CHAPTER 5

Applications of Newton's Laws

The soles of hiking shoes are designed to stick, not slip, on rocky surfaces. In this chapter we'll learn about the interactions that give good traction.

By the end of this chapter, you will be able to:

1. Draw a free-body diagram showing the forces acting on an individual object.
2. Solve for unknown quantities (such as magnitudes of forces or accelerations) using Newton's second law in problems involving an individual object or a system of objects connected to each other.
3. Relate the force of friction acting on an object to the normal force exerted on an object in Newton's second law problems.
4. Use Hooke's law to relate the magnitude of the spring force exerted by a spring to the distance from the equilibrium position the spring has been stretched or compressed.

Newton's three laws of motion, the foundation of classical mechanics, can be stated very simply, as we have seen. But applying these laws to situations such as a locomotive, a suspension bridge, a car rounding a banked curve, or a toboggan sliding down a hill requires specific problem-solving skills. Although we will not introduce any new principles in this chapter, we will help you develop the skills you will need in order to solve problems with Newton's laws of motion.

We begin with equilibrium problems, concentrating on systems at rest. Then we generalize our problem-solving techniques to include systems that are not in equilibrium, for which we need to deal precisely with the relationships between forces and motion. We'll learn how to describe and analyze the contact force that acts on an object when it rests or slides on a surface, as well as the elastic forces that are present when a solid object is deformed. Finally, we take a brief look at the fundamental nature of force and the kinds of forces found in the physical universe.

5.1 Equilibrium of a Particle

We learned (in Chapter 4) that an object is in **equilibrium** when it is at rest or moving with constant velocity in an inertial frame of reference. A hanging lamp, a rope-and-pulley setup for hoisting heavy loads, a suspension bridge—all these are examples of equilibrium situations. In this section, we consider only the equilibrium of an object that can be modeled as a particle. (Later, in Chapter 10, we'll consider the additional principles needed when an object can't be represented adequately as a particle.) The essential physical principle is Newton's first law: **When an object is at rest or moving with constant velocity in an inertial frame of reference, the vector sum of all the forces acting on it must be zero.** That is (as discussed in Chapter 4), we have the following principle:

Necessary condition for equilibrium of an object

For an object to be in equilibrium, the vector sum of the forces acting on it must be zero:

$$\sum \vec{F} = 0. \quad (4.3)$$

This condition is sufficient only if the object can be treated as a particle, which we assume throughout the remainder of this chapter.

Notes:

- An object in equilibrium must have $\vec{a} = 0$.
- An object in equilibrium may be at rest or moving with a constant velocity.
- Equation 4.3 is always applied to a single object on which the forces are acting.

Equation 4.3 is actually more useful in component form when we work specific problems:

Equilibrium conditions in component form

An object is in equilibrium if the sum of the components of the force in each axis direction is zero:

$$\sum F_x = 0, \qquad \sum F_y = 0. \quad (4.4)$$

Notes:

- You are free to choose any coordinate system for determining the components of the forces.
- The components of the forces can be either positive or negative depending on the coordinate system and the direction of the respective force vectors.

▲ **Application**
Easy does it!

This crane is unloading a 50 ton, 8.2 meter mirror for the European Southern Observatory in Antofagasta, Chile. It is one of four such mirrors that make up an instrument known as the Very Large Telescope, one of the world's biggest, most advanced optical telescopes. The crane holds the mirror steady, so that it remains in equilibrium with the vector sum of the forces on it equal to zero. However, that does not ensure the mirror's safety because it is not a particle. The forces acting on different *parts* of it are not the same. Warping or deforming would be disastrous. The mirror is heavy in part because it is heavily reinforced.

The primary objective of this chapter is to help you develop the skills necessary for solving force problems. We strongly recommend that you study carefully the strategies that follow, look for their applications in the worked-out examples, and then try to apply them when you solve problems.

PROBLEM-SOLVING STRATEGY 5.1 Solving problems involving an object in equilibrium

SET UP

1. Draw a simple sketch of the apparatus or structure, showing all relevant dimensions and angles.
2. Identify the object or objects in equilibrium that you will consider.
3. Draw a free-body diagram for each object identified in step 2.
   a. Assuming that the object can be modeled as a particle, you can represent it by a large dot. Do not include other objects (such as a surface the object may be resting on or a rope pulling on it) in your free-body diagram.
   b. Identify all the ways in which other things interact with the object, either by touching it or via a noncontact force such as gravity. Draw a force vector to represent each force *acting on the object.* Do not include forces exerted *by* the

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object. For each force, make sure that you can answer the question "What other object interacts with the chosen object to cause that force?" If you can't answer that question, you may be imagining a force that isn't there.

c. Label each force as a vector using a symbol that represents the *magnitude* of the force (w for the object's weight, n for a normal force, T for a tension force, etc.); indicate the directions with appropriate angles. (If the object's mass is given, use $w = mg$ to find its weight.)

4. Choose a set of coordinate axes, and represent each force acting on the object in terms of its components along those axes. Cross out each force that has been replaced by its components, so that you don't count it twice. You can often simplify the problem by using a particular choice of coordinate axes. For example, when an object rests or slides on a plane surface, it is usually simplest to take the axes in the directions parallel and perpendicular to this surface, even when the plane is tilted.

SOLVE

5. For each object, set the algebraic sum of all x components of force equal to zero. In a separate equation, set the algebraic sum of all y components equal to zero. (Never add x and y components in a single equation.) You can then solve these equations for up to two unknown quantities, which may be force magnitudes, components, or angles.
6. If you are dealing with two or more objects that interact with each other, use Newton's third law to relate the forces they exert on each other. You need to find as many independent equations as the number of unknown quantities. Then solve these equations to obtain the values of the unknowns. This part is algebra, not physics, but it's an essential step.

REFLECT

7. Whenever possible, look at your results and ask whether they make sense. When the result is a symbolic expression or formula, try to think of special cases (particular values or extreme cases for the various quantities) for which you can guess what the results ought to be. Check to see that your formula works in these particular cases. Think about what the problem has taught you that you can apply to other problems in the future.

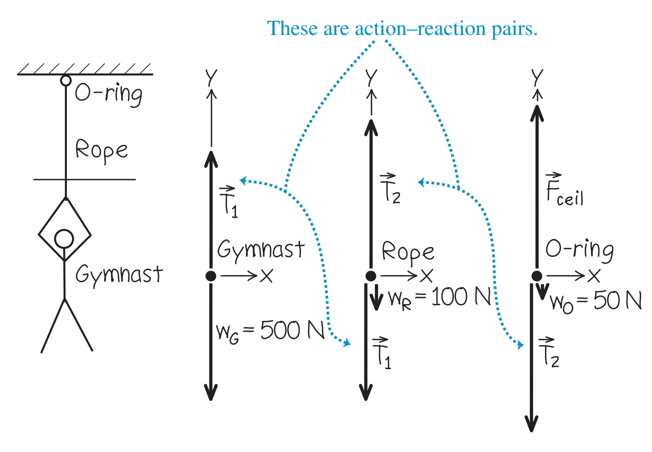
EXAMPLE 5.1 One-dimensional equilibrium

Video Tutor Solution

Now let's apply the problem-solving strategy outlined above to a simple one-dimensional equilibrium problem. A gymnast hangs from the lower end of a rope connected to an O-ring that is bolted to the ceiling. The weights of the gymnast, the rope, and the O-ring are 500 N, 100 N, and 50 N, respectively. What are the magnitudes of the tensions at both ends of the rope?

SOLUTION

SET UP First we sketch the situation (Figure 5.1a). Then we draw three free-body diagrams, for the gymnast, the rope, and the O-ring (Figure 5.1b, c, and d). The forces acting on the gymnast are her weight (magnitude 500 N) and the upward force (magnitude T_1) exerted on her by the rope. We *don't* include the downward force she exerts on the rope because it isn't a force that acts *on* her. We take the y axis to be directed vertically upward, the x axis horizontally. There are no x components of force; that's why we call this a one-dimensional problem.



▶ **FIGURE 5.1** Our diagrams for this problem. We sketch the situation (a), plus free-body diagrams for the gymnast (b), rope (c), and O-ring (d).

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SOLVE The gymnast is motionless, so we know that she is in equilibrium. Since we know her weight, we can use Equation 4.4, $\sum F_y = 0$, to find the magnitude of the upward tension T_1 on her. This force pulls in the positive y direction. Her weight acts in the negative y direction, so its y component is the *negative* of the magnitude—that is, -500 N. Thus,

$$\sum F_y = 0,$$
$$T_1 + (-500\text{ N}) = 0 \quad \text{(equilibrium of gymnast)},$$
$$T_1 = 500\text{ N}.$$

The two forces acting on the gymnast are *not* an action–reaction pair because they act on the same object.

Next we need to consider the forces acting on the rope (Figure 5.1c). Newton's third law tells us that the gymnast exerts a force on the rope that is equal and opposite to the force it exerts on her. In other words, she pulls down on the rope with a force whose magnitude T_1 is 500 N. As you probably expect, this force equals the gymnast's weight. The other forces on the rope are its own weight (magnitude 100 N) and the upward force (magnitude T_2) exerted on its upper end by the O-ring. The equilibrium condition $\sum F_y = 0$ for the rope gives

$$\sum F_y = T_2 + (-100\text{ N}) + (-500\text{ N}) = 0 \quad \text{(equilibrium of rope)},$$
$$T_2 = 600\text{ N}.$$

REFLECT The tension is 100 N greater at the top of the rope (where it must support the weights of both the rope and the gymnast) than at the bottom (where it supports only the gymnast).

Practice Problem: What is the magnitude of the force exerted by the bolt on the O-ring as the gymnast hangs from the bottom end of the rope? *Answer:* 650 N.

TABLE 5.1 Approximate breaking strengths

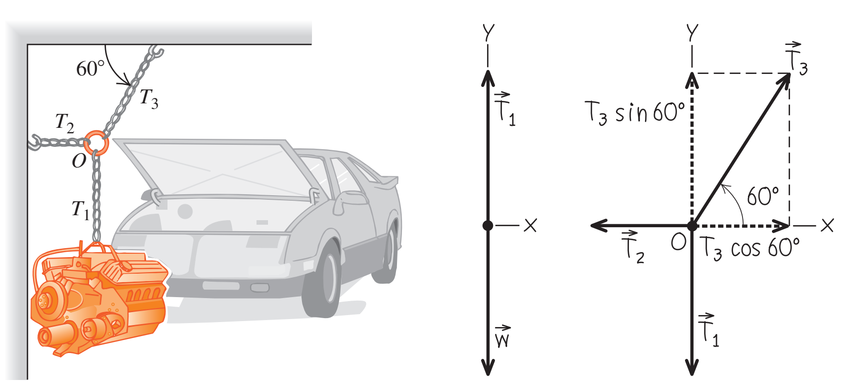
Thin white string	50 N
$\frac{1}{4}$ in. nylon clothesline rope	4000 N
11 mm Perlon mountaineering rope	3×10^4 N
$1\frac{1}{4}$ in. manila climbing rope	6×10^4 N
$\frac{1}{4}$ in. steel cable	6×10^4 N

The strength of the rope in Example 5.1 probably isn't a major concern. The breaking strength of a string, rope, or cable is described by the maximum tension it can withstand without breaking. Some typical breaking strengths are listed in Table 5.1.

EXAMPLE 5.2 Two-dimensional equilibrium

Now we will look at an example where several forces are acting on an object, but in this case the forces are not all along the same axis. As shown in Figure 5.2a, a car engine with weight w hangs from a chain that is linked at point O to two other chains, one fastened to the ceiling and the other to the wall. Find the tension in each of the three chains, assuming that w is given and the weights of the chains themselves are negligible.

Video Tutor Solution



(a) Engine, chains, and ring

(b) Free-body diagram for engine

(c) Free-body diagram for ring O

▲ **FIGURE 5.2**

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SOLUTION

SET UP Figure 5.2b is our free-body diagram for the engine. Without further ceremony, we can conclude that $T_1 = w$. (Because we ignore the weights of the chains, the tension is the same throughout the length of each chain.) The horizontal and slanted chains do not exert forces on the engine itself because they are not attached to it, but they do exert forces on the ring (point O), where the three chains join. So let's consider the *ring* as a particle in equilibrium; the weight of the ring itself is negligible.

In our free-body diagram for the ring (Figure 5.2c), we have three forces T_1, T_2, and T_3; their directions are specified by the vectors in the diagram. We add an x-y coordinate system and resolve the force with magnitude T_3 into its x and y components. Note that the downward force with magnitude T_1 of the chain acting on the ring and the upward force with magnitude T_1 of the chain acting on the engine are not an action–reaction pair.

SOLVE We now apply the equilibrium conditions *for the ring,* writing separate equations for the x and y components. (Note that x and y components are *never* added together in a single equation.) We find that

$$\sum F_x = 0, \qquad T_3 \cos 60° + (-T_2) = 0,$$

$$\sum F_y = 0, \qquad T_3 \sin 60° + (-T_1) = 0.$$

Because $T_1 = w$, we can rewrite the second equation as

$$T_3 = \frac{T_1}{\sin 60°} = \frac{w}{\sin 60°} = 1.155\, w.$$

We can now use this result in the first equation:

$$T_2 = T_3 \cos 60° = (1.155\, w) \cos 60° = 0.577\, w.$$

So we can express all three tensions as multiples of the weight w of the engine, which we assume is known. To summarize, we have

$$T_1 = w,$$

$$T_2 = 0.577\, w,$$

$$T_3 = 1.155\, w.$$

If the engine's weight is $w = 2200$ N (about 500 lb), then

$$T_1 = 2200\ \text{N},$$

$$T_2 = (0.577)(2200\ \text{N}) = 1270\ \text{N},$$

$$T_3 = (1.155)(2200\ \text{N}) = 2540\ \text{N}.$$

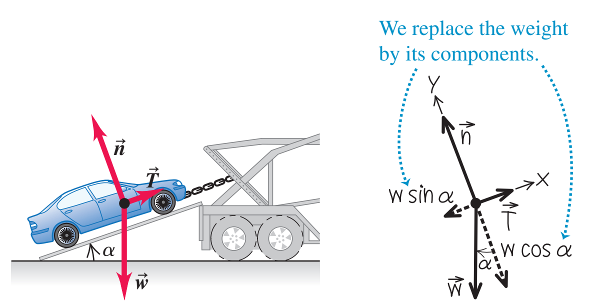
REFLECT Each of the three tensions is proportional to the weight w; if we double w, all three tensions double. Note that T_3 is greater than the weight of the engine. If this seems strange, observe that T_3 must be large enough for its vertical component to be equal in magnitude to w, so T_3 itself must have a magnitude *larger* than w.

Practice Problem: If we change the angle of the upper chain from 60° to 45°, determine the new expressions for the three tensions. *Answers:* $T_1 = w$, $T_2 = w \cos 45°/\sin 45° = w$, $T_3 = w/\sin 45° = \sqrt{2}w$.

EXAMPLE 5.3 Car on a ramp

In this example we will see how to handle the case of an object resting on an inclined plane. A car with a weight of 1.76×10^4 N rests on the ramp of a trailer (Figure 5.3a). The car's brakes and transmission lock are released; only the cable prevents the car from rolling backward off the trailer. The ramp makes an angle of 26.0° with the horizontal. Find the tension in the cable and the force with which the ramp pushes on the car's tires.

Video Tutor Solution



▶ **FIGURE 5.3** **(a)** Car on ramp **(b)** Free-body diagram for car

SOLUTION

SET UP Figure 5.3b shows our free-body diagram for the car. The three forces exerted on the car are its weight (magnitude w), the tension in the cable (magnitude T), and the normal force with magnitude n exerted by the ramp. (Because we treat the car as a particle, we can lump the normal forces on the four wheels together as a single force.)

We orient our coordinate axes parallel and perpendicular to the ramp, and we replace the weight force by its components.

SOLVE The car is in equilibrium, so we first find the components of each force in our axis system and then apply Newton's first law. To find the components of the weight, we note that the angle α between

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the ramp and the horizontal is equal to the angle α between the weight vector and the normal to the ramp, as shown. The angle α is *not* measured in the usual way, counterclockwise from the $+x$ axis. To find the components of the weight (w_x and w_y), we use the right triangles in Figure 5.3b. We find that $w_x = -w \sin\alpha$ and $w_y = -w \cos\alpha$. The equilibrium conditions then give us

$$\sum F_x = 0, \qquad T + (-w \sin\alpha) = 0,$$
$$\sum F_y = 0, \qquad n + (-w \cos\alpha) = 0.$$

Be sure you understand how the signs are related to our choice of coordinate axis directions. Remember that, by definition, T, w, and n are *magnitudes* of vectors and are therefore positive.

Solving these equations for T and n, we find

$$T = w \sin\alpha,$$
$$n = w \cos\alpha.$$

Finally, inserting the numerical values $w = 1.76 \times 10^4$ N and $\alpha = 26°$, we obtain

$$T = (1.76 \times 10^4\ \text{N})(\sin 26°) = 7.72 \times 10^3\ \text{N},$$
$$n = (1.76 \times 10^4\ \text{N})(\cos 26°) = 1.58 \times 10^3\ \text{N}.$$

REFLECT To check some special cases, note that if the angle α is zero, then $\sin\alpha = 0$ and $\cos\alpha = 1$. In this case, the ramp is horizontal; no cable tension T is needed to hold the car, and the magnitude of the total normal force n is equal to the car's weight. If the angle is 90° (the ramp is vertical), then $\sin\alpha = 1$ and $\cos\alpha = 0$. In that case, the cable tension T equals the weight w and the normal force n is zero.

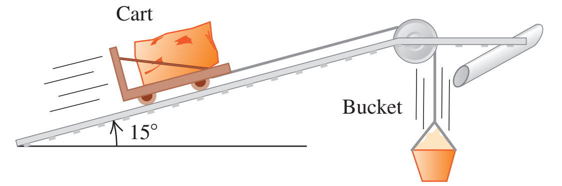
We also note that our results would still be correct if the car were on a ramp on a car transport trailer traveling down a straight highway at a constant speed of 65 mi/h. Do you see why?

