STUDENT WORKED SOLUTIONS

NEW SENIOR MATHEMATICS

EXTENSION 1

FOR YEARS 11 & 12

THIRD EDITION

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NSW
STAGE 6

CONTENTS

YEAR 11

YEAR 12

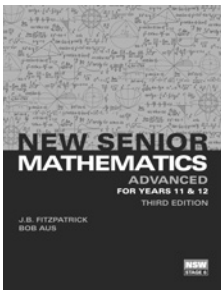
NEWSENIOR MATHEMATICS
THIRD EDITION

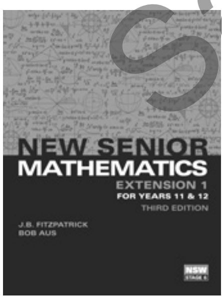
New Senior Mathematics Extension 1 for Years 11 & 12 is part of a new edition of the well-known Mathematics series for New South Wales. The series has been updated to address all requirements of the new Stage 6 syllabus. We have maintained our focus on mathematical rigour and challenging student questions, while providing new opportunities for students to consolidate their understanding of concepts and ideas with the aid of digital resources and activities.

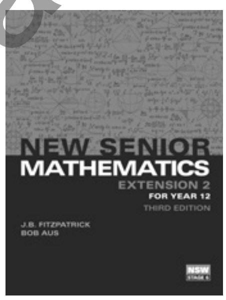
Student Book

The first three chapters of the first student book contain revision material that provides the necessary foundation for the development of senior mathematics concepts. In the new edition you'll also find:

- content built on a rigorous, academic approach that promotes excellence and prepares students for higher education
- a simple, convenient approach with Year 11 and 12 content in one book for Advanced and Extension 1, with colour coding to distinguish year levels
- digital technology activities that promote a deeper understanding, allowing students to make connections, and visualise and manipulate data in real time

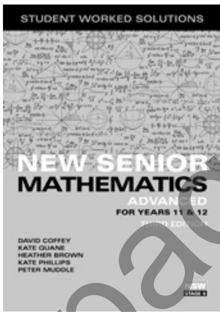


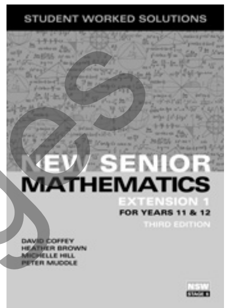


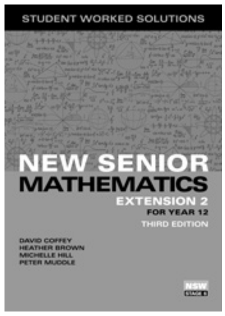


Student Worked Solutions

The *New Senior Mathematics Extension 1 for Years 11 & 12 Student Worked Solutions* contain the fully worked solutions for every second question in *New Senior Mathematics Extension 1 for Years 11 & 12*.







Reader+

Reader+, our next generation eBook, features content and digital activities, with technology such as graphing software and spreadsheets, to help students engage on their devices.

There are also teacher support materials, such as practice exams, question banks, investigation assignments and fully worked solutions to cover all internal and external assessment items and save you time.

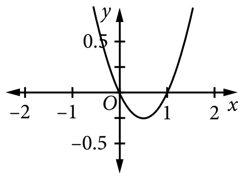
Pearson Places

Pearson

CHAPTER 1
Further work with functions

EXERCISE 1.1

1 $x(x-1) \le 0$



$\therefore\ 0 \le x \le 1$

3 $4x^2 - 12x + 10 > 0$

$2x^2 - 6x + 5 > 0$

$\Delta = 36 - 4 \times 5 \times 2$

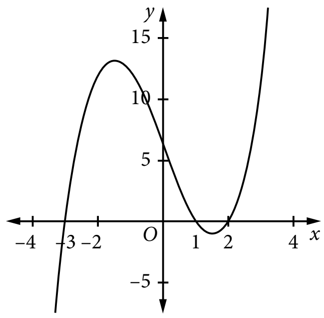
$= -4$

$-4 < 0$

$\therefore$ No real solutions; always above the x-axis ($a > 0$).

