900\text{ km/h}^2$$

Applying the relative-acceleration equation gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

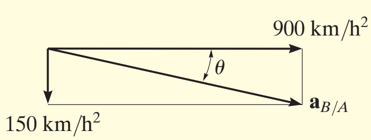
$$900\mathbf{i} - 100\mathbf{j} = 50\mathbf{j} + \mathbf{a}_{B/A}$$

Thus,

$$\mathbf{a}_{B/A} = \{900\mathbf{i} - 150\mathbf{j}\}\text{ km/h}^2$$

From Fig. 12–44c, the magnitude and direction of $\mathbf{a}_{B/A}$ are therefore

$$a_{B/A} = 912\text{ km/h}^2 \quad \theta = \tan^{-1}\frac{150}{900} = 9.46^\circ \ \searrow \qquad \textit{Ans.}$$



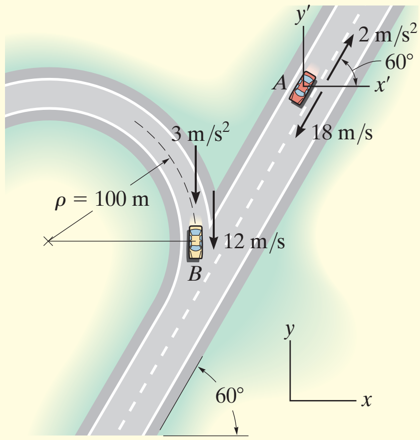
(c)

Fig. 12–44

NOTE: The solution to this problem was possible using a translating frame of reference, since the pilot in plane A is "translating." Observation of the motion of plane A with respect to the pilot of plane B, however, must be obtained using a *rotating* set of axes attached to plane B. (This assumes, of course, that the pilot of B is fixed in the rotating frame, so he does not turn his eyes to follow the motion of A.) The analysis for this case is given in Example 16.21.

EXAMPLE 12.27

At the instant shown in Fig. 12–45*a*, cars A and B are traveling with speeds of 18 m/s and 12 m/s, respectively. Also at this instant, A has a decrease in speed of 2 m/s^2, and B has an increase in speed of 3 m/s^2. Determine the velocity and acceleration of B with respect to A.



(a)

SOLUTION

Velocity. The fixed x, y axes are established at an arbitrary point on the ground and the translating x', y' axes are attached to car A, Fig. 12–45*a*. Why? The relative velocity is determined from $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. What are the two unknowns? Using a Cartesian vector analysis, we have

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-12\mathbf{j} = (-18\cos 60°\mathbf{i} - 18\sin 60°\mathbf{j}) + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{9\mathbf{i} + 3.588\mathbf{j}\}\ \text{m/s}$$

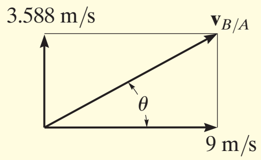
Thus,

$$v_{B/A} = \sqrt{(9)^2 + (3.588)^2} = 9.69\ \text{m/s} \qquad \textit{Ans.}$$

Noting that $\mathbf{v}_{B/A}$ has $+\mathbf{i}$ and $+\mathbf{j}$ components, Fig. 12–45*b*, its direction is

$$\tan\theta = \frac{(v_{B/A})_y}{(v_{B/A})_x} = \frac{3.588}{9}$$

$$\theta = 21.7° \qquad \textit{Ans.}$$



(b)

Acceleration. Car B has both tangential and normal components of acceleration. Why? The magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(12\ \text{m/s})^2}{100\ \text{m}} = 1.440\ \text{m/s}^2$$

Applying the equation for relative acceleration yields

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(-1.440\mathbf{i} - 3\mathbf{j}) = (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j}) + \mathbf{a}_{B/A}$$

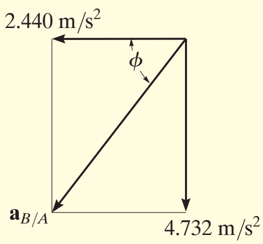
$$\mathbf{a}_{B/A} = \{-2.440\mathbf{i} - 4.732\mathbf{j}\}\ \text{m/s}^2$$

Here $\mathbf{a}_{B/A}$ has $-\mathbf{i}$ and $-\mathbf{j}$ components. Thus, from Fig. 12–45*c*,

$$a_{B/A} = \sqrt{(2.440)^2 + (4.732)^2} = 5.32\ \text{m/s}^2 \qquad \textit{Ans.}$$

$$\tan\phi = \frac{(a_{B/A})_y}{(a_{B/A})_x} = \frac{4.732}{2.440}$$

$$\phi = 62.7° \qquad \textit{Ans.}$$



(c)

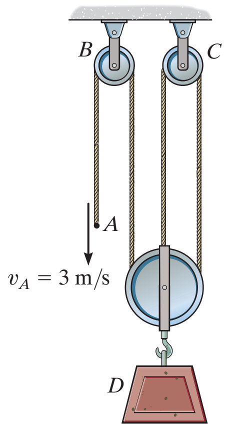
Fig. 12–45

NOTE: Is it possible to obtain the relative acceleration of $\mathbf{a}_{A/B}$ using this method? Refer to the comment made at the end of Example 12.26.

12

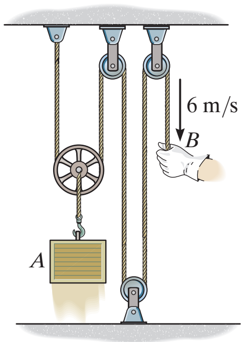
FUNDAMENTAL PROBLEMS

F12–39. Determine the velocity of block D if end A of the rope is pulled down with a speed of $v_A = 3$ m/s.



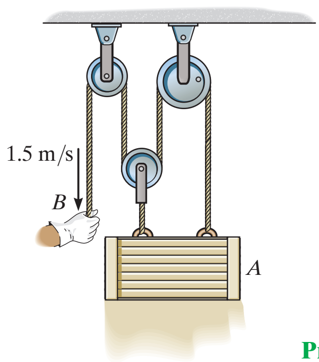
Prob. F12–39

F12–40. Determine the velocity of block A if end B of the rope is pulled down with a speed of 6 m/s.