Practice Problem: What ramp angle would be needed in order for the cable tension to equal one-half of the car's weight? *Answer:* 30°.

EXAMPLE 5.4 Lifting granite and dumping dirt

Video Tutor Solution

Let us again look at an inclined plane problem, but this time one involving two objects that are tied together by a cable. Blocks of granite, each with weight w_1, are being hauled up a 15° slope out of a quarry (Figure 5.4). For environmental reasons, dirt is also being dumped into the quarry to fill up old holes. You have been asked to find a way to use this dirt to move the granite out more easily. You design a system that lets the dirt (weight w_2, including the weight of the bucket) that drops vertically into the quarry pull out a granite block on a cart with steel wheels (total weight w_1), rolling on steel rails. Ignoring the weight of the cable and friction in the pulley and wheels, determine how the weights w_1 and w_2 must be related for the system to move with constant speed.



▲ **FIGURE 5.4**

SOLUTION

SET UP Figure 5.5a shows an idealized model for the system. We draw two free-body diagrams, one for the dirt and bucket (Figure 5.5b) and one for the granite block on its cart (Figure 5.5c). In drawing the coordinate axes for each object, we're at liberty to orient them differently for the two objects; the choices shown are the most convenient ones. (But we must *not* change the orientation of the objects themselves.) We represent the weight of the granite block in terms of its components in the chosen axis system. The tension T in the cable is the same throughout because we are assuming that the cable's weight is negligible. (The pulley changes the *directions* of the cable forces, but if it is frictionless and massless, it can't change their *magnitudes*.)

SOLVE Each object is in equilibrium. Applying $\sum F_y = 0$ to the bucket (Figure 5.5b), we find that

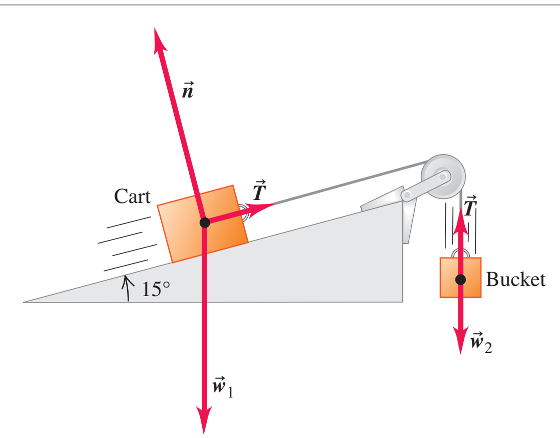
$$T + (-w_2) = 0, \qquad T = w_2.$$

Applying $\sum F_x$ to the cart (Figure 5.5c), we get

$$T + (-w_1 \sin 15°) = 0, \qquad T = w_1 \sin 15°.$$

Equating the two expressions for T yields

$$w_2 = w_1 \sin 15° = 0.26\, w_1.$$



(a) Idealized model

▲ **FIGURE 5.5**

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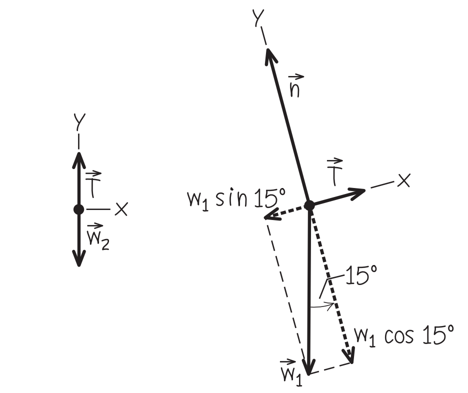

(b) Free-body diagram for bucket

(c) Free-body diagram for cart

REFLECT If the weight w_2 of the dirt and bucket totals 26% of the weight w_1 of the granite block and cart, the system can move in either direction with constant speed. In the real world, where friction tends to hold back the motion, would w_2 need to be greater or less than $0.26\,w_1$ for the granite block to move up with constant speed?

Practice Problem: The system you have designed could also be used to remove dirt from the quarry by letting the cart slide back down the incline. If the cart slides back down the incline at a constant speed, determine the total normal force n that the tracks apply to the wheels of the granite cart, as a fraction of w_1. *Answer:* $n = w_1\cos 15° = 0.97\,w_1$.

◀ **FIGURE 5.5** (continued)

5.2 Applications of Newton's Second Law

We're now ready to discuss problems in **dynamics,** showing applications of Newton's second law to systems that are *not* in equilibrium. Here's a restatement of that law:

Newton's second law

An object's acceleration equals the vector sum of the forces acting on it, divided by its mass. In vector form, we write this statement as

$$\sum \vec{F} = m\vec{a}. \qquad (4.7)$$

However, we'll usually use this relationship in its component form:

$$\sum F_x = ma_x, \qquad \sum F_y = ma_y. \qquad (4.8)$$

The following problem-solving strategy is similar to our strategy for solving equilibrium problems, presented in Section 5.1:

PROBLEM-SOLVING STRATEGY 5.2 **Using Newton's second law**

SET UP

1. Draw a sketch of the physical situation, and identify the moving object or objects to which you will apply Newton's second law.
2. Draw a free-body diagram for each chosen object, showing all the forces acting *on* that object, as described in the strategy for Newton's first law (Problem-Solving Strategy 5.1). Label each force as a vector with the symbol that you will use to represent its magnitude. Usually, one of the forces will be the object's weight; it is generally best to label this as *mg*. If a numerical value of mass is given, you can compute the corresponding weight.
3. Show your coordinate axes explicitly in each free-body diagram, and then determine the components of forces with reference to these axes. If you know the direction of the acceleration, it is usually best to take that direction as one of the axes. When you represent a force in terms of its components, cross out the original force so as not to include it twice. When there are two or more objects, you can use a separate axis system for each object; you don't have to use the same axis system for all the objects. But, in the equations for each object, the signs of the components *must* be consistent with the axes you have chosen for that object.

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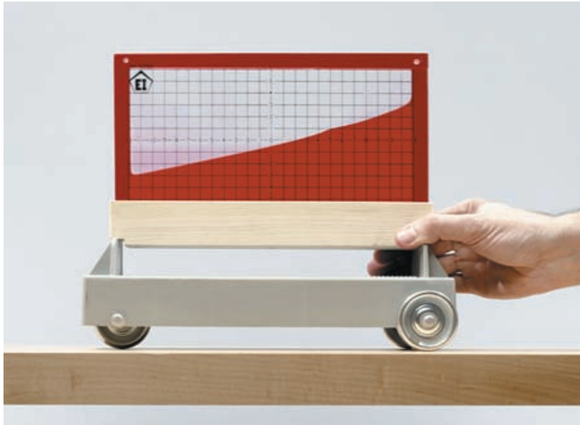


▲ **Application**
Gravity-defying liquid?

Although this container is on a level tabletop, the liquid inside is at a slant. How can that be? You may have guessed that the photo shows a demonstration of a simple liquid-filled accelerometer. As the container is accelerated to the left, the surface of the liquid forms an angle with the horizontal. The tangent of the angle is proportional to the acceleration. Can you verify that if the container is 19.6 cm wide, the height of the liquid at the right end (in cm) above its level when the apparatus is at rest will give the acceleration of the container itself, in meters per second squared?

SOLVE

4. Write the equations for Newton's second law in component form: $\sum F_x = ma_x$ and $\sum F_y = ma_y$ (Equations 4.8). Be careful not to add x and y components in the same equation.
5. If more than one object is involved, repeat step 4 for each object.
6. Use what you know about the motion to identify any components of the acceleration that must have a certain value, such as zero. For example, a block sliding on a horizontal surface cannot have a vertical acceleration; therefore, you should set the vertical component of the acceleration to zero.
7. Solve the equations to find the required unknowns.

REFLECT

8. Check particular cases or extreme values of quantities, when possible, and compare the results for these particular cases with your intuitive expectations. Ask yourself, "Does this result make sense?" Think about what the problem has taught you that you can apply to other problems in the future.

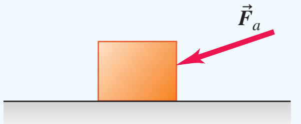
QUANTITATIVE ANALYSIS 5.1

Pushing a box across the floor

You push a box across a horizontal floor with a force $\vec{F}_a$ that is angled below the horizontal, as shown in Figure 5.6. How does the magnitude of the normal force from the floor compare to the weight of the box as you push it?

A. The normal force from the floor is less than the box's weight.
B. The normal force from the floor is equal to the box's weight.
C. The normal force from the floor is greater than the box's weight.

SOLUTION As the box slides horizontally across the floor, it doesn't move vertically. Therefore, the vertical component of the acceleration of the box must be zero. This means that the upward-pointing normal force must cancel out both the weight and the vertical component of $\vec{F}_a$, which point downward. Therefore, the correct answer is C. (Notice that it is not clear from the problem whether or not the horizontal component of the box's acceleration is zero, but that doesn't affect our answer for this problem, since we are concerned only with what is going on vertically.)

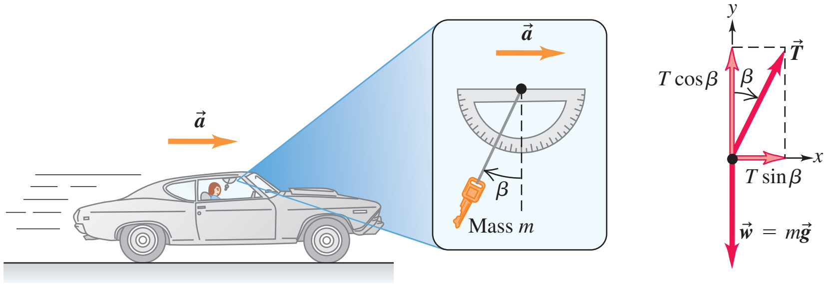


▲ **FIGURE 5.6**

EXAMPLE 5.5 **A simple accelerometer**

In this example we will use Newton's second law to calculate the acceleration of a key hanging in a car. You tape one end of a piece of string to the ceiling light of your car and hang a key with mass m on the other end of the string (Figure 5.7a). A protractor taped to the light allows you to measure the angle the string makes with the vertical. Your friend drives the car while you make measurements. When the car has a constant acceleration with magnitude a toward the right, the string hangs at rest (relative to the car), making an angle β with the vertical. **(a)** Derive an expression for the acceleration a in terms of the mass m and the measured angle β. **(b)** In particular, what is a when $\beta = 45°$? When $\beta = 0$?

Video Tutor Solution



(a) Low-tech accelerometer

(b) Free-body diagram for the key

▲ **FIGURE 5.7**

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SOLUTION

SET UP Our free-body diagram is shown in Figure 5.7b. The forces acting on the key are its weight $w = mg$ and the string tension T. We direct the x axis to the right (in the direction of the acceleration) and the y axis vertically upward.

SOLVE **Part (a)**: This problem may look like an equilibrium problem, but it isn't. The string and the key are at rest with respect to the car, but the car, string, and key are all accelerating in the $+x$ direction. Thus, there must be a horizontal component of force acting on the key, and so we use Newton's second law in component form: $\sum F_x = ma_x$ and $\sum F_y = ma_y$.

We find the components of the string tension, as shown in Figure 5.7b. The sum of the horizontal components of force is

$$\sum F_x = T \sin\beta,$$

and the sum of the vertical components is

$$\sum F_y = T\cos\beta + (-mg).$$

The x component of the acceleration of the car, string, and key is $a_x = a$, and the y component of acceleration is zero, so

$$\sum F_x = T\sin\beta = ma_x,$$
$$\sum F_y = T\cos\beta + (-mg) = ma_y = 0.$$

Rearranging terms, we obtain

$$T\sin\beta = ma_x, \qquad T\cos\beta = mg.$$

When we divide the first equation by the second, we get

$$a_x = g\tan\beta.$$

The acceleration a_x is thus proportional to the tangent of the angle β.

Part (b): When $\beta = 0$, the key hangs vertically and the acceleration is zero; when $\beta = 45°$, $a_x = g$.

REFLECT We note that β can never be 90° because that would require an infinite acceleration. We note also that the relationship between a_x and β doesn't depend on the mass of the key, but it *does* depend on the acceleration due to gravity.

Practice Problem: What is the angle β if the acceleration is $g/2$? *Answer:* 26.6°.

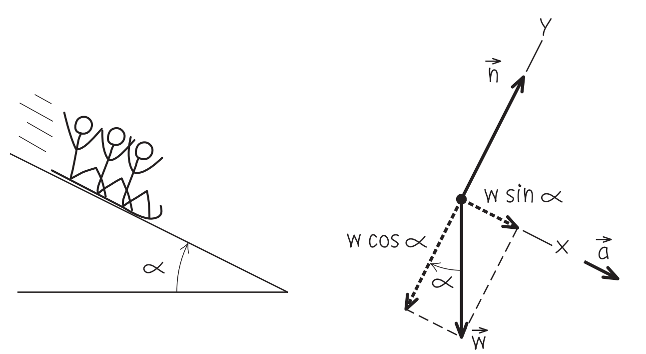
EXAMPLE 5.6 Acceleration down a hill

Video Tutor Solution

Now we are going to look at the problem of an object sliding down a frictionless inclined plane. Suppose a toboggan loaded with vacationing students (total weight w) slides down a long, snow-covered slope. The hill slopes at a constant angle α, and the toboggan is so well waxed that there is virtually no friction. Find the toboggan's acceleration and the magnitude n of the normal force the hill exerts on the toboggan.

SOLUTION

SET UP Figure 5.8a is our sketch for this problem. The only forces acting on the toboggan are its weight w and the normal force n (Figure 5.8b). The direction of the weight is straight downward, but the direction of the normal force is perpendicular to the surface of the hill, at an angle α with the vertical. We take axes parallel and perpendicular to the surface of the hill and resolve the weight into x and y components.



(a) The situation **(b)** Free-body diagram for toboggan

▲ **FIGURE 5.8** Our diagrams for this problem.

SOLVE There is only one x component of force, so

$$\sum F_x = w\sin\alpha.$$

From $\sum F_x = ma_x$, we have

$$w\sin\alpha = ma_x,$$

and since $w = mg$, the acceleration is

$$a_x = g\sin\alpha.$$

The y component equation gives $\sum F_y = n + (-mg\cos\alpha)$. We know that the y component of acceleration is zero because there is no motion in the y direction. So $\sum F_y = 0$ and $n = mg\cos\alpha$.

REFLECT The mass m does not appear in the expression for a_x; this means that *any* toboggan, regardless of its mass or number of passengers, slides down a frictionless hill with an acceleration of $g\sin\alpha$. In particular, when $\alpha = 0$ (a flat surface with no slope at all), the acceleration is $a_x = 0$, as we should expect. When the surface is vertical, $\alpha = 90°$ and $a_x = g$ (free fall).

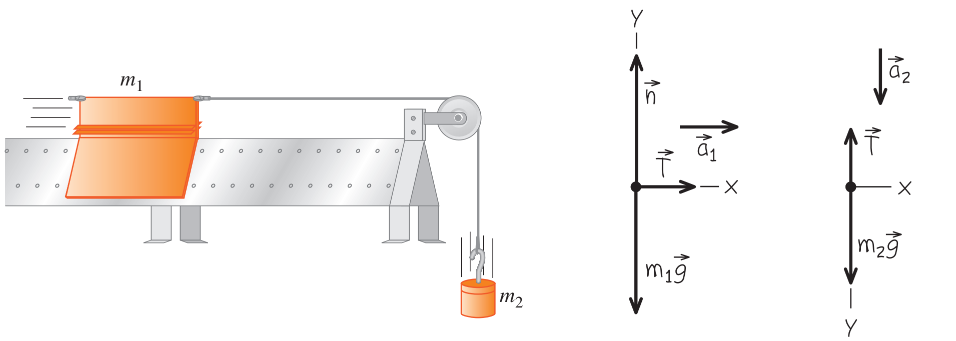
Note that the magnitude n of the normal force exerted on the toboggan by the surface of the hill ($n = mg\cos\alpha$) is *proportional* to the magnitude mg of the toboggan's weight; the two are *not* equal, except in the special case where $\alpha = 0$.

Practice Problem: At what angle does the hill slope if the acceleration is $g/2$? *Answer:* 30°.

EXAMPLE 5.7 An air track in a physics lab

Video Tutor Solution

Now we will work a problem involving two objects tied together by a string. Figure 5.9a shows a glider with mass m_1 that moves on a level, frictionless air track in a physics lab. It is connected by a string passing over a small frictionless pulley to a hanging weight with total mass m_2. The string is light and flexible, and it doesn't stretch. Find the acceleration of each object and the tension in the string.



▶ **FIGURE 5.9** **(a)** Apparatus **(b)** Free-body diagram for glider **(c)** Free-body diagram for weight

SOLUTION

SET UP The two objects have different motions, so we need to draw a separate free-body diagram and coordinate system for each, as shown in Figure 5.9b and c. We are free to use different coordinate axes for the two objects; in this case, it's convenient to take the $+x$ direction to the right for the glider and the $+y$ direction downward for the hanging weight. Then the glider has only an x component of acceleration (i.e., $a_{1x} = a_1$, $a_{1y} = 0$), and the weight has only a y component (i.e., $a_{2x} = 0$, $a_{2y} = a_2$). There is no friction in the pulley, and we consider the string to be massless, so the tension T in the string is the same throughout; it applies a force with magnitude T to each object. The weights are $m_1 g$ and $m_2 g$.