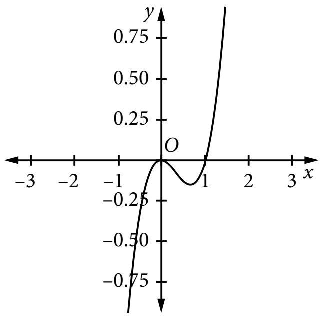
$\therefore$ Solution is: All real values of x.

5 $(x-1)(x+3)(x-2) < 0$



$x < -3,\ 1 < x < 2$

7 $x^2(x-1) \le 0$



$\therefore\ x \le 1$

9 $2^{2x} - 5(2^x) + 4 \le 0$

Let $a = 2^x$.

$a^2 - 5a + 4 \le 0$

$(a-4)(a-1) \le 0$

$\therefore\ 1 \le a \le 4$

If $a = 1$:

$1 = 2^x$

$x = 0$

If $a = 4$:

$4 = 2^x$

$x = 2$

So, $0 \le x \le 2$.

11 $1 - x < 2x + 1 < x + 4$

$1 < 3x + 1 < 2x + 4$

Consider $1 < 3x + 1$.

$-3x < 0$

$x > 0$

Consider $3x + 1 < 2x + 4$.

$x < 3$

Putting these together gives: $0 < x < 3$

EXERCISE 1.2

1 **(a)** Student D

(b) $\dfrac{2}{x-1} \le \dfrac{1}{2}$

Student A: Incorrect, because $x - 1$ could be positive or negative, so we don't know whether the inequality should change direction when we multiply across the inequality.

Student B: This is correct except it needs to show that $x \ne 1$.

Student C: Incorrect, because $x - 1$ could be positive or negative so we don't know whether the inequality should change direction when we take the reciprocal.

Student D: Correct, because that student multiplied by $2(x-1)^2$, which is positive, and noted that $x \ne 1$.

3 $\dfrac{x-2}{x+3} > -2$

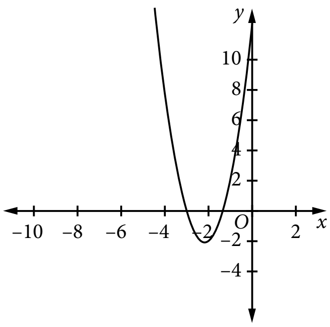
$\dfrac{x-2}{x+3} \times (x+3)^2 > -2(x+3)^2$

$(x-2)(x+3) > -2(x+3)^2$

$x^2 + x - 6 > -2x^2 - 12x - 18$

$3x^2 + 13x + 12 > 0$

$(x+3)(3x+4) > 0$



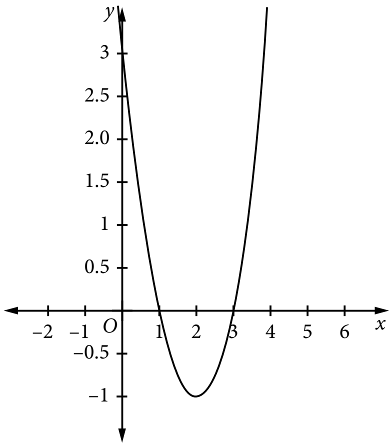
$x < -3, x > -\frac{4}{3}$

5 $\frac{2}{1-x} + 1 > 0$

$\therefore \frac{3-x}{1-x} > 0$

$\therefore (1-x)(3-x) > 0$

$\therefore$ Zeros are 1, 3.



