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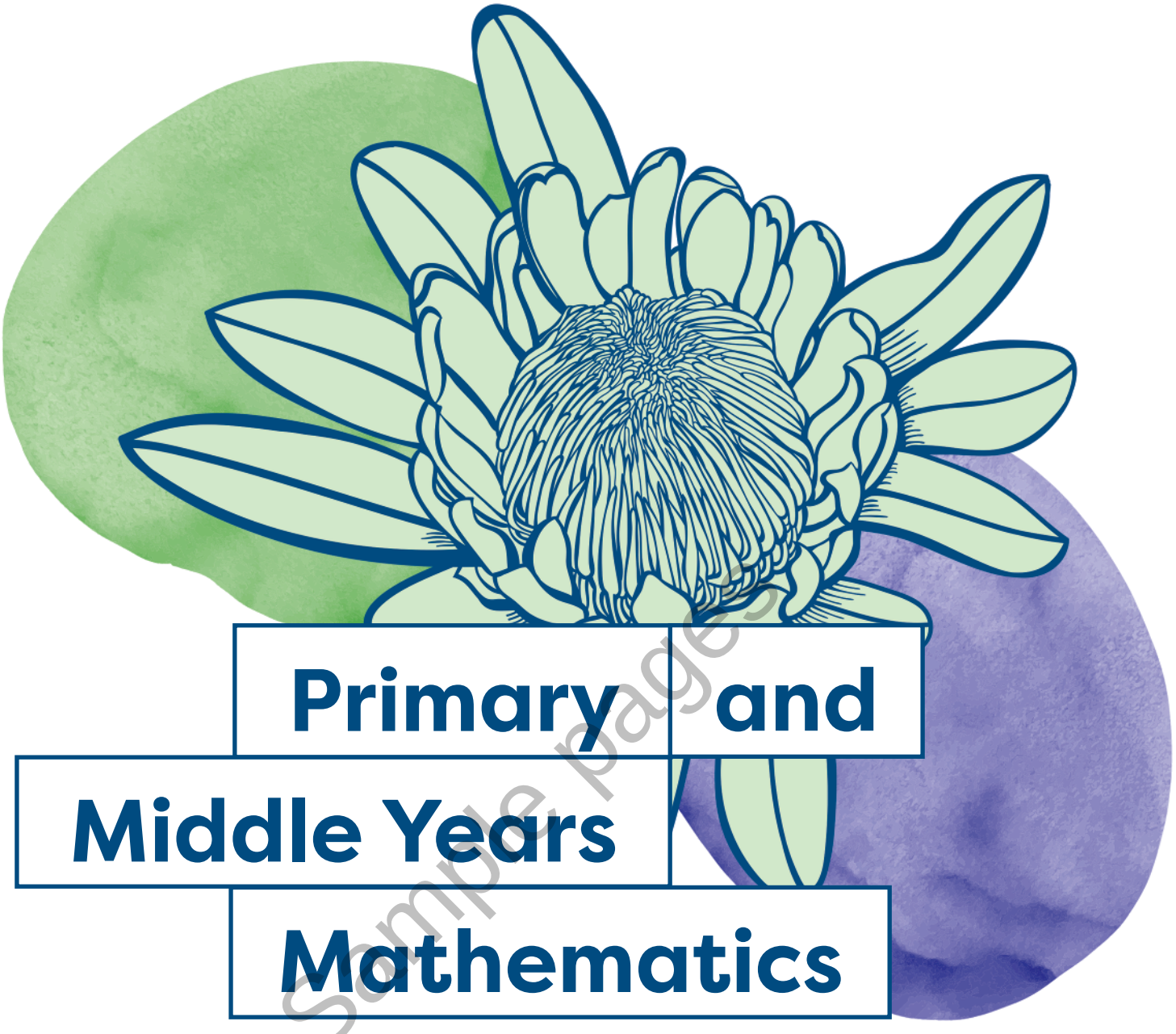

Primary and Middle Years Mathematics

Teaching Developmentally

SECOND AUSTRALIAN EDITION

CONTENTS

SECTION 1 TEACHING MATHEMATICS: FOUNDATIONS AND PERSPECTIVES

The fundamental core of effective instruction of mathematics combines an understanding of how students learn, how to promote that learning by teaching through problem-solving and how to plan for and assess that learning on a daily basis. Introductory chapters in this section provide perspectives on trends in mathematics education and the process of doing mathematics. These chapters develop the core ideas of learning, teaching, planning and assessment. Additional perspectives on mathematics for students with diverse backgrounds are also emphasised.

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CHAPTER 3

SECTION 2 DEVELOPMENT OF MATHEMATICAL CONCEPTS AND PROCEDURES

This section serves as the application of the core ideas discussed in Section 1. Here you will find chapters on major content areas in the Foundation to Year 9 Australian Curriculum: Mathematics (Version 9.0). Numerous problem-based activities to engage students are interwoven with a discussion of the mathematical content and how students develop their understanding of that content. At the outset of each chapter, you will find a listing of 'Big ideas', the mathematical umbrella for the chapter. Also included are ideas for incorporating children's literature, tips for including technology and formative assessment notes. These chapters are designed to help you develop pedagogical strategies and to serve as a resource for your mathematics instruction now and in the future.

