PEARSON

PHYSICS

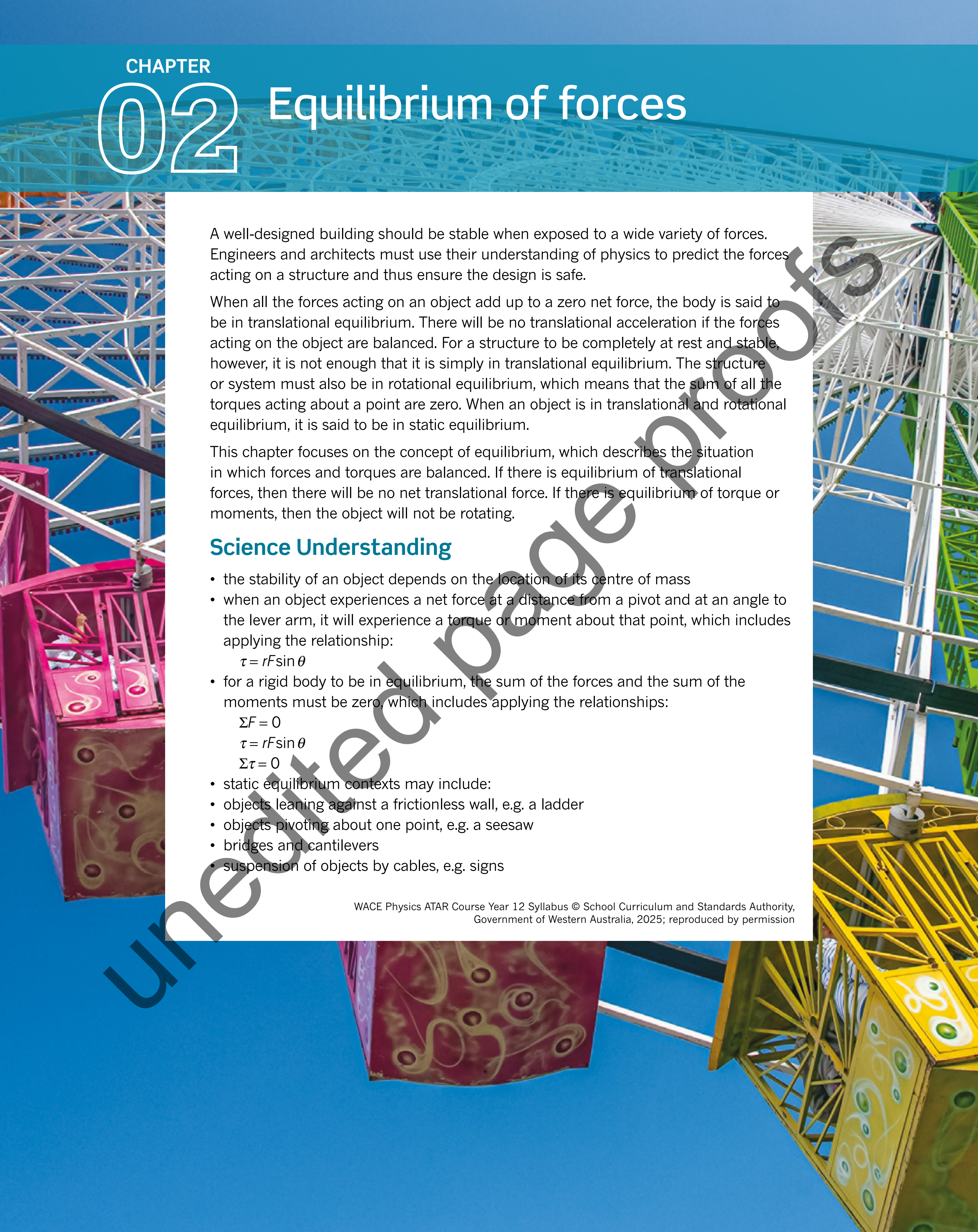
WESTERN AUSTRALIA

12

2ND EDITION

CHAPTER 02

Equilibrium of forces



A well-designed building should be stable when exposed to a wide variety of forces. Engineers and architects must use their understanding of physics to predict the forces acting on a structure and thus ensure the design is safe.

When all the forces acting on an object add up to a zero net force, the body is said to be in translational equilibrium. There will be no translational acceleration if the forces acting on the object are balanced. For a structure to be completely at rest and stable, however, it is not enough that it is simply in translational equilibrium. The structure or system must also be in rotational equilibrium, which means that the sum of all the torques acting about a point are zero. When an object is in translational and rotational equilibrium, it is said to be in static equilibrium.

This chapter focuses on the concept of equilibrium, which describes the situation in which forces and torques are balanced. If there is equilibrium of translational forces, then there will be no net translational force. If there is equilibrium of torque or moments, then the object will not be rotating.

Science Understanding

- the stability of an object depends on the location of its centre of mass
- when an object experiences a net force at a distance from a pivot and at an angle to the lever arm, it will experience a torque or moment about that point, which includes applying the relationship:
  $\tau = rF\sin\theta$
- for a rigid body to be in equilibrium, the sum of the forces and the sum of the moments must be zero, which includes applying the relationships:
  $\Sigma F = 0$
  $\tau = rF\sin\theta$
  $\Sigma\tau = 0$
- static equilibrium contexts may include:
- objects leaning against a frictionless wall, e.g. a ladder
- objects pivoting about one point, e.g. a seesaw
- bridges and cantilevers
- suspension of objects by cables, e.g. signs

2.1 Torque

Many real-life situations involve objects that rotate about a pivot point, such as closing a door, using a spanner or turning a steering wheel (Figure 2.1.1). In these situations, a force acts to provide a turning effect, known as a **torque** (τ). Newton's laws use the concept of force to help explain the motion of a body in a straight line. The concept of torque is used in exactly the same way to explain changes in the rotational motion of an object.

FIGURE 2.1.1 Applying a torque to the rim or arms of a steering wheel will cause it to turn.

UNDERSTANDING TORQUE

When a turning effect is applied to an object, a number of factors must work together to cause that object to turn. First, there must be a **pivot point** around which the object will rotate; this is referred to as the **axis of rotation**. There must also be a force (F) applied to the object in such a way as to cause the object to rotate. To allow rotation, the force must not be applied through the pivot point, but rather at some perpendicular distance from it. The force applied will have a **line of action**, which must not pass through the pivot point either.

EXTENSION

Torque and moment of a force

The terms 'torque' (τ) and 'moment of a force' (M), which is usually shortened to 'moment', are used interchangeably in physics at the high school level.

The difference between these terms is in the conventions of their use. Typically, torque is used for dynamic problems in which there is an angular acceleration, and so the speed at which an object rotates is changing. Moments are typically used in static problems, often as reaction forces or internal forces in an object, and are balanced so that there is no angular acceleration. An example of how a torque is used would be in the force applied by an axle to the wheel of a car to increase its speed of rotation. A moment might be used when calculating the reaction force for a beam that is attached to a wall at one end and has a force applied on the other.