$\therefore x < 1, x > 3$

7 $\frac{1}{(x-1)(x-3)} \le -1$

$\frac{1}{(x-1)(x-3)} + 1 \le 0$

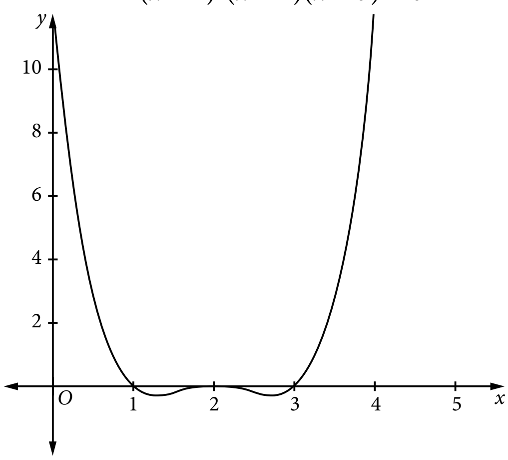
$\frac{1+(x-1)(x-3)}{(x-1)(x-3)} \le 0$

$\frac{1+x^2-4x+3}{(x-1)(x-3)} \le 0$

$\frac{x^2-4x+4}{(x-1)(x-3)} \le 0$

$\frac{(x-2)^2}{(x-1)(x-3)} \times [(x-1)(x-3)]^2 \le 0$

$(x-2)^2(x-1)(x-3) \le 0$



$\therefore$ Solution is $1 < x < 3$.

Alternatively:

Note that $x \ne 1, x \ne 3$.

The solution of $\frac{1}{(x-1)(x-3)} = -1$ is $x = 2$.

$\therefore$ Critical values are 1, 2, 3.

Test $x = 0$: Is $\frac{1}{(0-1)(0-3)} \le 1$? No

Test $x = 1.5$: Is $\frac{1}{(1.5-1)(1.5-3)} \le 1$? Yes

Test $x = 2.5$: Is $\frac{1}{(2.5-1)(2.5-3)} \le 1$? Yes

Test $x = 4$: Is $\frac{1}{(4-1)(4-3)} \le 1$? No

$\therefore$ Solution is $1 < x < 3$.

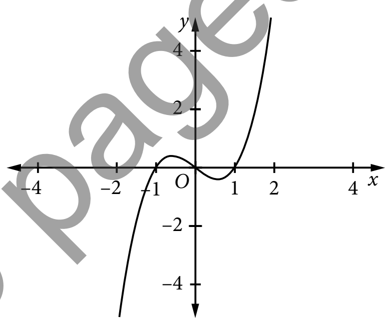
9 $\frac{x}{x^2-1} < 0$

$\frac{x(x^2-1)^2}{x^2-1} < 0$

$x(x^2-1) < 0$

$x(x-1)(x+1) < 0$

$\therefore$ Zeros are 0, 1, −1.

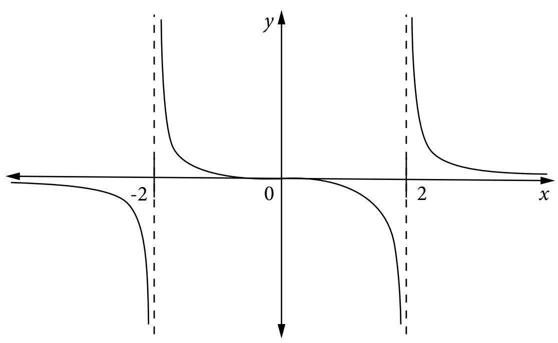


$\therefore x < -1, 0 < x < 1$

11 **(a)** $f(-x) = \frac{-x}{(-x)^2-4} = -f(x)$

$\therefore f(x)$ is an odd function.

(b)



(c) $\frac{x}{x^2-4} < \frac{1}{3}$

$3x < x^2 - 4$

$x^2 - 3x - 4 > 0$

$(x-4)(x+1) > 0$

$x > -1, 4$

From the graph and inequality equation you can see that $f(x) < \frac{1}{3}$ for $x < -2$, $-1 < x < -2$, $x > 4$.