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CHAPTER 1

Teaching mathematics for the 21st century

LEARNING OBJECTIVES

After reading this chapter and engaging in the embedded activities and reflections, you should be able to:

LO 1.1 Summarise the factors that influence the teaching of mathematics. (AITSL 1.2, 2.1)

LO 1.2 Describe the important elements that comprise the Australian Curriculum: Mathematics. (AITSL 2.1, 2.3, 2.5)

LO 1.3 Explore the qualities needed to learn and grow as a professional teacher of mathematics. (AITSL 6.4, 7.2, 7.4)

Someday soon you will find yourself in front of a class of students, or perhaps you are already teaching. What general ideas will guide the way you teach mathematics? This text will help you become comfortable with the content of the Foundation–Year 9 Australian Curriculum: Mathematics, particularly the content of Foundation–Year 6. You will also learn about research-based strategies for helping students come to understand mathematics and be confident in their ability to do mathematics. It is essential that you have a thorough personal knowledge of mathematics along with an understanding of how students learn mathematics. Combined, these are the most important tools you can acquire to be successful in encouraging all students to become life-long learners of mathematics. Mathematics understanding is vital for many career paths and is important if people are to be fully functioning participants in personal, social and civic life.

Becoming an effective teacher of mathematics

Before we get started, think back to when you were in Foundation–Year 9 classrooms as a student. What are your memories of learning mathematics? Here are some thoughts from in-service and pre-service teachers of whom we asked the same question. Which description resonates with you?

> I was good at maths in lower primary year levels, but because I never understood why maths works, it made it very difficult to embrace the concepts as I moved into higher year levels. I started believing I wasn't good at maths, so I didn't get upset when my grades reflected that. *Kathryn*
>
> I had a wonderful teacher who made maths fun and relevant. We were encouraged to use materials and to talk about our thinking. Everyone's ideas were shared and respected. I loved maths! I also had a teacher who just wanted everything done the way they said. It didn't matter if you had an alternative way of thinking that was correct. If it wasn't their way, then it was wrong. I hated maths then. *Ainslee*
>
> As a student, I always felt lost during mathematics lessons. It was as if everyone around me had a magic key or code that I missed out on getting. *Ahmed*
>
> I remember maths being very challenging, intimidating and capable of making me literally sick to my stomach. Maths was a bunch of rules and formulas I was expected to memorise, but not to understand. *Hung*

I consider myself to be good at maths and I enjoy mathematics-related activities, but I often wonder if I would have been GREAT at maths and had a completely different career if I had cared about maths as much as I do now. Sometimes I feel robbed. *Jennet*

Maths went from engaging, interactive instruction that I excelled at and loved, to lecture-style instruction that I struggled with. I could not seek outside help, even though I tried, because the teacher's way was so different from the way of the people trying to help me. I went from being in the top group to struggling without knowing how the change happened. *Janelle*

Maths class was full of elimination games where students competed against each other to see who could answer maths facts the fastest. Because I have a good memory I did well, but I hated every moment. It was such a nerve-wracking experience and for the longest time that is what I thought maths was. *George*

Maths was never a problem because it was logical, everything made sense. *Tula*

As you can see, these memories reflect a range of emotions and experiences. The question now becomes, what do you hope your students will say as they think back to your mathematics teaching? The challenge is to get all your students to learn mathematics with understanding and enthusiasm. Would you relish hearing your students, 15 years after leaving your classroom, state that you encouraged them to be mathematically minded, curious about solving new problems, self-motivated, able to critically think about both correct and incorrect strategies, and that you nurtured them to be risk-takers willing to try and to persevere with challenging tasks? What will your legacy be?

As part of your personal desire to build successful learners of mathematics, you might recognise that mathematics is sometimes seen as the subject that people love to hate. At social events of all kinds – even at family conferences with teachers – other adults will respond to the fact that you are a teacher of mathematics with comments such as, 'I could never do maths' or 'I can't even balance my bank account'. Instead of dismissing these disclosures, consider what positive action you can take. Would people confide that they don't read and hadn't read a book in years? Not likely. Families' and teachers' attitudes towards mathematics may enhance or detract from students' ability to do maths. It is important for you and for students' families to know that mathematics ability is not inherited – anyone can learn mathematics. Moreover, learning mathematics is an essential life skill. You need to find ways of countering these statements, especially if they are stated in the presence of students, pointing out that it is a myth that only some people can be successful in learning mathematics. Only in that way can the chain of passing apprehension from family member to child, or in rare cases from teacher to student, be broken. There is much joy to be had in solving mathematical problems, and you need to model this excitement and nurture that passion in your students.

Your students need to think of themselves as mathematicians in the same way that many of them think of themselves as readers. As students interact with our increasingly mathematical and technological world, they need to construct, modify, communicate or integrate new information in many forms. Solving novel problems and approaching circumstances with a mathematical perspective should come as naturally as reading new materials to comprehend facts, insights or news. Consider how important this is to interpreting and successfully surviving in our economy and in our environment.

The goal of this text is to help you understand the mathematics methods that will make you an effective teacher for the 21st century. As you consider the information in this text, your vision and confidence will grow.

What influences the mathematics we teach?

LO 1.1 Summarise the factors that influence the teaching of mathematics. (AITSL 1.2, 2.1)

How you are teaching mathematics should not look anything like the teaching of mathematics 50 years ago or even 20 years ago. Many factors influence these changes. Here, we briefly discuss four major reasons.

The world is changing

In his book *The world is flat* (2007), Thomas Friedman discusses the need for people to have lasting skills that will survive the ever-changing landscape of available jobs. People with these skills sit within a group called 'untouchables' as, regardless of the shifting landscape of career options, they will be successful in finding jobs. Friedman points out that in a world that is digitised and surrounded by algorithms, mathematics lovers will always have career opportunities and options. The current skills gap in science, technology, engineering and mathematics (STEM) careers means these jobs take more than twice as long to fill as other jobs in the marketplace (Rothwell, 2014). This is also aligned with the thinkers who believe students need to not just be university ready but innovation ready (Wagner, 2012).

It is the job of every teacher of mathematics to equip students with skills for both their potential careers and their personal lives, and to develop a 'love of maths' in students. Lynn Arthur Steen, a well-known mathematician and educator, stated: 'As information becomes ever more quantitative and as society relies increasingly on computers and the data they produce, an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time' (1997, p. xv).