Both torque and moment are calculated using the equation $\tau = rF \sin\theta$.

Force and the pivot point

When analysing a rotating system, the position of the pivot point or axis of rotation must be considered (Figure 2.1.2). A wheel, for example, moves in a circular path around its axle. The imaginary line along the length of the axle is the axis of rotation.

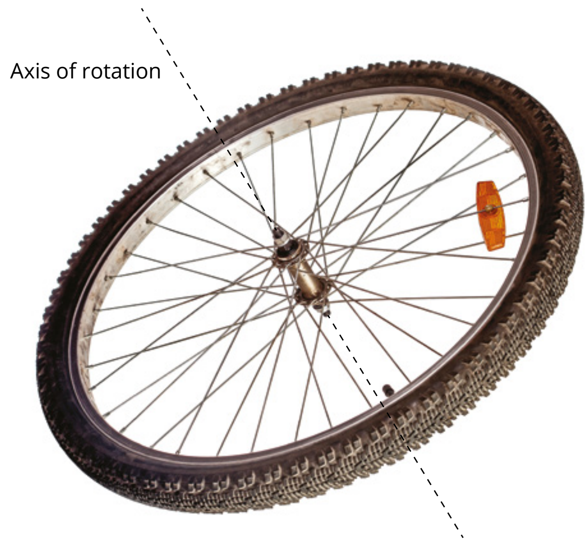


FIGURE 2.1.2 The axis of rotation.

A force applied directly towards or away from the axis of rotation will not create a turning effect on the wheel. This is because the line of action of the force passes through the pivot point (Figure 2.1.3).

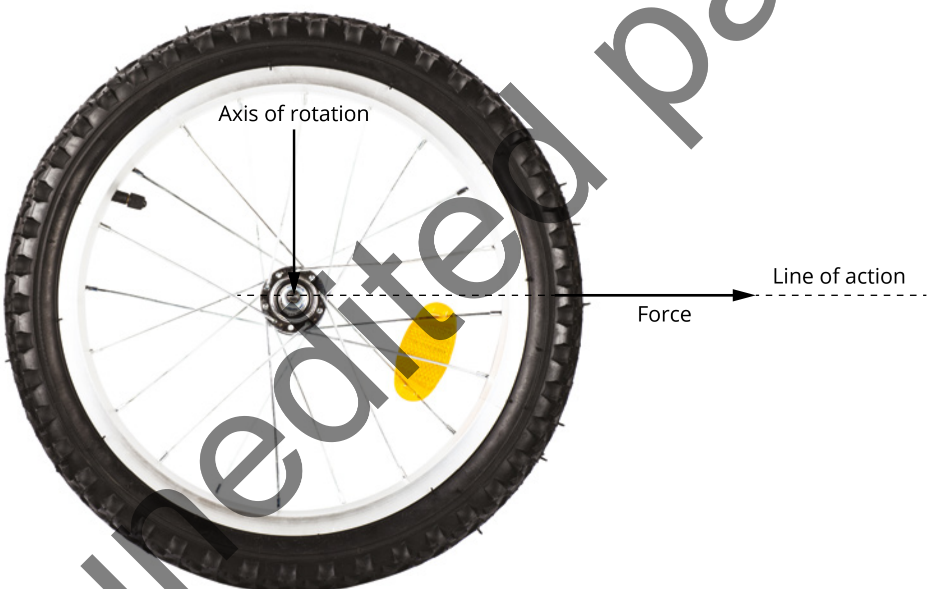


FIGURE 2.1.3 When the line of action of the force passes through the pivot point, the wheel will not experience a torque.

Torque is achieved by applying a force on the wheel at a point where the line of action of the force does not pass through the axis of rotation or pivot point. The maximum effect is achieved when the force applied is at 90° to a line drawn from the axis of rotation to the point at which the force is applied (Figure 2.1.4).

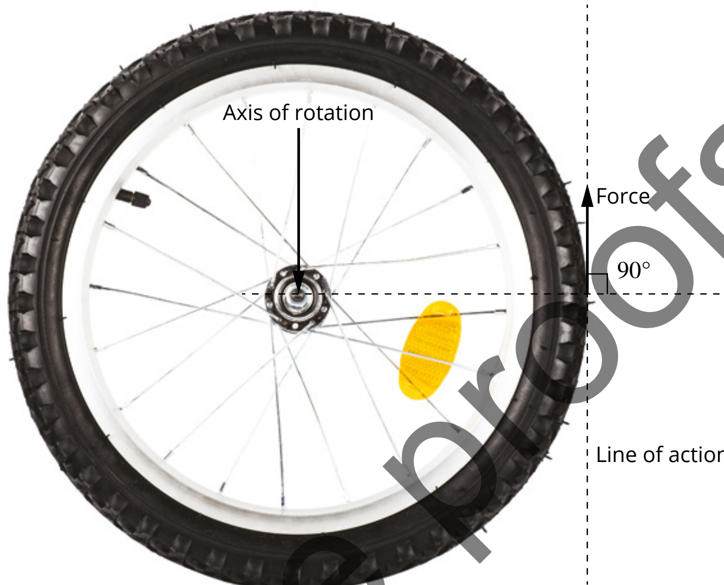


FIGURE 2.1.4 Maximum torque occurs when the force applied is perpendicular (90°) to the axis of rotation.

Magnitude of the force and the torque

The torque (τ) on an object is directly proportional to the magnitude of the force (F). If all other things are equal, a larger force will result in a larger torque (Figure 2.1.5).