SOLVE We apply Newton's second law, in component form, to each object in turn. For the glider on the track, Newton's second law gives

$$\sum F_x = T = m_1 a_{1x},$$
$$\sum F_y = n + (-m_1 g) = m_1 a_{1y} = 0,$$

and for the hanging weight,

$$\sum F_y = m_2 g + (-T) = m_2 a_{2y}.$$

Now, because the string doesn't stretch, the two objects must move equal distances in equal times, and their *speeds* at any instant must be equal. When the speeds change, they change by equal amounts in a given time, so the accelerations of the two bodies must have the same magnitude a. With our choice of coordinate systems, if a_{1x} is positive, a_{2y} will also be positive, and if a_{1x} is negative, so will be a_{2y}. We can express this relationship as

$$a_{1x} = a_{2y} = a.$$

(The directions of the two accelerations are different, of course.) The two Newton's second law equations are then

$$T = m_1 a,$$
$$m_2 g + (-T) = m_2 a.$$

We'd like to get separate expressions for T and a in terms of the masses and g.

An easy way to get an expression for a is to replace $-T$ in the second equation by $-m_1 a$ and rearrange the result; when we do so, we obtain $m_2 g + (-m_1 a) = m_2 a$, $m_2 g = (m_1 + m_2)a$, and, finally,

$$a = \frac{m_2}{m_1 + m_2} g.$$

Then, to get an expression for T, we substitute this expression for a back into the first equation. The result is

$$T = \frac{m_1 m_2}{m_1 + m_2} g = \frac{m_1}{m_1 + m_2} m_2 g.$$

REFLECT It's always a good idea to check general symbolic results such as this for particular cases where we can guess what the answer ought to be. For example, if the mass of the glider is zero ($m_1 = 0$), we expect that the hanging weight (mass m_2) will fall freely with acceleration g and there will be no tension in the string. When we substitute $m_1 = 0$ into the preceding expressions for T and a, they do give $T = 0$ and $a = g$, as expected. Also, if $m_2 = 0$, there is nothing to create tension in the string or accelerate either mass. For this case, the equations give $T = 0$ and $a = 0$. Thus, in these two special cases, the results agree with our intuitive expectations.

We also note that, in general, the tension T is *not* equal to the weight $m_2 g$ of the hanging mass m_2, but is *less* by a factor of $m_1/(m_1 + m_2)$. If T *were* equal to $m_2 g$, then m_2 would be in equilibrium, but it isn't.

Practice Problem: To set up for another run, you pull the glider back up the track at a constant speed with a horizontal force $\vec{F}_a$. Determine the magnitude of $\vec{F}_a$ in terms of the two masses in the problem. *Answer*: $m_2 g$.

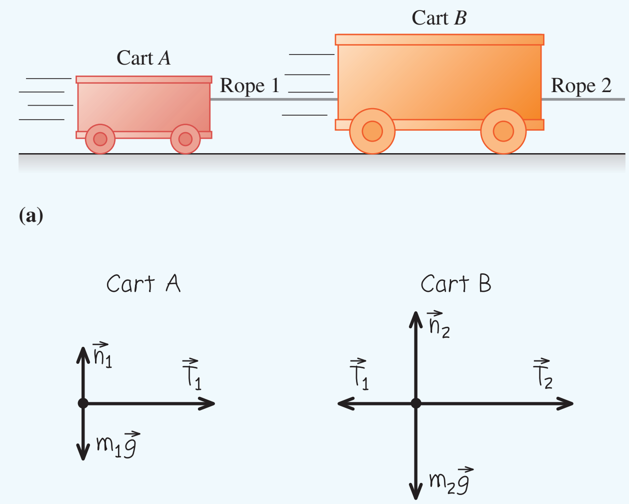
QUANTITATIVE ANALYSIS 5.2

A two-cart train

The carts in Figure 5.10a are speeding up as they are pulled to the right with increasing speed across a frictionless surface. The ropes have negligible mass. We can conclude that

A. the pull of rope 1 on cart *A* has greater magnitude than the pull of rope 1 on cart *B*.
B. the pull of rope 2 on cart *B* has greater magnitude than the pull of rope 1 on cart *B*.
C. the pull of rope 1 on cart *A* has greater magnitude than the pull of rope 2 on cart *B*.

SOLUTION The correct answer is B: For cart *B* to accelerate, there must be a net force to the right. Answer A cannot be right; if the rope is massless, the forces acting on its two ends must add to zero. Nor can answer C be right: The two carts have the same acceleration; therefore, the tension in rope 2 must be great enough to give this acceleration to both carts, while the tension in rope 1 is just great enough to give cart *A* the same acceleration, as shown in the free-body diagrams in Figure 5.10b.



(a)

(b) Free-body diagrams of the two carts

▲ **FIGURE 5.10**

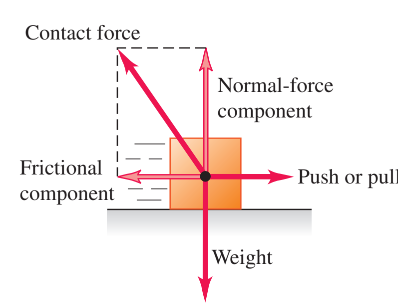
5.3 Contact Forces and Friction

We've seen several problems in which an object rests or slides on a surface that exerts forces on the object. Previously (at the beginning of Chapter 4), we introduced the terms *normal force* and *friction force* to describe these forces. Whenever two objects interact by direct contact (touching) of their surfaces, we call the interaction forces *contact forces.* Normal and friction forces are both contact forces. Friction is important in many aspects of everyday life—for both facilitating and preventing the slipping of an object that is in contact with a surface. For example, the oil in a car engine facilitates sliding between moving parts, but friction between the tires and the road prevents slipping. Similarly, air drag decreases automotive fuel economy, but makes parachutes work. Without friction, nails would pull out, light bulbs and bottle caps would unscrew effortlessly, and riding a bicycle would be hopeless. A "frictionless surface" is a useful concept in idealized models of mechanical systems in which the friction forces are negligibly small. In the real world, such an idealization is unattainable, although driving a car on wet ice comes fairly close!

Let's consider an object sliding across a surface. When you try to slide a heavy box of books across the floor, the box doesn't move at all until you reach a certain critical force. Then the box starts moving, and you can usually keep it moving with less force than you needed to get it started. If you take some of the books out, you need less force than before to get the box started or keep it moving. What general statements can we make about this behavior?

First, when an object rests or slides on a surface, we can always represent the contact force exerted by the surface on the object in terms of the components of force perpendicular and parallel to the surface (Figure 5.11). We call the perpendicular component the *normal force,* denoted by $\vec{n}$. (*Normal* is a synonym for *perpendicular.*) The component parallel to the surface is the *friction force,* denoted by $\vec{f}$. By definition, $\vec{n}$ and $\vec{f}$ are always perpendicular to each other. When an object is sliding with respect to a surface, the associated friction force is called a *kinetic-friction* force, with magnitude f_k. When there is no relative motion, we speak of a *static-friction* force, with magnitude f_s. We'll discuss these two possibilities separately.

The frictional and normal forces are really components of a single contact force.



▲ **FIGURE 5.11** The frictional and normal forces for a block sliding on a rough surface.

▲ **Application**
Friction can be fun.

As this rock climber shows, frictional forces can be important. Without them, a climber could not move or maintain an equilibrium position. Notice how the climber is demonstrating one of the key concepts of frictional forces: The friction force is proportional to the normal force. Therefore, a good climber tries to contact the rock at an angle as close as feasible to 90°. This maximizes the normal force and therefore also maximizes the friction force.

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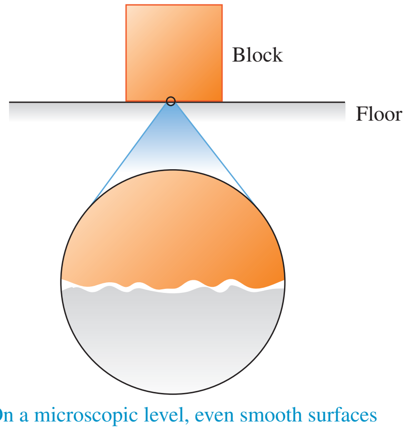


▲ **FIGURE 5.12** The origin of frictional forces.

When one object is moving relative to a surface, the *direction* of the kinetic-friction force on the object is always opposite to the direction of motion of that object relative to the surface. For example, when a book slides from left to right along a tabletop, the friction force on it acts to the left, as in Figure 5.11. According to Newton's third law, the book simultaneously applies a force to the table that is equal in magnitude, but directed to the right; in effect, the book tries to drag the table along with it.

The *magnitude* f_k of a kinetic-friction force usually increases when the normal-force magnitude n increases. Thus, more force is needed to slide a box full of books across the floor than to slide the same box when it is empty. This principle is also used in automobile braking systems: The harder the brake pads are squeezed against the rotating brake disks, the greater is the braking effect. In some cases, the magnitude of the sliding friction force f_k is found to be approximately *proportional* to the magnitude n of the normal force. In such cases, we call the ratio f_k/n the **coefficient of kinetic friction,** denoted as μ_k.

Relationship between kinetic-friction force and normal force

When the magnitude of the sliding friction force f_k is roughly proportional to the magnitude n of the normal force, the two are related by a constant μ_k called the coefficient of kinetic friction:

$$f_k = \mu_k n. \tag{5.1}$$

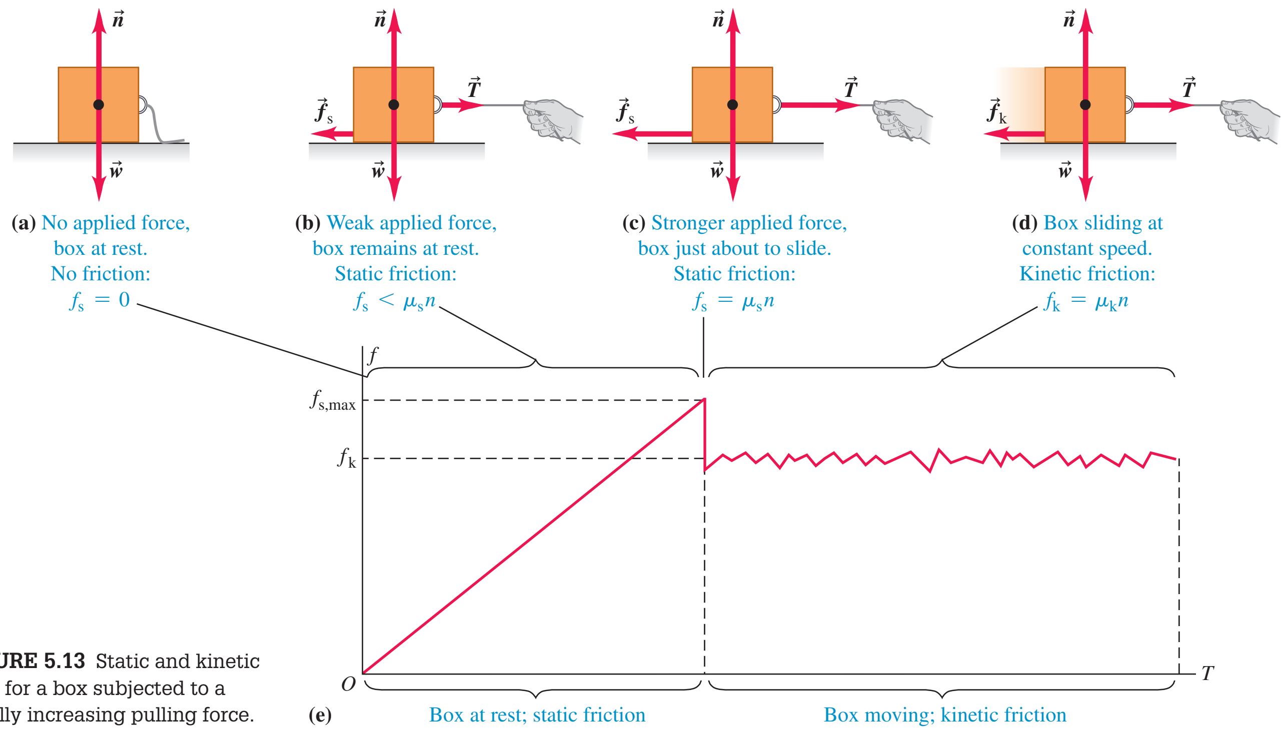
Units: Because μ_k is the ratio of two force magnitudes, it has no units.
Notes:

- The friction force and the normal force are always perpendicular, so Equation 5.1 is *not* a vector equation but a relationship between the *magnitudes* of the two forces.
- A frictionless surface corresponds to $\mu_k = 0$.

The more slippery the surface, the smaller the value of the coefficient of friction. The specific numerical value of the coefficient of kinetic friction for any two surfaces depends on the materials and the surfaces. For Teflon on steel, μ_k is about 0.04. For rubber tires on rough dry concrete (or sneakers on a dry gym floor), μ_k can be as large as 1.0 to 1.2. For two smooth metal surfaces, it is typically 0.4 to 0.8.

Although some of these numbers are given to two significant figures, they are only approximate values; indeed, Equation 5.1 is just an *approximate* representation of a complex phenomenon. On a microscopic level, friction and normal forces result from the intermolecular forces (fundamentally electrical in nature) between two rough surfaces at high points where they come into contact. Figure 5.12 suggests the nature of the interaction. The actual area of contact of two rough surfaces is usually much smaller than the total surface area. However, when two smooth surfaces of the same metal are brought together, these forces can cause a "cold weld" where the metal surfaces actually bond together. This can be disastrous for a complex mechanical system such as an engine. Without lubrication the metal surfaces of an engine's bearings, bushings, and sleeves will bond together and greatly limit its operational lifespan. It is interesting that the success of lubricating oils depends on the fact that they maintain a film between the surfaces that prevents them from coming into actual contact. Friction forces can also depend on the *velocity* of the object relative to the surface. We'll ignore that complication for now in order to concentrate on the simplest cases.

Friction forces may also act when there is *no* relative motion between the surfaces of contact. If you try to slide that box of books across the floor, the box may not move at all if you don't push hard enough, because the floor exerts an equal and opposite friction force on the box. This force is called a **static-friction force** $\vec{f}_s$. In Figure 5.13a, the box is at rest, in equilibrium, under the action of its weight $\vec{w}$ and the upward normal force $\vec{n}$, which is equal in magnitude to the weight and exerted on the box by the floor. Now we tie a rope to the box (Figure 5.13b) and gradually increase the tension T in the rope. At first the box remains at rest because, as T increases, the force of static friction f_s also increases (staying equal in magnitude to T).



▶ **FIGURE 5.13** Static and kinetic friction for a box subjected to a gradually increasing pulling force.

At some point, however, T becomes greater than the maximum friction force f_s that the surface can exert; the box then "breaks loose" and starts to slide. Figure 5.13c shows the force diagram when T is at this critical value, and Figure 5.13d is the case when T exceeds this value and the box begins to move. Figure 5.13e shows how the friction force varies if we start with no applied force $(T = 0)$ and gradually increase the force until the object starts to slide. Note that the force f_k of kinetic friction is somewhat less than the maximum force $f_{s,max}$ of static friction: Less force is needed to keep the box sliding with constant speed than to start it moving initially.

For a given pair of surfaces, the maximum value of f_s depends on the normal force. In some cases, the maximum value of f_s is approximately *proportional* to n; we call the proportionality factor μ_s the **coefficient of static friction.** In a particular situation, the actual force of static friction can have any magnitude between zero (when there is no other force parallel to the surface) and a maximum value $f_{s,max}$ given by $f_{s,max} = \mu_s n$.

Relationship between normal force and maximum static-friction force

When the maximum magnitude of the static-friction force can be represented as proportional to the magnitude of the normal force, the two are related by a constant μ_s called the coefficient of static friction:

$$f_s \leq \mu_s n. \tag{5.2}$$

Units: μ_s has no units.
Notes:

- The static-friction force is an "adjustable" force that always adjusts itself so as to keep two surfaces from sliding against each other.
- The static-friction force has a maximum possible magnitude, which is given by the equality sign in Equation 5.2.
- The coefficient of static friction is usually less than the corresponding coefficient of kinetic friction for any two particular surfaces.

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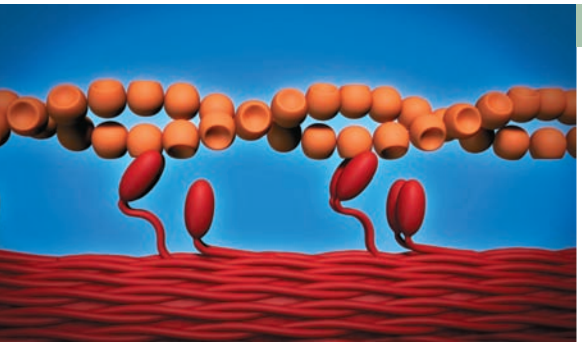
▲ **Application**
Friction can be a problem.

If you're trying to saw through a concrete roadbed, friction is not necessarily a good thing. If this saw blade were not lubricated, friction would make it hard to draw the saw through the cut, and the heat from friction could warp or melt the blade. Therefore, the blade of this roadbed saw is continuously showered with water to minimize frictional forces and carry away heat. Even so, a cut across the entire width of the road will take some time to complete.



◀ BIO Application Molecular motors.

Nature has engineered a small number of molecular motors that convert the chemical energy of an ATP molecule into mechanical force. These nanomachines rely on a kind of friction to direct this force to their loads. A two-headed motor protein called myosin (in red) binds to a structural track called actin (in orange), along which it applies force through electrical interactions that are the small-scale version of friction. During the power stroke, when the myosin is applying force to the actin, each myosin head applies 1–10 pN of force to the actin, depending on the type of myosin. The aggregate force of many such molecules working in concert is the basis of the forces generated by animal skeletal muscles.

The equal sign holds only when the applied force T, parallel to the surface, has reached the critical value at which motion is about to start (Figure 5.13c). When T is *less* than this value (Figure 5.13b), the inequality sign holds. In that case, we have to use the equilibrium conditions $\sum \vec{F} = 0$ to find f_s. For any given pair of surfaces, the coefficient of kinetic friction is usually *less* than the coefficient of static friction. As a result, when sliding starts, the friction force usually *decreases,* as Figure 5.13e shows.

In some situations, the surfaces alternately stick (static friction is operative) and slip (kinetic friction arises); this alternation is what causes the horrible squeak made by chalk held at the wrong angle while you're writing on a blackboard. Another slip-and-stick phenomenon is the squeaky noise your windshield-wiper blades make when the glass is nearly dry; still another is the outraged shriek of tires sliding on asphalt pavement. A positive example is the sound produced by the motion of a violin bow against the string.