The changing world influences what should be taught in Foundation–Year 9 mathematics classrooms. As we prepare students for careers that may not currently exist, we can predict that there are few jobs for people where they just do simple computation. This is the task of digital devices. We can also predict that there will be work that requires interpreting complex data, designing algorithms to make predictions and using STEM skills to approach new problems in a variety of ways.

As you prepare to help students learn mathematics for the future and to become lifelong learners of mathematics, it is important to have some perspective on the forces that effect change in the mathematics classroom. This chapter addresses the leadership that you, the teacher, will develop as you shape the mathematics experience for your students. Your beliefs about what it means to know and do mathematics, and about how students make sense of mathematics, will affect how you approach teaching and the understandings and skills your students take from the classroom.

Factors to consider

For more than two decades, mathematics education has been constantly changing. There have been significant reforms that reflect the technological and informational needs of our society, research on how students learn mathematics and the importance of providing opportunities to learn for all students, and ideas on how and what to teach from an international perspective. Just as we would not expect doctors to be using the same techniques and medicines that were prevalent when you were a child, so too are mathematics teachers' methods evolving and transforming via a powerful collection of expert knowledge about how the mind functions and how to design effective instruction (Wiggins, 2013).

There are several significant factors in this transformation. One factor is the public or political pressure for change in mathematics education due largely to information about student performance in national and international studies. These large-scale comparisons of student performance continue to make headlines, provoke public opinion and put pressure on governments to call for tougher standards backed by testing. The demands of testing policies exerted on schools and, ultimately, on teachers may have an impact on teaching. These studies are important because international and national assessments provide strong evidence that mathematics teaching *must* change if our students are to be competitive in the global market and able to understand the complex issues they must confront as responsible citizens.

National Assessment Program – Literacy and Numeracy (NAPLAN). NAPLAN is an annual assessment conducted for all students in Years 3, 5, 7 and 9 (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016a). The development and delivery of the tests is administered by ACARA (Australian Curriculum, Assessment and Reporting Authority),

and since 2016 the tests have been aligned to the Australian Curriculum. The first NAPLAN tests took place in 2008 (ACARA, 2016a) and this was the first time all students in Australia in Years 3, 5, 7 and 9 were assessed in literacy and numeracy using the same year-level tests. NAPLAN replaced the different tests administered by Australian states and territories and improved the comparability of students' results across all states and territories (ACARA, 2016a).

In 2022, NAPLAN tests were administered online for the first time, and in 2023 were rescheduled to take place in March. NAPLAN Online is a tailored or adaptive assessment that provides opportunities not possible with paper tests. For example, students will answer the same first question and, depending on their response (correct or incorrect), the next question is either more difficult (for a correct response) or less difficult (for an incorrect response). This results in a more effective assessment approach and more efficient reporting of student results (National Assessment Program, 2022).

Data about student performance on the tests are provided to schools and families. Students' results reflect the achievement of the student in comparison to the national median scores across 10 bands. Data from NAPLAN tests are powerful tools for teachers and school leadership teams as they examine how to improve the learning of their students. Moving NAPLAN into Term 1 ensures student data and information will be available to teachers and school leaders sooner. They will have more time to analyse the data and tailor mathematics teaching to better meet the needs of students. This shift can ultimately lead to more effective teaching and learning, and better academic results for students across primary and secondary schools.

The numeracy NAPLAN test consists of multiple choice and constructed response questions. Calculators are not allowed on NAPLAN tests in the primary years, but in Year 7 and Year 9, there are two tests: one where calculators are allowed and one where they are not (ACARA, 2016b). Content is assessed across all six strands of the Australian Curriculum: Mathematics. There are also questions that cover the curriculum's first key consideration, proficiency in mathematics (understanding, fluency, reasoning, problem-solving). Example tests and previous tests can be downloaded at the National Assessment Program (www.nap.edu.au) website.

Trends in International Mathematics and Science Study (TIMSS). In the mid-1990s, 41 nations participated in the Third International Mathematics and Science Study, the largest study of mathematics and science education ever conducted. Data were gathered in Years 4, 8 and 12 from 500,000 students as well as from teachers. The results revealed that Australian students performed above the international average of the TIMSS countries in Years 4 and 8.

TIMSS studies have been repeated every four years since 1995, with the most recent occurring in 2019. The results showed that in both Year 4 and Year 8, Australia performs in the middle of the scale internationally, and there was no change in the mean score from 2015 to 2019. In particular, Australian Year 4 students performed relatively better in the content domain of *Data* but were relatively weaker in *Number*. The score for *Measurement and geometry* was not significantly different to the overall mathematics score. Australian Year 8 students performed relatively better in the content domains of *Number* and *Data and probability* but were relatively weaker in *Algebra* and *Geometry* (Thomson et al., 2020).

Australian Association of Mathematics Teachers (AAMT). The Australian Association of Mathematics Teachers (AAMT) was formed in 1966 with the key aims of supporting and enhancing the work of teachers, promoting the learning of mathematics, and representing and promoting interests in mathematics education (Australian Association of Mathematics Teachers [AAMT], n.d.). Comprising a federation of associations from each state and territory, the AAMT played a major part in the development of the Australian Curriculum: Mathematics and continues to play a role in other major curriculum and assessment projects. The association offers a range of services and products. In particular, the AAMT website provides a link to resources for Australian mathematics teachers (www.mathseducation.org.au/). There are further links to *Top Drawer Teachers*, which has some excellent resources for teaching at all year levels; *Maths300*, which offers a library of inquiry-based lessons for teaching at all year levels; *Problemo*,an online mathematics problem-solving resource; and *re(Solve)*, which offers lesson plans that support the teaching of the content of the Australian Curriculum: Mathematics.