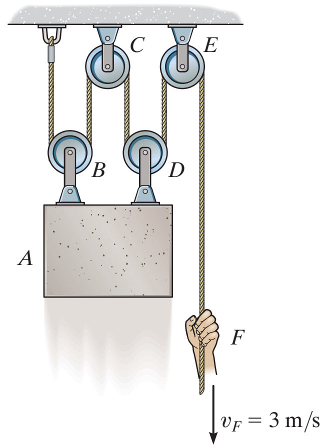
Prob. F12–40

F12–41. Determine the velocity of block A if end B of the rope is pulled down with a speed of 1.5 m/s.



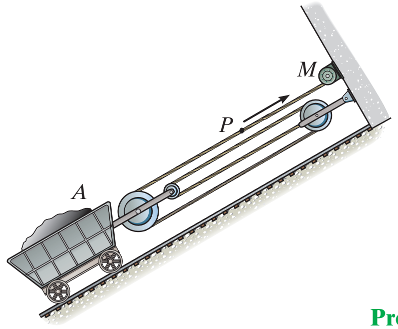
Prob. F12–41

F12–42. Determine the velocity of block A if end F of the rope is pulled down with a speed of $v_F = 3$ m/s.



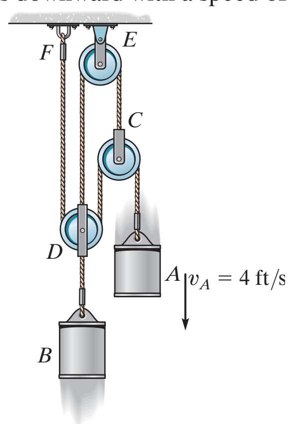
Prob. F12–42

F12–43. Determine the velocity of car A if point P on the cable has a speed of 4 m/s when the motor M winds the cable in.



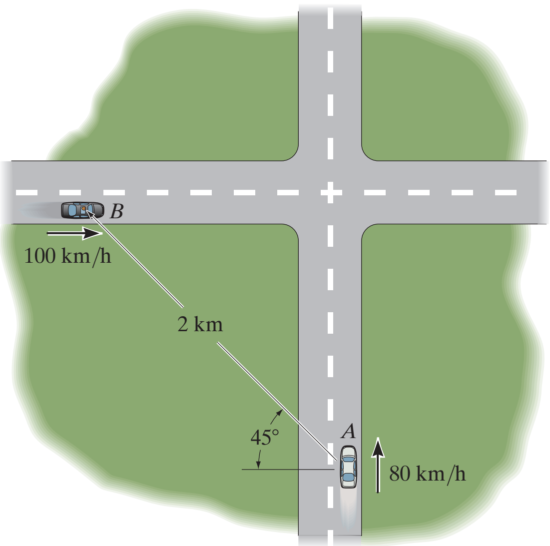
Prob. F12–43

F12–44. Determine the velocity of cylinder B if cylinder A moves downward with a speed of $v_A = 4$ ft/s.



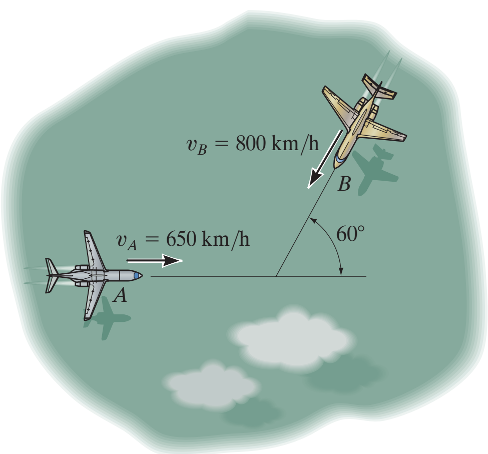
Prob. F12–44

F12–45. Car A is traveling with a constant speed of 80 km/h due north, while car B is traveling with a constant speed of 100 km/h due east. Determine the velocity of car B relative to car A.



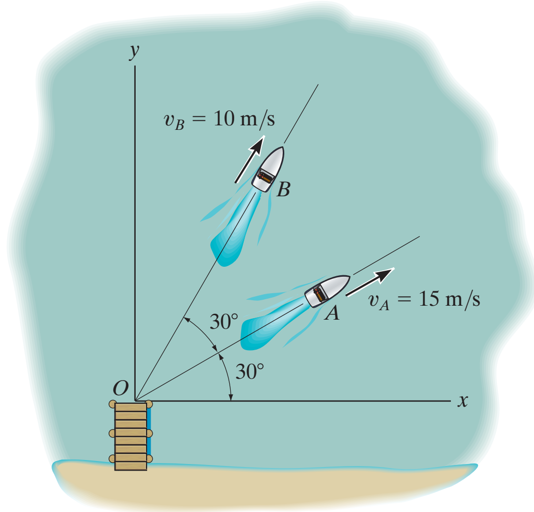
Prob. F12–45

F12–46. Two planes A and B are traveling with the constant velocities shown. Determine the magnitude and direction of the velocity of plane B relative to plane A.



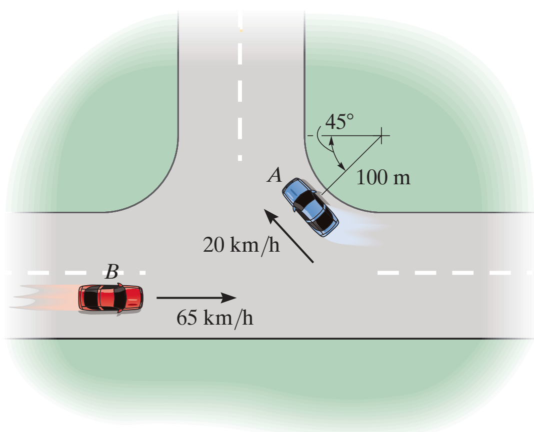
Prob. F12–46

F12–47. The boats A and B travel with constant speeds of $v_A = 15$ m/s and $v_B = 10$ m/s when they leave the pier at O at the same time. Determine the distance between them when $t = 4$ s.



