Gravity is, quite literally, the force that drives the universe. It was gravity that first caused particles to coalesce into atoms, and atoms to congregate into nebulas, planets and stars. An understanding of gravity is fundamental to understanding the universe.

This chapter centres on Newton’s law of universal gravitation. This will be used to predict the size of the force experienced by an object at various locations on the Earth and other planets. It will also be used to develop the idea of a gravitational field. Since the field concept is also used to describe other basic forces such as electromagnetism and the strong and weak nuclear forces, this will provide an important foundation for further study in physics.

Content

INQUIRY QUESTION

How does the force of gravity determine the motion of planets and satellites?

By the end of this chapter you will be able to:

• apply qualitatively and quantitatively Newton’s law of universal gravitation to:
  - determine the force of gravity between two objects \( F = \frac{GMm}{r^2} \)
  - investigate the factors that affect the gravitational field strength \( g = \frac{GM}{r^2} \)
  - predict the gravitational field strength at any point in a gravitational field, including at the surface of a planet (ACSPH094, ACSPH095, ACSPH097)
• investigate the orbital motion of planets and artificial satellites when applying the relationships between the following quantities: (ICT N)
  - gravitational force
  - centripetal force
  - centripetal acceleration
  - mass
  - gravitational field
  - orbital radius
  - orbital velocity
  - orbital period
• predict quantitatively the orbital properties of planets and satellites in a variety of situations, including near the Earth and geostationary orbits, and relate these to their uses (ACSPH101)
• investigate the relationship of Kepler’s Laws of Planetary Motion to the forces acting on, and the total energy of, planets in circular and non-circular orbits using: (ACSPH101)
  - \( v_0 = \frac{2\pi r}{T} \)
  - \( r^l = \frac{GM}{4\pi^2} \)
• derive quantitatively and apply the concepts of gravitational force and gravitational potential energy in radial gravitational fields to a variety of situations, including but not limited to: (ICT N)
  - the concept of escape velocity \( v_{esc} = \sqrt{\frac{2GM}{r}} \)
  - total potential energy of a planet or satellite in its orbit \( U = -\frac{GMm}{r} \)
  - total energy of a planet or satellite in its orbit \( U + K = -\frac{GMm}{2r} \)
  - energy changes that occur when satellites move between orbits (ACSPH096)
  - Kepler’s Laws of Planetary Motion (ACSPH101).
4.1 Gravity

In 1687, Sir Isaac Newton (Figure 4.1.1) published a book that changed the world. Entitled *Philosophiæ Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy), Newton’s book (shown in Figure 4.1.2) used a new form of mathematics now known as calculus and outlined his famous laws of motion.

The *Principia* also introduced Newton’s law of universal gravitation. This was particularly significant because, for the first time in history, it scientifically explained the motion of the planets. This led to a change in humanity’s understanding of its place in the universe.

**FIGURE 4.1.2** The *Principia* is one of the most influential books in the history of science.

**NEWTON’S LAW OF UNIVERSAL GRAVITATION**

Newton’s law of universal gravitation states that any two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Mathematically, Newton’s law of universal gravitation can be expressed as:

\[ \vec{F} = \frac{GMm}{r^2} \]

where

- \( \vec{F} \) is the gravitational force (N)
- \( M \) is the mass of object 1 (kg)
- \( m \) is the mass of object 2 (kg)
- \( r \) is the distance between the centres of objects 1 and 2 (m)
- \( G \) is the gravitational constant, \( 6.67 \times 10^{-11} \text{ N m}^2\text{kg}^{-2} \)

The gravitational force value is always an attractive force. The gravitational constant, \( G \), was first accurately measured by the British scientist Henry Cavendish in 1798, over a century after Newton’s death. As its name suggests, the law of universal gravitation predicts that any two objects that have mass will attract each other. However, because the value of \( G \) is so small, the gravitational force between two everyday objects is too small to be noticed.

The fact that \( r \) appears in the denominator of Newton’s law of universal gravitation indicates an inverse relationship. Since \( r \) is also squared, this relationship is known as an inverse square law. The implication is that as \( r \) increases, \( F \) will decrease dramatically. This law will reappear again later in the chapter when gravitational fields are examined in detail.
Worked example 4.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

A man with a mass of 90 kg and a woman with a mass of 75 kg have a distance of 80 cm between their centres. Calculate the force of gravitational attraction between them.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall the formula for Newton’s law of universal gravitation.</td>
<td>( F = \frac{GMm}{r^2} )</td>
</tr>
</tbody>
</table>
| Identify the information required, and convert values into appropriate units when necessary. | \( G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \)  
\( M = 90 \text{ kg} \)  
\( m = 75 \text{ kg} \)  
\( r = 80 \text{ cm} = 0.80 \text{ m} \) between the man and the woman |
| Substitute the values into the equation.                                | \( F = 6.67 \times 10^{-11} \times \frac{90 \times 75}{0.80^2} \)         |
| Solve the equation.                                                    | \( F = 7.0 \times 10^{-7} \text{ N} \) towards one another.             |

Worked example: Try yourself 4.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Two bowling balls are sitting next to each other on a shelf so that the centres of the balls are 60 cm apart. Ball 1 has a mass of 7.0 kg and ball 2 has a mass of 5.5 kg. Calculate the force of gravitational attraction between them.

For the gravitational force to become significant, at least one of the objects must have a very large mass—for example, a planet (Figure 4.1.3).

![Gravitational forces become significant when at least one of the objects has a large mass, for example the Earth and the Moon.](image-url)
**Worked example 4.1.2**

**GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS**

Calculate the force of gravitational attraction between the Sun and the Earth given the following data:

\[ m_{\text{Sun}} = 2.0 \times 10^{30} \text{ kg} \]
\[ m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg} \]
\[ r_{\text{Sun-Earth}} = 1.5 \times 10^{11} \text{ m} \]

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall the formula for Newton’s law of universal gravitation.</td>
<td>[ F = \frac{GMm}{r^2} ]</td>
</tr>
</tbody>
</table>
| Identify the information required. | \[ G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \]
\[ M = 2.0 \times 10^{30} \text{ kg} \]
\[ m = 6.0 \times 10^{24} \text{ kg} \]
\[ r = 1.5 \times 10^{11} \text{ m} \] between the Sun and the Earth |
| Substitute the values into the equation. | \[ F = 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.0 \times 10^{24}}{(1.5 \times 10^{11})^2} \] |
| Solve the equation. | \[ F = 3.6 \times 10^{22} \text{ N} \] between the Sun and the Earth. |

**Worked example: Try yourself 4.1.2**

**GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS**

Calculate the force of gravitational attraction between the Earth and the Moon, given the following data:

\[ m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg} \]
\[ m_{\text{Moon}} = 7.3 \times 10^{22} \text{ kg} \]
\[ r_{\text{Moon-Earth}} = 3.8 \times 10^{8} \text{ m} \]

The forces in Worked example 4.1.2 are much greater than those in Worked example 4.1.1, illustrating the difference in the gravitational force when at least one of the objects has a very large mass.

**Multi-body systems**

So far, only gravitational systems involving two objects have been considered, such as the Moon and the Earth. In reality, objects experience gravitational force from every other object around them. Usually, most of these forces are negligible and only the gravitational effect of the largest object nearby (i.e. the Earth) needs to be considered.

When there is more than one significant gravitational force acting on a body, the gravitational forces must be added together as vectors to determine the net gravitational force (Figure 4.1.4). The direction and relative magnitude of the net gravitational force in a multi-body system depends entirely on the masses and positions of the attracting objects (i.e. \( m_1 \), \( m_2 \) and \( m_3 \) in Figure 4.1.4).

**FIGURE 4.1.4** For the three masses \( m_1 = m_2 = m_3 \), the gravitational forces acting on the central red ball are shown by the green arrows. The vector sum of the green arrows is shown by the blue arrow. This will be the direction of the net (or resultant) gravitational force on the red ball due to the other three masses.
EFFECT OF GRAVITY

According to Newton’s third law of motion, forces occur in action–reaction pairs. An example of such a pair is shown in Figure 4.1.5. The Earth exerts a gravitational force on the Moon and, conversely, the Moon exerts an equal and opposite force on the Earth. Using Newton’s second law of motion, you can see that the effect of the gravitational force of the Moon on the Earth will be much smaller than the corresponding effect of the Earth on the Moon. This is because of the Earth’s larger mass.

Worked example 4.1.3

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between the Moon and the Earth is approximately \(2.0 \times 10^{20}\) N. Calculate the accelerations of the Earth and the Moon caused by this attraction. Compare these accelerations by calculating the ratio \(\frac{a_{\text{Moon}}}{a_{\text{Earth}}}\).