National Council of Teachers of Mathematics (NCTM). One transformative factor is the professional leadership of the National Council of Teachers of Mathematics (NCTM). The NCTM has more than 60,000 members and is the world's largest mathematics education organisation. This organisation, which is based in the United States, holds an influential role in the support of teachers and places an emphasis on what is best for learners. The research published in its journals and the resources produced and endorsed by NCTM can support the Australian Curriculum (ACARA, 2022) and complement the teaching and learning of Australian students. NCTM publish a peer-reviewed teacher journal, *Mathematics Teacher: Learning and Teaching PK-12*, which has high-quality teaching ideas for extending teachers' understanding of practice and theory. Visit the NCTM website (www.nctm.org). Other resources include the NCTM Illuminations website (https://illuminations.nctm.org), which provides over 700 lesson plans for teachers and more than 50 interactive virtual tools for your students.

The Australian Curriculum: Mathematics

LO 1.2 Describe the important elements that comprise the Australian Curriculum: Mathematics. (AITSL 2.1, 2.3, 2.5)

Education in Australia has historically been a state or territory responsibility. Traditionally, while there were similarities across all learning areas, there were also some key differences. These differences comprised a range of issues between states and territories, such as the starting age of primary school students and the year students commenced secondary school, which caused problems when students moved states.

In December 2008, the Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA) released the *Melbourne Declaration on Educational Goals for Young Australians* (2008), which set the foundation for the development of an Australian Curriculum. Since its introduction in 2008, the Australian Curriculum has been reviewed and revised multiple times. The most recent iteration, Version 9, was endorsed for use in April 2022.

Version 9 of the Australian Curriculum (ACARA, 2022) affirms the vision of the Education Council's *Alice Springs (Mparntwe) Education Declaration* (2019), which was signed by all Australian education ministers at the time. The Declaration provides a vision for education in Australia, with the aim of improving educational outcomes for young Australians. For example:

> Every student must develop strong literacy and numeracy skills in their earliest years of schooling, and go on to develop broad and deep knowledge across a range of curriculum areas. (p. 2)

As part of the Declaration, all Australian governments agreed to a range of actions that encompass 'promoting world-class curriculum and assessment' (p. 9), including the development of knowledge and understanding in 10 learning areas, with mathematics being one of these (Education Council, 2019).

The Australian Curriculum: Mathematics includes expectations for the compulsory years (approximately Foundation–Year 10), although ACARA acknowledges that while the content of the curriculum is described in levels that coincide with the years of schooling, at any year level there will be students with a range of understanding. Similarly, the *Alice Springs (Mparntwe) Education Declaration* recognises a need for the Australian Curriculum to set learning goals for students while supporting them to progress, regardless of their starting point. All learning areas, including the general capabilities, are designed to equip students with knowledge and skills to contribute to society and Australia's economic prosperity (Education Council, 2019).

In summary, the Australian Curriculum: Mathematics (ACARA, 2022) seeks to make certain that students:

- become confident, proficient and effective users and communicators of mathematics, who can investigate, represent and interpret situations in their personal and work lives, think critically, and make choices as active, engaged, numerate citizens
- develop proficiency with mathematical concepts, skills, procedures and processes, and use them to demonstrate mastery in mathematics as they pose and solve problems, and reason with number, algebra, measurement, space, statistics and probability

- make connections between areas of mathematics and apply mathematics to model situations in various fields and disciplines
- foster a positive disposition towards mathematics, recognising it as an accessible and useful discipline to study
- acquire specialist mathematical knowledge and skills that underpin numeracy development and lead to further study in mathematics and other disciplines. (ACARA, 2022)

The Australian Curriculum aims to provide students with essential mathematical skills and knowledge along with the 'numeracy capabilities that all students need in their personal, work and civic lives, and provides the fundamentals on which mathematical specialties and professional applications of mathematics are built' (ACARA, 2022). The curriculum emphasises that mathematics is for everyone, and students can engage in mathematics through creativity and efficiency as they develop mathematical understanding. Further information about the Australian Curriculum can be found at the ACARA website (https://v9.australiancurriculum.edu.au).

Structure of the Australian Curriculum: Mathematics

The Australian Curriculum: Mathematics is structured under six strands: number, algebra, measurement, space, statistics and probability (see Figure 1.1). Although the strands are structured by year level, there is no intention that every student in a particular year will have the same level of competence in a given strand; there will be many students above the level described for the year and many below it. Classroom programs and student learning activities should be designed with this in mind.

Within the Australian Curriculum, each year level includes *content descriptions*, which are organised by each of the six strands. The content descriptions outline the concepts and processes that are the mathematical focus of the year level. *Elaborations* for each content description, which provide further clarification of the intent of the content, are also provided. Icons within the curriculum provide the connections to relevant *general capabilities* – literacy, numeracy, critical and creative thinking, digital literacy and ethical understanding – for each content description. Additionally, there are links to *related content* which connect the content description with other learning areas.

There is also an *achievement standard* at each year level, which is a statement outlining the key concepts and processes that summarise the level. The achievement standards follow the order of the six strands in each year level and integrate the relevant expectations for aspects of proficiency in mathematics (understanding, fluency, reasoning and problem-solving).