Prob. F12–47

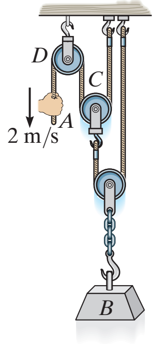
F12–48. At the instant shown, cars A and B are traveling at the speeds shown. If B is accelerating at 1200 km/h^2 while A maintains a constant speed, determine the velocity and acceleration of A with respect to B.



Prob. F12–48

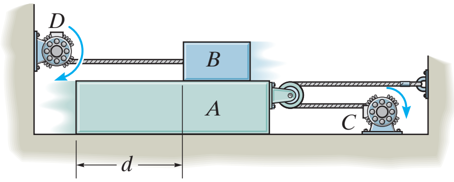
PROBLEMS

12–195. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



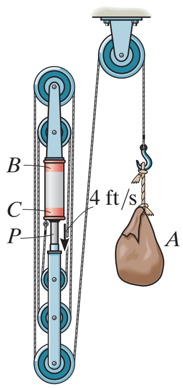
Prob. 12–195

***12–196.** The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



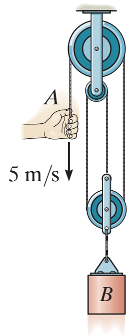
Prob. 12–196

12–197. The pulley arrangement shown is designed for hoisting materials. If BC *remains fixed* while the plunger P is pushed downward with a speed of 4 ft/s, determine the speed of the load at A.



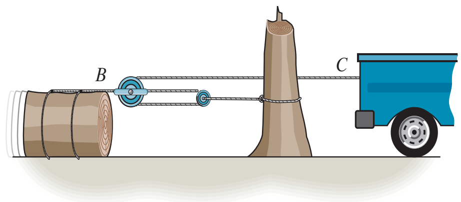
Prob. 12–197

12–198. If the end of the cable at A is pulled down with a speed of 5 m/s, determine the speed at which block B rises.



Prob. 12–198

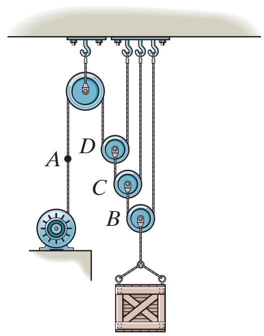
12–199. Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



Prob. 12–199

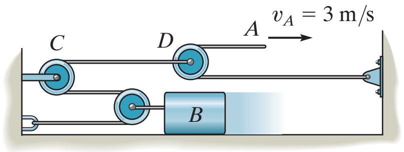
***12–200.** Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load 6 m in 1.5 s.

12–201. Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



Probs. 12–200/201

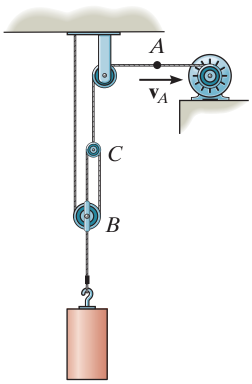
12–202. If the end A of the cable is moving at $v_A = 3\text{ m/s}$, determine the speed of block B.



Prob. 12–202

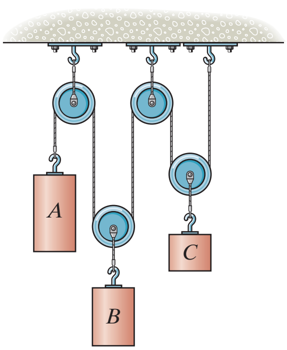
12–203. Determine the time needed for the load at B to attain a speed of 10 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 3 m/s^2.

***12–204.** The cable at A is being drawn toward the motor at $v_A = 8\text{ m/s}$. Determine the velocity of the block.



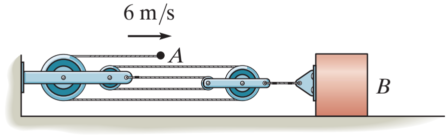
Probs. 12–203/204

12–205. If block A of the pulley system is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the relative velocity of block B with respect to C.



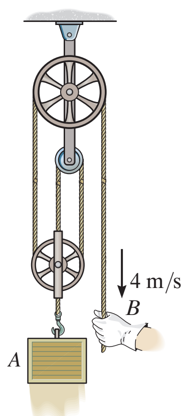
Prob. 12–205

12–206. Determine the speed of the block at B.



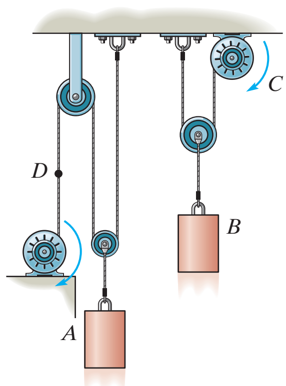
Prob. 12–206

12–207. Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



Prob. 12–207

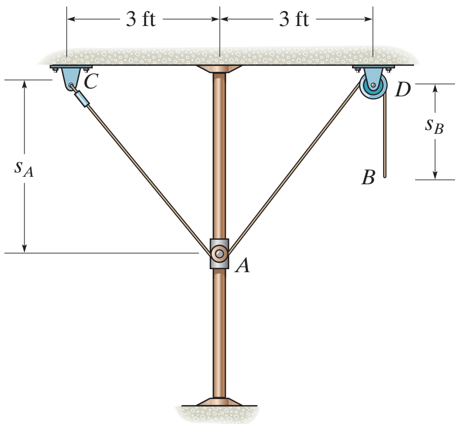
***12–208.** The motor draws in the cable at C with a constant velocity of $v_C = 4\text{ m/s}$. The motor draws in the cable at D with a constant acceleration of $a_D = 8\text{ m/s}^2$. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



Prob. 12–208

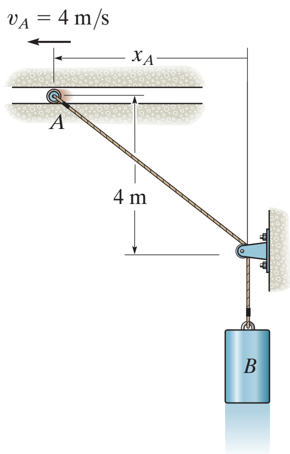
12–209. The cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant $s_A = 4$ ft the collar is moving upward at 5 ft/s, which is decreasing at 2 ft/s^2.