Liquids and gases also show frictional effects. Friction between two solid surfaces separated by a layer of liquid or gas is determined primarily by the *viscosity* of the fluid. In a car engine, the pistons are separated from the cylinder walls by a thin layer of oil. When an object slides on a layer of gas, friction can be made very small. For instance, in the familiar linear air track used in physics labs (like the one in Example 5.7), the gliders are supported on a layer of air. The frictional force is dependent on velocity, but at typical speeds the effective coefficient of friction is on the order of 0.001. A device similar to the air track is the frictionless air table, on which pucks are supported by an array of small air jets about 2 cm apart.

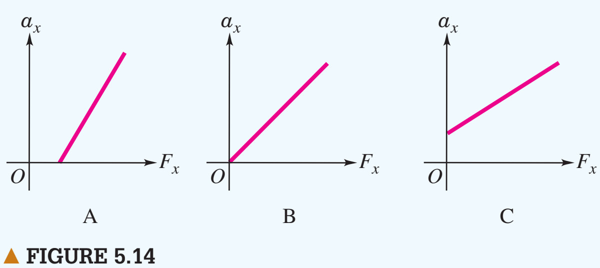
For a wheeled vehicle, we can define a **coefficient of rolling friction** μ_r which is the horizontal force needed for constant speed on a flat surface, divided by the upward normal force exerted by the surface. Typical values of μ_r are 0.002 to 0.003 for steel wheels on steel rails and 0.01 to 0.02 for rubber tires on concrete. These values are one reason that railroad trains are, in general, much more fuel efficient than highway trucks.

QUANTITATIVE ANALYSIS 5.3

A friction experiment

A block is pulled along a horizontal surface (with friction) by a constant force with magnitude F_x, and the acceleration a_x is measured. The experiment is repeated several times with different values of F_x, and a graph of acceleration as a function of force is plotted. For the various trials, which of the graphs of acceleration versus force shown in Figure 5.14 is most nearly correct?

SOLUTION No motion occurs unless the force F_x exceeds the maximum static-friction force; when it does, the block starts to move under the action of F_x and the smaller kinetic-friction force. Only answer A shows no acceleration at small values of the applied force.



▲ **FIGURE 5.14**

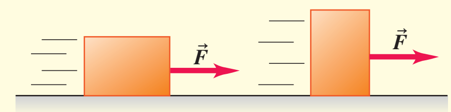
CONCEPTUAL ANALYSIS 5.4

Turning things over

You are pushing the heavy plastic block in Figure 5.15 across the room with constant velocity. You decide to turn the block on end, reducing the surface area in contact with the floor by half. In this new orientation, to push this same block across the same floor with the same speed as before, the magnitude of force that you must apply is

A. twice as great as before.
B. the same as before.
C. half as great as before.

SOLUTION For many surfaces, dry sliding friction depends only on the normal force with which the object presses against the surface—in this case, a force equal to the weight of the block. The friction force does not depend on the area of contact. Since the friction force is the same in both orientations, the applied force needed is the same. The correct answer is B.



▲ **FIGURE 5.15**

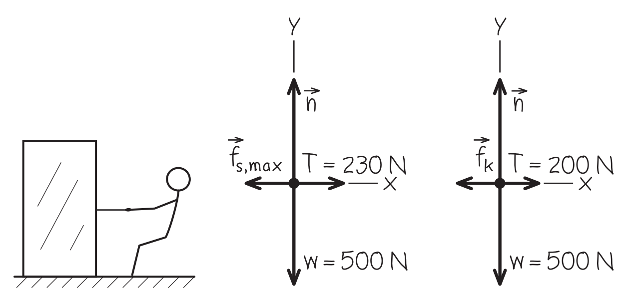
EXAMPLE 5.8 Delivering the goods

In this example we will calculate the coefficients of static and kinetic friction using Equations 5.1 and 5.2 along with Newton's second law of motion. Suppose that a delivery company has just unloaded a 500 N crate full of home exercise equipment in your level driveway. You find that to get it started moving toward your garage, you have to pull with a horizontal force of magnitude 230 N. Once the crate "breaks loose" and starts to move, you can keep it moving at constant velocity with only 200 N of force. What are the coefficients of static and kinetic friction?

Video Tutor Solution

SOLUTION

SET UP As shown in Figure 5.16, the forces acting on the crate are its weight (magnitude w), the force applied by the rope (magnitude T), and the normal and frictional components (magnitudes n and f, respectively) of the contact force the driveway surface exerts on the crate. We draw free-body diagrams for the crate at the instant it starts to move (when the static-friction force is maximum) and while it is in motion.



(a) Pulling a crate

(b) Free-body diagram for crate just before it starts to move

(c) Free-body diagram for crate moving at constant speed

▲ **FIGURE 5.16**

SOLVE Whether the crate is moving or not, the friction force opposes your pull because it tends to prevent the crate from sliding relative to the surface. An instant before the crate starts to move, the static-friction force has its maximum possible value, $f_{s,max} = \mu_s n$. The state of rest and the state of motion with constant velocity are both equilibrium conditions. Remember that w, n, and f are the *magnitudes* of the forces; some of the components have negative signs. For example, the magnitude of the weight is 500 N, but its y component is -500 N. We write Newton's second law in component form, $\sum F_x = 0$ and $\sum F_y = 0$, for the crate. Then

$$\sum F_y = n + (-w) = n - 500\text{ N} = 0, \qquad n = 500\text{ N},$$
$$\sum F_x = T + (-f_s) = 230\text{ N} - f_s = 0, \qquad f_s = 230\text{ N},$$
$$f_{s,\,max} = \mu_s n \qquad \text{(motion about to start)},$$
$$\mu_s = \frac{f_{s,\,max}}{n} = \frac{230\text{ N}}{500\text{ N}} = 0.46.$$

After the crate starts to move, the friction force becomes kinetic friction (Figure 5.16c). The crate moves with constant velocity, so it is still in equilibrium. Thus, the vector sum of the forces is still zero, and we have

$$\sum F_y = n + (-w) = n - 500\text{ N} = 0, \qquad n = 500\text{ N},$$
$$\sum F_x = T + (-f_k) = 200\text{ N} - f_k = 0, \qquad f_k = 200\text{ N},$$
$$f_k = \mu_k n \qquad \text{(moving)}.$$

The coefficient of kinetic friction is

$$\mu_k = \frac{f_k}{n} = \frac{200\text{ N}}{500\text{ N}} = 0.40.$$

REFLECT There is no vertical motion, so $a_y = 0$ and $n = w$. It is easier to keep the crate moving than to start it moving from rest because $\mu_k < \mu_s$. Note, however, that just as the crate begins to move, the acceleration is nonzero while the velocity changes from zero to its final, constant value.

Practice Problem: Suppose you wet down your driveway prior to delivery, so that $\mu_s = 0.30$ and $\mu_k = 0.25$. What horizontal force is required to start the crate moving? What force is required to keep it sliding at constant velocity? *Answers:* 150 N, 125 N.

EXAMPLE 5.9 Not pulling hard enough

Video Tutor Solution

Now suppose that the horizontal force in Example 5.8 is not large enough to move the crate. What is the friction force if a horizontal force of 50 N is applied to it?

SOLUTION

SET UP AND SOLVE The setup is the same as for Example 5.8. From the equilibrium conditions, we have

$$\sum F_x = T + (-f_s) = 50\text{ N} - f_s = 0,$$
$$f_s = 50\text{ N}.$$

REFLECT In this case, $f_s < \mu_s n$. The friction force can prevent motion for any horizontal force less than 230 N.

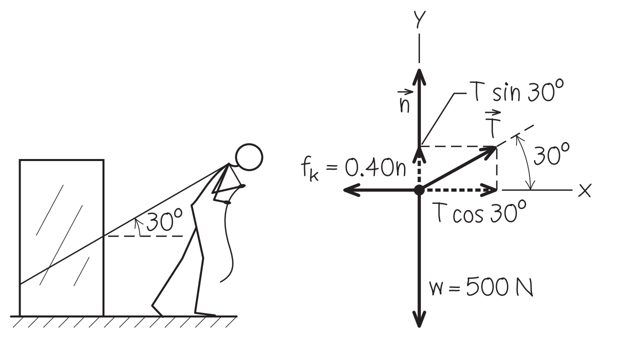
EXAMPLE 5.10 Moving the exercise equipment again

Video Tutor Solution

Now let's see what happens if you pull at an angle on the crate in Example 5.8. Suppose you try to move the crate by pulling upward on the rope at an angle of 30° above the horizontal. How hard do you have to pull to keep the crate moving with constant velocity? Is this easier or harder than pulling horizontally? Recall that $w = 500$ N and $\mu_k = 0.40$ for this crate.

SOLUTION

SET UP Figure 5.17 shows the situation and our free-body diagram. We use the same coordinate system as in Example 5.8. The magnitude f_k of the friction force is still equal to $\mu_k n$, but now the magnitude n of the normal force is *not* equal in magnitude to the weight of the crate. Instead, the force exerted by the rope has an additional vertical component that tends to lift the crate off the floor; this component makes n less than w.



(a) Pulling a crate at an angle **(b)** Free-body diagram for moving crate

▲ **FIGURE 5.17**

SOLVE The crate is moving with constant velocity, so it is still in equilibrium. Applying $\sum \vec{F} = 0$ in component form, we find

$$\sum F_x = T\cos 30° + (-f_k) = T\cos 30° - 0.40n = 0,$$
$$\sum F_y = T\sin 30° + n + (-500\text{ N}) = 0.$$

These are two simultaneous equations for the two unknown quantities T and n. To solve them, we can eliminate one unknown and solve for the other. There are many ways to do this. Here is one way: Rearrange the second equation to the form

$$n = 500\text{ N} - T\sin 30°.$$

Then substitute this expression for n back into the first equation:

$$T\cos 30° - 0.40(500\text{ N} - T\sin 30°) = 0.$$

Finally, solve this equation for T, and then substitute the result back into either of the original equations to obtain n. The results are

$$T = 188\text{ N}, \qquad n = 406\text{ N}.$$

REFLECT The normal-force magnitude n is *less* than the weight ($w = 500$ N) of the box, and the tension required is a little less than the force that was needed (200 N) when you pulled horizontally.

Practice Problem: How hard do you have to pull on the crate initially to get it moving if you pull upward on the rope at an angle of 30° above the horizontal? *Answers:* $T = 210$ N, $n = 430$ N.

EXAMPLE 5.11 Toboggan ride with friction

Video Tutor Solution

Let's go back to the toboggan we looked at in Example 5.6. The wax has worn off, and there is now a coefficient of kinetic friction μ_k. The slope has just the right angle to make the toboggan slide with constant velocity. Derive an expression for the slope angle in terms of w and μ_k.

CONTINUED

SOLUTION

SET UP Figure 5.18 shows our sketch and free-body diagram. The slope angle is α. The forces on the toboggan are identified by their magnitudes: its weight (w) and the normal (n) and friction (f_k) components of the contact force exerted on it by the sloping surface. We take axes perpendicular and parallel to the surface and represent the weight in terms of its components in these two directions as shown.

SOLVE The toboggan is moving with constant velocity and is therefore in equilibrium. The equilibrium conditions are

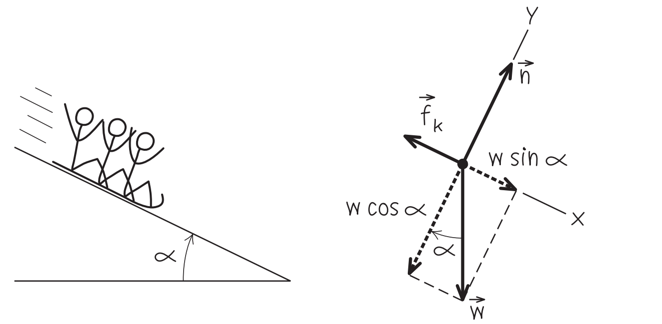
$$\sum F_x = w \sin\alpha + (-f_k) = w \sin\alpha - \mu_k n = 0,$$
$$\sum F_y = n + (-w \cos\alpha) = 0.$$

Rearranging terms, we get

$$\mu_k n = w \sin\alpha, \qquad n = w \cos\alpha.$$

Note that the normal force n is less than the weight w. When we divide the first of these equations by the second, we find

$$\mu_k = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha.$$



(a) The situation **(b)** Free-body diagram for toboggan

▲ **FIGURE 5.18** Our diagrams for this problem.

REFLECT The normal force is *not* equal in magnitude to the weight; it is always smaller. The weight w doesn't appear in the expression for μ_k; *any* toboggan, regardless of its weight, slides down an incline with constant speed if the tangent of the slope angle of the hill equals the coefficient of kinetic friction. The greater the coefficient of friction, the steeper the slope has to be for the toboggan to slide with constant velocity. This behavior is just what we should expect.

EXAMPLE 5.12 A steeper hill

Video Tutor Solution

Suppose the toboggan in Example 5.11 goes down a steeper hill. The coefficient of friction is the same, but this time the toboggan accelerates. Derive an expression for the acceleration in terms of g, α, μ_k, and w.

SOLUTION

SET UP Our free-body diagram (Figure 5.19) is almost the same as for Example 5.11, but the toboggan is no longer in equilibrium: a_y is still zero, but a_x is not.

SOLVE We apply Newton's second law in component form. From $\sum F_x = ma_x$ and $\sum F_y = ma_y$, we get the two equations

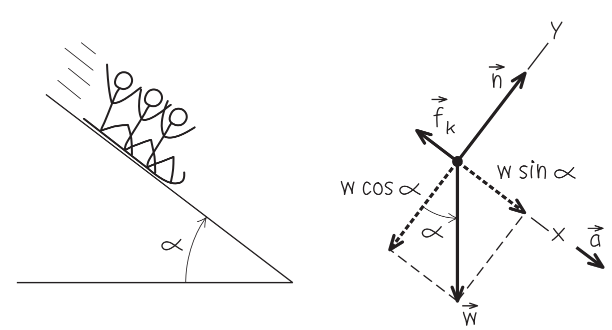
$$\sum F_x = mg \sin\alpha + (-f_k) = ma_x,$$
$$\sum F_y = n + (-mg \cos\alpha) = 0.$$

From the second equation, we find that $n = mg \cos\alpha$; then, from Equation 5.1 $(f_k = \mu_k n)$, we get $f_k = \mu_k n = \mu_k mg \cos\alpha$. That is, the magnitude of the friction force equals μ_k times the component of the weight normal to the surface. We substitute this expression for f_k back into the x component equation. The result is

$$mg \sin\alpha + (-\mu_k mg \cos\alpha) = ma_x,$$
$$a_x = g(\sin\alpha - \mu_k \cos\alpha).$$

REFLECT Does this result make sense? Let's check some special cases. First, if the hill is *vertical,* $\alpha = 90°$, then $\sin\alpha = 1$, $\cos\alpha = 0$, and $a_x = g$. This is free fall, just what we would expect. Second, on a hill with angle α with *no* friction, $\mu_k = 0$. Then $a_x = g \sin\alpha$. The situation is the same as in Example 5.6, and we get the same result; that's encouraging! Next, suppose that $a_x = 0$. Then there is just enough friction to make the toboggan move with constant velocity. In that case, $a_x = 0$, and it follows that

$$\sin\alpha = \mu_k \cos\alpha \qquad \text{and} \qquad \mu_k = \tan\alpha.$$



(a) The situation **(b)** Free-body diagram for toboggan

▲ **FIGURE 5.19** Our diagrams for the toboggan on a steeper hill.

This agrees with our result from Example 5.11; good! Finally, note that there may be *so much* friction that $\mu_k \cos\alpha$ is actually greater than $\sin\alpha$. In that case, a_x is negative. If we give the toboggan an initial downhill push, it will slow down and eventually stop.

What if we give the toboggan an initial push *up* the hill? The direction of the friction force is now reversed, so the acceleration is different from the downhill value. It turns out that the expression for a_x is the same as before, except that the *minus* sign in the expression becomes a *plus* sign.

In this chapter, we started with a simple toboggan problem and then extended it to more and more general situations. Note that our most general result includes all the earlier ones as special cases. We employ this strategy throughout the text—starting with a simple scenario and applying it to several situations.

EXAMPLE 5.13 Fluid friction

Video Tutor Solution

In this example we are going to take a look at a different type of friction, one that is not described by Equations 5.1 and 5.2. When an object moves through a fluid (such as water or air), it exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the object, in a direction opposite to the object's velocity relative to the fluid, always opposing the object's motion and usually increasing with speed. In high-speed motion through air, the resisting force is approximately proportional to the square of the object's speed v; it's called a *drag force,* or simply *drag*. We can represent its magnitude F_D by

$$F_D = Dv^2,$$

where D is a proportionality constant that depends on the shape and size of the object and the density of air.

When an object falls vertically through air, the drag force opposing the object's motion increases and the downward acceleration decreases. Eventually, the object reaches a *terminal velocity:* Its acceleration approaches zero and the velocity becomes nearly constant. Derive an expression for the terminal-velocity magnitude v_T in terms of D and the weight mg of the object.

SOLUTION

SET UP As shown in Figure 5.20, we take the positive y direction to be downward, and we ignore any force associated with buoyancy in the fluid. The net vertical component of force is $mg - Dv^2$.