Use the following data:
- \(m_{\text{Earth}} = 6.0 \times 10^{24}\) kg
- \(m_{\text{Moon}} = 7.3 \times 10^{22}\) kg

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall the formula for Newton’s second law of motion.</td>
<td>( \vec{F}_{\text{net}} = m \vec{a} )</td>
</tr>
<tr>
<td>Transpose the equation to make (\vec{a}) the subject.</td>
<td>( \vec{a} = \frac{\vec{F}_{\text{net}}}{m} )</td>
</tr>
<tr>
<td>Substitute values into this equation to find the accelerations of the Moon and the Earth.</td>
<td>( a_{\text{Earth}} = \frac{2.0 \times 10^{20}}{6.0 \times 10^{24}} = 3.3 \times 10^{-5}) Nkg(^{-1})</td>
</tr>
<tr>
<td></td>
<td>( a_{\text{Moon}} = \frac{2.0 \times 10^{20}}{7.3 \times 10^{22}} = 2.7 \times 10^{-3}) Nkg(^{-1})</td>
</tr>
<tr>
<td>Compare the two accelerations.</td>
<td>( \frac{a_{\text{Moon}}}{a_{\text{Earth}}} = \frac{2.7 \times 10^{-3}}{3.3 \times 10^{-5}} = 82 )</td>
</tr>
<tr>
<td></td>
<td>The acceleration of the Moon is 82 times greater than the acceleration of the Earth.</td>
</tr>
</tbody>
</table>

Worked example: Try yourself 4.1.3

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between the Sun and the Earth is approximately \(3.6 \times 10^{22}\) N. Calculate the accelerations of the Earth and the Sun caused by this attraction. Compare these accelerations by calculating the ratio \(\frac{a_{\text{Sun}}}{a_{\text{Earth}}}\).

Use the following data:
- \(m_{\text{Earth}} = 6.0 \times 10^{24}\) kg
- \(m_{\text{Sun}} = 2.0 \times 10^{30}\) kg

Gravity in the solar system

Although the accelerations caused by gravitational forces in Worked example 4.1.3 are small, over billions of years they created the motion of the solar system.

In the Earth–Moon system, the acceleration of the Moon is many times greater than that of the Earth, which is why the Moon orbits the Earth. Although the Moon’s gravitational force causes a much smaller acceleration of the Earth, it does have other significant effects, such as the tides.

Similarly, the Earth and other planets orbit the Sun because their masses are much smaller than the Sun’s mass. However, the combined gravitational effect of the planets of the solar system (and Jupiter in particular) causes the Sun to wobble slightly as the planets orbit it.

Extrasolar planets

In recent years, scientists have been interested in discovering whether other stars have planets like those in our own solar system. One of the ways in which these ‘extrasolar planets’ (or ‘exoplanets’) can be detected is from their gravitational effect.

When a large planet (i.e. Jupiter-sized or larger) orbits a star, it causes the star to wobble. This causes variations in the star’s appearance, which can be detected on Earth. Hundreds of exoplanets have been discovered using this technique.
GRAVITATIONAL FIELDS

In the 18th century, to simplify the process of calculating the effect of simultaneous gravitational forces, scientists developed a mental construct known as the gravitational field. In the following centuries, the idea of a field was also applied to other forces and has become a very important concept in physics.

A gravitational field is a region in which a gravitational force is exerted on all matter within that region. Complex systems like the solar system involve a number of objects (i.e. the Sun and planets shown in Figure 4.1.6) that are all exerting attractive forces on each other at the same time. Every physical object has an accompanying gravitational field. For example, the space around your body contains a gravitational field because any other object that comes into this region will experience a (small) force of gravitational attraction towards your body.

The gravitational field around a large object like a planet is much more significant than that around a small object. The Earth's gravitational field exerts a significant influence on objects on its surface and even up to thousands of kilometres into space.

Representing gravitational fields

Over time, scientists have developed a commonly understood method of representing fields using a series of arrows known as field lines (Figure 4.1.7). For gravitational fields, these are constructed as follows:

- the direction of the arrowhead indicates the direction of the gravitational force
- the space between the arrows indicates the relative magnitude of the field:
  - closely spaced arrows indicate a strong field
  - widely spaced arrows indicate a weaker field
  - parallel field lines indicate constant or uniform field strength.

An infinite number of field lines could be drawn, so only a few are chosen to represent the rest. The size of the gravitational force acting on a mass in the region of a gravitational field is determined by the strength of the field, and the force acts in the direction of the field.

GRAVITATIONAL FIELD STRENGTH

In theory, gravitational fields extend infinitely out into space. However, since the magnitude of the gravitational force decreases with the square of distance, eventually these fields become so weak as to become negligible.

The constant for the acceleration due to gravity $g$ can be derived directly from the dimensions of the Earth. An object with mass $m$ sitting on the surface of the Earth is a distance of $6.4 \times 10^6$ m from the centre of the Earth.

Given that the Earth has a mass of $6.0 \times 10^{24}$ kg, then:

\[
 weight = gravitational \ force
\]

\[
 mg = G \frac{M_{\text{Earth}} m}{(r_{\text{Earth}})^2}
\]

\[
 g = G \frac{M_{\text{Earth}}}{(r_{\text{Earth}})^2}
\]

\[
 = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6.4 \times 10^6)^2}
\]

\[
 = 9.8 \text{ m s}^{-2} \text{ towards the centre of the Earth}
\]

So, the rate of acceleration of objects near the surface of the Earth is a result of the Earth’s mass and radius. A planet with a different mass and/or different radius will therefore have a different value for $g$.

The variable $g$ can also be used as a measure of the strength of the gravitational field. When understood in this way, it is written with the equivalent units of N kg$^{-1}$ rather than m s$^{-2}$. This means $g_{\text{Earth}} = 9.8 \text{ N kg}^{-1}$. 

**FIGURE 4.1.6** The solar system is a complex gravitational system.

**FIGURE 4.1.7** The arrows in this gravitational field diagram indicate that objects will be attracted towards the mass in the centre; the spacing of the lines shows that force will be strongest at the surface of the central mass and weaker further away from it.
These units indicate that an object on the surface of the Earth experiences 9.8 N of gravitational force for every kilogram of its mass.

Accordingly, the familiar equation $F = mg$ can be transposed so that the gravitational field strength $g$ can be calculated: 

$$ g = \frac{F}{m} = \frac{GMm}{r^2} \times \frac{1}{m} $$

where

- $g$ is gravitational field strength (N/kg$^{-1}$)
- $M$ is the mass of an object in the field (kg)
- $G$ is the gravitational constant, $6.67 \times 10^{-11}$ N m$^2$ kg$^{-2}$
- $r$ is the distance from the centre of $M$ (m)

If the surface of the Earth is considered a flat surface as it appears in everyday life, then the gravitational field lines are approximately parallel, indicating a uniform field (Figure 4.1.8).

However, when the Earth is viewed from a distance as a sphere, it becomes clear that the Earth’s gravitational field is not uniform at all (Figure 4.1.9). The increasing distance between the field lines indicates that the field becomes progressively weaker out into space.

This is because gravitational field strength, like gravitational force, is governed by the inverse square law given above.

The gravitational field strength at different altitudes can be calculated by adding the altitude to the radius of the Earth to calculate the distance of the object from Earth’s centre (Figure 4.1.10).
Variations in gravitational field strength of the Earth

The gravitational field strength of the Earth $g$ is usually assigned a value of 9.81 N kg$^{-1}$. However, the field strength experienced by objects on the surface of the Earth can actually vary between 9.76 N kg$^{-1}$ and 9.83 N kg$^{-1}$, depending on the location.

The Earth’s gravitational field strength is not the same at every point on the Earth’s surface. As the Earth is not a perfect sphere (Figure 4.1.11), objects near the equator are slightly further from the centre of the Earth than objects at the poles. This means that the Earth’s gravitational field is slightly stronger at the poles than at the equator.

Geological formations can also create differences in gravitational field strength, depending on their composition. Geologists use a sensitive instrument known as a gravimeter that detects small local variations in gravitational field strength to indicate underground features. Rocks with above-average density, such as those containing mineral ores, create slightly stronger gravitational fields, whereas less-dense sedimentary rocks produce weaker fields.

WORKED EXAMPLE 4.1.4

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Calculate the strength of the Earth’s gravitational field at the top of Mt Everest using the following data:

- $r_{\text{Earth}} = 6.38 \times 10^6$ m
- $m_{\text{Earth}} = 5.97 \times 10^{24}$ kg
- Height of Mt Everest = 8850 m

**Thinking**

Recall the formula for gravitational field strength.

$$ g = \frac{GM}{r} $$

**Working**

Add the height of Mt Everest to the radius of the Earth.

$$ r = 6.38 \times 10^6 + 8850 \text{ m} $$
$$ r = 6.389 \times 10^6 \text{ m} $$

Substitute the values into the formula.

$$ g = \frac{GM}{r^2} $$
$$ = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.389 \times 10^6)^2} $$
$$ = 9.76 \text{ N kg}^{-1} \text{ towards the centre of the Earth} $$

WORKED EXAMPLE: TRY YOURSELF 4.1.4

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Commercial airlines typically fly at an altitude of 11 000 m. Calculate the gravitational field strength of the Earth at this height using the following data:

- $r_{\text{Earth}} = 6.38 \times 10^6$ m
- $m_{\text{Earth}} = 5.97 \times 10^{24}$ kg
GRAVITATIONAL FIELD STRENGTHS OF OTHER PLANETS

The gravitational field strength on the surface of the Moon is much less than on Earth, at approximately 1.6 N kg\(^{-1}\). This is because the Moon’s mass is smaller than the Earth’s.

The formula \( g = \frac{GM}{r^2} \) can be used to calculate the gravitational field strength on the surface of any astronomical object, such as the Moon (Figure 4.1.12).

![Figure 4.1.12](image)

**Figure 4.1.12** The gravitational field strength on the surface of the Moon is different to the gravitational field strength on the surface of the Earth.