12–210. The 16-ft-long cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. When $s_B = 6$ ft, the end of the cord at B is pulled downward with a velocity of 4 ft/s and is given an acceleration of 3 ft/s^2. Determine the velocity and acceleration of the collar at this instant.



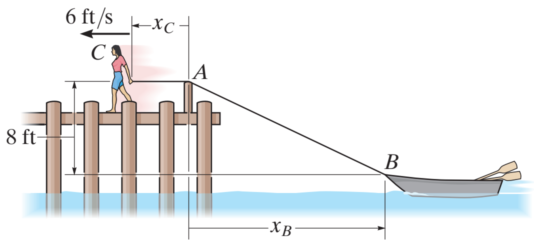
Probs. 12–209/210

12–211. The roller at A is moving with a velocity of $v_A = 4$ m/s and has an acceleration of $a_A = 2$ m/s^2 when $x_A = 3$ m. Determine the velocity and acceleration of block B at this instant.



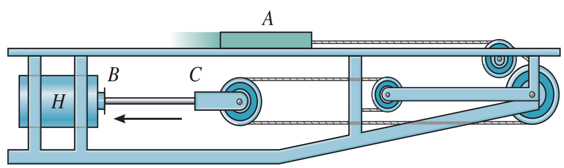
Prob. 12–211

***12–212.** The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



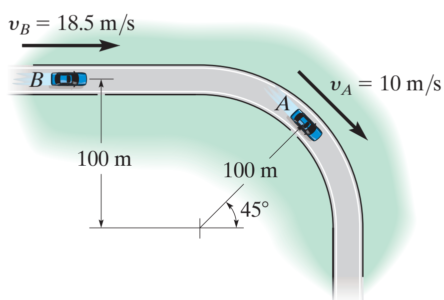
Prob. 12–212

12–213. If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.



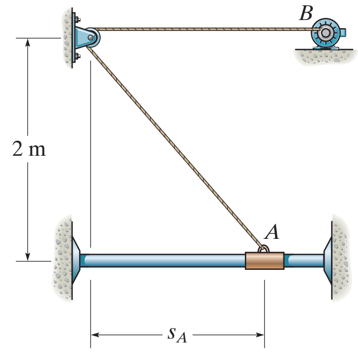
Prob. 12–213

12–214. At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



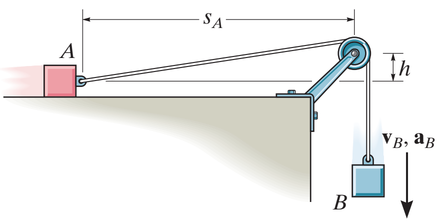
Prob. 12–214

12–215. The motor draws in the cord at B with an acceleration of $a_B = 2\ \text{m/s}^2$. When $s_A = 1.5$ m, $v_B = 6$ m/s. Determine the velocity and acceleration of the collar at this instant.



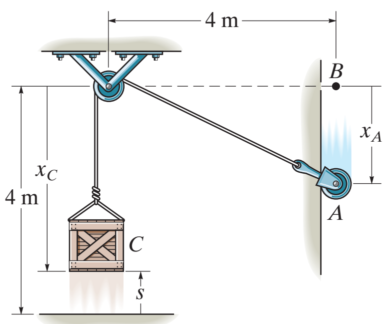
Prob. 12–215

***12–216.** If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



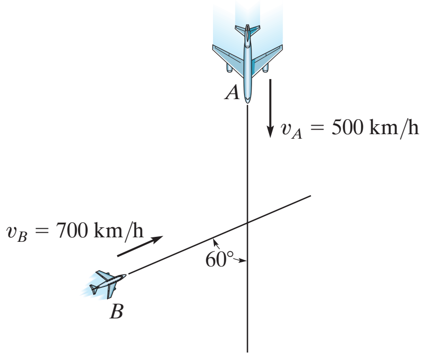
Prob. 12–216

12–217. The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint*: Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



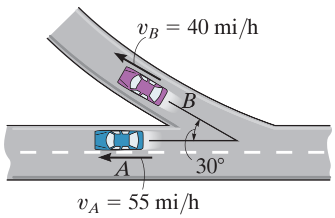
Prob. 12–217

12–218. Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 500$ km/h and $v_B = 700$ km/h such that the angle between their straight-line courses is $\theta = 60°$, determine the velocity of plane B with respect to plane A.



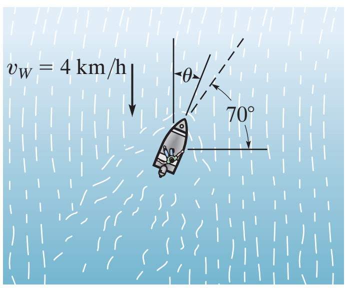
Prob. 12–218

12–219. At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



Prob. 12–219

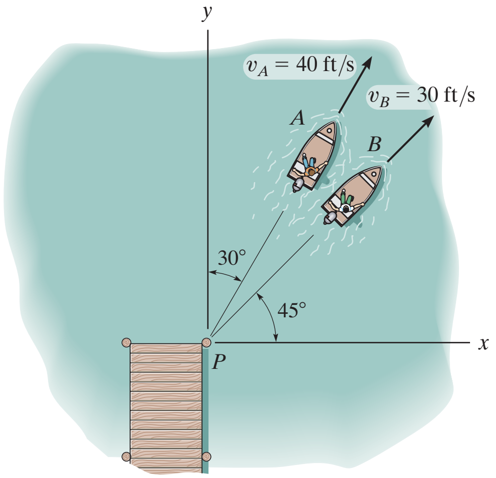
***12–220.** The boat can travel with a speed of 16 km/h in still water. The point of destination is located along the dashed line. If the water is moving at 4 km/h, determine the bearing angle θ at which the boat must travel to stay on course.