SOLVE Newton's second law gives

$$mg - Dv_y^2 = ma_y.$$

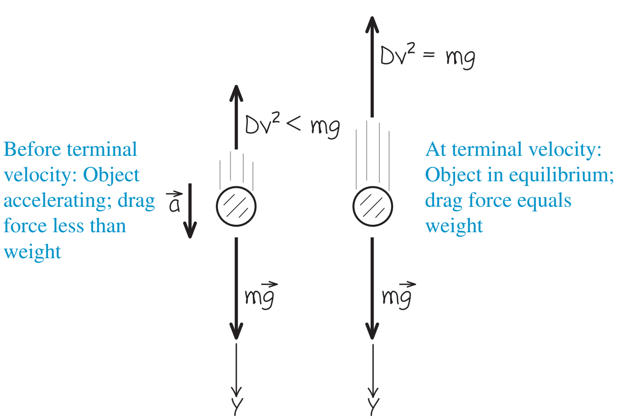
When the object has reached terminal velocity v_T, the acceleration a_y is zero and

$$mg - Dv_T^2 = 0 \quad \text{or} \quad v_T = \sqrt{\frac{mg}{D}}.$$

REFLECT For a skydiver in the spread-eagle position, the value of the constant D is found experimentally to be about 0.25 kg/m. (Does this number have the correct units?) If the skydiver's mass is 80 kg, the terminal-velocity magnitude is

$$v_T = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(80\ \text{kg})(9.8\ \text{m/s}^2)}{0.25\ \text{kg/m}}} = 56\ \text{m/s}.$$

This magnitude is about 125 mi/h. Does that seem reasonable?



▲ **FIGURE 5.20** An object falling through air, before and after it reaches its terminal velocity.

Practice Problem: An average-sized cat of 4.5 kg falls with a terminal velocity of approximately 27 m/s. What is the value of the proportionality constant D for the cat? *Answer:* 0.062 kg/m.

QUANTITATIVE ANALYSIS 5.5

Skydivers everywhere

As we just saw, objects falling through the air at high speeds are acted upon by a drag force proportional in magnitude to v^2. A falling object speeds up until the magnitude of the force from air drag equals the magnitude of the object's weight. If a 120 lb woman using a parachute falls with terminal velocity v, a 240 lb man using an identical chute will fall with terminal velocity

A. v. B. $\sqrt{2}v$. C. $2v$.

SOLUTION Since both people use the same chute, the constant D is the same for both. The mass of the man is twice the mass of the woman, so he requires twice the drag force to produce zero acceleration. The drag force is proportional to v^2; hence, it doubles if the velocity increases by a factor of $\sqrt{2}$. The correct answer is B.

Rolling vehicles also experience air drag, which usually increases proportionally to the square of the speed. Air drag is often negligible at low speeds, but at highway speeds it is comparable to or greater than rolling friction.

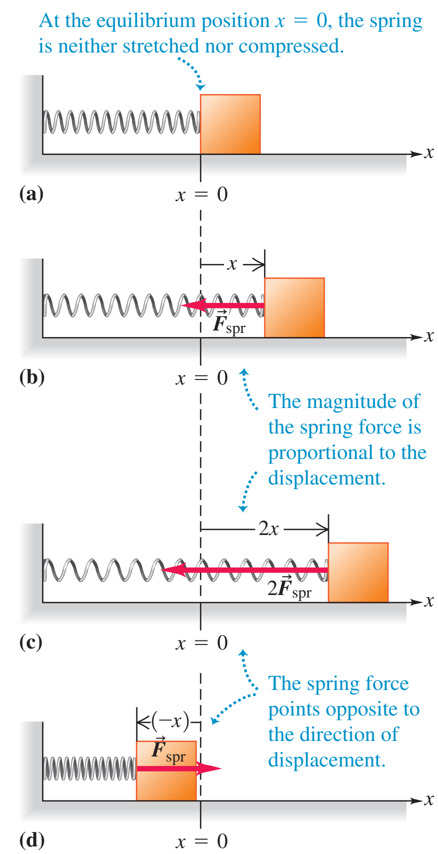
5.4 Elastic Forces

In Section 4.1, when we introduced the concept of *force,* we mentioned that when forces are applied to a solid object, they usually deform the object. In particular, we discussed the use of a coil spring in a device called a spring balance that can be used to measure the magnitudes of forces.

Let's look at the relationship of force to deformation in a little more detail. Figure 5.21 shows a coil spring with one end attached to an anchor point and the other end attached to an object that can slide without friction along the x axis. The x coordinate describes the position of the right end of the spring. When $x = 0$, the spring is neither stretched nor compressed (Figure 5.21a). When the object is moved a distance x to the right, the spring is stretched by an amount x (Figure 5.21b). It then exerts a force with magnitude F_{spr} on the object and an "equal and opposite" force on the anchor point. When the string is stretched farther, greater forces are required. For many common materials, such as steel coil springs, experiments show that stretching the spring by an amount $2x$ requires forces with magnitude $2F_{spr}$; that is, the magnitude of force required is *directly proportional* to the amount of stretch (Figure 5.21c).

If the spring has spaces between its coils, as in the figure, then it can be compressed as well as stretched (Figure 5.21d). Again, experiments show that the amount of compression is directly proportional to the magnitude of the force. In both stretching and compressing, the sign of the x coordinate of the right end of the spring is always opposite the sign of the x component of force the spring applies to the object at its right end. When x is positive, the spring is stretched and applies a force that pulls to the left (the negative x direction) on the object. When x is negative, the spring is compressed and pushes to the right (the positive x direction) on the object.

This proportionality of force to stretching and compressing can be summarized in a simple equation called *Hooke's law* (after Robert Hooke, a contemporary of Newton):



▲ **FIGURE 5.21** Some experiments exploring spring forces.

Elastic behavior of springs (Hooke's law)

The *magnitude* of the spring force F_{spr} is approximately proportional to the distance ΔL by which the spring is stretched or compressed:

$$F_{spr} = k\Delta L. \quad (5.3)$$

Units: k has units of N/m and ΔL has units of meters.
Notes:

- k is a positive constant called the *spring constant*. It is a measure of the "stiffness" of the spring.
- The spring always pushes back toward its equilibrium position when compressed, and pulls back toward its equilibrium position when stretched. We use these two rules to determine the direction of the spring force.

A very stiff spring requires a large force for a little deformation, corresponding to a large value of the constant k; a weak spring, made with thin wire, requires only a small force for the same deformation, corresponding to a smaller value of k.

Although this relationship is called Hooke's *law,* it is only an approximate relationship. Most metallic springs obey it quite well for deformations that are small compared with the overall length of the spring, but rubber bands don't obey it for even small deformations. So it is safer to call it "Hooke's rule of thumb" (without intending any disparagement of Hooke, one of the great pioneers of late-17th-century science).

EXAMPLE 5.14 Fishy business

Video Tutor Solution

In this example we will consider the forces that come into play in a spring balance. Suppose that a spring balance used to weigh fish is built with a spring that stretches 1.00 cm when a 12.0 N weight is placed in the pan. When the 12.0 N weight is replaced with a 1.50 kg fish, what distance does the spring stretch?

SOLUTION

SET UP Figure 5.22a and b shows how the spring responds to the 12.0 N weight and to the 1.50 kg fish. The spring and fish are in equilibrium. We draw a free-body diagram for the fish (Figure 5.22c), showing the downward force of gravity and the upward force exerted by the stretched spring. We point the y axis vertically upward, as usual.

SOLVE We want to use Hooke's law ($F_{spr} = k\Delta L$) to relate the stretch of the spring to the force, but first we need to find the force constant k. The problem states that forces of magnitude 12.0 N are required to stretch the spring $\Delta L = 1.00$ cm $(= 1.00 \times 10^{-2}$ m$)$, so

$$k = \frac{12.0\ \text{N}}{1.00 \times 10^{-2}\ \text{m}} = 1200\ \text{N/m}.$$

The weight of the 1.50 kg fish is

$$w = mg = (1.50\ \text{kg})(9.80\ \text{m/s}^2) = 14.7\ \text{N}.$$

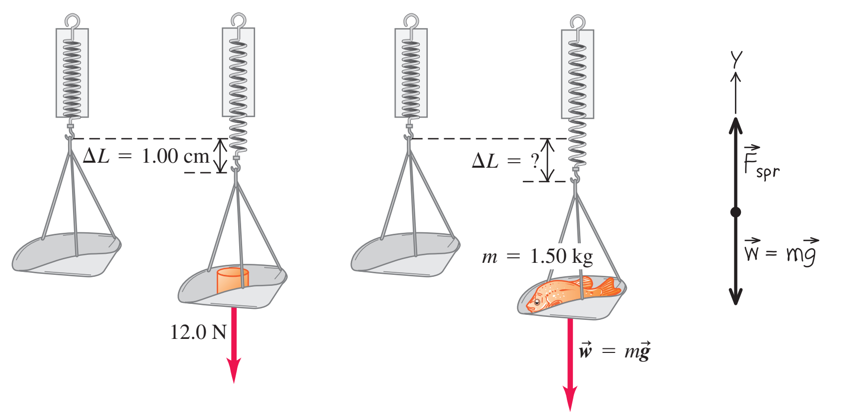
The equilibrium condition for the fish is

$$\Sigma F_y = 0, \qquad -mg + F_{spr} = 0,$$
$$-14.7\ \text{N} + (1200\ \text{N/m})\Delta L = 0,$$

and finally, $\Delta L = 0.0123\ \text{m} = 1.23\ \text{cm}$.

REFLECT The weight of the fish is greater than the amount of force needed to stretch the spring 1.00 cm, so we expect the fish to stretch the spring more than 1.00 cm.

Practice Problem: If, instead of the fish, we place a 3.00 kg rock in the pan, how much does the spring stretch? *Answer:* twice as much, 2.45 cm.



▶ **FIGURE 5.22** Weighing a fish.

(a) The scale stretched by a known weight

(b) The scale stretched by a known mass

(c) Free-body diagram for the fish

EXAMPLE 5.15 An innerspring mattress

Video Tutor Solution

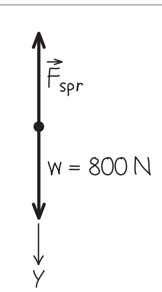
You have just bought an innerspring mattress that contains coil springs in a rectangular array 20 coils wide and 40 coils long. You estimate that when you lie on the mattress, your weight is supported by about 200 springs (about one-fourth of the total number of springs in the mattress). You observe that the springs compress about 2.6 cm when you lie on the mattress. Assuming that your weight of 800 N is supported equally by 200 springs, find the force constant of each spring.

SOLUTION

SET UP You are the object in equilibrium, so start with a free-body diagram of yourself (Figure 5.23). Observe that the spring forces are *additive,* in the sense that the total upward force they exert on your body is 200 times the force of each individual spring. Your body is in equilibrium under the action of the total spring force and your weight.

SOLVE For equilibrium, the total upward force of all the supporting springs must be 800 N, so $(200)k(2.6 \times 10^{-2}\ \text{m}) = 800$ N, and

$$k = 154\ \text{N/m} = 1.54\ \text{N/cm}.$$



▲ **FIGURE 5.23** Free-body diagram for you on the mattress.

CONTINUED

REFLECT To borrow a term from electric circuit analysis, the springs are *in parallel*: The deformation is the same for each, and the total force is the sum of the individual spring forces. Of course, the assumption that all the deformations are the same is probably not realistic.

Practice Problem: Suppose you lay a thin, lightweight sheet of plywood on top of the mattress, so that your weight is spread over all 800 springs. How much does each spring compress? *Answer:* 0.65 cm.

5.5 Forces in Nature

The historical development of our understanding of the forces (or interactions) found in nature has traditionally placed them into four distinct classes (Figure 5.24). Two are familiar from our everyday experience. The other two involve fundamental particle interactions that we cannot observe with our unaided senses.

Of the two familiar classes, **gravitational interactions** were the first to be studied in detail. The *weight* of an object results from the earth's gravitational pull acting on it. The sun's gravitational attraction for the earth keeps the earth in its nearly circular orbit around the sun. Newton recognized that the motions of the planets around the sun and the free fall of objects on earth are both the result of gravitational forces. We'll study gravitational interactions in greater detail in Chapter 6 and analyze their vital role in the motions of planets and satellites.

The second familiar class of forces, **electromagnetic interactions,** includes electric and magnetic forces. When you run a comb through your hair, you can then use the comb to pick up bits of paper or fluff; this interaction is the result of an *electric* charge on the comb, which also causes the infamous "static cling." *Magnetic* forces occur in interactions between magnets or between a magnet and a piece of iron. These forces may seem to form a different category, separate from electric forces, but magnetic interactions are actually the result of electric charges in motion. In an electromagnet, an electric current in a coil of wire causes magnetic interactions. One of the great achievements of 19th-century physics was the *unification* of theories of electric and magnetic interactions into a single theoretical framework.

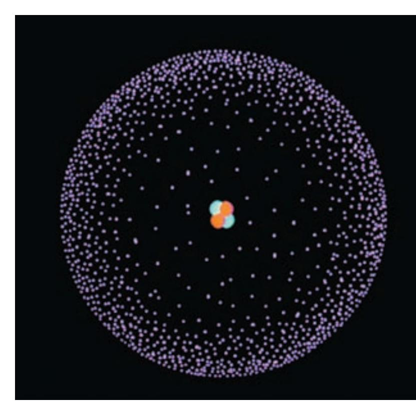
Gravitational and electromagnetic interactions differ enormously in their strength. The electrical repulsion between two protons at a given distance is stronger than their gravitational attraction by a factor on the order of 10^{35}. Gravitational forces play no significant role in atomic or molecular structure. But in bodies of astronomical size, positive and negative charges are usually present in nearly equal amounts, and the resulting electrical interactions nearly cancel out. Gravitational interactions are the dominant influence in the motions of planets and also in the internal structures of stars.

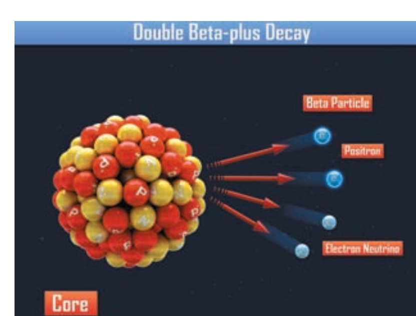
The other two classes of interactions are less familiar. One, the **strong interaction,** is responsible for holding the nucleus of an atom together. Nuclei contain electrically neutral and positively charged particles. The charged particles repel each other, and a nucleus could not be stable if it were not for the presence of an *attractive* force of a different kind that counteracts the repulsive electrical interactions. In this context, the strong interaction is also called the *nuclear force*. It has a much shorter range than electrical interactions, but within its range it is much stronger. The strong interaction is also responsible for the creation of unstable particles in high-energy particle collisions.

Finally, there is the **weak interaction.** This force plays no direct role in the behavior of ordinary matter, but it is of vital importance in the behavior of fundamental particles. The weak interaction is responsible for beta decay (one kind of nuclear radiation) as well as the decay of many unstable particles produced in high-energy collisions of fundamental particles. It also plays a significant role in the evolution of stars.


(a)


(b)


(c)



(d)

▶ **FIGURE 5.24** Examples of the four fundamental forces. (a) Gravitational interactions hold the earth and the moon in orbit around each other. (b) Electromagnetic interactions hold together the atoms of a DNA molecule. (c) The strong nuclear interaction holds the nucleus together, even though there is a strong electrical repulsion between the protons in a nucleus. (d) The weak nuclear interaction plays an important role in radioactive decay. In particular, it can cause a neutron to break up into a proton plus an electron.

One of the most ambitious goals of physics, which is still being pursued today by physicists all over the world, is to unite all four of the fundamental forces of nature in a single all-encompassing theory, sometimes half-jokingly referred to as a *theory of everything* (TOE). Theories of this type are intimately related to the very early history of the universe, following the Big Bang. Such theories are still highly speculative and raise many unanswered questions.

FREQUENTLY ASKED QUESTIONS

Q: *How is working an equilibrium problem different from working a dynamic problem?*

A: Both types of problems are, in fact, very similar. In each you have to choose a coordinate system, put all the forces in component form, and then put these components into Newton's second law equation. The difference is that for an equilibrium problem you know that each component of the acceleration must be zero. In a dynamic problem, one or both components of the acceleration are not zero.

Q: *Since $F = ma$, can't I just replace some of the forces in the problem with ma?*

A: No! Actually $F = ma$ is not an accurate statement of Newton's second law because it assumes that only one force is acting on the object. Futhermore, it wrongly suggests that force is not a vector. You must use the relationships in Equations 4.8, with the force components on the left-hand side of the equation and the components of the acceleration on the right-hand side.

Q: *In some of the example dynamic problems, one component of the acceleration is set to zero at the beginning. Why?*

A: In dynamic problems, the coordinate system is usually chosen so that the object moves along one of the axes. This means that there can be no motion along the other axis. Therefore, the acceleration along this other axis must be set to zero in Equations 4.8.

Q: *Can I assume that the normal force n that appears in the friction Equations 5.1 and 5.2 is simply equal to the weight of the object, mg?*

A: No! Often you need to calculate the normal force using Equations 4.8. If the object is on a horizontal surface with only horizontal forces acting on it, then $n = mg$. But if the surface is inclined, or if there are forces with vertical components, then generally $n \neq mg$.

Q: *The inequality in the equation for static friction (Equation 5.2) is confusing. What should I do with it?*

A: Remember that static friction is an adjustable force. If it can, it will always adjust itself so as to keep the object from sliding. However, it has a limit, and if the other forces are too great, then static friction will not be able to prevent the object from sliding. This limit, or maximum value, is obtained by using the equal sign in Equation 5.2. The case in which the static-friction force is less than its maximum value corresponds to the inequality sign in Equation 5.2. You do not use this inequality sign directly in working problems. If the static-friction force is less than its maximum value, then you have to calculate the static-friction force using Newton's second law.

Q: *Is the kinetic-friction force also an "adjustable" force?*

A: No. Its magnitude is simply given by Equation 5.1.

Q: *So, if a friction problem states that an object is just on the verge of breaking free and sliding, can I assume that the magnitude of the static-friction force is $f_s = \mu_s n$ and both components of the acceleration are zero?*

A: Yes. You should work a problem with static friction that asks about when an object "just begins to move" exactly the same way you work a problem asking about the last point at which an object remains stationary. In both cases, we need the maximum value of static friction and no acceleration.