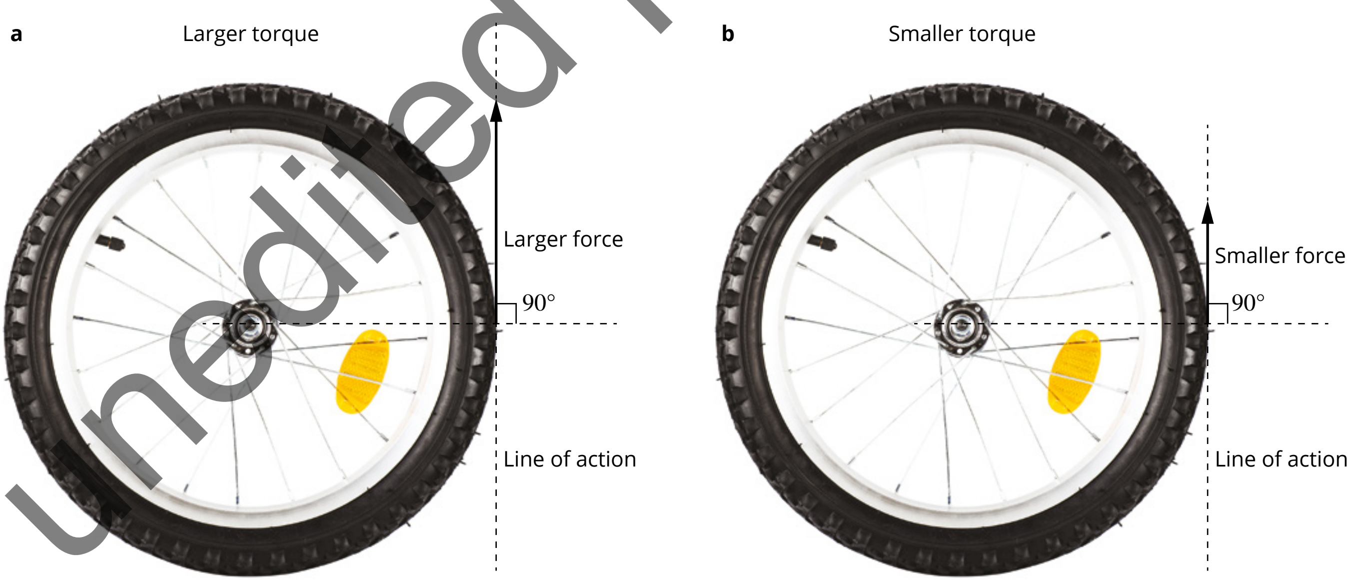


FIGURE 2.1.5 The magnitude of a force affects the torque on an object. The wheel in (a) will experience a larger torque than the wheel in (b).

Distance from the pivot point and torque

In addition to the size of the force and the angle at which it is acting, the amount of torque created is also directly proportional to the perpendicular distance between the axis of rotation and the line of action of the force. This perpendicular distance is called the **force arm** and is given the symbol $r_{\perp}$. Given that everything else is constant, the larger the force arm or perpendicular distance ($r_{\perp}$), the larger the torque (τ) (Figure 2.1.6).

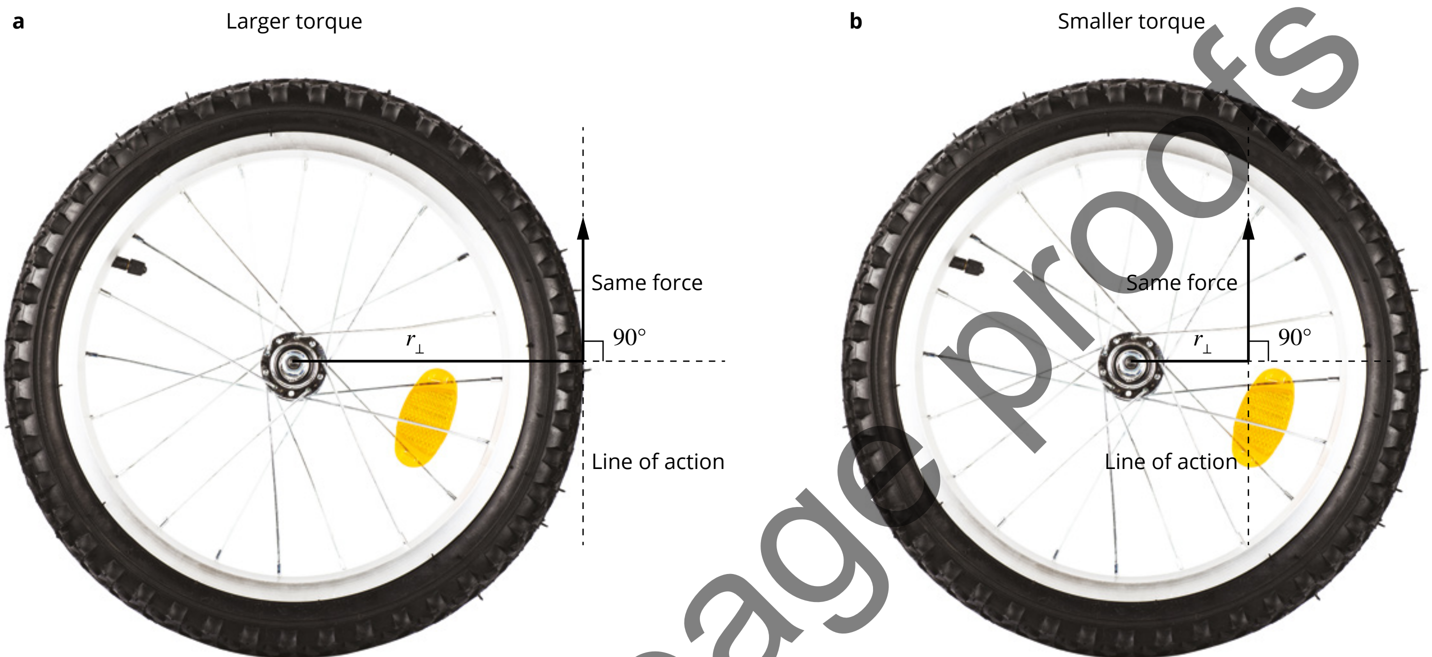


FIGURE 2.1.6 The perpendicular distance from the pivot point to the line of action of the force affects the torque on an object. The wheel in (a) will experience a larger torque than the wheel in (b).

The torque equation

The magnitude of the torque (τ) increases or decreases as the force (F) increases or decreases. The torque also increases or decreases as the force arm or the perpendicular distance from the pivot to the line of action of the force ($r_{\perp}$) increases or decreases.

Hence, the formula for calculating the magnitude of the torque is:

$\tau = r_{\perp}F$

where τ is torque (N m)

$r_{\perp}$ is the perpendicular distance from pivot to the force's line of action (m)

F is force (N).

A rotating body moves either clockwise or anticlockwise. You can determine the direction of the rotation by considering the context of the question. When two or more torques are acting, a sign convention can be applied to each torque where clockwise is considered to be positive and anticlockwise is considered to be negative. In this way, the torques can be added to determine the resultant torque.

Worked example 2.1.1

CALCULATING TORQUE

A bus driver applies a force of 45.0 N on the steering wheel of a bus as it turns a right-hand corner. The radius of the steering wheel is 30.0 cm. If the force is applied at 90.0° to the radius, calculate the torque on the steering wheel.	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$\tau = ?$ $r_\perp = 0.300\,\text{m}$ $F = 45.0\,\text{N}$
Apply the equation for torque. Rearrange as necessary.	$\tau = r_\perp F$ $\tau = (0.300)(45.0)$ $\tau = 13.500$
State the answer to the correct number of significant figures and include the appropriate direction (clockwise or anticlockwise).	$\tau = 13.5\,\text{N m}$ clockwise

Worked example: Try yourself 2.1.1

CALCULATING TORQUE

A force of 255 N is required to apply a torque on the steering wheel of a sports car as it turns left. The force is applied at 90.0° to the 15.5 cm radius of the steering wheel. Calculate the torque on the steering wheel.

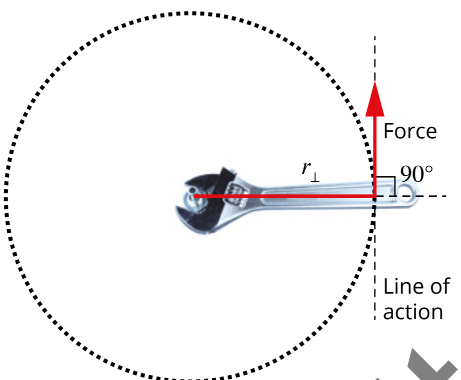


FIGURE 2.1.7 An adjustable spanner applies a torque to the nut of a nut-and-bolt system.