EXERCISE 1.3

1 $\frac{1}{|x|} < 3$

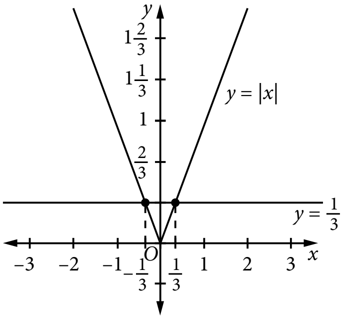
$|x| > \frac{1}{3}$

For positive x: $x > \frac{1}{3}$

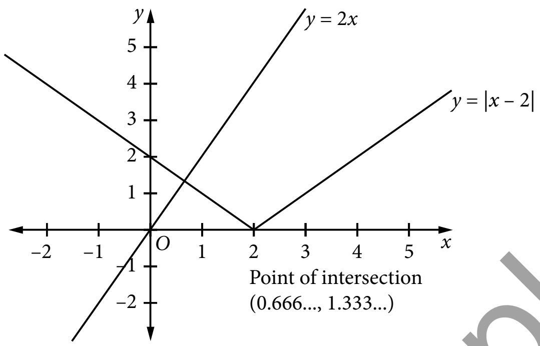
For negative x: $-x > \frac{1}{3}$

$x < -\frac{1}{3}$

$\therefore\ x < -\frac{1}{3},\ x > \frac{1}{3}$



3 **(a)**



(b) Where they intersect, $y = |x-2|$ is the ray $y = -x - 2$.

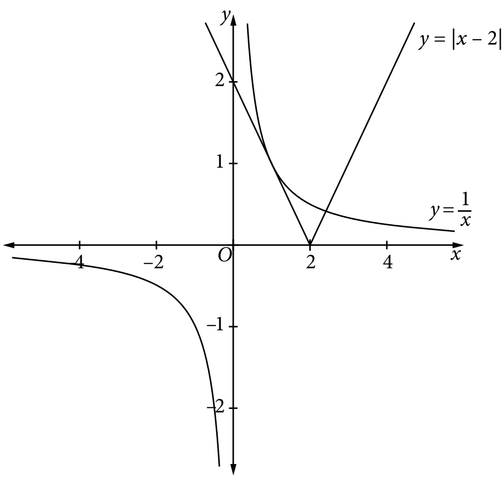
$\therefore\ 2x = -x + 2$ (solving simultaneously)

$3x = 2$

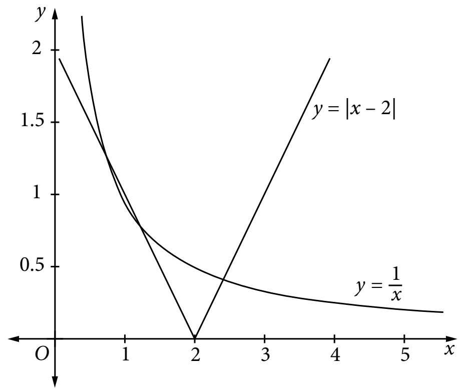
$x = \frac{2}{3}$

So $|x-2| < 2x$ when $x > \frac{2}{3}$.

5 **(a)**



A closer look at the parts of interest:



(b) The intersection point on the left occurs when $y = -x + 2$.

$\therefore\ \frac{1}{x} = -x + 2$

$1 = -x^2 + 2x$

$0 = -x^2 + 2x - 1$

$0 = x^2 - 2x + 1$

$0 = (x-1)^2$

$x = 1$

The intersection point on the right occurs when $y = x - 2$.

$\therefore\ \frac{1}{x} = x - 2$

$1 = x^2 - 2x$

$0 = x^2 - 2x - 1$

Use the quadratic formula: $x = \frac{2 \pm \sqrt{8}}{2}$

We have $x = \frac{2 \pm 2\sqrt{2}}{2} = 1 + \sqrt{2}$

So $|x-2| > \frac{1}{x}$ where $x < 0,\ x > 1 + \sqrt{2}$.

7 $\left|\frac{1-x}{2x+1}\right| \geq 1$

$\frac{|1-x|}{|2x+1|} \geq 1$

$|1-x| \geq |2x+1|$, because $|2x+1| > 0$

$\therefore\ 2x + 1 \neq 0$

$x \neq -\frac{1}{2}$

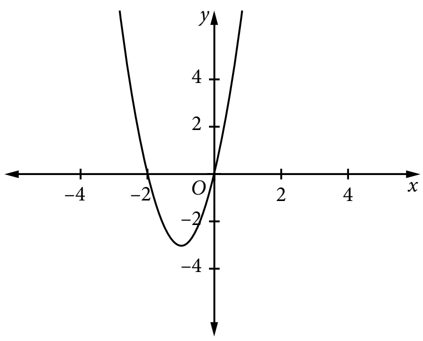
$(1-x)^2 \geq (2x+1)^2$, because both sides are positive