The six strands and the achievement standard are connected to five key considerations:

- proficiency in mathematics (understanding, fluency, reasoning, problem-solving)
- mathematical processes (mathematical modelling, computational thinking, statistical investigation, probability experiments and simulations)

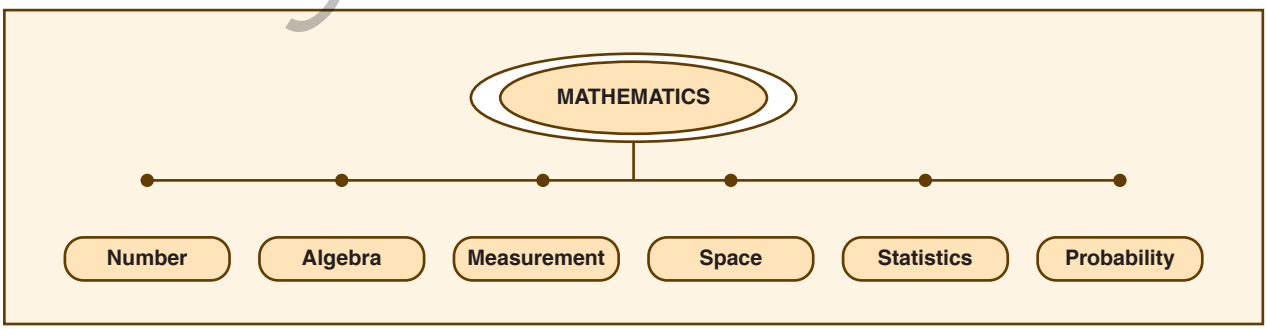

FIGURE 1.1 The six strands of the Australian Curriculum: Mathematics

Source: Copyright © 2010 to present by the Australian Curriculum, Assessment and Reporting Authority (ACARA).

- computation, algorithms and the use of digital tools
- protocols for engaging First Nations Australians
- meeting the needs of diverse learners.

Teachers should keep these key considerations in mind when planning for teaching and student learning.

Proficiency in mathematics, the first of the five key considerations, collates the four proficiency strands (understanding, fluency, reasoning, problem-solving) from the earlier versions of the Australian Curriculum. These are embedded in the content descriptions and achievement standards for the strands and should be considered when planning and teaching each strand. This approach aims to develop students' mathematical proficiency more broadly.

The strands

Number. Fundamental to the number strand is students' understanding and ability to work flexibly with our number system. This includes initially working with whole numbers and moving to decimal and common fractions in the upper primary levels. Within this strand, students count, order and represent numbers as well as investigate the importance and structure of the base-ten place-value system. Calculation strategies, both mental and written, are explored through reasoning strategies and formal algorithms. Estimation skills are also developed, and the modelling of real-world situations is encouraged.

Algebra. The algebra strand develops ways to think about relationships. This includes reasoning about and describing patterns, making generalisations and articulating relationships. As such, students are encouraged to use 'symbols and symbolic representations to think and reason about relationships in both mathematical and real-world contexts' (ACARA, 2022). Students connect symbolic, graphic and numeric representations, develop increasingly sophisticated concepts of variables and functions, and solve equations and inequalities as they progress through this strand.

Measurement. In this strand, students quantify aspects of shapes and objects in the human and physical world (ACARA, 2022). Selecting what to measure, understanding how to measure and selecting units for measuring are relevant and important features of this strand. Additionally, measurement as the foundation for decision-making in different contexts is emphasised. Within this strand, students estimate and measure attributes, and reason to formulate rules for derived units such as area and volume. Students use their fluency with measurement to investigate and solve real-world problems, and it is imperative that students are involved with hands-on activities in this strand – this is important in all strands but essential when students learn measurement concepts. The measurement aspects particularly allow students to consolidate their fluency with number.

Space. The space strand focuses on 'visualising, representing and working with the location, direction, shape, placement, proximity and transformation of objects' (ACARA, 2022). Objects in both natural and constructed worlds are considered within this strand, and movement in space and how it affects the properties of shapes and objects are investigated. Students also explore ideas of direction, location, dimension, symmetry, congruence and similarity through the space strand.

Statistics. Data pervades our world, and it is imperative that students can examine data and critically assess interpretations of data in a variety of forms. Within this strand, students investigate different ways to collect, represent, describe and interpret data. In doing so, students develop statistical literacy, which 'requires an understanding of statistical information and processes, including an awareness of data and the ability to estimate, interpret, evaluate and communicate with respect to variation in the real world' (ACARA, 2022).

Probability. In this strand, students investigate situations that involve uncertainty and chance. They solve problems that require interpretation of chance and make predictions. Through this strand, students investigate how likely events are to occur in experimental and theoretical ways.

This allows 'students to understand contexts involving chance and to build mathematical models surrounding risk and decision-making in a range of areas of human endeavour' (ACARA, 2022).

The key considerations. There are five key considerations in the Australian Curriculum (ACARA, 2022) that teachers should keep in mind when planning for teaching and student learning. These key considerations are briefly described in this section.

Proficiency in mathematics. Being proficient in mathematics helps students to use mathematics to make informed decisions and respond to various situations (ACARA, 2022). Aspects of proficiency in mathematics include understanding, fluency, reasoning and problem-solving. These aspects are described in detail in the section 'Proficiency in mathematics'.

Mathematical processes. The mathematical processes help foster mathematical thinking, problem-solving and investigating with students. There are four mathematical processes included in this key consideration: mathematical modelling, computational thinking, statistical investigation and probability experiments and simulations. These processes develop students' abilities to make assumptions; analyse, generalise and communicate mathematical results; identify patterns; investigate statistical questions; and evaluate their reasoning (ACARA, 2022).

Computation, algorithms and the use of digital tools in mathematics. The Australian Curriculum emphasises the various ways in which computations are used in mathematics, including calculations, graphs and diagrams. Further, the curriculum stresses that as students 'develop a conceptual understanding of how an algorithm works and fluency with using algorithms appropriately, they can reason and solve problems using algorithms as part of a computational thinking process' (ACARA, 2022). Additionally, when considering computational thinking, students should be able to use digital tools purposefully and effectively.

Protocols for engaging First Nations Australians. Recognising and respecting the cultures, teachings and learnings of First Nations Australians is central to this key consideration (ACARA, 2022). There are ideas for making connections to First Nations Australians' stories, games and teachings throughout the content elaborations in the Australian Curriculum.