**Worked example 4.1.5**

**GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON**

Calculate the strength of the gravitational field on the surface of the Moon given that the Moon's mass is 7.35 \( \times \) 10\(^{22} \) kg and its radius is 1740 km. Give your answer correct to three significant figures.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall the formula for gravitational field strength.</td>
<td>( g = \frac{GM}{r^2} )</td>
</tr>
<tr>
<td>Convert the Moon's radius to m.</td>
<td>( r = 1740 \text{ km} ) ( \rightarrow ) ( 1740 \times 1000 \text{ m} ) ( \rightarrow ) ( 1.74 \times 10^6 \text{ m} )</td>
</tr>
<tr>
<td>Substitute values into the formula.</td>
<td>( g = \frac{GM}{r^2} ) ( \rightarrow ) ( 6.67 \times 10^{-11} \times \frac{7.35 \times 10^{22}}{(1.74 \times 10^6)^2} ) ( \rightarrow ) ( 1.62 \text{ N kg}^{-1} ) towards the centre of the Moon</td>
</tr>
</tbody>
</table>

**Worked example: Try yourself 4.1.5**

**GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON**

Calculate the strength of the gravitational field on the surface of Mars. \( m_{\text{Mars}} = 6.42 \times 10^{23} \text{ kg} \) \( r_{\text{Mars}} = 3390 \text{ km} \) Give your answer correct to three significant figures.
4.1 Review

**SUMMARY**

- All objects with mass attract one another with a gravitational force.
- The gravitational force acts equally on each of the masses.
- The magnitude of the gravitational force is given by Newton’s law of universal gravitation:
  \[ F = \frac{GMm}{r^2} \]
- Gravitational forces are usually negligible unless one of the objects is massive, e.g. a planet.
- The weight of an object on the Earth’s surface is due to the gravitational attraction of the Earth.
- A gravitational field is a region in which a gravitational force is exerted on all matter within that region.
- A gravitational field can be represented by a gravitational field diagram:
  - The arrowheads indicate the direction of the gravitational force.
  - The spacing of the lines indicates the relative strength of the field. The closer the line spacing, the stronger the field.
- The strength of a gravitational field can be calculated using the following formulae:
  \[ g = \frac{F}{m} \text{ or } g = \frac{GM}{r^2} \]
- Gravitational forces are usually negligible unless one of the objects is massive, e.g. a planet.
- The weight of an object on the Earth’s surface is due to the gravitational attraction of the Earth.
- A gravitational field is a region in which a gravitational force is exerted on all matter within that region.
- A gravitational field can be represented by a gravitational field diagram:
  - The arrowheads indicate the direction of the gravitational force.

**KEY QUESTIONS**

1. What are the proportionalities in Newton’s law of universal gravitation?
2. Calculate the force of gravitational attraction between the Sun and Mars given the following data:
   - \( m_{\text{Sun}} = 2.0 \times 10^{30} \text{ kg} \)
   - \( m_{\text{Mars}} = 6.4 \times 10^{23} \text{ kg} \)
   - \( r_{\text{Sun-Mars}} = 2.2 \times 10^{11} \text{ m} \)
3. The force of gravitational attraction between the Sun and Mars is \( 1.8 \times 10^{21} \text{ N} \). Calculate the acceleration of Mars given that \( m_{\text{Mars}} = 6.4 \times 10^{23} \text{ kg} \).
4. On 14 April 2014, Mars came within 93 million km of Earth. Its gravitational effect on the Earth was the strongest it had been for over 6 years. Use the following data to answer the questions below.
   - \( m_{\text{Sun}} = 2.0 \times 10^{30} \text{ kg} \)
   - \( m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg} \)
   - \( m_{\text{Mars}} = 6.4 \times 10^{23} \text{ kg} \)
   a. Calculate the gravitational force between the Earth and Mars on 14 April 2014.
   b. Calculate the force of the Sun on the Earth if the distance between them was 153 million km.
   c. Compare your answers to parts (a) and (b) above by expressing the Mars–Earth force as a percentage of the Sun–Earth force.
5. A gravitational field, \( g \), is measured as \( 5.5 \text{ N kg}^{-1} \) at a distance of 400 km from the centre of a planet. The distance from the centre of the planet is then increased to 1200 km. What would the ratio of the magnitude of the gravitational field be at this new distance compared to the original measurement?
6. On 12 November 2014, the Rosetta spacecraft landed a probe on the comet 67P/Churyumov–Gerasimenko. Assuming this comet is a roughly spherical object with a mass of \( 1 \times 10^{13} \text{ kg} \) and a diameter of 1.8 km, calculate the gravitational field strength on its surface.
7. The masses and radii of three planets are given in the following table.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass (kg)</th>
<th>Radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>( 3.30 \times 10^{23} )</td>
<td>( 2.44 \times 10^6 )</td>
</tr>
<tr>
<td>Saturn</td>
<td>( 5.69 \times 10^{26} )</td>
<td>( 6.03 \times 10^7 )</td>
</tr>
<tr>
<td>Jupiter</td>
<td>( 1.90 \times 10^{27} )</td>
<td>( 7.15 \times 10^7 )</td>
</tr>
</tbody>
</table>

Calculate the magnitude of the gravitational field strength, \( g \), at the surface of each planet.
4.2 Satellite motion

**PHYSICS INQUIRY**

**Satellite Motion**

How does the force of gravity determine the motion of planets and satellites?

**COLLECT THIS...**
- string
- washer
- retort stand or tape to attach string to ceiling
- ball
- support for ball (beaker, roll of tape or rolled paper)

**DO THIS...**

1. Place the ball on the support. Tie the string to the washer, and fix the string so that it is directly above the centre of the ball as shown below, and falls on the centre line or equator of the ball.

2. Starting with the washer against the ball, tap the washer. Try different directions and strengths of the tap to try to get the washer to orbit around the ball for the longest time.

**RECORD THIS...**
Describe your observations of the orbit for different initial forces.

Present a free-body diagram of the washer orbiting the ball. Refer back to the conical pendulums in Chapter 3 on page XX for additional assistance if required.

**REFLECT ON THIS...**

How does the force of gravity determine the motion of planets and satellites? What do your observations tell you about how satellites are launched into orbit?
Isaac Newton’s development of the law of universal gravitation built on work previously done by Nicolaus Copernicus, Johannes Kepler and Galileo Galilei. Copernicus had proposed a Sun-centred (heliocentric) solar system. Galileo had developed laws relating to motion near the Earth’s surface and Kepler had devised rules concerned with the motion of the planets. Kepler’s laws on the motion of planets were published 80 years before Newton published his law of universal gravitation.

In this section, you will look at how Newton synthesised the work of Galileo and Kepler and proposed that the force that was causing an apple to fall to the Earth was the same force that was keeping the Moon in its orbit. Newton was the first to propose that satellites could be placed in orbit around Earth, almost 300 years before it was technically possible to do this. Now, thousands of artificial satellites are in orbit around Earth and are an essential part of modern life (Figure 4.2.1).

**NEWTON’S THOUGHT EXPERIMENT**

A satellite is an object in a stable orbit around another object. Isaac Newton developed the notion of satellite motion while working on his theory of gravitation. He was comparing the motion of the Moon with the motion of a falling apple and realised that it was the gravitational force of attraction towards the Earth that determined the motion of both objects (Figure 4.2.2). He reasoned that if this force of gravity was not acting on the Moon, the Moon would move at constant speed in a straight line at a tangent to its orbit.

Newton proposed that the Moon, like the apple, was also falling. It was continuously falling to the Earth without actually getting any closer to the Earth. He devised a thought experiment in which he compared the motion of the Moon with the motion of a cannonball fired horizontally from the top of a high mountain.

Newton’s thought experiment is illustrated in Figure 4.2.3. In this thought experiment, if the cannonball was fired at a low speed, it would not travel a great distance before gravity pulled it to the ground (see the shortest dashed line in Figure 4.2.3b). If it was fired with a greater velocity, it would follow a less curved path and land a greater distance from the mountain (see the next two dashed lines in Figure 4.2.3b). Newton reasoned that, if air resistance was ignored and if the cannonball was fired fast enough, it could travel around the Earth and reach the place from where it had been launched (shown by the solid circular line in Figure 4.2.3b). At this speed, it would continue to circle the Earth indefinitely even though the cannonball has no propulsion system.

In reality, satellites could not orbit the Earth at low altitudes, because of air resistance. Nevertheless, Newton had proposed the notion of an artificial satellite hundreds of years before one was actually launched. Any object placed at the right altitude with enough speed would simply continue in its orbit.
Apparent weightlessness

Your apparent weight is a contact reaction force that acts upwards on you from a surface because gravity is pulling you down on that surface. So if you are not standing on a surface, then you will experience zero apparent weight or apparent weightlessness. This means that you will experience apparent weightlessness the moment you step off the top platform of a diving pool or as you skydive from a plane.

Astronauts also experience apparent weightlessness in the International Space Station, which orbits about 370 km above the surface of the Earth (about the horizontal distance from Sydney to the town of Gundagai).

Whenever you are in free fall, you experience apparent weightlessness. It follows then that whenever you experience apparent weightlessness, you must be in free fall. When astronauts experience apparent weightlessness, they are not floating in space as they orbit the Earth. They are actually in free fall. Astronauts and their spacecraft are both falling, but not directly towards the Earth. The astronauts are actually moving horizontally, as shown in Figure 4.2.4. Astronauts are moving at a velocity relative to the Earth so they are moving across the sky at the same rate as they are falling. The combined effect is that they fall in a curved path that exactly mirrors the curve of the Earth. So they fall, but continually miss the Earth as the surface of the Earth curves away from their path.