Torque on different objects

A torque does not need to be acting on a circular object. Any object can rotate about a point if a force is applied where the line of action of the force is not acting through the pivot point. For example, spanners can apply a torque to a nut or bolt when the pivot point is the bolt and a force is applied at right angles to the spanner (Figure 2.1.7).

Longer spanners can apply a greater torque on a nut than shorter spanners. This can be useful when trying to remove tight nuts or bolts, such as the wheel nuts on a car (Figure 2.1.8). Some wheel-nut spanners have extendable handles that increase the torque for loosening very tight nuts or tightening the nuts sufficiently so that they won't come loose.

FIGURE 2.1.8 Loosening the wheel nuts with an extended handle spanner will increase the torque on the nut.

Worked example 2.1.2

CALCULATING PERPENDICULAR DISTANCE

A car driver can apply a maximum force of 845 N on a wheel-nut spanner that is adjustable up to 30.0 cm in length. The force is applied at 90.0° to the radius. If the wheel nuts need a torque of 224 N m to remove them, what is the minimum length of the adjustable spanner at which the nuts can be loosened? State whether the spanner is long enough.	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$\tau = 224\,\text{N m}$ $r_\perp = ?$ $F = 845.0\,\text{N}$
Apply the equation for torque. Rearrange as necessary.	$\tau = r_\perp F$ $r_\perp = \dfrac{\tau}{F}$ $r_\perp = \dfrac{(224)}{(845)}$ $r_\perp = 0.265\,09$
State the answer to the appropriate number of significant figures and with comparable units to the question.	$r_\perp = 26.5\,\text{cm}$
Compare the answer with the length of the spanner and state whether it is appropriate for this task.	As the spanner can be extended to 30.0 cm, it can be made long enough to provide the minimum perpendicular distance of 26.5 cm.

Worked example: Try yourself 2.1.2

CALCULATING PERPENDICULAR DISTANCE

A truck driver can apply a maximum force of 1020 N on a large truck wheel-nut spanner that has a length of 80.0 cm. The force is applied at 90.0° to the radius. If the truck's wheel nuts need a torque of 635 N m to make them secure, determine if the length of this spanner is sufficient for the job.

Doors are also good examples of torque in action when the hinges form the axis of rotation and the force is applied to the door at a perpendicular distance (Figure 2.1.9).

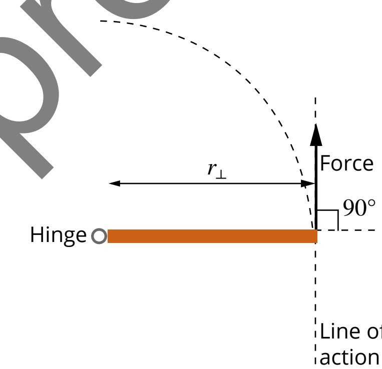


FIGURE 2.1.9 A door can have a torque applied to it, as long as the line of action of the force is not through the axis of rotation.

NON-PERPENDICULAR CALCULATIONS OF TORQUE

When the force causing a torque acts along a line that is at an angle other than 90.0° to an object, such as a door, the torque is reduced (Figure 2.1.10). In these circumstances, you can calculate the torque by using one of two approaches: either by finding the component of the force acting perpendicular to the door, or by finding the perpendicular distance from the pivot point to the line of action of the force.

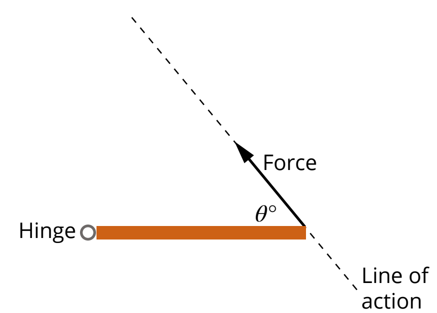


FIGURE 2.1.10 When the force causing a torque is not perpendicular to the door, the torque is reduced.

Recall that the formula for torque (τ) on an object is:

$$\tau = r_\perp F$$

This equation calculates the torque (τ) when the force (F) and the distance from the pivot point to the line of action of the force (r) are perpendicular to each other. It doesn't matter whether the radius is perpendicular to the line of action of the force, or if the force is perpendicular to the radius. You will explore both methods and see that they are equivalent.

Calculating torque using perpendicular force

The component of any force can be calculated using trigonometry. To find the component of the force that is perpendicular to a door, for example, you can use the magnitude of the force and the angle between the door and the line of action of the force (Figure 2.1.11).

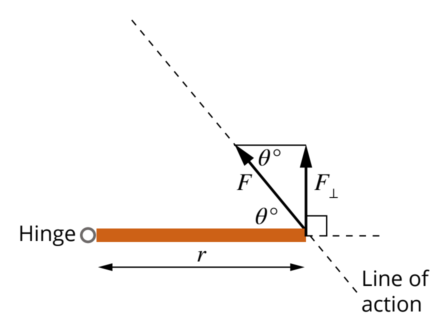


FIGURE 2.1.11 Determining the components of a force.

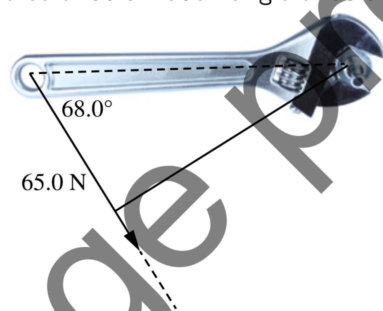
In this case, $\tau = rF_{\perp}$ and $F_{\perp} = F\sin\theta$.
Combining the two equations gives:
$\tau = rF\sin\theta$

When solving this problem, you can either calculate the perpendicular component of the force and then apply the torque equation, or simply use the combined equation. It is recommended that you begin by calculating the perpendicular force, and then use the torque equation. Once you are comfortable with that strategy, you can try using the combined equation.

Worked example 2.1.3

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF FORCE

A student uses a 42.0 cm long adjustable spanner to loosen a nut on their bike. The student applies a force of 65.0 N at an angle of 68.0° to the spanner.



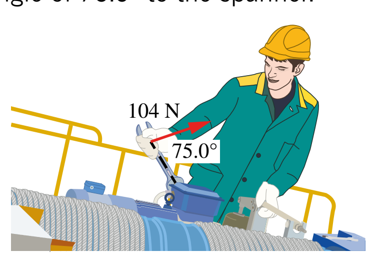
Calculate the anticlockwise torque that the student applies to the nut.