Prob. 12–220

12

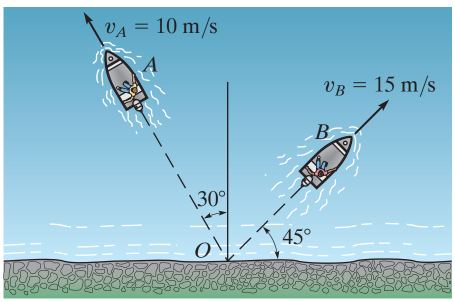
12–221. Two boats leave the pier P at the same time and travel in the directions shown. If $v_A = 40$ ft/s and $v_B = 30$ ft/s, determine the velocity of boat A relative to boat B. How long after leaving the pier will the boats be 1500 ft apart?



Prob. 12–221

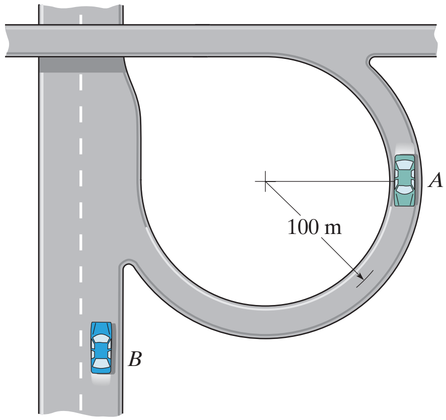
12–222. A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is coming from the east. If the car's speed is 80 km/h, the instrument indicates that the wind is coming from the northeast. Determine the speed and direction of the wind.

12–223. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10$ m/s and $v_B = 15$ m/s, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 600 m apart?



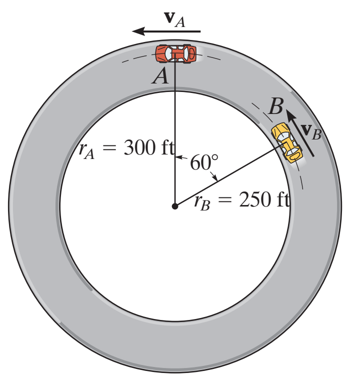
Prob. 12–223

***12–224.** At the instant shown, car A has a speed of 20 km/h, which is being increased at the rate of 300 km/h^2 as the car enters the expressway. At the same instant, car B is decelerating at 250 km/h^2 while traveling forward at 100 km/h. Determine the velocity and acceleration of A with respect to B.



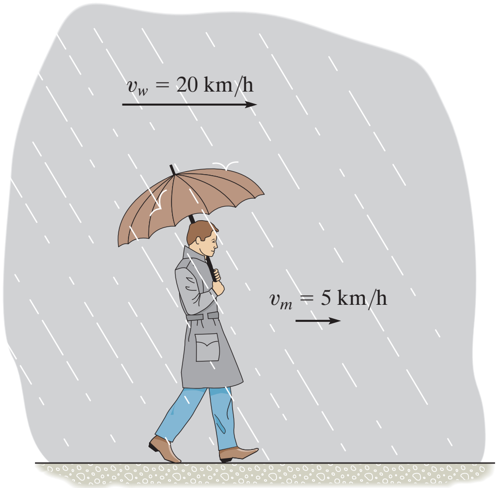
Prob. 12–224

12–225. Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



Prob. 12–225

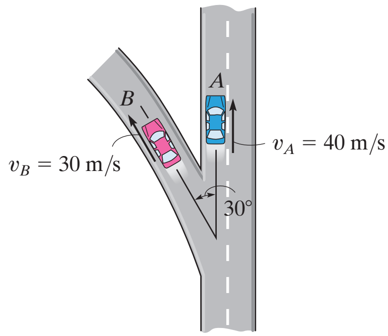
12–226. A man walks at 5 km/h in the direction of a 20 km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine direction in which the drops appear to fall with respect to the man.



Prob. 12–226

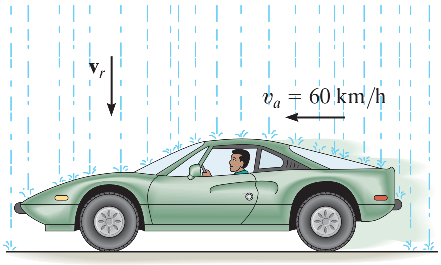
12–227. At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If B is increasing its velocity by 2 m/s^2, while A maintains a constant velocity, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.

***12–228.** At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If A is increasing its speed at 4 m/s^2, whereas the speed of B is decreasing at 3 m/s^2, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



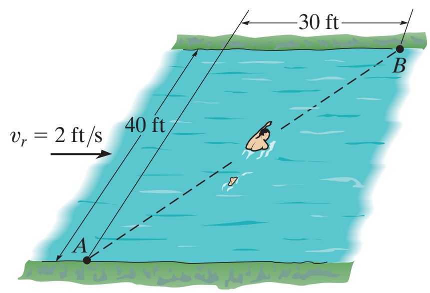
Probs. 12–227/228

12–229. A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



Prob. 12–229

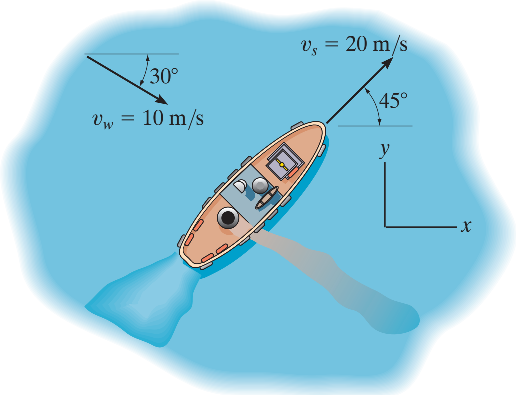
12–230. A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself toward point B to reach this point. Why?