Importantly there is a significant difference between apparent weightlessness and true weightlessness. True weightlessness only occurs when the gravitational field strength is zero. This only occurs in deep space, far enough away from any planets that their gravitational effect is zero. Apparent weightlessness, however, can occur when still under the influence of a gravitational field.

NATURAL SATELLITES

Natural satellites have existed throughout the universe for billions of years. The planets and asteroids of the solar system are natural satellites of the Sun (Figure 4.2.5).

The Earth has one natural satellite: the Moon. The largest planets—Jupiter and Saturn—have more than 60 natural satellites each in orbit around them. Most of the stars in the Milky Way galaxy have planets and more of these exoplanets are being discovered each year.

FIGURE 4.2.4 Astronauts are in free fall while orbiting the Earth.

FIGURE 4.2.5 The planets are natural satellites of the Sun.
ARTIFICIAL SATELLITES

Since the Space Age began in 1957 with the launch of Sputnik, about 6000 artificial satellites have been launched into orbit around the Earth. Today there are around 4000 still in orbit, although only around 1200 of these are operational.

Satellites in orbit around the Earth are classified as low, medium or high orbit.
- **Low orbit:** 180 km to 2000 km altitude. Most satellites orbit in this range (an example is shown in Figure 4.2.6). These include the Hubble Space Telescope, which is used by astronomers to view objects right at the edge of the universe.
- **Medium orbit:** 2000 km to 36000 km altitude. The most common satellites in this region are the Global Positional System (GPS) satellites used to run navigation systems.
- **High orbit:** 36000 km altitude or greater. Australia uses the Optus satellites for communications, and deep-space weather pictures come from the Japanese MTSAT-1R satellite. The satellites that sit at an altitude of about 36000 km and orbit with a period of 24 hours are known as geostationary satellites (or geosynchronous satellites). Most communications satellites are geostationary.

**FIGURE 4.2.6** A low-orbit satellite called the Soil Moisture and Ocean Salinity (SMOS) probe was launched in August 2014. Its role is to measure water movements and salinity levels on Earth as a way of monitoring climate change. It was launched from northern Russia by the European Space Agency (ESA).

Earth satellites can have different orbital paths depending on their function:
- equatorial orbits, where the satellite always travels above the equator
- polar or near-polar orbits, where the satellite travels over or close to the north and south poles as it orbits
- inclined orbits, which lie between equatorial and polar orbits.

Satellites are used for a multitude of different purposes, with 60 per cent used for communications.

As discussed in Chapter 3, all objects travelling in circular motion require a centripetal force. Artificial and natural satellites are not propelled by rockets or engines. They orbit in free fall and the only force acting on them is the gravitational attraction between themselves and the body about which they orbit. This means the gravitational attraction is equivalent to the centripetal force of the satellite’s motion. The satellites therefore have a centripetal acceleration that is equal to the gravitational field strength at their location (Figure 4.2.7). Centripetal acceleration is covered in more detail in Chapter 3.

Artificial satellites are often equipped with tanks of propellant that are squirted in the appropriate direction when the orbit of the satellite needs to be adjusted.
Chapter 4  |  Motion in Gravitational Fields

Space junk

Today there are around 1200 satellites that are still in operation. There are also around 2800 satellites that have reached the end of their operational life or have malfunctioned but are still in orbit.

In 2007, a Chinese satellite was deliberately destroyed by a missile, creating thousands of pieces of debris. In 2009, a collision between the defunct Russian Cosmos 2251 and operational US Iridium 33 created even more debris. This debris and the defunct satellites are classified as space junk (Figure 4.2.8).

The presence of this fast-moving space junk puts the other satellites and the International Space Station at risk from collision. Currently around 22,000 pieces of space junk are being tracked and monitored. There have been a number of occasions where satellites have been moved to avoid collisions with space junk.

The UN has passed a resolution to remove defunct satellites from low-Earth orbits by placing them in much higher orbits, or bringing them back to Earth and allowing them to burn up in the atmosphere.

Kepler’s Laws

Johannes Kepler, a German astronomer (depicted in Figure 4.2.9), published his three laws regarding the motion of planets in 1609. This was about 80 years before Newton’s law of universal gravitation was published. Kepler analysed the motion of the planets in orbit around the Sun, but Kepler’s laws can be used for any satellite in orbit around any central mass.

Kepler’s laws are as follows:

1. The planets move in elliptical orbits with the Sun at one focus.
2. The line connecting a planet to the Sun sweeps out equal areas in equal intervals of time (Figure 4.2.10).
3. For every planet, the ratio of the cube of the average orbital radius, \( r \), to the square of the period, \( T \), of revolution is the same, i.e. \( \frac{r^3}{T^2} = \text{constant, } k \).

See the International Space Station (ISS) and other satellites

It is easy to see low-orbit satellites if you are away from city lights. The best time to look is just after sunset. If you can, go outside and look for any slow-moving objects passing across the star background.

There are also many websites that will allow you to track and predict the real-time paths of satellites. You can use the NASA ‘Spot the Station’ website to see when the ISS is passing over your part of the planet. The ISS is so bright that it is easy to see from most locations.

Figure 4.2.8 An exaggerated map showing the location of space debris and abandoned satellites in near-Earth orbits.

Figure 4.2.9 Johannes Kepler, was the first to work out that the planets do not travel in circular paths, but rather in elliptical paths.
Kepler’s first two laws proposed that planets move in elliptical paths and that they move faster when they are closer to the Sun. It took Kepler many months of laborious calculations to arrive at his third law. Newton used Kepler’s laws to justify the inverse square relationship that he used in his law of universal gravitation.

CALCULATING THE ORBITAL PROPERTIES OF SATELLITES

The speed, $v$, of a satellite can be calculated from its motion for one revolution. For simplicity, we will assume that the orbital paths are approximately circular. This means that a satellite will travel a distance equal to the circumference of the orbit, $2\pi r$, in the time of one period, $T$.

The speed, $v$, of a satellite in a circular orbit is given by:

$$v = \frac{2\pi r}{T}$$

where

- $r$ is the radius of the orbit (m)
- $T$ is the time for one revolution, or the period (s)

The centripetal acceleration of a satellite can be determined from the gravitational field strength at its location. Satellites are in free fall; therefore, the only force acting is gravity, $F_g$. For example, the International Space Station (ISS) is in orbit at a distance from Earth where $g$ is $8.8 \text{ N kg}^{-1}$, and so it orbits with a centripetal acceleration of $8.8 \text{ m s}^{-2}$.

The centripetal acceleration, $a_c$, of the satellite can also be calculated by considering its circular motion. The equation for speed given above can be substituted into the centripetal acceleration formula:

$$a_c = \frac{v^2}{r}$$

Since $v = \frac{2\pi r}{T}$ then $\frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$.

Since the centripetal acceleration of the satellite is equal to the gravitational field strength at the location of its orbit, and using the gravitational field strength equation from earlier, we can give the following expression.

The centripetal acceleration, $a_c$, of a satellite in circular orbit is given by:

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} = g$$

where

- $v$ is the speed of the satellite (m s$^{-1}$)
- $r$ is the radius of the orbit (m)
- $T$ is the period of orbit (s)
- $M$ is the central mass (kg)
- $g$ is the gravitational field strength at $r$ (N kg$^{-1}$)
- $G$ is the gravitational constant, $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

Remember, acceleration can be a vector or a scalar quantity. From Chapter 3, if you are to write the centripetal acceleration as a vector, the direction will always be towards the centre of the circular motion—the same direction as the net force.

These relationships can be manipulated to determine any feature of a satellite’s motion: its speed, radius of orbit or period of orbit. They can also be used to find the mass, $M$, of the central body around which the satellite orbits.

In the same way as with freely falling objects at the Earth’s surface, the mass of the satellite itself has no effect on any of these orbital properties.
Worked example 4.2.1

WORKING WITH KEPLER’S LAWS

Determine the orbital speed of the Moon, assuming it is in a circular orbit of radius 384,000 km around the Earth. Take the mass of the Earth to be 5.97 × 10^24 kg and use G = 6.67 × 10^{-11} N m^2 kg^{-2}.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ensure that the variables are in their standard units.</td>
<td>( r = 384000 \text{ km} = 3.84 \times 10^8 \text{ m} )</td>
</tr>
</tbody>
</table>
| Choose the appropriate relationship between the orbital speed, \( v \), and the data that has been provided. | \( a_c = g = \frac{GM}{r^2} = \frac{v^2}{r} \)  
\[ \therefore \frac{GM}{r} = \frac{v^2}{r} \]  
\[ \therefore \frac{GM}{r} = v^2 \] |
| Make \( v \), the orbital speed, the subject of the equation. | \( v = \sqrt{\frac{GM}{r}} \) |
| Substitute in values and solve for the orbital speed, \( v \). | \( v = \sqrt{\frac{GM}{r}} \)  
\[ = \sqrt{\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{3.84 \times 10^8}} \]  
\[ = 1.02 \times 10^3 \text{ m s}^{-1} \] |

Worked example: Try yourself 4.2.1

WORKING WITH KEPLER’S LAWS

Determine the orbital speed of a satellite, assuming it is in a circular orbit of radius of 42 100 km around the Earth. Take the mass of the Earth to be 5.97 × 10^{24} kg and use \( G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \).