Thinking	Working
Use the trigonometric relationship $F_{\perp} = F\sin\theta$ to determine the force perpendicular to the spanner.	$F_{\perp} = F\sin\theta$ $F_{\perp} = (65.0)\sin(68.0°)$ $F_{\perp} = 60.267$ N
Convert variables to their standard units.	$r = 42.0$ cm $r = 0.420$ m
Apply the equation for torque: $\tau = rF_{\perp}$	$\tau = rF_{\perp}$ $\tau = (0.420)(60.267)$ $\tau = 25.312$
State the answer to the appropriate number of significant figures and with the appropriate units, including the direction since torque is a vector.	$\tau = 25.3$ N m anticlockwise

Worked example: Try yourself 2.1.3

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF FORCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.



Calculate the clockwise torque that the mechanic applies to the nut.

Calculating torque using perpendicular radius

The perpendicular component of any distance can be calculated using Pythagoras' theory or trigonometry. To find the component of a length that is perpendicular to the line of action of the force acting on a door, construct a line from the pivot point to the line of action of the force so that it intersects the line of action at right angles (Figure 2.1.12).

In this case

$\tau = r_{\perp}F$ and $r_{\perp} = r\sin\theta$

combine to give

$\tau = r\sin\theta F$

The equation $\tau = rF\sin\theta$ is identical to $\tau = r\sin\theta F$, so either method would be appropriate for calculating the torque when the force is not acting at right angles to the object. In both cases, the component of the distance or the component of the force is always going to be less than the distance or the force itself. This decrease in value will result in a smaller torque being applied to the object. The maximum torque will always be achieved when the line of action of the force is perpendicular to the maximum distance from the pivot point to the line of action.

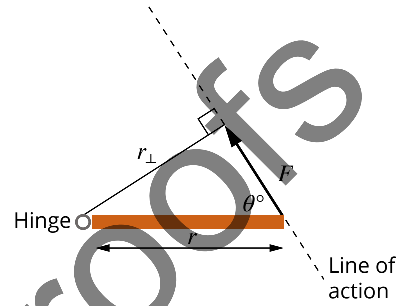
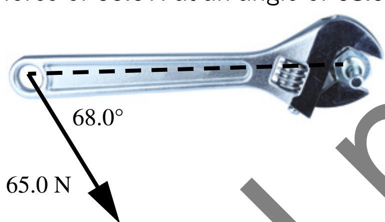


FIGURE 2.1.12 Determining the components of a distance.

Worked example 2.1.4

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF DISTANCE

A student uses a 42.0 cm long adjustable spanner to loosen a nut on their bike. The student applies a force of 65.0 N at an angle of 68.0° to the spanner.



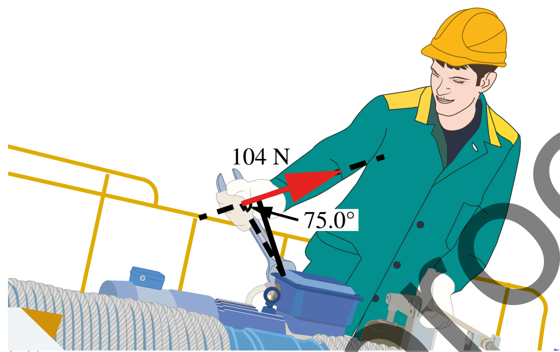
Using the perpendicular distance, calculate the anticlockwise torque that the student applies to the nut.

Thinking	Working
Convert variables to their standard units.	$r = 42.0$ cm $r = 0.420$ m
Use the trigonometric relationship $r_{\perp} = r\sin\theta$ to determine the perpendicular distance from the pivot point to the line of action of the force.	$r_{\perp} = r\sin\theta$ $r_{\perp} = (0.420)\sin(68.0°)$ $r_{\perp} = 0.38942$ m
Apply the equation for torque: $\tau = r_{\perp}F$.	$\tau = r_{\perp}F$ $\tau = (0.38942)(65.0)$ $\tau = 25.312$
State the answer with the appropriate unit, direction and significant figures. Note that this is the same as the answer to Worked example 2.1.3.	$\tau = 25.3$ N m anticlockwise

Worked example: Try yourself 2.1.4

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF DISTANCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.



Using the perpendicular distance, calculate the clockwise torque that the mechanic applies to the nut. Give your answer to three significant figures.

PHYSICS IN ACTION

The torque wrench

The extent to which a nut or bolt is tightened can be critical to the safe operation of machinery or motors, a good example being the wheel nuts on a car. If a nut or bolt is too loose then it could fall out, but if it is too tight then it could either distort the part or the bolt could break off. Both of these situations could result in expensive repairs. To avoid nuts and bolts being too loose or too tight, manufacturers use different tools and methods to estimate the amount of torque required to tighten a nut or bolt to the correct tightness.

Figure 2.1.13 illustrates different types of torque wrenches. The beam wrench is the simplest type of torque wrench; it has a flexible lever arm with a bar and scale that separates the wrench head and handle. When a torque is applied, a pointer on the scale moves to indicate the amount of torque in newton metres (N m). The click-type torque wrench can be set to apply a fixed amount of torque. When the required torque has been achieved, the wrench 'clicks' and releases itself, preventing any further tightening.

More recently, electronic torque wrenches have been developed. A force beam within the wrench converts the applied compressive or tensile force to an electrical signal that is calibrated to show the force in newton metres on a digital readout. Measurements can also be stored within the instrument's memory and transferred to a computer. The force sensors you will most likely use in physics operate on this same principle.

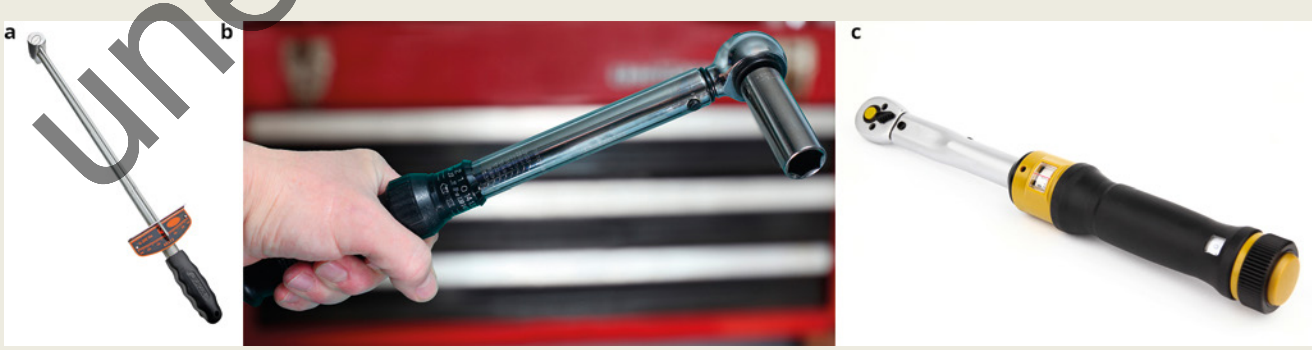


FIGURE 2.1.13 Three types of wrenches commonly used to measure the torque applied to a nut or bolt: (a) beam torque wrench, (b) click-type torque wrench, (c) digital torque wrench.