Prob. 12–230

12

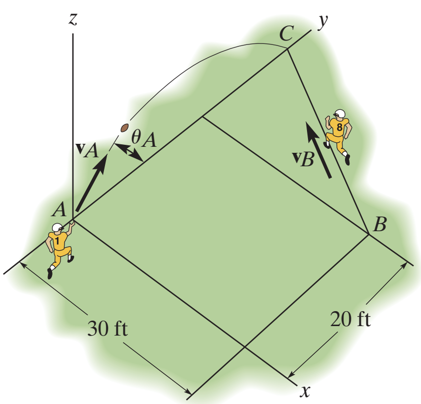
12–231. The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



Prob. 12–231

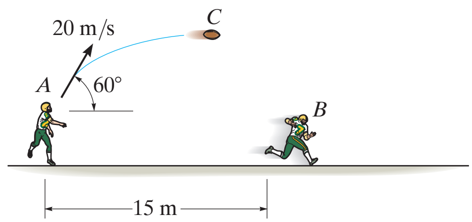
***12–232.** The football player at A throws the ball in the y–z plane at a speed $v_A = 50$ ft/s and an angle $\theta_A = 60°$ with the horizontal. At the instant the ball is thrown, the player is at B and is running with constant speed along the line BC in order to catch it. Determine this speed, v_B, so that he makes the catch at the same elevation from which the ball was thrown.

12–233. The football player at A throws the ball in the y–z plane with a speed $v_A = 50$ ft/s and an angle $\theta_A = 60°$ with the horizontal. At the instant the ball is thrown, the player is at B and is running at a constant speed of $v_B = 23$ ft/s along the line BC. Determine if he can reach point C, which has the same elevation as A, before the ball gets there.



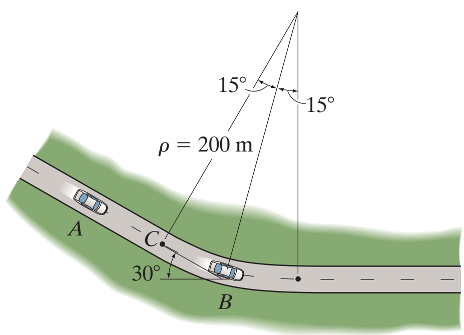
Probs. 12–232/233

12–234. At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



Prob. 12–234

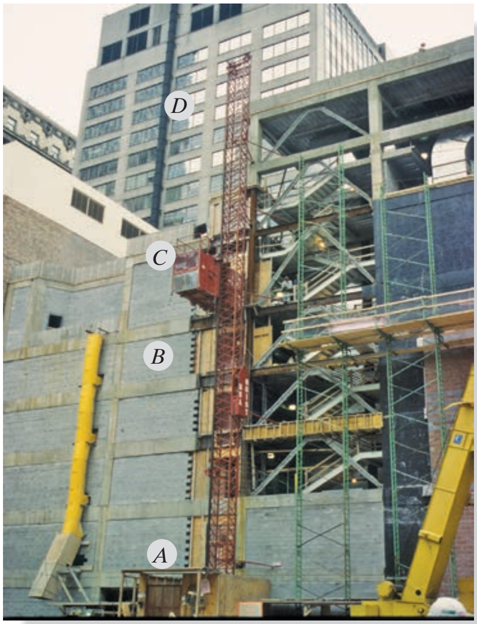
12–235. At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.



Prob. 12–235

CONCEPTUAL PROBLEMS

C12–1. If you measured the time it takes for the construction elevator to go from A to B, then B to C, and then C to D, and you also know the distance between each of the points, how could you determine the average velocity and average acceleration of the elevator as it ascends from A to D? Use numerical values to explain how this can be done.



Prob. C12–1 (© R.C. Hibbeler)

C12–2. If the sprinkler at A is 1 m from the ground, then scale the necessary measurements from the photo to determine the approximate velocity of the water jet as it flows from the nozzle of the sprinkler.

Prob. C12–2 (© R.C. Hibbeler)

C12–3. The basketball was thrown at an angle measured from the horizontal to the man's outstretched arm. If the basket is 3 m from the ground, make appropriate measurements in the photo and determine if the ball located as shown will pass through the basket.

Prob. C12–3 (© R.C. Hibbeler)

C12–4. The pilot tells you the wingspan of her plane and her constant airspeed. How would you determine the acceleration of the plane at the moment shown? Use numerical values and take any necessary measurements from the photo.

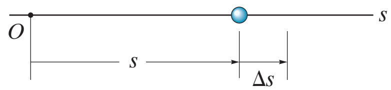
Prob. C12–4 (© R.C. Hibbeler)

12

CHAPTER REVIEW

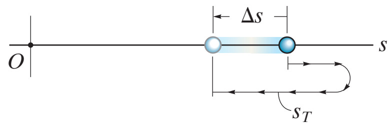
Rectilinear Kinematics

Rectilinear kinematics refers to motion along a straight line. A position coordinate s specifies the location of the particle on the line, and the displacement Δs is the change in this position.



The average velocity is a vector quantity, defined as the displacement divided by the time interval.

$$v_{avg} = \frac{\Delta s}{\Delta t}$$



The average speed is a scalar, and is the total distance traveled divided by the time of travel.

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

The time, position, velocity, and acceleration are related by three differential equations.

$$a = \frac{dv}{dt}, \qquad v = \frac{ds}{dt}, \qquad a\,ds = v\,dv$$

If the acceleration is known to be constant, then the differential equations relating time, position, velocity, and acceleration can be integrated.

$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Graphical Solutions

If the motion is erratic, then it can be described by a graph. If one of these graphs is given, then the others can be established using the differential relations between a, v, s, and t.

$$a = \frac{dv}{dt},$$

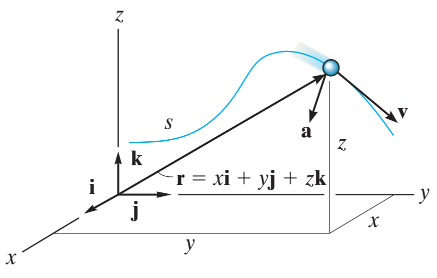
$$v = \frac{ds}{dt},$$

$$a\,ds = v\,dv$$

Curvilinear Motion, x, y, z

Curvilinear motion along the path can be resolved into rectilinear motion along the x, y, z axes. The equation of the path is used to relate the motion along each axis.