HOW NEWTON DERIVED KEPLER’S THIRD LAW USING ALGEBRA

It took Kepler many months of trial-and-error calculations to arrive at his third law:

\[ \frac{r^3}{T^2} = \text{constant.} \]

Newton was able to use some clever algebra to derive this from his law of universal gravitation. Using Newton’s law for the magnitude of the gravitational force:

\[ F_g = mg = m \frac{4\pi^2 r}{T^2} = \frac{GMm}{r^2} \]

\[ \therefore \frac{r^3}{T^2} = \frac{GM}{4\pi^2} \]

For any central mass, \( M \), the term \( \frac{GM}{4\pi^2} \) is constant and the ratio \( \frac{r^3}{T^2} \) is equal to this constant value for all of its satellites (Figure 4.2.12).

So, for example, if you know the orbital radius, \( r \), and period, \( T \), of one of the moons of Saturn, you could calculate \( \frac{r^3}{T^2} \) and use this as a constant value for all of Saturn’s moons. If you knew the period, \( T \), of a different satellite of Saturn, it would then be straightforward to calculate its orbital radius, \( r \).

FIGURE 4.2.12 These three satellites are at different distances from Earth and hence according to Kepler’s third law will have different orbital periods. For all three, the ratio of \( \frac{r^3}{T^2} \) will equal the same constant value.
**Worked example 4.2.2**

**SATELLITES IN ORBIT**

Ganymede is the largest of Jupiter’s moons. It has a mass of \(1.66 \times 10^{23}\) kg, an orbital radius of \(1.07 \times 10^6\) km and an orbital period of 7.15 days (\(6.18 \times 10^5\) s).

**a** Use Kepler’s third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
</table>
| Note down the values for the known satellite. You can work in days and km as this question involves a ratio. | Ganymede:  
\[ r = 1.07 \times 10^6\text{km} \]  
\[ T = 7.15 \text{ days} \] |
| For all satellites of a central mass, \(\frac{r^3}{T^2} = \text{constant} \). Work out this ratio for the known satellite. | \[ \frac{r^3}{T^2} = \text{constant} \]  
\[ \frac{(1.07 \times 10^6)^3}{7.15^2} = 2.40 \times 10^{16} \] |
| Use this constant value with the ratio for the satellite in question. Make sure \(T\) is in days to match the ratio calculated in the previous step. | Europa:  
\[ T = 3.55 \text{ days}, r = ? \]  
\[ \frac{r^3}{3.55^2} = 2.40 \times 10^{16} \] |
| Make \(r^3\) the subject of the equation. | \[ r^3 = 2.40 \times 10^{16} \times 3.55^2 \]  
\[ = 3.02 \times 10^{17} \] |
| Solve for \(r\). The unit for \(r\) is km as the original ratio was calculated using km. | \[ r = \sqrt[3]{3.02 \times 10^{17}} \]  
\[ = 6.71 \times 10^5\text{km} \]  
Note: Europa has a shorter period than Ganymede so you should expect Europa to have a smaller orbit than Ganymede. |

**b** Use the orbital data for Ganymede to calculate the mass of Jupiter.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
</table>
| Note down the values for the known satellite. You must work in SI units to find the mass value in kg. | Ganymede/Jupiter:  
\[ r = 1.07 \times 10^9\text{m} \]  
\[ T = 6.18 \times 10^5\text{s} \]  
\[ m = 1.66 \times 10^{23}\text{kg} \]  
\[ G = 6.67 \times 10^{-11}\text{N}\text{m}^2\text{kg}^{-2} \]  
\( M = ? \) |
| Select the expressions from the equation for centripetal acceleration that best suit your data. |  
\[ a_c = \frac{v^2}{r} = \frac{4\pi^2r}{T^2} = GM \]  
Use the 3rd and 4th terms of the expression.  
\[ \frac{4\pi^2r^3}{T^2} = \frac{GM}{r} \]  
These two expressions use the given variables \(r\) and \(T\), and the constant \(G\), so that a solution may be found for \(M\). |
| Transpose the equation to make \(M\) the subject. | \[ M = \frac{4\pi^2r^3}{GT^2} \] |
| Substitute values and solve. | \[ M = \frac{4\pi^2(1.07 \times 10^6)^3}{6.67 \times 10^{-11} \times (6.18 \times 10^5)^2} \]  
\[ = 1.90 \times 10^{27}\text{kg} \] |
c Calculate the orbital speed of Ganymede in km s\(^{-1}\).

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note values you will need to use in the equation ( v = \frac{2\pi r}{T} ).</td>
<td>Ganymede: ( r = 1.07 \times 10^6 \text{ km} ) ( T = 6.18 \times 10^5 \text{ s} ) ( v = ? )</td>
</tr>
<tr>
<td>Substitute values and solve. The answer will be in km s(^{-1}) if ( r ) is expressed in km.</td>
<td>( v = \frac{2\pi r}{T} ) ( = \frac{2\pi \times 1.07 \times 10^6}{6.18 \times 10^5} ) ( = 10.9 \text{ km s}^{-1} )</td>
</tr>
</tbody>
</table>

**Worked example: Try yourself 4.2.2**

**SATELLITES IN ORBIT**

Callisto is the second largest of Jupiter’s moons. It is about the same size as the planet Mercury. Callisto has a mass of \( 1.08 \times 10^{23} \text{ kg} \), an orbital radius of \( 1.88 \times 10^6 \text{ km} \) and an orbital period of \( 1.44 \times 10^6 \text{ s} \) (16.7 days).

a Use Kepler’s third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.

b Use the orbital data for Callisto to calculate the mass of Jupiter.

c Calculate the orbital speed of Callisto in km s\(^{-1}\).

**PHYSICSFILE ICT**

**SuitSat1**

One of the more unusual satellites was launched from the International Space Station on 3 February 2006. It was an obsolete Russian spacesuit into which the astronauts had placed a radio transmitter, batteries and some sensors. Its launch involved simply being pushed off by one of the astronauts while on a spacewalk. SuitSat1 was meant to transmit signals that would be picked up by ham radio operators on Earth for a few weeks, but transmissions ceased after just a few hours. The spacesuit burned up in the atmosphere over Western Australia in September 2006.

SuitSat2 was launched in August 2011 and contained experiments created by school students. It re-entered Earth’s atmosphere in January 2012 after 5 months in orbit.

**FIGURE 4.2.11** This photograph does not show an astronaut drifting off to certain death in space. This is SuitSat1, one of the strangest satellites ever launched, at the start of its mission.
Three satellites

Geostationary Meteorological Satellite MTSAT-1R
The Japanese MTSAT-1R satellite was launched in February 2005, and orbits at approximately 35,800 km directly over the equator. At its closest point to the Earth, known as the perigee, its altitude is 35,776 km. At its furthest point from the Earth, known as the apogee, it is at 35,798 km. MTSAT-1R orbits at a longitude of 140° E, so it is just to the north of Cape York and ideally located for use by Australia’s weather forecasters. It has a period of 24 hours, so is in a geostationary orbit.

Signals from MTSAT-1R are transmitted every 2 hours and are received by a satellite dish on the roof of the head office at the Bureau of Meteorology in Perth. Infrared images show the temperature variations in the atmosphere and are invaluable in weather forecasting. MTSAT-1R is box-like and measures about 2.6 m along each side. It has a mass of 1250 kg and is powered by solar panels that, when deployed, take its overall length to over 30 m.

Hubble Space Telescope (HST)
This cooperative venture between NASA and the European Space Agency (ESA) was launched by the crew of the space shuttle Discovery on 25 April 1990. Hubble is a permanent unoccupied space-based observatory with a 2.4 m–diameter reflecting telescope, spectrographs and a faint-object camera. It orbits above the Earth’s atmosphere, producing images of distant stars and galaxies far clearer than those from ground-based observatories (Figure 4.2.13). The HST is in a low-Earth orbit inclined at 28° to the equator. Its expected life span was originally around 15 years, but service and repair missions have extended its life and it is still in use today.

This is an image of the spiral galaxy known as NGC 3344.

FIGURE 4.2.13 The Hubble Space Telescope has produced spectacular images of stars that are clearer than from any ground-based telescope. This is an image of the spiral galaxy known as NGC 3344.

National Oceanic and Atmospheric Administration Satellite (NOAA-19)
Many of the US-owned and operated NOAA satellites are located in low-altitude near-polar orbits. This means that they pass close to the poles of the Earth as they orbit. NOAA-19 was launched in February 2009 and orbits at an inclination of 99° to the equator. Its low altitude means that it captures high-resolution pictures of small bands of the Earth. The data is used in local weather forecasting as well as to provide enormous amounts of information for monitoring global warming and climate change.