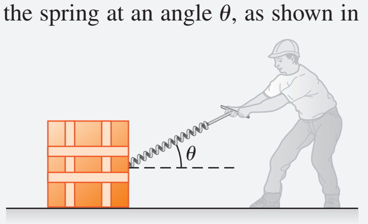
Q: *Do static- and kinetic-friction forces always oppose the motion of an object?*

A: No, not in all cases. For instance, if you place a box on a moving conveyor belt, it is friction that causes the box to begin moving. Similarly, when a red light turns green, it's the friction force between the tires and the road that causes a car to accelerate from rest.

Bridging Problem

Video Tutor Solution

A dockworker attaches a spring of unstretched length L and spring constant k to a crate of weight W. He attempts to drag the crate along the dock platform by pulling on the spring at an angle θ, as shown in Figure 5.25. The coefficient of static friction between the crate and the dock platform is μ_s and the coefficient of kinetic friction is μ_k. Assume that $\mu_s > \mu_k$. (a) Derive an expression for the distance the spring will stretch ΔL before the crate begins to slide. Your expression should be in terms of μ_s, θ, k, and W. (b) Derive an expression for the acceleration of the crate the moment after it breaks loose. In this case your expression should be in terms of μ_k, θ, k, and W.



▲ **FIGURE 5.25** Bridging Problem

Set Up

- Choose an appropriate coordinate system for this problem.
- Draw a free-body diagram for the crate.
- Calculate the x and y components of the forces acting on the crate.
- Write down Newton's second law for each direction.

CONTINUED

Solve

- Write down the magnitude of the spring force in terms of ΔL.
- Derive an expression for the normal force using Newton's second law.
- Solve for ΔL.
- Repeat your analysis for the case in which the crate is moving by replacing the static-friction force with the kinetic-friction force.

Reflect

- Check that your expressions make sense by considering the simplest case where $\theta = 0$.
- Suppose the dockworker pulled the crate straight up at a constant speed using the spring. What would ΔL be in that case? Does that ΔL agree with your expression for ΔL when you plug in $\theta = 90°$?
- If the dockworker pulled the crate along the platform at a constant speed, would ΔL be greater or smaller than it was when the crate broke free?
- Suppose the dock platform were frictionless. How much would the spring stretch if the dockworker pulled the crate at a constant speed?

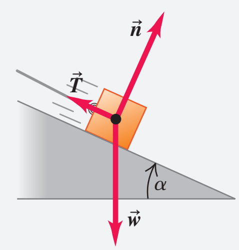
CHAPTER 5 SUMMARY

Equilibrium of a Particle

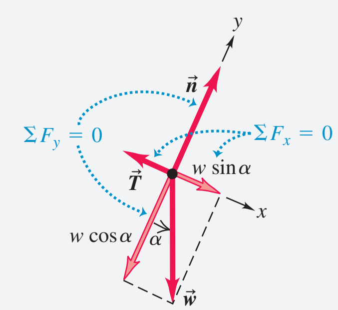
(Section 5.1) When an object is in **equilibrium,** the vector sum of the forces acting on it must be zero: $\sum\vec{F} = 0$ (Equation 4.3). In component form,

$$\sum F_x = 0, \qquad \sum F_y = 0. \tag{4.4}$$

Free-body diagrams are useful in identifying the forces that act on the object being considered. Newton's third law is also frequently needed in equilibrium problems. The two forces in an action–reaction pair never act on the same object.



An object moving at constant velocity down a frictionless ramp is in equilibrium: $\sum\vec{F} = 0$.

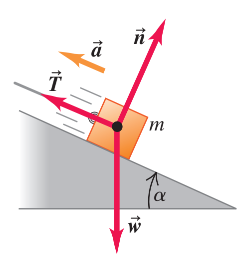


A free-body diagram and coordinate system for the object. The weight vector is replaced by its components.

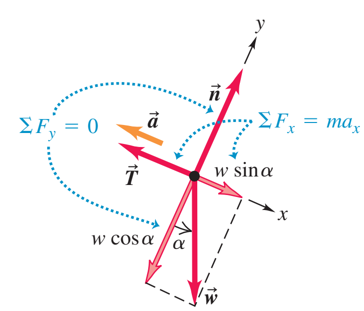
Applications of Newton's Second Law

(Section 5.2) When the vector sum of the forces on an object is not zero, the object has an acceleration determined by Newton's second law, $\sum\vec{F} = m\vec{a}$ (Equation 4.7). In component form,

$$\sum F_x = ma_x, \qquad \sum F_y = ma_y. \tag{4.8}$$



$\sum\vec{F} = m\vec{a}$



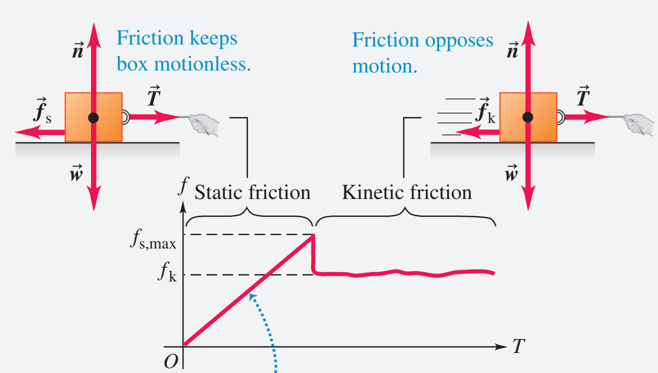
A free-body diagram for the object

Contact Forces and Friction

(Section 5.3) The **contact force** between two objects can always be represented in terms of a normal component n perpendicular to the surface of interaction and a friction component f parallel to the surface. When sliding occurs, the kinetic-friction force f_k is often approximately proportional to n. Then the proportionality constant is μ_k, the **coefficient of kinetic friction:** $f_k = \mu_k n$ (Equation 5.1).

When there is no relative motion, the maximum possible friction force is approximately proportional to the normal force, and the proportionality constant is μ_s, the **coefficient of static friction.** The governing equation is $f_s \le \mu_s n$ (Equation 5.2).

The actual static-friction force may be anything from zero to the maximum value given by the equality in Equation 5.2, depending on the situation. Usually, μ_k is less than μ_s for a given pair of surfaces.

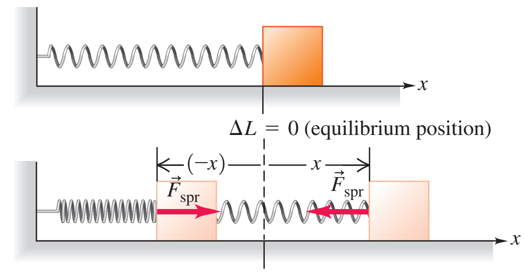


The static-friction force remains equal in magnitude to the tension force until its maximum value $f_{s,max}$ is exceeded.

Elastic Forces

(Section 5.4) When forces act on a solid object, the object usually deforms. In some cases, such as a stretched or compressed spring, the amount that the object is deformed ΔL is approximately proportional to the magnitude of the applied force. A stretched or compressed spring produces a force whose magnitude is given by **Hooke's law:**

$$F_{spr} = k\Delta L. \qquad (5.3)$$



When stretched or compressed from equilibrium, the spring exerts a force $\vec{F}_{spr}$ whose magnitude is approximately proportional to the displacement.

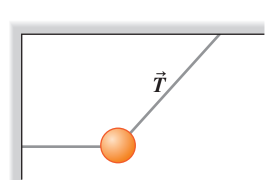
Forces in Nature

(Section 5.5) Historically, forces have been classified as **strong, electromagnetic, weak,** and **gravitational.** Present-day research includes intensive efforts to create a unified description of the forces in all these categories.

MP For assigned homework and other learning materials, go to MasteringPhysics®.

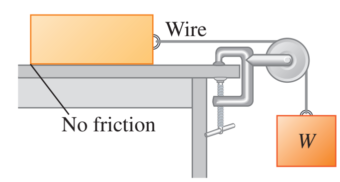
Conceptual Questions

1. Can a body be in equilibrium when only one force acts on it?
2. A clothesline is hung between two poles, and then a shirt is hung near the center of the line. No matter how tightly the line is stretched, it always sags a little at the center. Show why.
3. A man sits in a seat that is suspended from a rope. The rope passes over a pulley suspended from the ceiling, and the man holds the other end of the rope in his hands. What is the tension in the rope, and what force does the seat exert on the man? Draw a free-body force diagram for the man.
4. You push a box up a frictionless incline at a constant speed with a horizontal force $\vec{F}_a$. Which force acting on the box has the greatest magnitude: (a) the normal force from the incline, (b) the weight of the box, or (c) the horizontal force $\vec{F}_a$?
5. Why is it so much more difficult to walk on icy pavement than dry pavement?
6. A car accelerates gradually to the right with power on the two rear wheels. What is the direction of the friction force on these wheels? Show why. Is it static or kinetic friction?
7. Without doing any calculations, decide whether the tension T in Figure 5.26 will be equal to, greater than, or less than the weight of the suspended object.



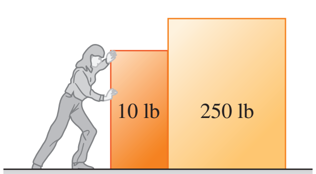
▲ **FIGURE 5.26** Question 7.

8. A box slides up an incline, comes to rest, and then slides down again, accelerating in both directions. Answer the following questions, using only a free-body diagram and *without* doing any calculations: (a) If there is no friction on the incline, how will the box's acceleration going up compare in magnitude *and* direction with its acceleration going down? (b) If there is kinetic friction on the incline, how will the box's acceleration going up compare in magnitude *and* direction with its acceleration going down? (c) What will be the box's acceleration at its highest point?
9. For the objects shown in Figure 5.27, will the tension in the wire be greater than, equal to, or less than the weight W? Decide *without* doing any calculations.



▲ **FIGURE 5.27** Question 9.

10. A woman is pushing horizontally on two boxes on a factory floor, as shown in Figure 5.28. Which is greater, the force the woman exerts on the 10 lb box or the force the 10 lb box exerts on the 250 lb box? Decide without doing any calculations.



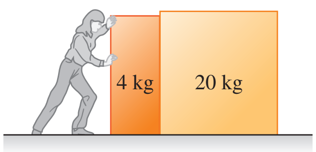
▲ **FIGURE 5.28** Question 10.

11. In a world *without* friction, could you (a) walk on a horizontal sidewalk, (b) climb a ladder, (c) climb a vertical pole, (d) jump into the air, (e) ride your bike, and (f) drive around a curve on a flat roadway? Explain your reasoning.
12. You can classify scales for weighing objects as those that use springs and those that use standard masses to balance unknown masses. Which group would be more accurate when you use it in an accelerating spaceship? When you use it on the moon?
13. When you stand with bare feet in a wet bathtub, the grip feels fairly secure, and yet a catastrophic slip is quite possible. Explain this in terms of the two coefficients of friction.

Multiple-Choice Problems

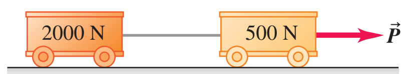
1. A horizontal force accelerates a box across a rough horizontal floor with friction present, producing an acceleration a. If the force is now tripled, but all other conditions remain the same, the acceleration will become
   A. greater than $3a$.
   B. equal to $3a$.
   C. less than $3a$.

2. You slide an 800 N table across the kitchen floor by pushing with a force of 100 N. If the table moves at a constant speed, the friction force with the floor must be
   A. 100 N.
   B. greater than 100 N but less than 800 N.
   C. 800 N.
   D. greater than 800 N.
3. A woman wearing spiked shoes pushes two crates across her frictionless, horizontal studio floor. (See Figure 5.29.) If she exerts a horizontal force of 36 N on the smaller crate, then the smaller crate exerts a force on the larger crate that is closest to
   A. 36 N. B. 30 N.
   C. 200 N. D. 240 N.



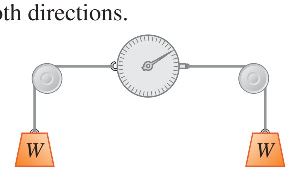
▲ **FIGURE 5.29** Multiple-Choice Problem 3.

4. A horizontal force with a magnitude P pulls two wagons over a horizontal frictionless floor, as shown in Figure 5.30. The tension in the light horizontal rope connecting the wagons is
   A. equal to P, by Newton's third law. B. equal to 2000 N.
   C. greater than P. D. less than P.



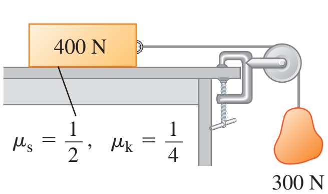
▲ **FIGURE 5.30** Multiple-Choice Problem 4.

5. A crate slides up an inclined ramp and then slides down the ramp after momentarily stopping near the top. This crate is acted upon by friction on the ramp and accelerates both ways. Which statement about this crate's acceleration is correct?
   A. The acceleration going up the ramp is greater than the acceleration going down.
   B. The acceleration going down the ramp is greater than the acceleration going up.
   C. The acceleration is the same in both directions.
6. A weightless spring scale is attached to two equal weights as shown in Figure 5.31. The reading in the scale will be
   A. 0. B. W. C. $2W$.



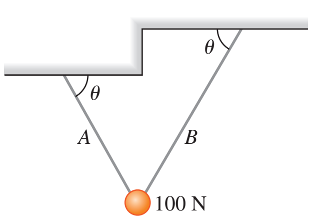
▲ **FIGURE 5.31** Multiple-Choice Problem 6.

7. Two objects are connected by a light wire as shown in Figure 5.32, with the wire pulling horizontally on the 400 N object. After this system is released from rest, the tension in the wire will be
   A. less than 300 N.
   B. 300 N.
   C. 200 N.
   D. 100 N.



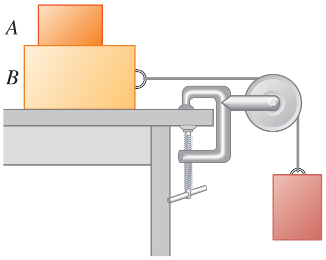
▲ **FIGURE 5.32** Multiple-Choice Problem 7.

8. A 100 N weight is supported by two weightless wires A and B as shown in Figure 5.33. What can you conclude about the tensions in these wires if the value of θ is unknown?
   A. The tensions are equal to 50 N each.
   B. The tensions are equal, but less than 50 N each.
   C. The tensions are equal, but greater than 50 N each.
   D. The tensions are equal to 100 N each.



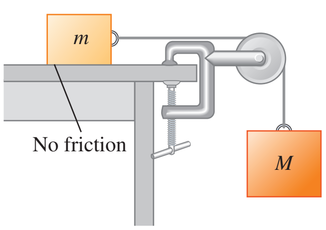
▲ **FIGURE 5.33** Multiple-Choice Problem 8.

9. The system shown in Figure 5.34 is released from rest, there is no friction between B and the tabletop, and all of the objects move together. What must be true about the friction force on A?
   A. It is zero.
   B. It acts to the right on A.
   C. It acts to the left on A.
   D. We cannot tell whether there is any friction force on A because we do not know the coefficients of friction between A and B.



▲ **FIGURE 5.34** Multiple-Choice Problem 9.

10. In the system shown in Figure 5.35, $M > m$, the surface of the bench is horizontal and frictionless, and the connecting string pulls horizontally on m. As more and more weight is gradually added to m, which of the following statements best describes the behavior of the system after it is released?
   A. The acceleration remains the same in all cases, since there is no friction and the pull of gravity on M is the same.
   B. The acceleration becomes zero when enough weight is added so that $m = M$.
   C. The velocity becomes zero when $m = M$.
   D. None of the preceding statements is correct.

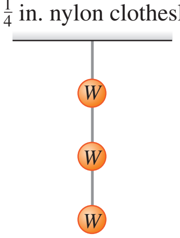


▲ **FIGURE 5.35** Multiple-Choice Problem 10.

Problems

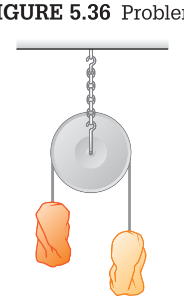
5.1 Equilibrium of a Particle

1. | A 15.0 N bucket is to be raised at a constant speed of 50.0 cm/s by a rope. According to the information in Table 5.1, how many kilograms of cement can be put into this bucket without breaking the rope if it is made of (a) thin white string, (b) $\frac{1}{4}$ in. nylon clothesline, (c) $1\frac{1}{4}$ in. manila climbing rope?
2. | In a museum exhibit, three equal weights are hung with identical wires, as shown in Figure 5.36. Each wire can support a tension of no more than 75.0 N without breaking. Start each of the following parts with an appropriate free-body diagram. (a) What is the maximum value that W can be without breaking any wires? (b) Under these conditions, what is the tension in each wire?



▲ **FIGURE 5.36** Problem 2.

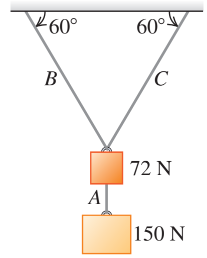
3. | Two 25.0 N weights are suspended at opposite ends of a rope that passes over a light, frictionless pulley. The pulley is attached to a chain that is fastened to the ceiling. (See Figure 5.37.) Start solving this problem by making a free-body



▲ **FIGURE 5.37** Problem 3.

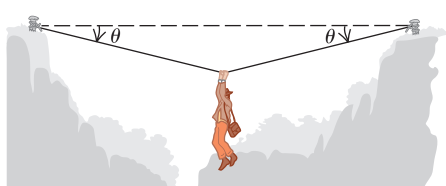
diagram of each weight. (a) What is the tension in the rope? (b) What is the tension in the chain?

4. | Two weights are hanging as shown in Figure 5.38. (a) Draw a free-body diagram of each weight. (b) Find the tension in cable *A*. (c) Find the tension in cables *B* and *C*.



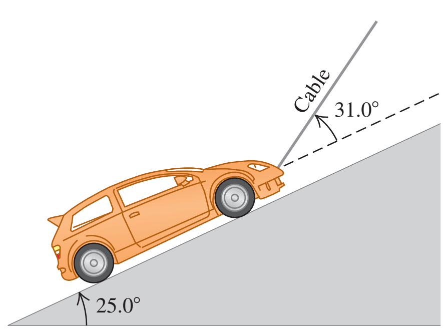
▲ **FIGURE 5.38** Problem 4.