2.1 Review

SUMMARY

- Torque is a measurement of the amount of force applied to an object to make it rotate around an axis.
- The formula for calculating the magnitude of the torque is $\tau = r_{\perp}F$.
- Torque occurs when the acting force is not applied directly through the pivot point of the object.
- Maximum torque occurs when the acting force is applied at a perpendicular angle of 90° to the force arm.
- The larger the force acting on the object, the larger the torque will be.
- The longer the force arm, the greater the torque will be.
- If torque is generated by a force that is not perpendicular to the lever arm of the object, one of the two methods can be used:
  - Use the component of the force perpendicular to the length of the object to calculate torque $\tau = rF_{\perp}$
  - Use the distance from the pivot point perpendicular to the line of action of the force to calculate torque $\tau = r_{\perp}F$.
- Both methods for determining torque from non-perpendicular situations equate to $\tau = rF\sin\theta$.

KEY QUESTIONS

1 Select the correct option below to complete the following statement about torque.
When a force acts such that the line of action of the force is not directed through the pivot point of the object then:
A a torque will result
B no torque will result

2 Which of the options listed below best explains the factors affecting the torque acting on an object?
A Torque is only increased by increasing the force applied to the object.
B Torque increases when either the force or the force arm increases, or both increase.
C Torque only increases when the force arm decreases.
D Torque only increases when the force arm increases.

3 Calculate the torque applied to a bolt by a 20.0 cm long spanner that has a force of 25.0 N acting at 90.0° to its length and at the end of the spanner.

4 Calculate the radius of the wheel on a pressure valve that supplies a torque on the valve of magnitude 3.47 N m when a force of 12.0 N is applied.

5 When opening a 1.00 m wide door, the torque of the greatest magnitude is provided by:
A pushing with a force of 33.0 N at right angles to the door, in the middle of the door
B pushing with a force of 25.0 N at right angles to the door, 25.0 cm from the handle edge of the door
C pushing with a force of 50.0 N at right angles 25.0 cm from the hinges of the door
D pulling with a force of 25.0 N at right angles to the door, at the handle edge of the door

6 Demolishers wish to knock over a concrete wall. They plan to use a wrecking ball that exerts 5250 N as it hits the wall. If it hits at a point that is 3.05 m above the ground, calculate the magnitude of the torque that is developed on the wall if the wall pivots at its base.

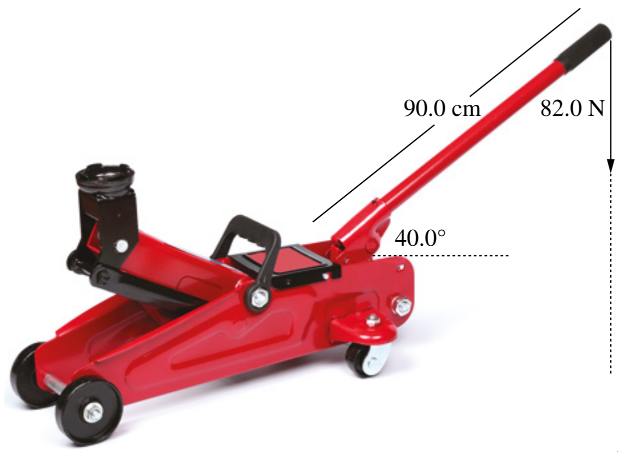
7 Calculate the length of a spanner that is used to tighten a nut to a torque of magnitude 32.1 N m when a force of 24.0 N is applied at right angles to the spanner, at the end of the spanner.

8 A camper ties a rope from the top of a 2.00 m tent pole to a peg on the ground. The rope is tightened so that the rope applies a 30.0 N force at an angle of 40.0° to the pole. Calculate the magnitude of the torque that is developed on the tent pole due to the rope if it pivots at its base.

2.1 Review *continued*

9 Penny is assembling some flat-pack furniture with an Allen key. Calculate the torque supplied on a screw by a 7.00 cm long Allen key that has a force of 8.50 N acting at 90.0° to its length, and at the end of the Allen key.

10 A mechanic uses a trolley jack to jack up a car by pushing vertically down on the end of a 90.0 cm lever with a force of 82.0 N, as shown in the diagram below. The lever is shown at an angle of 40.0° up from horizontal. Calculate the magnitude of the torque acting on the pivot.



The following information applies to questions 11 and 12.
A rope is attached at 30.0° to a freshly planted tree. The line of action of the force is along the same line as the rope, and the rope is attached 1.50 m from the base of the tree. Assume the base of the tree is the pivot point.

11 Calculate the length of the perpendicular force arm of the rope.

12 Calculate the torque on the tree if the force applied to the tree by the rope is 12.5 N and it is tensioned clockwise about the base of the tree.

2.2 Equilibrium of forces

Newton's first law states that an object will continue with its motion unless acted upon by an external unbalanced force. The situation in which the object continues with its motion, or doesn't change its velocity, occurs when the forces on the object are balanced. Balanced forces, like those in a tug of war (Figure 2.2.1), are said to be in translational equilibrium. When a tug of war starts, for example, there is an equilibrium of forces as both teams take the strain. Winning a tug of war match involves one team applying a greater force so that there is a net force on the rope, causing the rope and teams to accelerate in the winning team's direction. Once the rope and the teams are moving at a constant velocity, an equilibrium of forces exists once again.

FIGURE 2.2.1 The game 'tug of war' is an example of translational equilibrium.

CENTRE OF MASS AND STABILITY

Think about an athlete running in a 100 m sprint. In simple terms, the athlete runs in a straight line along the track, and the displacement and velocity at any time can be calculated using the principles discussed in Chapter 3 'Linear motion' in Year 11. However, the motion of the various parts of the athlete's body will differ significantly during the run. The athlete's arms and legs move in a complex manner that requires a significantly more complex analysis.

The analysis of the motion of complicated systems, such as a sprinter or high-jumper, can be simplified to the motion of a single point. In other words, the mass of the sprinter can be considered to be 'concentrated' into a single point that has travelled in a straight line. This single point is called the **centre of mass**. There is as much mass above the centre of mass as there is below it, as much mass to the left as there is to the right, and as much mass in front as there is behind it.

If an object is uniform in one dimension only (e.g. a straight piece of wire), its centre of mass will be exactly halfway along its length. The centre of mass of an object in two dimensions (e.g. a flat piece of card), will be the point that is halfway along and halfway across the object (Figure 2.2.2). It is even possible for the centre of mass to lie outside the body; for example, the centre of mass of a doughnut is located at the centre of the hole. A person's centre of mass is typically in the lower part of the back, but it will vary with the positions of the arms and legs.