Meeting the needs of diverse learners. As teachers plan learning experiences for students, they should consider the diversity of their students' languages, cultures, learning styles, abilities and talents. 'The Australian Curriculum values diversity by providing for multiple means of representation, action, expression and engagement, and allows schools the flexibility to respond to the diversity of learners within their community' (ACARA, 2022). Responding to diversity in the mathematics classroom can involve making connections between mathematics and familiar experiences for students as well as providing opportunities for all students to deepen their mathematical understanding.

Proficiency in mathematics

As described earlier, the first of the key considerations in the Australian Curriculum is proficiency in mathematics (ACARA, 2022). The components of proficiency in mathematics – understanding, fluency, reasoning and problem-solving – describe the actions in which students can engage when learning and using the mathematics content. While not all mathematical concepts apply to every content description, the four components indicate the breadth of mathematical actions that teachers can incorporate into their teaching and student learning to develop proficiency in mathematics. Here, we provide more detail about the components of proficiency in mathematics.

Understanding. Understanding focuses on students making meaning of mathematics through making connections, interpreting mathematical information and representing mathematical ideas in different ways. To have true understanding, students can answer questions such as: Why does this work? How is multiplication related to addition or to division? Why do the denominators have to be the same when you add and subtract fractions? Why can a square also be described as a quadrilateral or a rectangle? What is the difference between having a zero chance as opposed to having an unlikely chance?

This component of proficiency in mathematics is fundamental to all other components. Students cannot be fluent if they are blindly following rules they do not understand. Problem-solving requires understanding so that problems can be interpreted and the mathematics can be employed to solve these problems. Understanding is also essential for students to reason as they explain their thinking.

Fluency. Fluency is about students learning to carry out procedures flexibly, accurately and appropriately; it is not so much about following the formal algorithms. For example, 29×31 can be thought of as $30 \times 31 - 1 \times 31$, which is much more appropriate if mental strategies are being employed. Fluency is dependent on students understanding the structure of number and relationships between the operations. Building a knowledge of critical facts and having the ability to construct others from known facts is essential for fluency. For example, if students know the multiplication facts for 2 and 3, these facts can be used to generate multiplication facts for 4, 6, 8 and 9.

Reasoning. Reasoning aims to develop an increasingly sophisticated capacity for justifying and explaining results as students develop an ever-increasing ability to explain their mathematical thinking. Examples of reasoning in the different content strands might be drawing on known number facts to deduce and describe patterns, explaining general properties of classes of shapes and objects, and justifying interpretation of data or why a game is fair or not fair.

Problem-solving. Problem-solving involves students working mathematically to solve problems and conduct investigations. Students learn to construct questions and try out strategies to solve the problems. They also evaluate the success of their strategies and verify the accuracy of their solutions. As part of the problem-solving component of proficiency in mathematics, students learn how to use strategies as well as how to suggest and evaluate strategies of their own.

At each level of the curriculum, these components of proficiency in mathematics are embedded within the *achievement standards*. For example, in Year 3, problem-solving is embedded through using 'mathematical modelling to solve practical problems' and through the statistical investigations expected at this year level (ACARA, 2022). Additionally, reasoning is embedded in the achievement standard in Year 3 as students 'identify and describe outcomes and the likelihood of everyday events explaining reasoning' (ACARA, 2022).

Pause & reflect

Think about your own experiences in mathematics classrooms. How were the components of proficiency in mathematics emphasised in those classrooms? As you think about your own mathematics teaching, what connections can you make to develop students' proficiency in mathematics? How can you emphasise the actions and thinking that make up proficiency in mathematics in your mathematics classroom?

An invitation to learn and grow

LO 1.3 Explore the qualities needed to learn and grow as a professional teacher of mathematics. (AITSL 6.4, 7.2, 7.4)

The mathematics education described in this text may not be the same as the mathematics content and teaching you experienced in primary school and early secondary school. As a practising or prospective teacher preparing to teach mathematics using a problem-solving approach, this text may require you to confront some of your personal beliefs – beliefs about what it means to *do mathematics*, how one goes about *learning mathematics*, how to *teach mathematics* and what it means to *assess mathematics*. Success in mathematics is not merely about speed or the notion that there is 'one right answer'. Thinking and talking about mathematics as a means to sense-making is a strategy that will serve us well in becoming a society where all citizens are confident in their ability to do mathematics.

Becoming a teacher of mathematics

This text is critical to your professional teaching career as either a prospective or in-service teacher. The mathematics education course you are taking now as a pre-service teacher or the professional development you are experiencing as a teacher or leader in mathematics aims to immerse you in the content you will teach while providing you with research-based knowledge about how students learn the Australian Curriculum. The authors of this text take that seriously, as we know you do. Therefore, this section describes some of the requirements for gaining accreditation as a teacher as well as some of the overall characteristics, habits of thought, skills and dispositions you will need to succeed as a teacher of mathematics.

Literacy and Numeracy Test for Initial Teacher Education Students (LANTITE). The Literacy and Numeracy Test for Initial Teacher Education Students (LANTITE) is an online test, delivered and managed by Australian Council for Education Research (ACER), that all students in an initial teacher education course must pass before graduation (ACER, 2018b). LANTITE assesses personal literacy and numeracy skills to ensure that all teachers graduating from undergraduate and postgraduate teaching courses have a high level of knowledge and understanding of literacy and numeracy. The numeracy test consists of two sections: the longer one where an online calculator is available and a shorter section where no calculator is available (ACER, 2018a). More information, including sample and practice tests, can be found on the ACER website (www.acer.org).