$1 - 2x + x^2 \geq 4x^2 + 4x + 1$

$0 \geq 3x^2 + 6$

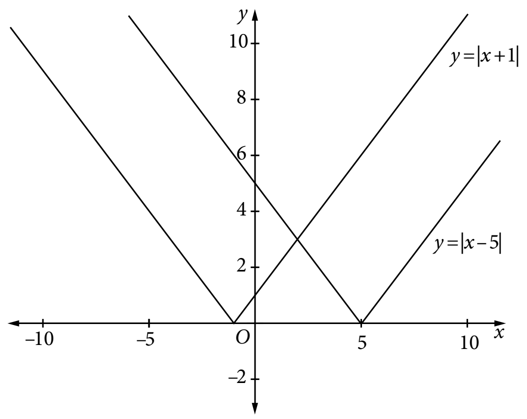
$3x(x+2) \leq 0$

$\therefore$ Zeros are 0, −2.

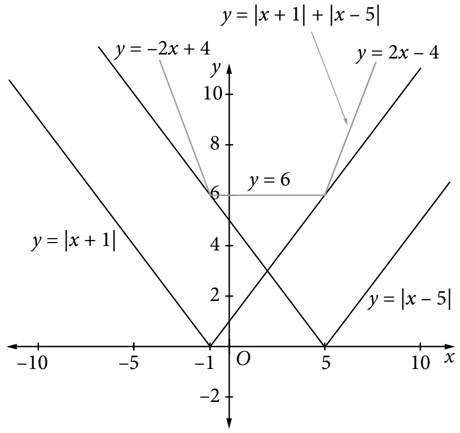


$-2 \leq x \leq 0$

9 **(a)**



(b)



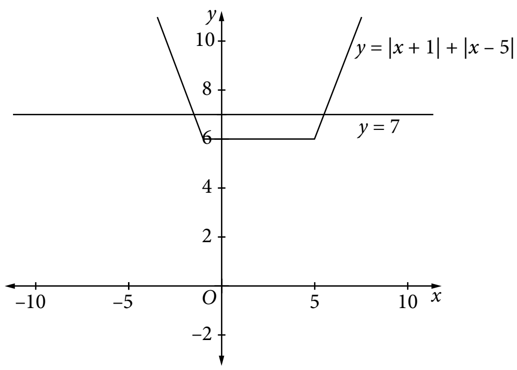
Note:

$x < -1, y = -(x + 1) - (x - 5) = -2x + 4$

$x > 5, y = x + 1 + x - 5 = 2x - 4$

$-1 < x < 5, y = x + 1 - (x - 5) = 6$

(c)



The intersection point on the left occurs when $y = -2x + 4$.

$-2x + 4 = 7$

$-2x = 3$

$x = -\frac{3}{2}$

The intersection point on the right occurs when $y = 2x - 4$.

$2x - 4 = 7$

$2x = 11$

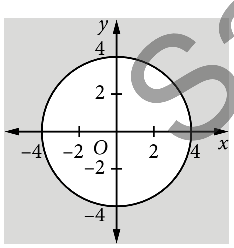
$x = \frac{11}{2}$

So $|x + 1| + |x - 5| > 7$ where $x < -\frac{3}{2},\ x > \frac{11}{2}$.

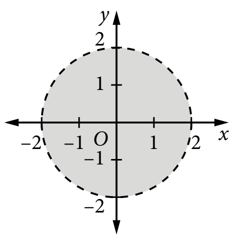
(d) From the graph: $-1 \le x \le 5$

EXERCISE 1.4

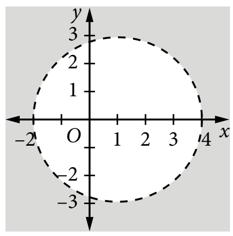
1 **(a)** Circle, centre $(0, 0)$, radius 4, the region outside the circle, including the boundary.