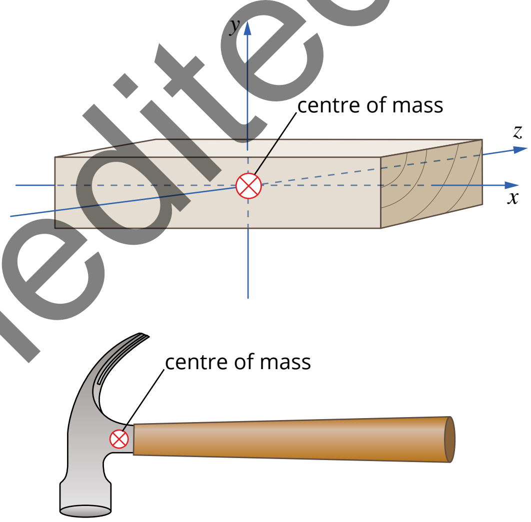


FIGURE 2.2.2 The centre of mass of a three-dimensional block of wood is shown with arrows drawn in the *x*, *y* and *z* dimensions illustrating the lines where there is equal mass on either side of the lines. The centre of mass of a non-uniform hammer shows that the centre of mass will be closer to the end with a higher density than the end with the lower density.

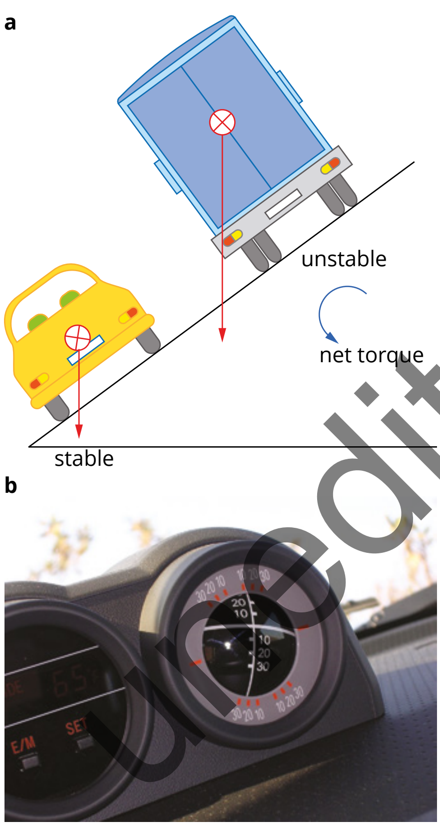


FIGURE 2.2.4 (a) The car on the incline is in stable equilibrium, while the heavily laden truck on the same incline could topple. The weight vector is outside the lower point of support for the truck, so there is no reaction force from the road to the higher wheel. (b) Modern four-wheel drives and tractors have inclinometers to warn the driver if the vehicle is in danger of tipping.

A concept that is closely related to centre of mass is **centre of gravity**. Instead of being a point particle whose motion equates to the whole extended body or system, the centre of gravity is the position from which the entire weight of the body or system is considered to act. As a consequence of this, the centre of gravity is the position at which the body will balance. For all practical purposes, the centre of gravity is exactly at the centre of mass. It is only when a body is so large that it is in a non-uniform gravitational field that the centre of gravity no longer coincides with the centre of mass.

There are three types of equilibrium related to the stability of an object:

- **stable equilibrium**—The object will return to its equilibrium position even when a force is applied to it, as long as the centre of mass is not moved outside the **base** of support.
- **unstable equilibrium**—The object will accelerate and will not return to its equilibrium position when a force is applied to it, since the centre of mass is readily moved outside the base of support.
- **neutral equilibrium**—The object will remain stationary no matter where it is placed, as any force applied has no effect on the relationship between the centre of mass and the base or point of support.

Figure 2.2.3 illustrates how this applies to real-life objects.

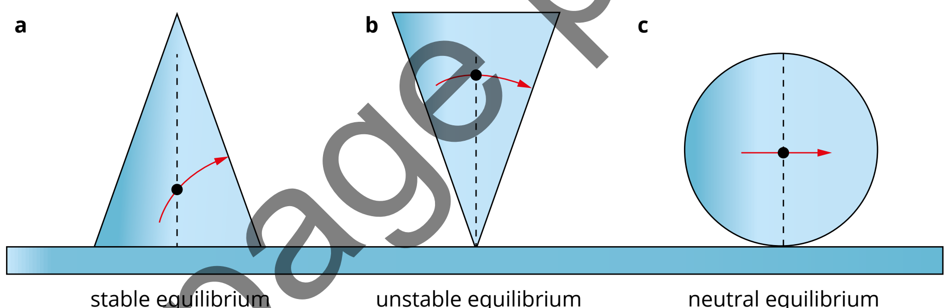


FIGURE 2.2.3 Three objects illustrating the difference between (a) stable, (b) unstable and (c) neutral equilibrium. As (a) has a broad base of support, it will return to its original position even when a force is applied to it, as long as the centre of mass is not moved outside the base of support.

When designing structures, engineers and architects want to ensure that balance and stability are maintained. Whether this occurs depends on the relative positions of the centre of gravity and the base. When a vertical line downwards from the centre of gravity passes through the base of support, the object is stable. The vertical line from the centre of gravity represents the direction of the force of gravity on the object.

In Figure 2.2.4(a), the weight of the car passes through the car's support base, between the wheels. The torque acting on the car is clockwise in this case and therefore does not cause the car to tip over. In the case of the truck, however, the weight is directed outside the truck's base of support, so an anticlockwise torque acts to tip the truck over.

The stability of an object or structure can be increased in a number of ways:

- the centre of gravity is lowered
- the width of the support base is increased
- the angle from the centre of gravity to the edge of the base is increased.

As a result of any of these, the object has to be tipped further to make the force of gravity act outside the support base. Racing cars have a very low centre of gravity to increase their stability when cornering at high speed. In a similar way, training wheels on a child's bicycle widen the support base, making it harder to tip the bicycle sideways.

These conditions also apply to objects that are supported from above. A child on a playground swing will be in stable equilibrium when the swing is hanging straight down. A gentle push will cause the swing to move but it will return to its original position once the outside force is removed.

If, on the other hand, the swing is raised to a significant height and held there, then it would be in unstable equilibrium. The swing is in equilibrium only because the forces on it are balanced while it is being held. If the holding force (say a parent's hands) is removed, then the swing will start to move, swinging across the point of stable equilibrium before eventually coming to rest in a position of stable equilibrium.

TRANSLATIONAL EQUILIBRIUM

A **translational equilibrium** of forces occurs when the sum of the forces acting through the centre of mass of an object add to give a zero resultant or zero net translational force. As a net translational force causes acceleration in one direction, a zero net translational force causes no acceleration of the object. This condition is the defining aspect of a translational equilibrium of forces.

When analysing a situation involving more than one force acting on an object, translational equilibrium will exist if the sum of the forces is equal to zero: $\Sigma F = 0$. This can also be written as $F_{\text{net}} = 0$.

Vector diagrams of an equilibrium of forces in one dimension

Vector diagrams can be drawn of the forces applied to an object when they are acting in one dimension. For example, if three people are pulling to the right and three people are pulling to the left in a game of tug of war, as shown in Figure 2.2.5, then the forces are all in one dimension: left and right. You can add these forces using a vector diagram by drawing all the forces from each person head to tail, as described in Chapter 2 'Scalars and vectors' in the Year 11 student book. If the tug of war is in translational equilibrium, then all the forces should add to give a zero net force.