5. | An adventurous archaeologist crosses between two rock cliffs by slowly going hand over hand along a rope stretched between the cliffs. He stops to rest at the middle of the rope (Figure 5.39). The rope will break if the tension in it exceeds 2.50×10^4 N. Our hero's mass is 90.0 kg. (a) If the angle θ is 10.0°, find the tension in the rope. Start with a free-body diagram of the archaeologist. (b) What is the smallest value the angle θ can have if the rope is not to break?



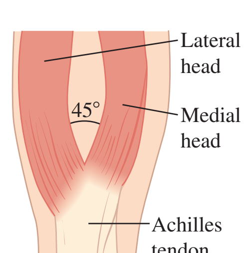
▲ **FIGURE 5.39** Problem 5.

6. || A 1130 kg car is being pulled up a frictionless ramp at a constant speed, as shown in Figure 5.40. The cable makes an angle of 31.0° above the surface of the ramp, and the ramp itself rises at 25.0° above the horizontal. (a) Draw a free-body diagram for the car. (b) Find the tension in the cable. (c) How hard does the surface of the ramp push on the car?



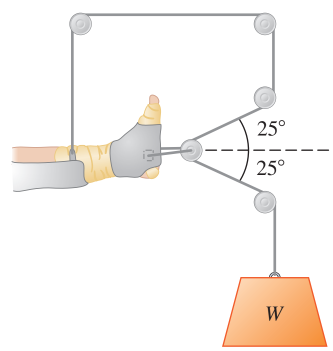
▲ **FIGURE 5.40** Problem 6.

7. || **Muscles and tendons.** Muscles are attached to bones by means **BIO** of tendons. The maximum force that a muscle can exert is directly proportional to its cross-sectional area *A* at the widest point. We can express this relationship mathematically as $F_{max} = \sigma A$, where σ (sigma) is a proportionality constant. Surprisingly, σ is about the same for the muscles of all animals and has the numerical value of 3.0×10^5 in SI units. The gastrocnemius muscle, in the back of the leg, has two portions, known as the medial and lateral heads. Assume that they attach to the Achilles tendon as shown in Figure 5.41. The cross-sectional area of each of these two muscles is typically 30 cm^2 for many adults. What is the maximum tension they can produce in the Achilles tendon?



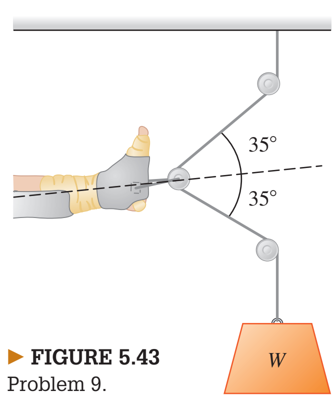
▲ **FIGURE 5.41** Problem 7.

8. | **Traction apparatus.** In order to prevent muscle contraction from **BIO** misaligning bones during healing (which can cause a permanent limp), injured or broken legs must be supported horizontally and at the same time kept under tension (traction) directed along the leg. One version of a device to accomplish this aim, the Russell traction apparatus, is shown in Figure 5.42. This system allows the apparatus to support the full weight of the injured leg and at the same time provide the traction along the leg. If the leg to be supported weighs 47.0 N, (a) what must be the weight of *W* and (b) what traction force does this system produce along the leg?



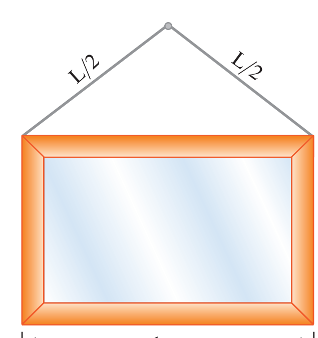
▲ **FIGURE 5.42** Problem 8.

9. | **A broken thigh bone.** When the thigh is fractured, the patient's **BIO** leg must be kept under traction. One method of doing so is a variation on the Russell traction apparatus. (See Figure 5.43.) If the physical therapist specifies that the traction force directed along the leg must be 25 N, what must *W* be?



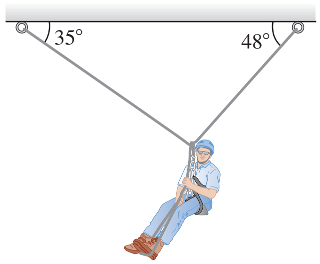
▶ **FIGURE 5.43** Problem 9.

10. || A heavy mirror that has a width of 1 m is to be hung on a wall as shown in Figure 5.44. The mirror weighs 500 N, and the wire used to hang it will break if the tension exceeds 500 N (breaking strength). What is the shortest length of wire *L* that can be used to hang the mirror without the wire breaking? (Ignore any friction forces between the mirror and the wall.)



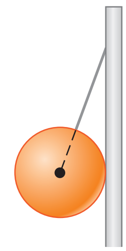
▲ **FIGURE 5.44** Problem 10.

11. ‖ In a rescue, the 73 kg police officer is suspended by two cables, as shown in Figure 5.45. (a) Sketch a free-body diagram of him. (b) Find the tension in each cable.



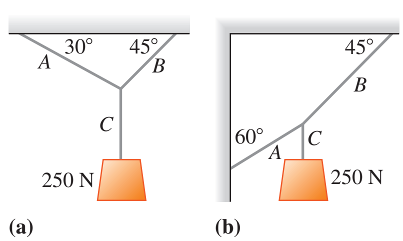
▲ **FIGURE 5.45** Problem 11.

12. ‖ A tetherball leans against the smooth, frictionless post to which it is attached. (See Figure 5.46.) The string is attached to the surface of the ball such that a line along the string passes through the center of the ball. The string is 1.40 m long and the ball has a radius of 0.110 m with mass 0.270 kg. (a) Make a free-body diagram of the ball. (b) What is the tension in the rope? (c) What is the force the pole exerts on the ball?



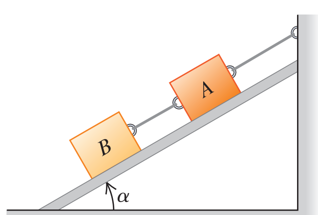
▲ **FIGURE 5.46** Problem 12.

13. ‖ Find the tension in each cord in Figure 5.47 if the weight of the suspended object is 250 N.



▲ **FIGURE 5.47** Problem 13.

14. ‖ Two blocks, each with weight w, are held in place on a frictionless incline as shown in Figure 5.48. In terms of w and the angle α of the incline, calculate the tension in (a) the rope connecting the blocks and (b) the rope that connects block A to the wall. (c) Calculate the magnitude of the force that the incline exerts on each block. (d) Interpret your answers for the cases $\alpha = 0$ and $\alpha = 90°$.



▲ **FIGURE 5.48** Problem 14.

15. ‖ A man pushes on a piano of mass 180 kg so that it slides at a constant velocity of 12.0 cm/s *down* a ramp that is inclined at 11.0° above the horizontal. No appreciable friction is acting on the piano. Calculate the magnitude and direction of this push (a) if the man pushes parallel to the incline, (b) if the man pushes the piano *up* the plane instead, also at 12.0 cm/s parallel to the incline, and (c) if the man pushes horizontally, but still with a speed of 12.0 cm/s.

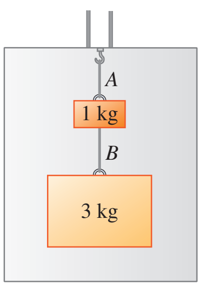
5.2 Applications of Newton's Second Law

16. ‖ **Forces during chin-ups.** People who do chin-ups raise their chin just over a bar (the chinning bar), supporting themselves only by their arms. Typically, the body below the arms is raised by about 30 cm in a time of 1.0 s, starting from rest. Assume that the entire body of a 680 N person who is chinning is raised this distance and that half the 1.0 s is spent accelerating upward and the other half accelerating downward, uniformly in both cases. Make a free-body diagram of the person's body, and then apply it to find the force his arms must exert on him during the accelerating part of the chin-up. **BIO**

17. ‖ **Force on a tennis ball.** The record speed for a tennis ball that is served is 73.14 m/s. During a serve, the ball typically starts from rest and is in contact with the tennis racquet for 30.00 ms. Assuming constant acceleration, what was the average force exerted on the tennis ball during this record serve, expressed in terms of the ball's weight W?

18. ‖ **Force during a jump.** An average person can reach a maximum height of about 60 cm when jumping straight up from a crouched position. During the jump itself, the person's body from the knees up typically rises a distance of around 50 cm. To keep the calculations simple and yet get a reasonable result, assume that the *entire body* rises this much during the jump. (a) With what initial speed does the person leave the ground to reach a height of 60 cm? (b) Make a free-body diagram of the person during the jump. (c) In terms of this jumper's weight W, what force does the ground exert on him or her during the jump? **BIO**

19. ‖ Two weights are hanging from the ceiling of an elevator as shown in Figure 5.49. (a) Draw a free-body diagram for each weight. (b) Find the tension in cables A and B when the elevator is not moving. (c) Find the tension in cables A and B when the elevator is accelerating downward at 1.8 m/s^2.

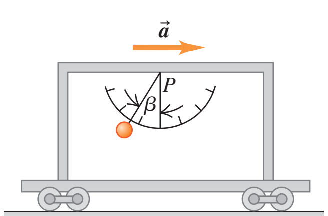


▲ **FIGURE 5.49** Problem 19.

20. ‖ A large fish hangs from a spring balance supported from the roof of an elevator. (a) If the elevator has an upward acceleration of 2.45 m/s^2 and the balance reads 60.0 N, what is the true weight of the fish? (b) Under what circumstances will the balance read 35.0 N? (c) What will the balance read if the elevator cable breaks?

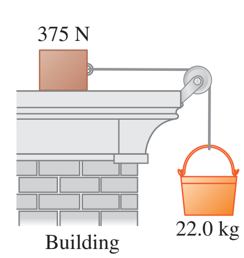
21. ‖ A 750.0 kg boulder is raised from a quarry 125 m deep by a long uniform chain having a mass of 575 kg. This chain is of uniform strength, but at any point it can support a maximum tension no greater than 2.50 times its weight without breaking. (a) What is the maximum acceleration the boulder can have and still get out of the quarry, and (b) how long does it take for the boulder to be lifted out at maximum acceleration if it started from rest?

22. ‖ Which way and by what angle does the accelerometer in Figure 5.50 deflect under the following conditions? (a) The cart is moving toward the right with speed increasing at 3.0 m/s^2. (b) The cart is moving toward the left with speed decreasing at 4.5 m/s^2. (c) The cart is moving toward the left with a constant speed of 4.0 m/s.



▲ **FIGURE 5.50** Problem 22.

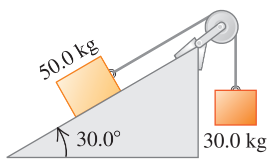
23. | At a construction site, a 22.0 kg bucket of concrete is connected over a very light frictionless pulley to a 375 N box on the roof of a building. (See Figure 5.51.) There is no appreciable friction on the box, since it is on roller bearings. The box starts from rest. (a) Make free-body diagrams of the bucket and the box. (b) Find the acceleration of the bucket. (c) How fast is



▲ **FIGURE 5.51** Problem 23.

the bucket moving after it has fallen 1.50 m (assuming that the box has not yet reached the edge of the roof)?

24. ‖ Two boxes are connected by a light string that passes over a light, frictionless pulley. One box rests on a frictionless ramp that rises at 30.0° above the horizontal (see Figure 5.52), and the system is released from rest. (a) Make free-body diagrams of each box. (b) Which way will the 50.0 kg box move, up the plane or down the plane? Or will it even move at all? Show why or why not. (c) Find the acceleration of each box.



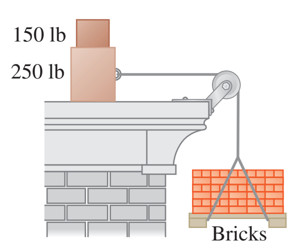
▲ **FIGURE 5.52** Problem 24.

5.3 Frictional Forces

25. | An 80 N box initially at rest is pulled by a horizontal rope on a horizontal table. The coefficients of kinetic and static friction between the box and the table are $\frac{1}{4}$ and $\frac{1}{2}$, respectively. What is the friction force on this box if the tension in the rope is (a) 0 N, (b) 25 N, (c) 39 N, (d) 41 N, (e) 150 N?

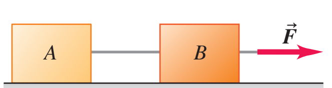
26. | A 2 kg book sits at rest on a horizontal table. The coefficient of static friction between the book and the surface is 0.40, and the coefficient of kinetic friction is 0.20. (a) What is the normal force acting on the book? (b) Is there a friction force on the book? (c) What minimum horizontal force would be required to cause the book to slide on the table? (d) If you give the book a strong horizontal push so that it begins sliding, what kind of force will cause it to come to rest? (e) What is the magnitude of this force?

27. | At a construction site, a pallet of bricks is to be suspended by attaching a rope to it and connecting the other end to a couple of heavy crates on the roof of a building, as shown in Figure 5.53. The rope pulls horizontally on the lower crate, and the coefficient of static friction between the lower crate and the roof is 0.666. (a) What is the weight of the heaviest pallet of bricks that can be supported this way? Start with appropriate free-body diagrams. (b) What is the friction force on the upper crate under the conditions of part (a)?



▲ **FIGURE 5.53** Problem 27.

28. ‖ Two crates connected by a rope of negligible mass lie on a horizontal surface. (See Figure 5.54.) Crate A has mass m_A and crate B has mass m_B. The coefficient of kinetic friction between each crate and the surface is μ_k. The crates are pulled to the right at a constant velocity of 3.20 cm/s by a horizontal force $\vec{F}$. In terms of m_A, m_B, and μ_k, calculate (a) the magnitude of the force $\vec{F}$ and (b) the tension in the rope connecting the blocks. Include the free-body diagram or diagrams you used to determine each answer.



▲ **FIGURE 5.54** Problem 28.

29. ‖ A hockey puck leaves a player's stick with a speed of 9.9 m/s and slides 32.0 m before coming to rest. Find the coefficient of friction between the puck and the ice.

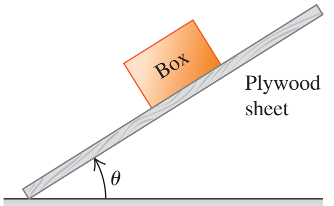
30. ‖ **Stopping distance of a car.** (a) If the coefficient of kinetic friction between tires and dry pavement is 0.80, what is the shortest distance in which you can stop an automobile by locking the brakes when traveling at 29.1 m/s (about 65 mi/h)? (b) On wet pavement, the coefficient of kinetic friction may be only 0.25. How fast should you drive on wet pavement in order to be able to stop in the same distance as in part (a)? (*Note:* Locking the brakes is *not* the safest way to stop.)

31. ‖ An 85 N box of oranges is being pushed across a horizontal floor. As it moves, it is slowing at a constant rate of 0.90 m/s each second. The push force has a horizontal component of 20 N and a vertical component of 25 N downward. Calculate the coefficient of kinetic friction between the box and floor.

32. ‖ A stockroom worker pushes a box with mass 11.2 kg on a horizontal surface with a constant speed of 3.50 m/s. The coefficients of kinetic and static friction between the box and the surface are 0.200 and 0.450, respectively. (a) What horizontal force must the worker apply to maintain the motion of the box? (b) If the worker stops pushing, what will be the acceleration of the box?

33. ‖ The coefficient of kinetic friction between a 40 kg crate and the warehouse floor is 70% of the corresponding coefficient of static friction. The crate falls off a forklift that is moving at 3 m/s and then slides along the warehouse floor for a distance of 2.5 m before coming to rest. What is the coefficient of static friction between the crate and the floor?

34. ‖ **Measuring the coefficients of friction.** One straightforward way to measure the coefficients of friction between a box and a wooden surface is illustrated in Figure 5.55. The sheet of wood can be raised by pivoting it about one edge. It is first raised to an angle θ_1 (which is measured) for which the box just begins to slide downward. The sheet is then immediately lowered to an angle θ_2 (which is also measured) for which the box slides with constant speed down the sheet. Apply Newton's second law to the box in both cases to find the coefficients of kinetic and static friction between it and the wooden sheet in terms of the measured angles θ_1 and θ_2.

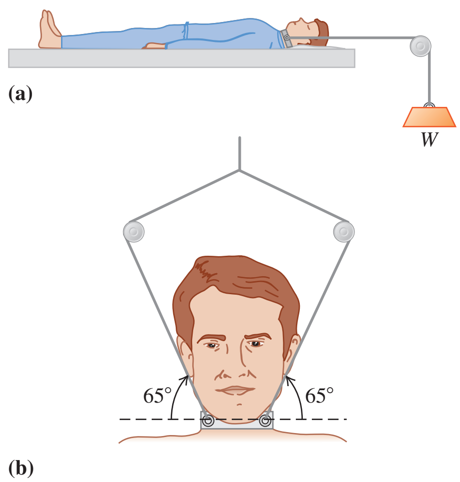


▲ **FIGURE 5.55** Problem 34.

35. | With its wheels locked, a van slides down an icy, frictionless hill. What angle does the hill make with the horizontal if the acceleration of the van is 4 m/s^2?

36. | **The Trendelberg position.** In emergencies involving major
BIO blood loss, the doctor will order the patient placed in the Trendelberg position, which is to raise the foot of the bed to get maximum blood flow to the brain. If the coefficient of static friction between the typical patient and the bedsheets is 1.2, what is the maximum angle at which the bed can be tilted with respect to the floor before the patient begins to slide?