The properties of these satellites are summarised in Table 4.2.1.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Orbit</th>
<th>Inclination</th>
<th>Perigee (km)</th>
<th>Apogee (km)</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTSAT-1R</td>
<td>equatorial</td>
<td>0°</td>
<td>35,776</td>
<td>35,798</td>
<td>1 day</td>
</tr>
<tr>
<td>Hubble</td>
<td>inclined</td>
<td>28°</td>
<td>591</td>
<td>599</td>
<td>96.6 min</td>
</tr>
<tr>
<td>NOAA-19</td>
<td>near polar</td>
<td>99°</td>
<td>846</td>
<td>866</td>
<td>102 min</td>
</tr>
</tbody>
</table>
**4.2 Review**

**SUMMARY**

- A satellite is an object that is in a stable orbit around a larger central mass.
- The only force acting on a satellite is the gravitational attraction between it and the central body.
- Satellites are in continual free fall. They move with a centripetal acceleration that is equal to the gravitational field strength at the location of their orbit.
- The speed of a satellite, $v$, is given by:

$$ v = \frac{2\pi r}{T} $$

- For a satellite in a circular orbit:

$$ a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = g $$

- The magnitude of the gravitational force acting on a satellite in a circular orbit is given by:

$$ F_g = \frac{mv^2}{r} = \frac{4\pi^2 m r^2}{T^2} = mg $$

- For any central body of mass, $M$:

$$ \frac{r^3}{T^2} = \frac{GM}{4\pi^2} = \text{constant, so knowing another satellite's orbital radius, } r, \text{ enables its period, } T, \text{ to be determined.} $$

**KEY QUESTIONS**

1. Which of the following is correct?
   - A Earth is a satellite of Mars.
   - B The Moon is a satellite of the Sun.
   - C Earth is a satellite of the Sun.
   - D The Sun is a satellite of Earth.

2. A geostationary satellite orbits above Singapore, which is on the equator. Which of the following statements about the satellite is correct?
   - A It is in a low orbit.
   - B It is in a high orbit.
   - C It passes over the north pole.
   - D It is not moving.

3. The gravitational field strength at the location where the Optus D1 satellite is in stable orbit around the Earth is equal to 0.22 N kg\(^{-1}\). The mass of this satellite is $2.3 \times 10^3$ kg.
   - a Using only the information given, calculate the magnitude of the acceleration of this satellite as it orbits.
   - b Calculate the net force acting on this satellite as it orbits.

4. One of Saturn’s moons is Atlas, which has an orbital radius of $1.37 \times 10^6$ km and a period of 0.60 days. The largest of Saturn’s moons is Titan. It has an orbital radius of $1.20 \times 10^6$ km. What is the orbital period of Titan in days?

5. Different types of satellite have different types of orbit, as shown in the table below. Calculate the strength of the Earth’s gravitational field in each orbit. Give all of your answers to three significant figures.

<table>
<thead>
<tr>
<th>Type of orbit</th>
<th>Altitude (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a low-Earth orbit</td>
<td>2000</td>
</tr>
<tr>
<td>b medium Earth orbit</td>
<td>10000</td>
</tr>
<tr>
<td>c semi-synchronous orbit</td>
<td>20200</td>
</tr>
<tr>
<td>d geosynchronous orbit</td>
<td>35786</td>
</tr>
</tbody>
</table>
4.3 Gravitational potential energy

The concept of gravitational potential energy should be familiar to you from Year 11 Physics. However, the nature of a gravitational field means that a more sophisticated understanding of gravitational potential energy is needed when considering the motion of objects like rockets or satellites (Figure 4.3.1).

**ENERGY IN A CONSTANT GRAVITATIONAL FIELD**

Up until now, our consideration of energy in gravitational fields has been simplified by the assumption that the Earth’s gravitational field is constant. Under this assumption, the gravitational potential energy of an object, $U$, is directly proportional to the mass of the object, $m$, its height above the surface of the planet, $h$, and the strength of the gravitational field, $g$.

$$U = mgh$$

where

- $U$ is the gravitational potential energy of an object (J)
- $m$ is the mass of the object (kg)
- $g$ is the magnitude of the gravitational field strength (N kg$^{-1}$; 9.80 N kg$^{-1}$ near the surface of the Earth)
- $h$ is the height of the object above a reference point (m)

**ENERGY IN A NON-CONSTANT GRAVITATIONAL FIELD**

Newton’s law of universal gravitation indicates that the strength of the Earth’s gravitational field changes with altitude: the field is stronger close to the ground and weaker at high altitudes (Figure 4.3.2).

$$K_k = \frac{1}{2}mv^2$$

where

- $K_k$ is the kinetic energy of an object (J)
- $m$ is the mass of the object (kg)
- $v$ is the speed of the object (m s$^{-1}$)

**FIGURE 4.3.2** As the distance from the surface of the Earth is increased from 0 to 40 000 km, the value for $g$ decreases rapidly from 9.8 N kg$^{-1}$, according to the inverse square law. The blue line on the graph gives the value of $g$ at various altitudes (h).
Clearly it is not sufficient to assume a constant value for the Earth’s gravitational field when considering objects like satellites or moons that orbit at high altitudes. In section 4.1, we saw that the magnitude of the gravitational field strength is given by the formula \( g = \frac{GM}{r^2} \). Substituting this into the formula for gravitational potential energy:

\[
U = mgh = m \frac{GM}{r^2}r
\]

Note that this formula contains a negative sign. Gravitational potential energy is measured against a reference level and a negative value means that it has moved below this level. By convention, the gravitational potential energy of a satellite is considered to be zero when it has escaped the gravitational field of the Earth. This only occurs when the satellite is a very large distance away (in theory, an infinite distance). As the satellite moves closer to the Earth, its distance from the Earth becomes less than the reference level and the potential energy of the satellite becomes a negative value.

**Worked example 4.3.1**

**GRAVITATIONAL POTENTIAL ENERGY IN A NON-CONSTANT FIELD**

A communication satellite with a mass of 800 kg is orbiting the Earth at an altitude of 35,800 km. Calculate the gravitational potential energy of the satellite. (Use \( r_{\text{Earth}} = 6.38 \times 10^6 \text{ m} \) and \( m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg} \).)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the radius of the satellite’s orbit.</td>
<td>( r = 3.58 \times 10^7 + 6.38 \times 10^6 ) ( = 4.22 \times 10^7 \text{ m} )</td>
</tr>
<tr>
<td>Recall the formula for the gravitational potential energy of a satellite.</td>
<td>( U = -\frac{GMm}{r} )</td>
</tr>
<tr>
<td>Substitute the values into the formula.</td>
<td>( U = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 800}{4.22 \times 10^7} ) ( = -7.55 \times 10^9 \text{ J} )</td>
</tr>
</tbody>
</table>
Worked example: Try yourself 4.3.1

GRAVITATIONAL POTENTIAL ENERGY IN A NON-CONSTANT FIELD

A 500 kg lump of space junk is plummeting towards the Moon (see the figure below). The Moon has a radius of 1.7 \times 10^6 m and a mass of 7.3 \times 10^{22} kg. Calculate the gravitational potential energy of the space junk when it is 2.7 \times 10^6 m away from the surface of the Moon.

TOTAL ENERGY IN A NON-CONSTANT GRAVITATIONAL FIELD

A satellite has two important forms of energy: gravitational potential energy, \( U \), and kinetic energy, \( K \). As you saw in Chapter 3, it is often useful to consider the sum of these two energies (sometimes known as total mechanical energy), i.e. \( E = U + K \).

The centripetal force of a satellite is equal to the force due to gravity. Combining this with magnitude of the force from Newton’s second law, and remembering the formula for the centripetal acceleration:

\[
F = \frac{GMm}{r^2} = \frac{mv^2}{r}
\]

\[
\therefore \quad m\frac{v^2}{r} = \frac{GMm}{r}
\]

Using the equation \( K = \frac{1}{2} mv^2 \), the kinetic energy is then equal to:

\[
K_k = \frac{1}{2} mv^2 = \frac{GMm}{2r}
\]

Combining this with the gravitational potential energy of a satellite, \( U = -\frac{GMm}{r} \), the total energy is then

\[
E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}
\]

\[
E = K + U = -\frac{GMm}{2r}
\]

where
- \( E \) is the total energy of an object (J)
- \( K \) is the kinetic energy of an object (J)
- \( U \) is the potential energy of an object (J)
- \( m \) is the mass of the object (kg)
- \( M \) is the mass of the Earth (or, more generally, the central body; kg)
- \( G \) is the gravitational constant, \( 6.67 \times 10^{-11} \text{N m}^2 \text{kg}^{-2} \)
- \( r \) is the radius of the orbit of the object (the sum of the radius of the central body and the altitude of the orbit; in m)
Worked example 4.3.2

TOTAL ENERGY OF A SATELLITE

A communications satellite with a mass of 1390 kg is orbiting the Earth at an altitude of 357 km. (Use \( r_{\text{Earth}} = 6.38 \times 10^6 \) m and \( m_{\text{Earth}} = 5.97 \times 10^{24} \) kg.)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate the total mechanical energy of this satellite.</td>
<td>Determine the radius of the satellite’s orbit.</td>
</tr>
<tr>
<td></td>
<td>( r = 357 \times 10^3 + 6.38 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td>= 6.783000</td>
</tr>
<tr>
<td></td>
<td>= 6.74 \times 10^6 ) m</td>
</tr>
<tr>
<td>Use the definition for total energy:</td>
<td>( E = -\frac{GMm}{2r} )</td>
</tr>
<tr>
<td></td>
<td>( = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1390}{2 \times 6.74 \times 10^6} )</td>
</tr>
<tr>
<td></td>
<td>= (-4.11 \times 10^{10} ) J</td>
</tr>
</tbody>
</table>

b Calculate the speed of the satellite.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall the equation for the kinetic energy of a satellite.</td>
<td>( K = \frac{GMm}{2r} )</td>
</tr>
<tr>
<td>Substitute the known values and solve for ( K ).</td>
<td>( K = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1390}{2 \times 6.74 \times 10^6} )</td>
</tr>
<tr>
<td></td>
<td>= 4.11 \times 10^{10} ) J</td>
</tr>
<tr>
<td>Remember that the kinetic energy of an object can also be calculated with the equation:</td>
<td>( K = \frac{1}{2}mv^2 )</td>
</tr>
<tr>
<td></td>
<td>(4.11 \times 10^{10} = \frac{1}{2} \times 1390 \times v^2 )</td>
</tr>
<tr>
<td></td>
<td>(v^2 = 5.91 \times 10^7 )</td>
</tr>
<tr>
<td></td>
<td>(v = 7690 ) m/s</td>
</tr>
</tbody>
</table>

