CHAPTER FOCUS

Students will solve linear, quadratic, simultaneous and literal equations in this chapter. They will simplify and solve linear equations involving algebraic fractions, and will substitute values to determine an unknown or to check an answer. Students will factorise and use inverse operations to solve linear equations and literal equations, including those derived from formulas and worded questions. They will also solve a variety of quadratic expressions using different techniques, including completing the square and the quadratic formula. Simultaneous equations involving linear and non-linear equations will be explored and solved using algebraic and graphical techniques and technology.

Outcomes

Equations [Stages 5.2, 5.3§]

MA5.2-1WM selects appropriate notations and conventions to communicate mathematical ideas and solutions

MA5.2-2WM interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems

MA5.2-3WM constructs arguments to prove and justify results

MA5.2-8NA solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques

MA5.3-1WM uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures

MA5.3-2WM generalises mathematical ideas and techniques to analyse and solve problems efficiently

MA5.3-3WM uses deductive reasoning in presenting arguments and formal proofs

MA5.3-7NA solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations

QUADRATIC EQUATIONS

2

Contents

Syllabus references (See pages x–xv for details.)

Number and Algebra

Selections from ***Equations*** [Stages 5.2, 5.3§]

- Solve simple quadratic equations using a range of strategies (ACMNA241)
- Solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)

Working Mathematically

- Communicating • Problem Solving • Reasoning • Understanding • Fluency

Key ideas

- A quadratic equation has the general form $ax^2 + bx + c = 0$ where $a \neq 0$ and the highest power of x is two.
- Quadratic equations can be solved by methods of factorisation, completing the square or using the quadratic formula. Choosing the most appropriate method is essential for proficiency.
- There can be two solutions to a quadratic equation, and each of these solutions represents the x-intercepts of the quadratic function.
- The value under the square root sign of the quadratic formula, $b^2 - 4ac$, determines how many solutions there are. This is called the discriminant: $\Delta = b^2 - 4ac$.
- There are many real-life situations that can be solved using a quadratic equation. Problem solving will require identifying key information, translating this into mathematical expressions, and forming and solving equations using the most appropriate technique.
- Some equations of a higher power can be solved using a suitable substitution. This method 'reduces' the equation so that it resembles a quadratic.

2:01 Solution using factors

PREP QUIZ 2:01

Factorise:

1 $x^2 - 3x$ 2 $x^2 + 7x$ 3 $x^2 + 3x + 2$ 4 $x^2 - 4x - 12$
5 $x^2 + x - 20$ 6 $x^2 - 8x + 7$

Solve for x:

7 $3x = 12$ 8 $7x = 0$ 9 $x - 4 = 0$ 10 $x + 6 = 0$

In a quadratic equation the highest power of the pronumeral is 2.

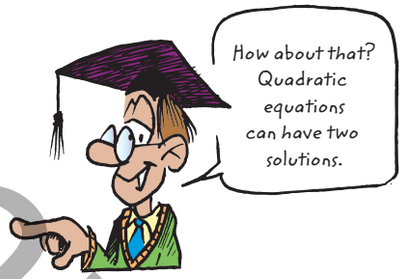
e.g. $x^2 = 9$ $5x^2 - 8 = 0$ $x^2 - 6x = 0$ $x^2 - 4x + 3 = 0$

Equations like the first two above can be solved directly but the second two require the expression to be factorised.

The term quadratic comes from *quadraticus*, which is the Latin word for square.

Equations of the form $ax^2 = c$

To solve $x^2 = 9$ we find the square root of both sides of the equation.
The square root of 9 is 3 or −3.



So if $x^2 = 9$, then $x = \pm 3$.
The equation has two solutions: $x = 3$ and $x = -3$

WORKED EXAMPLES

Solve these equations.

1 $x^2 - 16 = 0$ 2 $a^2 - 5 = 0$ 3 $3m^2 = 10$ 4 $k^2 + 4 = 0$

Solutions

1 $x^2 - 16 = 0$
$x^2 = 16$
$x = \pm\sqrt{16}$
$\therefore x = \pm 4$

2 $a^2 - 5 = 0$
$a^2 = 5$
$\therefore a = \pm\sqrt{5}$
(as decimal approximations $a = 2{\cdot}236$ or $-2{\cdot}236$)

3 $3m^2 = 10$
$m^2 = \frac{10}{3}$
$\therefore m = \pm\sqrt{\frac{10}{3}}$
(as decimal approximations $m = 1{\cdot}826$ or $-1{\cdot}826$)

4 $k^2 + 4 = 0$
$k^2 = -4$
The square of a real number is positive. So this equation has no real solutions!

2:01 Content statements

Solve simple quadratic equations using a range of strategies (ACMNA241) [Stage 5.2]

- solve simple quadratic equations of the form $ax^2 = c$, leaving answers in exact form and as decimal approximations
- solve quadratic equations of the form $ax^2 + bx + c = 0$, limited to $a = 1$, using factors

Solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269) [Stage 5.3]

- solve equations of the form $ax^2 + bx + c = 0$ by factorisation and by 'completing the square'

Answers

PREP QUIZ 2:01

1 $x(x - 3)$ 2 $x(x + 7)$
3 $(x + 1)(x + 2)$ 4 $(x - 6)(x + 2)$
5 $(x + 5)(x - 4)$ 6 $(x - 7)(x - 1)$
7 $x = 4$ 8 $x = 0$
9 $x = 4$ 10 $x = -6$

Lesson starter

Brainstorming

Ask students in groups to list all the forms that quadratic equations can take, e.g. $x^2 = c$, $ax^2 = c$, $ax^2 + bx + c = 0$. Have them try to remember the different methods used to factorise each form, such as perfect squares or completing the square. Have each group construct a comprehensive list of the methods including worked examples. This can form a starting point for this chapter as well as refresh students' factorisation knowledge.

Language

coefficient
complete the square
consecutive
constant
determinant
equation
expression
factorise
factors
formula
index
null factor
perfect square
power
product
quadratic
quadratic formula
reducible
second degree
solution
solve
substitution

Digital resources

eBook

- Foundation worksheet 2:01 Quadratic equations