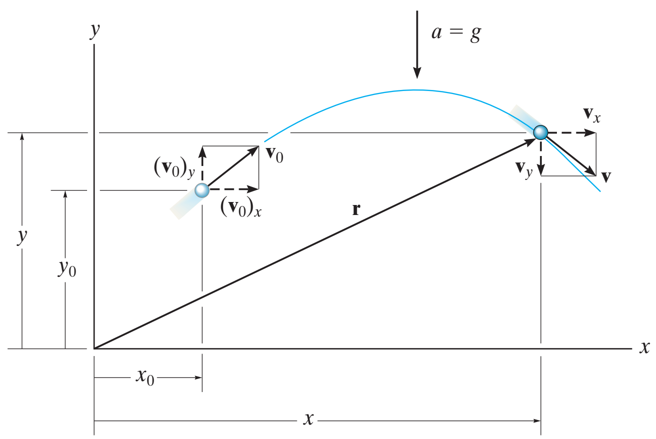
$$v_x = \dot{x} \qquad a_x = \dot{v}_x$$
$$v_y = \dot{y} \qquad a_y = \dot{v}_y$$
$$v_z = \dot{z} \qquad a_z = \dot{v}_z$$



Projectile Motion

Free-flight motion of a projectile follows a parabolic path. It has a constant velocity in the horizontal direction, and a constant downward acceleration of $g = 9.81\ \text{m/s}^2$ or $32.2\ \text{ft/s}^2$ in the vertical direction. Any two of the three equations for constant acceleration apply in the vertical direction, and in the horizontal direction only one equation applies.

$$(+\uparrow) \qquad v_y = (v_0)_y + a_c t$$
$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \tfrac{1}{2} a_c t^2$$
$$(+\uparrow) \qquad v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$
$$(\overset{+}{\rightarrow}) \qquad x = x_0 + (v_0)_x t$$



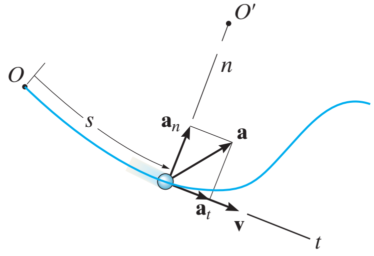
Curvilinear Motion n, t

If normal and tangential axes are used for the analysis, then $\mathbf{v}$ is always in the positive t direction.

The acceleration has two components. The tangential component, $\mathbf{a}_t$, accounts for the change in the magnitude of the velocity; a slowing down is in the negative t direction, and a speeding up is in the positive t direction. The normal component $\mathbf{a}_n$ accounts for the change in the direction of the velocity. This component is always in the positive n direction.

$$a_t = \dot{v} \qquad \text{or} \qquad a_t\,ds = v\,dv$$

$$a_n = \frac{v^2}{\rho}$$



Curvilinear Motion r, θ

If the path of motion is expressed in polar coordinates, then the velocity and acceleration components can be related to the time derivatives of r and θ.

To apply the time-derivative equations, it is necessary to determine $r, \dot{r}, \ddot{r}, \dot{\theta}, \ddot{\theta}$ at the instant considered. If the path $r = f(\theta)$ is given, then the chain rule of calculus must be used to obtain time derivatives. (See Appendix C.)

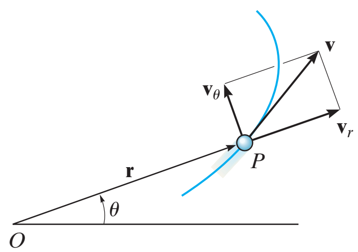
Once the data are substituted into the equations, then the algebraic sign of the results will indicate the direction of the components of $\mathbf{v}$ or $\mathbf{a}$ along each axis.

$$v_r = \dot{r}$$

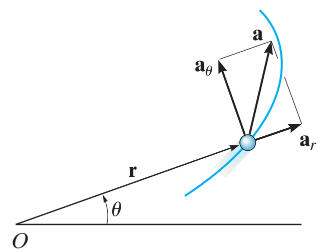
$$v_\theta = r\dot{\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$



Velocity



Acceleration

Absolute Dependent Motion of Two Particles

The dependent motion of blocks that are suspended from pulleys and cables can be related by the geometry of the system. This is done by first establishing position coordinates, measured from a fixed origin to each block. Each coordinate must be directed along the line of motion of a block.

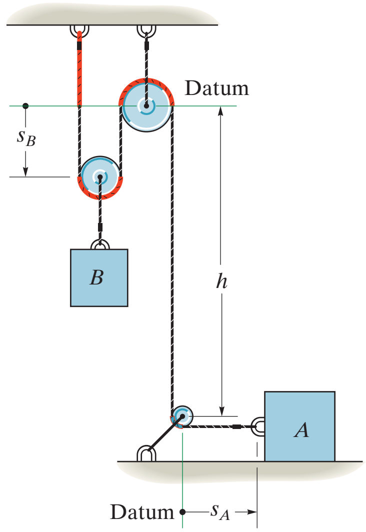
Using geometry and/or trigonometry, the coordinates are then related to the cable length in order to formulate a position coordinate equation.

$$2s_B + h + s_A = l$$

The first time derivative of this equation gives a relationship between the velocities of the blocks, and a second time derivative gives the relation between their accelerations.

$$2v_B = -v_A$$

$$2a_B = -a_A$$



Relative-Motion Analysis Using Translating Axes

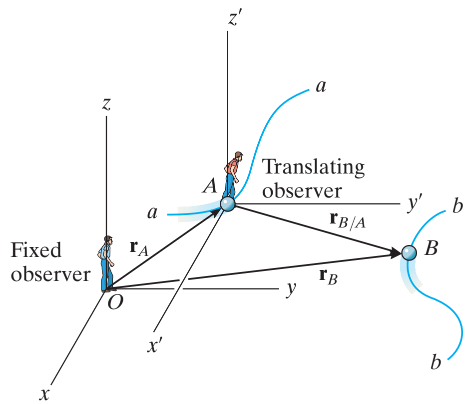
If two particles A and B undergo independent motions, then these motions can be related to their relative motion using a *translating set of axes* attached to one of the particles (A).