37. ‖ **Injuries to the spinal column.** In treating spinal injuries, it is
BIO often necessary to provide some tension along the spinal column to stretch the backbone. One device for doing this is the Stryker frame, illustrated in part (a) of Figure 5.56. A weight W is attached to the patient (sometimes around a neck collar, as shown in part (b) of the figure), and friction between the person's body and the bed prevents sliding. (a) If the coefficient of static friction between a 78.5 kg patient's body and the bed is 0.75, what is the maximum traction force along the spinal column that W can provide without causing the patient to slide? (b) Under the conditions of maximum traction, what is the tension in each cable attached to the neck collar?

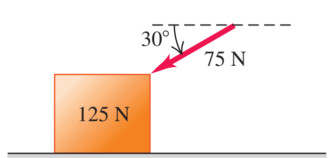


▲ **FIGURE 5.56** Problem 37.

38. ‖ A toboggan approaches a snowy hill moving at 11.0 m/s. The coefficients of static and kinetic friction between the snow and the toboggan are 0.40 and 0.30, respectively, and the hill slopes upward at 40.0° above the horizontal. Find the acceleration of the toboggan (a) as it is going up the hill and (b) after it has reached its highest point and is sliding down the hill.

39. ‖ A 25.0 kg box of textbooks rests on a loading ramp that makes an angle α with the horizontal. The coefficient of kinetic friction is 0.25, and the coefficient of static friction is 0.35. (a) As the angle α is increased, find the minimum angle at which the box starts to slip. (b) At this angle, find the acceleration once the box has begun to move. (c) At this angle, how fast will the box be moving after it has slid 5.0 m along the loading ramp?

40. ‖ A person pushes on a stationary 125 N box with 75 N at 30° below the horizontal, as shown in Figure 5.57. The coefficient of static friction between the box and the horizontal floor is 0.80. (a) Make a free-body diagram of the box. (b) What is the normal force on the box? (c) What is the friction force on the box? (d) What is the largest the friction force could be? (e) The person now replaces his push with a 75 N pull at 30° above the horizontal. Find the normal force on the box in this case.



▲ **FIGURE 5.57** Problem 40.

41. ‖ You are working for a shipping company. Your job is to stand at the bottom of an 8.0-m-long ramp that is inclined at 37° above the horizontal. You grab packages off a conveyor belt and propel them up the ramp. The coefficient of kinetic friction between the packages and the ramp is $\mu_k = 0.30$. (a) What speed do you need to give a package at the bottom of the ramp so that it has zero speed at the top of the ramp? (b) Your coworker is supposed to grab the packages as they arrive at the top of the ramp, but she misses one and it slides back down. What is its speed when it returns to you?

42. | An atmospheric drag force with magnitude $F_D = Dv^2$, where v is speed, acts on a falling 300 mg raindrop that reaches a terminal velocity of 11 m/s. (a) Show that the SI units of D are kg/m. (b) Find the value of D.

43. ‖ What is the acceleration of a raindrop that has reached half of its terminal velocity? Give your answer in terms of g.

44. | A bullet is fired horizontally from a high-powered rifle. If air drag is taken into account, is the magnitude of the bullet's acceleration after leaving the barrel greater than or less than g? Explain.

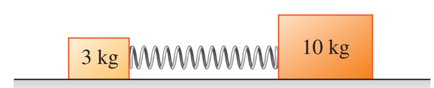
5.4 Elastic Forces

45. | You find that if you hang a 1.25 kg weight from a vertical spring, it stretches 3.75 cm. (a) What is the force constant of this spring in N/m? (b) How much mass should you hang from the spring so it will stretch by 8.13 cm from its original, unstretched length?

46. | An unstretched spring is 12.00 cm long. When you hang an 875 g weight from it, it stretches to a length of 14.40 cm. (a) What is the force constant (in N/m) of this spring? (b) What total mass must you hang from the spring to stretch it to a total length of 17.72 cm?

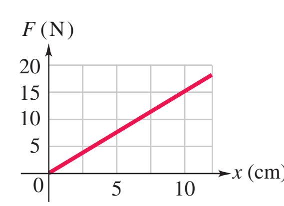
47. | **Heart repair.** A surgeon is using material from a donated heart to repair a patient's damaged aorta and needs to know the elastic characteristics of this aortal material. Tests performed on a 16.0 cm strip of the donated aorta reveal that it stretches 3.75 cm when a 1.50 N pull is exerted on it. (a) What is the force constant of this strip of aortal material? (b) If the maximum distance it will be able to stretch when it replaces the aorta in the damaged heart is 1.14 cm, what is the greatest force it will be able to exert there?
BIO

48. | A 3 kg mass and a 10 kg mass are attached to each other by a spring with spring constant $k = 500$ N/m and placed on a frictionless table, as shown in Figure 5.58. The masses are then pressed toward each other in such a way as to compress the spring 0.05 m. Calculate the magnitude and direction of the acceleration of each mass the moment after they are released.



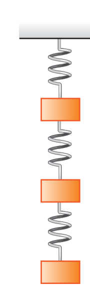
▲ **FIGURE 5.58** Problem 48.

49. ‖ A student measures the force required to stretch a spring by various amounts and makes the graph shown in Figure 5.59, which plots this force as a function of the distance the spring has stretched. (a) Does this spring obey Hooke's law? How do you know? (b) What is the force constant of the spring, in N/m? (c) What force would be needed to stretch the spring a distance of 17 cm from its unstretched length, assuming that it continues to obey Hooke's law?



▲ **FIGURE 5.59** Problem 49.

50. | Three identical 6.40 kg masses are hung by three identical springs, as shown in Figure 5.60. Each spring has a force constant of 7.80 kN/m and was 12.0 cm long before any masses were attached to it. (a) Make a free-body diagram of each mass. (b) How long is each spring when hanging as shown? (*Hint:* First isolate only the bottom mass. Then treat the bottom two masses as a system. Finally, treat all three masses as a system.)



▲ **FIGURE 5.60** Problem 50.

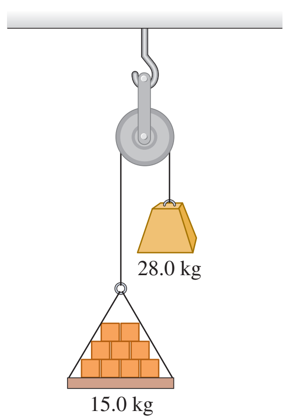
51. | A light spring having a force constant of 125 N/m is used to pull a 9.50 kg sled on a horizontal frictionless ice rink. If the sled has an acceleration of 2.00 m/s^2, by how much does the spring stretch if it pulls on the sled (a) horizontally, (b) at 30.0° above the horizontal?

52. ‖ In the previous problem, what would the answers in both cases be if there were friction and the coefficient of kinetic friction between the sled and the ice were 0.200?

General Problems

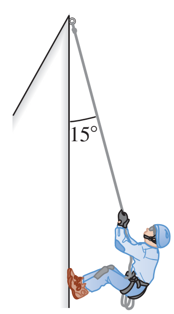
53. ‖ You've attached a bungee cord to a wagon and are using it to pull your little sister while you take her for a jaunt. The bungee's unstretched length is 1.3 m, and you happen to know that your little sister weighs 220 N and the wagon weighs 75 N. Crossing a street, you accelerate from rest to your normal walking speed of 1.5 m/s in 2.0 s, and you notice that while you're accelerating, the bungee's length increases to about 2.0 m. What's the force constant of the bungee cord, assuming it obeys Hooke's law?

54. ‖ **Atwood's machine.** A 15.0 kg load of bricks hangs from one end of a rope that passes over a small, frictionless pulley, called an Atwood's machine. A 28.0 kg counterweight is suspended from the other end of the rope, as shown in Figure 5.61. The system is released from rest. (a) Draw two free-body diagrams, one for the load of bricks and one for the counterweight. (b) What is the magnitude of the upward acceleration of the load of bricks? (c) What is the tension in the rope while the load is moving? How does the tension compare to the weight of the load of bricks? To the weight of the counterweight?



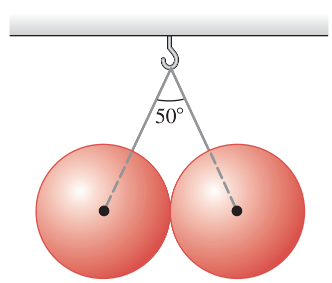
▲ **FIGURE 5.61** Problem 54.

55. | **Mountaineering.** Figure 5.62 shows a technique called *rappelling,* used by mountaineers for descending vertical rock faces. The climber sits in a rope seat, and the rope slides through a friction device attached to the seat. Suppose that the rock is perfectly smooth (i.e., there is no friction) and that the climber's feet push horizontally onto the rock. If the climber's weight is 600.0 N, find (a) the tension in the rope and (b) the force the climber's feet exert on the rock face. Start with a free-body diagram of the climber.



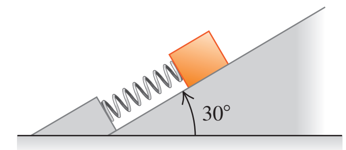
▲ **FIGURE 5.62** Problem 55.

56. ‖ Two identical, perfectly smooth 71.2 N bowling balls 21.7 cm in diameter are hung together from the same hook in the ceiling by means of two thin, light wires, as shown in Figure 5.63. Find (a) the tension in each wire and (b) the force the balls exert on each other.



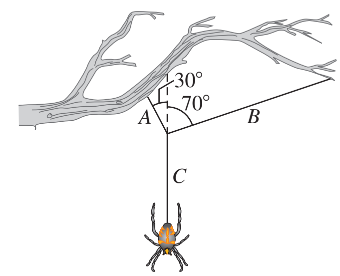
▲ **FIGURE 5.63** Problem 56.

57. ‖ A 2 kg block is launched up a frictionless inclined plane by a spring as shown in Figure 5.64. The plane is inclined at 30° and the spring constant is 1000 N/m. The block is initially pushed against the spring in order to compress the spring 0.1 m, and then it is released. (a) Calculate the magnitude and direction of the acceleration of the block the moment after it is released. (b) Calculate the acceleration when the spring reaches the point where its compression is 0.05 m. (c) What are the magnitude and direction of the acceleration when the spring reaches the point where its compression is zero?



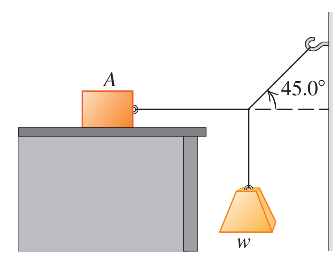
▲ **FIGURE 5.64** Problem 57.

58. ‖ **BIO** The stretchy silk of a certain species of spider has a force constant of 1.10 mN/cm. The spider, whose mass is 15 mg, has attached herself to a branch as shown in Figure 5.65. Calculate (a) the tension in each of the three strands of silk and (b) the distance each strand is stretched beyond its normal length.



▲ **FIGURE 5.65** Problem 58.

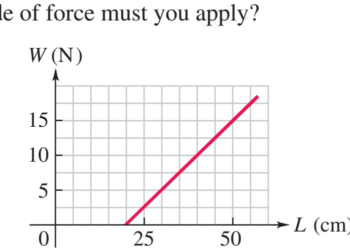
59. ‖ Block *A* in Figure 5.66 weighs 60.0 N. The coefficient of static friction between the block and the surface on which it rests is 0.25. The weight *w* is 12.0 N and the system remains at rest. (a) Find the friction force exerted on block *A*. (b) Find the maximum weight *w* for which the system will remain at rest.



▲ **FIGURE 5.66** Problem 59.

60. ‖ **Friction in an elevator.** You are riding in an elevator on the way to the 18th floor of your dormitory. The elevator is accelerating upward with $a = 1.90\ \text{m/s}^2$. Beside you is the box containing your new computer; the box and its contents have a total mass of 28.0 kg. While the elevator is accelerating upward, you push horizontally on the box to slide it at constant speed toward the elevator door. If the coefficient of kinetic friction between the box and the elevator floor is $\mu_k = 0.32$, what magnitude of force must you apply?

61. ‖ **BIO** A student attaches a series of weights to a tendon and measures the *total length* of the tendon for each weight. He then uses the data he has gathered to construct the graph shown in Figure 5.67, giving the weight as a function of the length of the tendon. (a) Does this tendon obey Hooke's law? How do you know? (b) What is the force constant (in N/m) for the tendon? (c) What weight should you hang from the tendon to make it stretch by 8.0 cm from its unstretched length?



▲ **FIGURE 5.67** Problem 61.

62. ‖ **BIO** A 65.0 kg parachutist falling vertically at a speed of 6.30 m/s hits the ground, which brings him to a complete stop in a distance of 0.92 m (roughly half of his height). Assuming constant acceleration after his feet first touch the ground, what is the average force exerted on the parachutist by the ground?

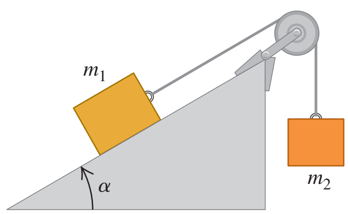
63. ‖ Block *A* in Figure 5.68 weighs 1.20 N and block *B* weighs 3.60 N. The coefficient of kinetic friction between all surfaces is 0.300. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block *B* to the left at a constant speed of 2.50 cm/s (a) if *A* rests on *B*

and moves with it (Figure 5.68a); (b) if *A* is held at rest by a string (Figure 5.68b). (c) In part (a), what is the friction force on block *A*?

(a) **(b)**

▲ **FIGURE 5.68** Problem 63.

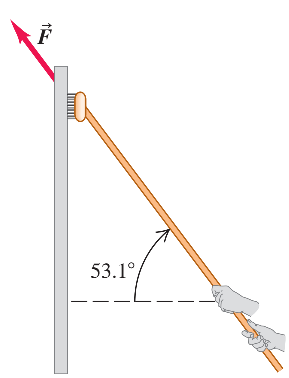
64. ‖ A block with mass m_1 is placed on an inclined plane with slope angle α and is connected to a second hanging block with mass m_2 by a cord passing over a small, frictionless pulley (Figure 5.69). The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k. (a) Find the mass m_2 for which block m_1 moves up the plane at constant speed once it is set in motion. (b) Find the mass m_2 for which block m_1 moves down the plane at constant speed once it is set in motion. (c) For what range of values of m_2 will the blocks remain at rest if they are released from rest?



▲ **FIGURE 5.69** Problem 64.

65. ‖ A pickup truck is carrying a toolbox, but the rear gate of the truck is missing, so the box will slide out if it is set moving. The coefficients of kinetic and static friction between the box and the bed of the truck are 0.355 and 0.650, respectively. Starting from rest, what is the shortest time in which this truck could accelerate uniformly to 30.0 m/s ($\approx$60 mi/h) without causing the box to slide? Include a free-body diagram of the toolbox as part of your solution. (*Hint:* First use Newton's second law to find the maximum acceleration that static friction can give the box, and then solve for the time required to reach 30.0 m/s.)

66. ‖ A window washer pushes his scrub brush up a vertical window at constant speed by applying a force $\vec{F}$ as shown in Figure 5.70. The brush weighs 12.0 N and the coefficient of kinetic friction is $\mu_k = 0.150$. Calculate (a) the magnitude of the force $\vec{F}$ and (b) the normal force exerted by the window on the brush.



▲ **FIGURE 5.70** Problem 66.

67. ‖ **DATA** An astronaut on the distant planet Xenon uses an adjustable inclined plane to measure the acceleration of gravity. The plane is frictionless and its angle of inclination can be varied. Here is a table of the data:

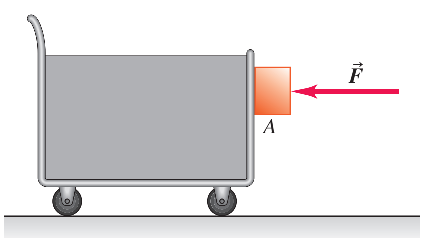
θ	a (m/s^2)
5.0°	1.20
10°	2.49
15°	3.59
20°	4.90
25°	5.95

Make a plot of the measured acceleration as a function of the sine of the angle of incline. Using a linear "best fit" to the data, determine the value of g on the planet Xenon.

68. ‖ **Elevator design.** You are designing an elevator for a hospital. The force exerted on a passenger by the floor of the elevator is not to exceed 1.60 times the passenger's weight. The elevator accelerates upward with constant acceleration for a distance of 3.0 m and then starts to slow down. What is the maximum speed of the elevator?

69. ‖ At night while it is dark, a driver inadvertently parks his car on a drawbridge. Some time later, the bridge must be raised to allow a boat to pass through. The coefficients of friction between the bridge and the car's tires are $\mu_s = 0.750$ and $\mu_k = 0.550$. Start each part of your solution to this problem with a free-body diagram of the car. (a) At what angle will the car just start to slide? (b) If the bridge attendant sees the car suddenly start to slide and immediately turns off the bridge's motor, what will be the car's acceleration after it has begun to move?

70. ‖ A block of mass m is placed against the vertical front of a cart of mass M as shown in Figure 5.71. Assume that the cart is free to roll without friction and that the coefficient of static friction between the block and the cart is μ_s. Derive an expression for the minimum horizontal force that must be applied to the block in order to keep it from falling to the ground.



▲ **FIGURE 5.71** Problem 70.

Passage Problems

Friction and climbing shoes. Shoes for the sports of bouldering and rock climbing are designed to provide a great deal of friction between the foot and the surface of the ground. On smooth rock these shoes might have a coefficient of static friction of 1.2 and a coefficient of kinetic friction of 0.90.

71. For a person wearing these shoes, what's the maximum angle (with respect to the horizontal) of a smooth rock that can be walked on without slipping?
A. 42°
B. 50°
C. 64°
D. Greater than 90°

72. If the person steps onto a smooth rock surface that's inclined at an angle large enough that these shoes begin to slip, what will happen?
A. She will slide a short distance and stop.
B. She will accelerate down the surface.
C. She will slide down the surface at constant speed.
D. We can't tell without knowing her mass.

73. A person wearing these shoes stands on a smooth horizontal rock. She pushes against the ground to begin running. What is the maximum horizontal acceleration she can have without slipping?
A. 0.20 *g* B. 0.75 *g* C. 0.90 *g* D. 1.2 *g*