Australian Professional Standards for Teachers. The Australian Institute for Teaching and School Leadership (AITSL) is the national body for teacher registration, and the Australian Professional Standards for Teachers set achievement expectations for every stage of your teaching career (Australian Institute for Teaching and School Leadership [AITSL], 2017). The seven standards of teaching are designed to help you to develop your practice for teaching across all learning areas of the Australian Curriculum, including mathematics teaching. They are:

- Know your students and how they learn.
- Know the content and how to teach it.
- Plan for and implement effective teaching and learning.
- Create and maintain supportive and safe learning environments.
- Assess, provide feedback and report on student learning.
- Engage in professional learning.
- Engage professionally with colleagues, parents and caregivers and the community.

Prior to completion of your teacher education course, you are required to meet the Graduate Teacher Standards (GTS) of the Australian Professional Standards for Teachers (AITSL, 2017). In January 2018, AITSL endorsed the Graduate Teacher Performance Assessment (GTPA). It is supported by researchers, teacher educators, education authorities, industry leaders and union representatives. This is an externally moderated assessment, which teacher candidates complete in their final year of their teacher education course, and is necessary for teacher registration in several jurisdictions (Institute for Learning Sciences and Teacher Education, n.d.).

This text can help support your work in meeting the AITSL standards and, if required, your preparation for the GTPA. Throughout this text, relevant AITSL standards are connected to the learning outcomes at the beginning of each chapter. Additionally, these standards and their connections within the text can be found in Appendix C. This can help you see how the AITSL standards are incorporated in instruction and the ways they can be used to develop your teaching practice in mathematics. Illustrations of practice for each of the Australian Professional Standards for Teachers can also be found on the AITSL website (www.aitsl.edu.au).

Characteristics, habits of thought, skills and dispositions. Here, we describe characteristics, habits of thought, skills and dispositions that you will continue to cultivate to reach success as an effective teacher of mathematics and that, we hope, you will embrace as you read this text.

Knowledge of mathematics. You will need to have a profound, flexible and adaptive knowledge of mathematics content (Ma, 1999). This statement is not intended to scare you if you feel that mathematics is not your strength, but it is meant to help you prepare for a serious semester of learning about mathematics and how to teach it. You cannot teach what you do not know. Teachers are the driving factors in students' school performance (Colvin & Edwards, 2018). An absence of high-quality opportunities for students to gain mathematics knowledge can result in students having economic challenges and little potential for social mobility (Organisation for Economic Co-operation and Development [OECD], 2016). These findings add to the seriousness of your responsibility because a student's learning for the year in

mathematics will likely come only from you. If you are not sure of a fraction concept or other aspect of mathematics content knowledge, now is the time to make changes in your depth of understanding and flexibility with mathematical ideas to best prepare you for your role as a teacher of mathematics. This text, along with discussions with mathematics teacher educators, school leaders, teachers or professional development sessions, will help you in that process.

Persistence. You need the ability to stave off frustration and to demonstrate persistence. Dweck (2007) describes the brain as being similar to a muscle – one that can be strengthened with a good workout! People are not just wired for learning mathematics; they must perform hard work and persevere to understand new ideas. As you move through this text and work out the problems yourself, you will learn methods and strategies that will help you anticipate barriers to students' learning and identify strategies to get them past these stumbling blocks. It is likely that what works for you as a learner will work for your students. As you conduct this mental 'workout', if you ponder, struggle, talk about your mathematical thinking and reflect on how these new ideas fit or don't fit with your prior knowledge, then you will enhance your repertoire as a teacher. Remember, as you model these characteristics for your students, they too will begin to value perseverance more than speed. In fact, Einstein did not describe himself as intelligent – instead, he suggested he was just someone who continued to work on problems longer than others. Having positive beliefs on the role of struggle in the mathematics classroom (Russo et al., 2020) and creating opportunities for your students to productively struggle is part of the learning process (Stigler & Hiebert, 2009; Warshauer, 2015) and are discussed in greater depth in the chapter 'Exploring what it means to know and do mathematics' (Chapter 2).

Positive disposition. Prepare yourself by developing a positive attitude towards the subject of mathematics. Research shows that teachers with positive attitudes teach mathematics in more successful ways that result in their students liking mathematics more (Karp, 1991), performing at higher levels (Palardy & Rumberger, 2008) and developing confidence and skills that are personally meaningful and support the community in which they live. If you have ever thought, 'I never liked mathematics', that mindset will be evident in your instruction (Ramirez et al., 2018). The good news is that research shows that changing attitudes towards mathematics is relatively easy (Tobias, 1995) and that attitude changes are long-lasting (Dweck, 2006). By expanding your mathematical knowledge and trying new ways to approach problems, you too can learn to enjoy doing and presenting mathematical activities. Not only can you acquire a positive attitude towards mathematics, but as a professional it is essential that you do so.

To explore your students' attitudes towards mathematics, consider using the **Student interview: Attitudes and environment** protocol. Here, you can explore how the classroom environment may affect their attitudes.

Readiness for change. Demonstrate a readiness for change, even for change so radical that it may cause disequilibrium. You may find that what is familiar will become unfamiliar and, conversely, what is unfamiliar will become familiar. For example, you may have always referred to 'reducing fractions' as the process of changing $\frac{2}{4}$ to $\frac{1}{2}$, but this is misleading as the fractions are not getting smaller. Such terminology can lead to mistaken connections. A careful look will point out that *reducing* is not the term to use; rather, you are writing an equivalent fraction that is simplified or expressed in the lowest terms. Even though you have used the language *reducing* for years, you need to become familiar with more precise language such as 'simplifying fractions'.

On the other hand, what is unfamiliar will become more comfortable. You may feel uncomfortable asking students if anyone solved the problem differently, especially if you are worried that you might not understand their approach. Yet this question is essential to effective teaching. As you bravely use this strategy, it will become familiar (and you will learn new strategies).