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

For planar motion, each vector equation produces two scalar equations, one in the x, and the other in the y direction. For solution, the vectors can be expressed in Cartesian form, or the x and y scalar components can be written directly.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$



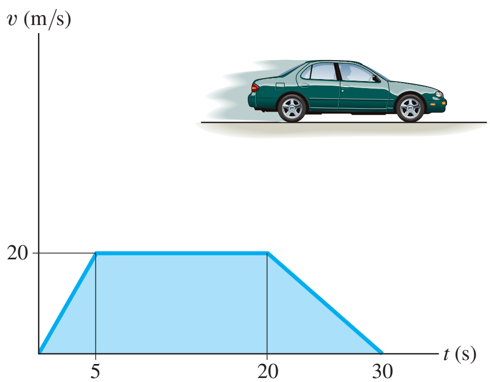
REVIEW PROBLEMS

R12–1. The position of a particle along a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \le t \le 10$ s.

R12–2. If a particle has an initial velocity $v_0 = 12$ ft/s to the right, and a constant acceleration of 2 ft/s^2 to the left, determine the particle's displacement in 10 s. Originally $s_0 = 0$.

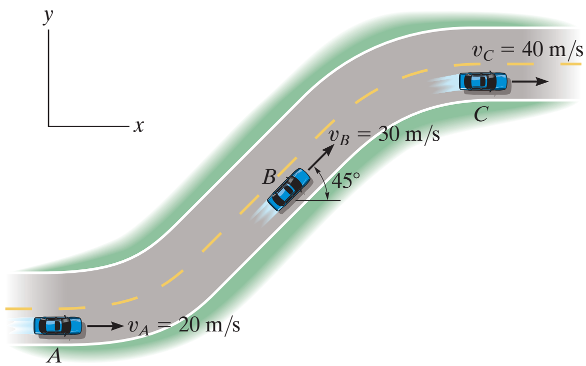
R12–3. A projectile, initially at the origin, moves along a straight-line path through a fluid medium such that its velocity is $v = 1800(1 - e^{-0.3t})$ mm/s where t is in seconds. Determine the displacement of the projectile during the first 3 s.

R12–4. The v–t graph of a car while traveling along a road is shown. Determine the acceleration when $t = 2.5$ s, 10 s, and 25 s. Also if $s = 0$ when $t = 0$, find the position when $t = 5$ s, 20 s, and 30 s.



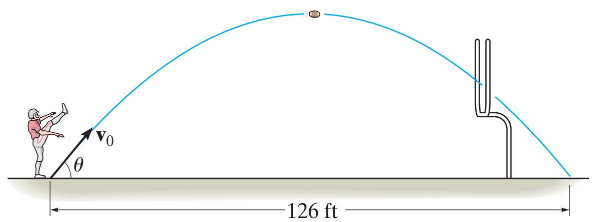
Prob. R12–4

R12–5. A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



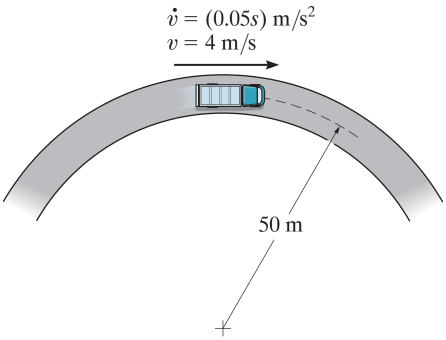
Prob. R12–5

R12–6. From a videotape, it was observed that a player kicked a football 126 ft during a measured time of 3.6 seconds. Determine the initial speed of the ball and the angle θ at which it was kicked.



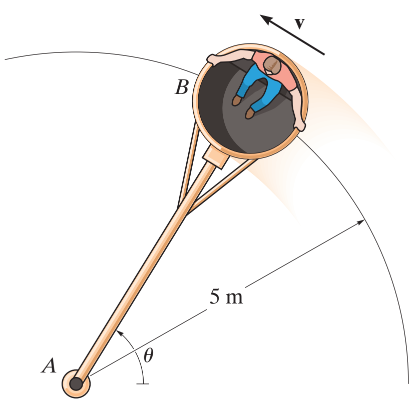
Prob. R12–6

R12–7. The truck travels in a circular path having a radius of 50 m at a speed of $v = 4$ m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Prob. R12–7

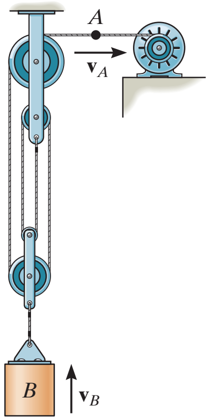
R12–8. Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)$ m/s^2, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car.



Prob. R12–8

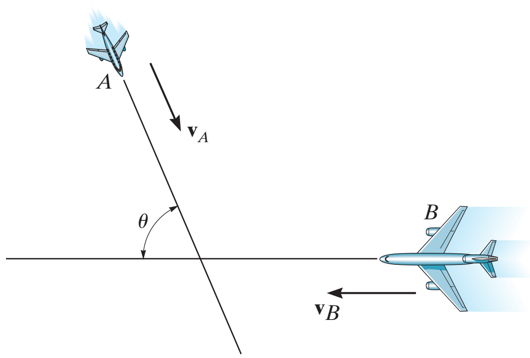
R12–9. A particle is moving along a circular path of 2-m radius such that its position as a function of time is given by $\theta = (5t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

R12–10. Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



Prob. R12–10

R12–11. Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



Prob. R12–11