Worked example: Try yourself 4.3.2

TOTAL ENERGY OF A SATELLITE

Sputnik 1 was the first artificial satellite to be put into orbit. It had a mass of 84.0 kg and orbited the Earth at an altitude of 577 km. (Use \( r_{\text{Earth}} = 6.38 \times 10^6 \) m and \( m_{\text{Earth}} = 5.97 \times 10^{24} \) kg.)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate the total mechanical energy of this satellite.</td>
<td>Calculate the speed of the satellite.</td>
</tr>
</tbody>
</table>
It is important to note that the total energy of a satellite remains constant throughout its orbit, even though its distance from the attracting object may vary. For example, Kepler’s first law states that planets travel in elliptical orbits around the Sun (Figure 4.3.3).

![Figure 4.3.3](image)

**FIGURE 4.3.3** The total energy of a planet remains constant throughout its orbit, even though its distance from the Sun changes.

A planet’s gravitational potential energy is greatest when it is at its **aphelion**, i.e. the point on its orbit when it is furthest from the Sun. As the planet moves towards its **perihelion** (i.e. point of closest approach to the Sun), its gravitational potential energy will decrease. However, according to Kepler’s second law, the velocity of the planet will increase as it approaches the Sun. This means that the kinetic energy of the planet will increase in a way that exactly balances the decrease in gravitational potential energy, keeping the total energy of the planet constant. This pattern holds true for all satellites.

**ENERGY CHANGES IN A NON-CONSTANT GRAVITATIONAL FIELD**

The familiar formula for change in gravitational potential energy $\Delta U = mg\Delta h$ is developed assuming that work is done against a constant force of gravity: $\Delta U = W = F \cdot s$. While this assumption holds true for objects close to the surface of a planet, it is not adequate for objects like satellites that move to altitudes at which the gravitational field of the planet becomes significantly diminished.

Consider the example of a 10kg meteor falling towards the Earth from deep space as shown in Figure 4.3.4. As the meteor gets closer to the Earth, it moves through regions of increasing gravitational field strength. So the gravitational force, $F_g$, on the meteor increases as it approaches Earth. Since the force is not constant, this means that the work done on the meteor (which corresponds to its change in gravitational potential energy) cannot be found by simply multiplying the gravitational force by the distance travelled; it must be calculated directly from the formula $U = -\frac{GMm}{r}$.

![Figure 4.3.4](image)

**FIGURE 4.3.4** As a meteor approaches Earth, it moves through an increasingly stronger gravitational field and so is acted upon by a greater gravitational force.
Note that the energy change of the meteor will be the same regardless of whether the meteor falls directly towards the planet (Figure 4.3.5a) or follows a more indirect path (Figure 4.3.5b).

![Diagram](image)

**FIGURE 4.3.5** The change in gravitational potential energy will be the same whether the satellite takes a direct path (a) or a curved path (b).

**Worked example 4.3.3**

**CHANGES IN GRAVITATIONAL POTENTIAL ENERGY**

A meteor with a mass of 10 kg is orbiting the Earth with an orbital radius of \(2 \times 10^7\) m. It moves into a lower orbit with a radius of \(1 \times 10^7\) m. Calculate the change in gravitational potential energy of the satellite.

(Use \(m_{\text{Earth}} = 5.97 \times 10^{24}\) kg.)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
</table>
| Use the formula for gravitational potential energy to calculate the initial gravitational potential energy, \(U_i\). | \[
U_i = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 10}{2 \times 10^7} = -2.0 \times 10^8\ J
\] |
| Use the formula for gravitational potential energy to calculate the final gravitational potential energy, \(U_f\). | \[
U_f = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 10}{1 \times 10^7} = -4.0 \times 10^8\ J
\] |
| Calculate the change in gravitational potential energy. | \[
U = U_f - U_i = -4.0 \times 10^8 - (-2.0 \times 10^8) = -2.0 \times 10^8\ J
\] |

**Worked example: Try yourself 4.3.3**

**CHANGES IN GRAVITATIONAL POTENTIAL ENERGY**

A satellite with a mass of 500 kg is orbiting the Earth an orbital radius of 7100 km. It moves into a lower orbit at an altitude of 6800 km. Calculate the change in gravitational potential energy of the satellite.

(Use \(m_{\text{Earth}} = 5.97 \times 10^{24}\) kg.)
Using the force–distance graph

When a free-falling body is acted upon by a varying gravitational force, the energy changes of the body can also be analysed by using a gravitational force–distance graph. As with other force–distance graphs, the area under the graph is equal to the work done, i.e. the energy change of the body. The area under the graph has units of newton metres (Nm), which are equivalent to joules (J).

The shaded area in Figure 4.3.6 represents the decrease in gravitational potential energy of the 10 kg meteor as it falls from a distance of \(2.0 \times 10^7\) m to \(1.0 \times 10^7\) m from the centre of the Earth. This area also represents the amount of kinetic energy that the meteor gains as it approaches Earth.

According to the law of conservation of mechanical energy, when an object loses gravitational potential energy, its kinetic energy must increase. This is also true for satellites.

Consider the 10 kg meteor discussed above. When it moves from a higher orbit to a lower orbit, it loses 200 MJ of gravitational potential energy. It will correspondingly gain 200 MJ of kinetic energy and therefore orbit at a higher speed.

Escape velocity

When an object like a rocket has enough kinetic energy to escape the Earth’s gravitational field, it is said to have reached escape velocity.

Since an object that has escaped a gravitational field has a potential energy of \(U = 0\), due to the conservation of mechanical energy the escape velocity occurs when the kinetic energy of the object is equal in magnitude to its gravitational potential energy. In other words, when \(K = U\):

\[
\frac{1}{2} m v_{\text{esc}}^2 = \frac{G M m}{r}
\]

\[
v_{\text{esc}}^2 = \frac{2 G M}{r}
\]

The escape velocity is given by the equation:

\[
v_{\text{esc}} = \sqrt{\frac{2 G M}{r}}
\]

where

- \(M\) is the mass of the Earth (or, more generally, the central body; in kg)
- \(G\) is the gravitational constant, \(6.67 \times 10^{-11}\) Nm\(^2\)kg\(^{-2}\)
- \(r\) is the radius of the orbit of the object (the sum of the radius of the central body and the altitude of the orbit; in m)

Note that escape velocity is independent of the mass of the object; i.e. regardless of how heavy the object is, its escape velocity will be the same.
Worked example 4.3.4

**ESCAPE VELOCITY**

Calculate the escape velocity for a 500 kg rocket being launched from the surface of the Earth. (Note: the Earth has a radius of $6.38 \times 10^6$ m and a mass of $5.97 \times 10^{24}$ kg.)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall the definition of the escape velocity.</td>
<td>$v_{esc} = \frac{\sqrt{GM}}{r}$</td>
</tr>
</tbody>
</table>
| Identify the information required, and convert values into appropriate units where necessary. | $G = 6.67 \times 10^{-11} \text{N m}^2 \text{kg}^{-2}$  
$M = 5.97 \times 10^{24} \text{kg}$  
$r = 6.38 \times 10^6 \text{m}$ |
| Substitute the values into the equation and solve for $v_{esc}$. | $v_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.38 \times 10^6}}$  
$= 1.12 \times 10^4 \text{m s}^{-1}$ |

Worked example: Try yourself 4.3.4

**ESCAPE VELOCITY**

Calculate the escape velocity for a 11900 kg space craft being launched from the surface of the Moon. (Note: the Moon has a radius of $1.74 \times 10^6$ m and a mass of $7.32 \times 10^{22}$ kg.)

**PHYSICS IN ACTION**

**ICT**

**Voyager space probes**

Two of the earliest human-made objects to achieve escape velocity are the Voyager space probes (Figure 4.3.7). These were launched in 1977 to explore the outer planets of the solar system. These probes have not only escaped the Earth’s gravitational field, but are also now escaping the Sun’s gravitational field as they travel out beyond the orbits of the most distant planets.

**FIGURE 4.3.7** The Voyager probes have achieved escape velocity from the Sun’s gravitational field.
4.3 Review

SUMMARY

• The gravitational potential energy formula \( U = mgh \) assumes that the Earth’s gravitational field is constant. This is approximately true for objects that are within a few kilometres of the Earth’s surface.

• The strength of the Earth’s gravitational field decreases as altitude increases.