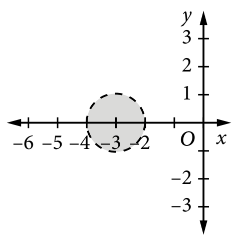
(b) Circle, centre $(0, 0)$, radius 2, the region inside the circle, not including the boundary.



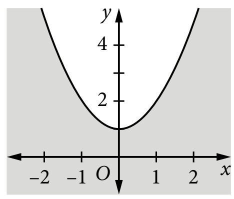
(c) Circle, centre $(1, 0)$, radius 3, the region outside the circle, not including the boundary.



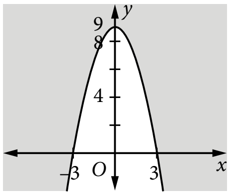
3 **(a)** Circle, centre $(-3, 0)$, radius 1, the region inside the circle, not including the boundary.



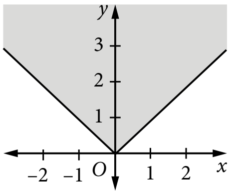
(b) Parabola, turning point $(0, 1)$, the region below the graph, including the graph.



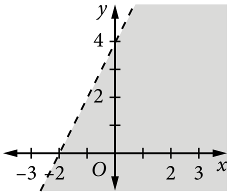
(c) Parabola, turning point $(0, 9)$, the region above the graph, including the boundary.



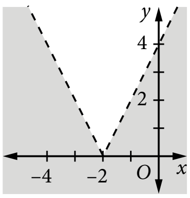
(d) The region above the graph of $y = |x|$, including the graph.



(e) The region below the graph of $y = 2x + 4$.



(f) The region below the graph of $y = |2x + 4|$.



5 **A**

$(x-3)^2 + (y+4)^2 \leq 25$

This is a circle centre $(3, -4)$, radius 5 and the region inside the circle.

On the circle, where $x = 0$, $9 + (y+4)^2 = 25$

$(y+4)^2 = 16$

$y = \pm 4 - 4$

$y = -8$ or $y = 0$; y-intercepts are $(0, 0)$, $(0, -8)$

At $y = 0$, $(x-3)^2 + 16 = 25$

$(x-3)^2 = 9$

$x = \pm 3 + 3$

$x = 0$ or $x = 6$; x-intercepts are $(0, 0)$, $(6, 0)$

A Correct. The x-intercepts are $(0, 0)$, $(6, 0)$ and the y-intercepts are $(0, 0)$, $(0, -8)$. The region inside the circle is shaded and the boundary is included.

B Incorrect. One of the x-intercepts is $(-6, 0)$, which is wrong.

C Incorrect. One of the y-intercepts is $(0, 8)$, which is wrong.

D Incorrect. One of the x-intercepts is $(6, 0)$, which is wrong. One of the y-intercepts is $(0, 8)$, which is wrong.

7 **(a)** Correct. The circle has centre $(0, 0)$ and radius 2 so the equation is $x^2 + y^2 = 4$. The shaded region is inside the circle and the boundary, which is given by $x^2 + y^2 \leq 4$. The graph of the straight lines is given by the equation $y = |x|$. The shaded region is below this line and including the line, which is given by the equation $y \leq |x|$.

(b) Incorrect. The statement should have stated that the points on the lines $y = |x|$ and $x^2 + y^2 = 4$ are included in the shaded region.

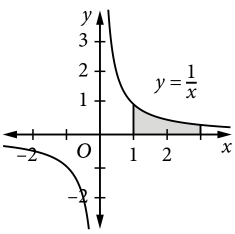
(c) Incorrect. The region indicated by these statements would be outside the circle and above the straight lines, including the boundaries.

(d) Correct. This is the region described in part **(a)**.

CHAPTER REVIEW 1

1 From the diagram, **C** is the correct answer.

3



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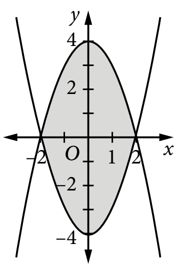

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