Another potentially difficult shift in practice is towards an emphasis on concepts as well as procedures. What happens in a procedure-focused classroom when a student does not understand division of fractions? A teacher with only procedural knowledge is often left to repeat the procedure louder and slower, 'Just change the division sign to multiplication, flip over the second fraction and multiply'. We know the use of a memorised approach does not work well if we want students to fully understand the process of dividing fractions, so let's consider an example using $3\frac{1}{2} \div \frac{1}{2} =$. You might start by relating this division problem to prior

knowledge of a whole number division problem, such as $25 \div 5 =$. A corresponding story problem might be, 'How many orders of 5 lamingtons are there in a group of 25 lamingtons?' Then ask students to put words around the fraction division problem, such as, 'You plan to serve each guest $\frac{1}{2}$ a lamington. If you have $3\frac{1}{2}$ lamingtons, how many guests can you serve?' Yes, there are seven halves in $3\frac{1}{2}$ and therefore 7 guests can be served. Are you surprised that you can do this division of fractions problem in your head once you learn this connection?

To respond to students' challenges, uncertainties and frustrations, you may need to unlearn and relearn mathematical concepts, developing comprehensive conceptual understanding and a variety of representations along the way. Supporting your mathematics content knowledge on solid, well-supported terrain is your best hope of making a lasting difference in your students' learning of mathematics – so be ready for change. What you already understand will provide you with many 'Aha' moments as you read this text and connect new information to your current mathematics knowledge.

A team player. The teachers at your school should work as a team when supporting student learning and plan in a coherent manner across the year levels. As a team, you should agree to use the same mathematical language, symbols, models and notations to provide students with a connected approach that ties the concepts and procedures together year after year.

Self-aware and reflective. As Steve Leinwand wrote, 'If you don't feel inadequate, you're probably not doing the job' (2007, p. 583). No matter whether you are a pre-service teacher or an experienced teacher, there is always more to learn about the content and methodology of teaching mathematics. The ability to examine oneself for areas that need improvement or to reflect on successes and challenges is critical for growth and development. The best teachers are always trying to improve their practice through reading the latest article, reading the newest texts, attending the most recent conference or signing up for the next series of professional development opportunities. These teachers don't say, 'Oh, that's what I am already doing'; instead, they identify and celebrate each new insight. Highly effective teachers never 'finish' learning nor do they exhaust the number of new mental connections they make and, as a result, they never see teaching as stale or stagnant. An ancient Chinese proverb states, 'The best time to plant a tree is twenty years ago; the second-best time is today'. Explore the self-reflection chart on **Professional growth** to list your strengths and indicate areas for continued growth.

Think back to the quotations from teachers at the beginning of this chapter. Again, what memories will you create for your students? As you begin this adventure, let's be reminded of what John Van de Walle said with every new edition, 'Enjoy the journey!'

REFLECTIONS ON CHAPTER 1

Writing to learn

Assess your understanding and application of chapter content by answering the following questions.

1. What is meant by a *content* strand as referred to in the Australian Curriculum: Mathematics? Give a brief description of each of the six content strands and an example of a concept that might be explored in each.
2. What components are involved in developing proficiency in mathematics? Give an example of an activity or learning experience that would develop at least one of these components.
3. Explain what NAPLAN is and why it was introduced.
4. Discuss why it is vital that teachers know both how to teach mathematics as well as how to do mathematics.

For discussion and exploration

- Examine a mathematics website (e.g., re(Solve)) or school mathematics curriculum at any year level of your choice – if possible, use an educational website. Look for signs of the Australian Curriculum: Mathematics content. (As Version 9 of the curriculum was published in 2022, websites might be using a previous version of the curriculum. If that is the case, you can expect to see references to the proficiency strands as the components of proficiency in mathematics.) To what extent are students who are being taught from that resource likely to be doing and learning mathematics in ways described by those concepts or practices?

RESOURCES FOR CHAPTER 1

Recommended readings

Articles

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2022). Understanding this learning area: Mathematics. Retrieved from https://v9.australiancurriculum.edu.au/teacher-resources/understand-this-learning-area/mathematics#accordion-32985f1628-item-67ae667d20.

This website provides information on the organisation and content of the Australian Curriculum: Mathematics. It describes the aims and structure of the curriculum, including the content strands and key considerations. It links to other subject areas, the general capabilities and the cross-curriculum priorities.

Hoffman, L., & Brahier, D. (2008). Improving the planning and teaching of mathematics by reflecting on research.

Mathematics Teaching in the Middle School, 13(7), 412–417. This article addresses how teachers' philosophies and beliefs influence their mathematics instruction. Using TIMSS and NAEP studies as a foundation, the authors discuss posing higher-level problems, asking thought-provoking questions, facing students' frustration and using mistakes to enhance understanding of concepts. They suggest reflective questions that are useful for self-assessment or discussions with peers.

Karp, K., Bush, S., & Dougherty, B. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18–25.

This article helps students move away from overgeneralisations and rules that cause confusion while considering developing terminology and notation that enhance student understanding.

Websites

Australian Association of Mathematics Teachers (AAMT): www.aamt.edu.au

Australian Council for Educational Research (ACER): www.acer.org

Australian Curriculum: https://v9.australiancurriculum.edu.au

Australian Curriculum, Assessment and Reporting Authority (ACARA): www.acara.edu.au

Australian Institute for Teaching and School Leadership (AITSL): www.aitsl.edu.au

Australian mathematics teacher resources: www.mathseducation.org.au/

Australian Professional Standards for Teachers: www.aitsl.edu.au/teach/standards

Graduate Teacher Performance Assessment: www.graduatetpa.com

LANTITE information: https://teacheredtest.acer.edu.au/about

National Assessment Program (NAP); NAPLAN information: www.nap.edu.au/naplan

Scootle: www.scootle.edu.au/ec/p/home