• The gravitational field potential energy of an object in a non-constant gravitational field is given by

\[
U = -\frac{GMm}{r}
\]

• The total energy of an object in a non-constant gravitational field is given by

\[
E = K + U = \frac{-GMm}{2r}
\]

• The total energy of a satellite remains constant throughout its orbit.

• Escape velocity is the velocity required for an object to escape a gravitational field:

\[
v_{\text{esc}} = \sqrt{\frac{2GM}{r}}
\]

KEY QUESTIONS

Where necessary, assume that the Earth has a radius of \( 6.38 \times 10^6 \) m and a mass of \( 5.97 \times 10^{24} \) kg.

1. Which one of the following statements is correct?
   A satellite in a circular orbit around the Earth will have:
   A varying potential energy as it orbits
   B varying kinetic energy as it orbits
   C constant kinetic energy and constant potential energy

2. The path of a meteor plunging towards the Earth is as shown. Ignore air resistance when answering these questions.

   a How does the gravitational field strength of the Earth change from point A to point D?
   b How will the acceleration of the meteor change as it travels along the path shown?
   c Which one or more of the following statements is correct?
      A The kinetic energy of the meteor increases as it travels from A to D.
      B The gravitational potential energy of the meteor decreases as it travels from A to D.
      C The total energy of the meteor remains constant.
      D The total energy of the meteor increases.

3. The Saturn V rocket that took the first astronauts to the Moon had a mass of 3000 tonnes. Its Stage I rockets fired for 6 minutes and took the rocket to an altitude of 67 km. Calculate the gravitational potential energy of the rocket at its final altitude.

4. A communications satellite of mass 240 kg is launched from a space shuttle that is in orbit 600 km above the Earth’s surface. The satellite travels directly away from the Earth and reaches a maximum distance of 8000 km from the centre of the Earth before stopping due to the influence of the Earth’s gravitational field. Calculate the change in gravitational potential energy of this satellite as it was launched.

5. Calculate the escape velocity (in \( \text{km} \text{s}^{-1} \)) for a spacecraft leaving Mars which has a radius of 3390 km and a mass of \( 6.42 \times 10^{23} \) kg.
Chapter review

KEY TERMS

acceleration due to gravity  
altitude  
aphelion  
apogee  
apparent weightlessness  
artificial satellite  
escape velocity  
field  
geostationary satellite  
gravimeter  
gravitational constant  
gravitational field  
gravitational field strength  
gravitational force  
gravitational potential energy  
inverse square law  
natural satellite  
Newton’s law of universal gravitation  
perigee  
perihelion  
satellite  
uniform

KEY QUESTIONS

Where necessary, assume that the Earth has a radius of $6.4 \times 10^6$ m and a mass of $6.0 \times 10^{24}$ kg.

1. Use Newton’s law of universal gravitation to calculate the gravitational force acting on a person with a mass of 75 kg.

2. The gravitational force of attraction between Saturn and Dione, a moon of Saturn, is equal to $2.79 \times 10^{20}$ N. Calculate the orbital radius of Dione. Use the following data:
   mass of Dione = $1.05 \times 10^{21}$ kg
   mass of Saturn = $5.69 \times 10^{26}$ kg

3. Of all the planets in the solar system, Jupiter exerts the largest force on the Sun: $4.2 \times 10^{23}$ N. Calculate the scalar acceleration of the Sun due to this force, using the following data: $m_{\text{Sun}} = 2.0 \times 10^{30}$ kg.

4. The acceleration of the Moon caused by the gravitational force of the Earth is much larger than the acceleration of the Earth due to the gravitational force of the Moon. What is the reason for this?

5. Calculate the acceleration due to gravity on the surface of Mars if it has a mass of $6.4 \times 10^{23}$ kg and a radius of 3400 km.

6. A comet of mass 1000 kg is plummeting towards Jupiter. Jupiter has a mass of $1.90 \times 10^{27}$ kg and a planetary radius of $7.15 \times 10^7$ m. If the comet is about to crash into Jupiter, calculate the:
   a. magnitude of the gravitational force that Jupiter exerts on the comet
   b. magnitude of the gravitational force that the comet exerts on Jupiter
   c. acceleration of the comet towards Jupiter
   d. acceleration of Jupiter towards the comet.

7. A person standing on the surface of the Earth experiences a gravitational force of 900 N. What gravitational force will this person experience at a height of two Earth radii above the Earth’s surface?
   A 900 N  
   B 450 N  
   C 100 N  
   D zero

8. Calculate the weight of a 65 kg cosmonaut standing on the surface of Mars, given that the planet has a mass of $6.4 \times 10^{23}$ kg and a radius of $3.4 \times 10^6$ m.

9. During a space mission, an astronaut of mass 80 kg initially accelerates at $30 \text{ m s}^{-2}$ upwards, then travels in a stable circular orbit at an altitude where the gravitational field strength is $8.2 \text{ N kg}^{-1}$.
   a. What is the total force acting on the astronaut during lift-off?
      A zero  
      B 660 N  
      C 780 N  
      D 3200 N
   b. During the lift-off phase, the astronaut will feel:
      A lighter than usual  
      B heavier than usual  
      C the same as usual
   c. During the orbit phase, the gravitational force acting on the astronaut is:
      A zero  
      B 660 N  
      C 780 N  
      D 3200 N

10. What are the main steps to follow when drawing gravitational field lines?

11. A group of students use a spring balance to measure the weight of a 150 g set of slotted masses to be 1.4 N. According to this measurement, what is the gravitational field strength in their classroom?

12. The Earth is a flattened sphere. Its radius at the poles is 6357 km compared to 6378 km at the equator. The Earth’s mass is $5.97 \times 10^{24}$ kg.
   a. Calculate the Earth’s gravitational field strength at the equator.
   b. Using the information in part (a), calculate how much stronger the gravitational field would be at the North Pole compared with the equator. Give your answer as a percentage of the strength at the equator.
13 Two stars of masses $M$ and $m$ are in orbit around each other. As shown in the following diagram, they are a distance $R$ apart. A spacecraft located at point $X$ experiences zero net gravitational force from these stars. Calculate the value of the ratio $\frac{M}{m}$.

![Diagram showing two stars and a spacecraft at point X](image)

14 Give the most appropriate units for measuring gravitational field strength.

15 There are bodies outside our solar system, such as neutron stars, that produce very large gravitational fields. A typical neutron star can have a mass of $3.0 \times 10^{30}$ kg and a radius of just 10 km. Calculate the gravitational field strength at the surface of such a star.

16 A newly discovered solid planet located in a distant solar system is found to be distinctly non-spherical in shape. Its polar radius is 5000 km, and its equatorial radius is 6000 km.

The gravitational field strength at the poles is $8.0 \text{ N kg}^{-1}$. How would the gravitational field strength at the poles compare with the strength at the equator?

17 An astronaut travels away from Earth to a region in space where the gravitational force due to Earth is only 1.0% of that at Earth’s surface. What distance, in Earth radii, is the astronaut from the centre of the Earth?

18 Which of the following are properties of a geostationary satellite?

A It is in a low orbit.

B It orbits the Earth once every 24 hours.

C It passes over the north pole.

D It is weightless.

19 Complete the table below, which contains information about the largest four moons of Jupiter:

<table>
<thead>
<tr>
<th>Name</th>
<th>Radius of orbit ($\times 10^3$ km)</th>
<th>Period of orbit (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io</td>
<td>422</td>
<td>42.5</td>
</tr>
<tr>
<td>Europa</td>
<td>671</td>
<td></td>
</tr>
<tr>
<td>Ganymede</td>
<td>1070</td>
<td></td>
</tr>
<tr>
<td>Callisto</td>
<td>1883</td>
<td></td>
</tr>
</tbody>
</table>

20 Calculate the gravitational field strength experienced by a satellite orbiting at an altitude of 19000 km.

21 Titan is the largest Moon of Saturn. It orbits Saturn once every 15.9 days at an average distance of $1.22 \times 10^6$ km.

a) Calculate the orbital speed of Titan.

b) Use these data to calculate the mass of Saturn.

22 a) Determine the gravitational potential energy of a 1.0 kg mass 100 km above the Earth’s surface.

b) Determine the total energy of a 1.0 kg mass 100 km above the Earth’s surface.

23 A 20 tonne remote-sensing satellite is in a circular orbit around the Earth at an altitude of 600 km. The satellite is moved to a new stable orbit with an altitude of 2600 km. Determine the increase in the gravitational potential energy of the satellite as it moved from its lower orbit to its higher orbit.

24 A 20 kg rock is speeding towards Mercury. The radius of Mercury is $2.4 \times 10^6$ m and its mass is $3.3 \times 10^{23}$ kg. Calculate the:

a) gravitational field strength at $3.0 \times 10^6$ m from the centre of Mercury

b) gravitational potential energy of the rock at $3.0 \times 10^6$ m from the centre of Mercury

c) decrease in gravitational energy of the rock as it moves to a point that is just $2.5 \times 10^6$ m from the centre of Mercury.

25 A wayward satellite of mass 1000 kg is drifting towards the Earth. How much kinetic energy does the satellite gain as it travels from an altitude of 600 km to an altitude of 200 km?

26 Calculate the escape velocity (in km s$^{-1}$) for a spacecraft leaving Mercury which has a radius of 2440 km and a mass of $3.30 \times 10^{23}$ kg.

27 After completing the activity on page XX, reflect on the inquiry question: How does the force of gravity determine the motion of planets and satellites?