FIGURE 2.2.5 Tug of war with a vector addition diagram showing a net force of zero, indicating an equilibrium of forces exists.

Calculating an equilibrium of forces in one dimension

Whether a situation is in translational equilibrium or not can be determined using a consistent sign convention to represent the direction of the force vectors in one dimension. Typically, in the *x*-dimension, left is treated as negative and right as positive; similarly in the *y*-dimension, up is treated as positive and down as negative. In the *z*-dimension, forwards is positive and backwards is negative. By applying a consistent sign convention to the forces acting on an object, the addition of those forces, with their signs, will give a vector sum of zero if the situation is in translational equilibrium.

That is:

$\Sigma F_{\text{left–right}} = 0$

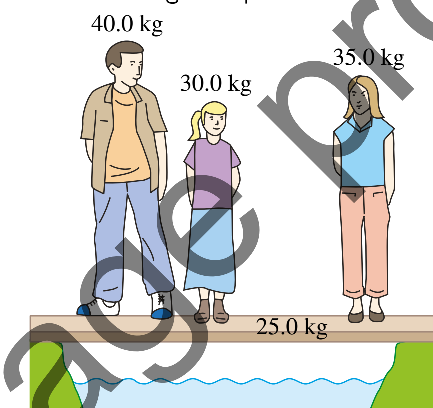
$\Sigma F_{\text{up–down}} = 0$

$\Sigma F_{\text{forwards–backwards}} = 0$

Worked example 2.2.1

CALCULATING TRANSLATIONAL EQUILIBRIUM IN ONE DIMENSION

Three students are standing on a plank that is bridging a small stream. The plank is supported at each end by the ground. The plank has a mass of 25.0 kg, and the students have masses of 40.0 kg, 30.0 kg and 35.0 kg. There is an upward force from the left bank on the plank of 725 N. If the plank is in translational equilibrium, calculate the force of the right bank on the plank. Use $g = 9.80\text{ N kg}^{-1}$ when answering this question.

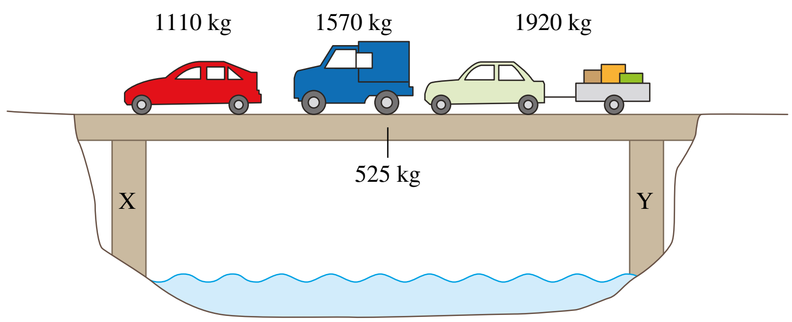


Thinking	Working
Identify the variables involved and state them with directions in their standard form.	$m_1 = 40.0\text{ kg}$ $m_2 = 30.0\text{ kg}$ $m_3 = 35.0\text{ kg}$ $m_P = 25.0\text{ kg}$ $F_{LB} = 725\text{ N up}$ $g = 9.80\text{ N kg}^{-1}$ down
Apply a sign convention to the vector data.	$F_{LB} = +725\text{ N}$ $g = -9.80\text{ N kg}^{-1}$
The object experiencing translational equilibrium is the plank.	$\Sigma F_{\text{up–down}} = 0$
Expand the equation to include each of the forces acting on the plank.	$F_1 + F_2 + F_3 + F_{WP} + F_{LB} + F_{RB} = 0$ $m_1g + m_2g + m_3g + m_Pg + F_{LB} + F_{RB} = 0$
Substitute the data into the equation and solve for the unknown.	$(40.0)(-9.80) + (30.0)(-9.80) + (35.0)(-9.80) + (25.0)(-9.80) + (725) + F_{RB} = 0$ $(-392) + (-294) + (-343) + (-245) + (725) + F_{RB} = 0$ $-549 + F_{RB} = 0$ $F_{RB} = +549$
State the answer to the appropriate number of significant figures and with the appropriate direction.	$F_{RB} = 549\text{ N up}$

Worked example: Try yourself 2.2.1

CALCULATING TRANSLATIONAL EQUILIBRIUM IN ONE DIMENSION

Three cars are parked on a beam bridge that has a mass of 525 kg. Pillar X applies a force of 2.00×10^4 N upwards. If the situation is in translational equilibrium, calculate the force provided by pillar Y. Use $g = 9.80\,\mathrm{N\,kg^{-1}}$ when answering this question. Car 1 (C_1) is red; car 2 (C_2) is blue; car 3 (C_3) is light green.



SOLVING EQUILIBRIUM IN TWO DIMENSIONS

If the forces involved in the equilibrium situation are in two dimensions, there are two strategies for determining if the sum of the forces is zero: using vector diagrams or vector components.

Vector diagrams

You can draw **vector diagrams** in which vectors are added in two dimensions. This was covered in Chapter 2 'Scalars and vectors' in the Year 11 student book. In any vector addition diagram, the individual vectors are added head to tail with the net vector drawn from the tail of the first vector to the head of the last vector. In a situation in which the forces are in equilibrium, the vector addition diagram should end up with the head of the last vector finishing at the tail of the first vector. This means that the vector addition diagram forms a closed loop and therefore there is no net force (Figure 2.2.6).

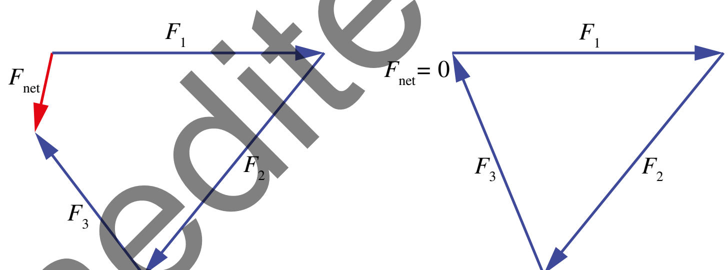


FIGURE 2.2.6 The vector diagram on the left shows three vectors added head to tail and the resultant net force vector F_{net} in red. The closed loop vector addition diagram on the right shows an equilibrium of forces, where $F_{net} = 0$.

Vector components

A vector diagram that results in a closed loop fulfils the conditions of a translational equilibrium of forces. If the three forces form a right triangle, you can use Pythagoras' theorem and trigonometry to determine the magnitude and direction of a third force that will be in equilibrium with two other forces. If the three forces are in any other triangle, then the unknown force can be found using the sine law or the cosine law, which is a more complex method. However, there is an easier approach that involves determining the **components** of the forces in two perpendicular dimensions. These force components are then added in each of their dimensions, which results in two perpendicular resultant vectors that can be added using Pythagoras' theorem and trigonometry (Figure 2.2.7).

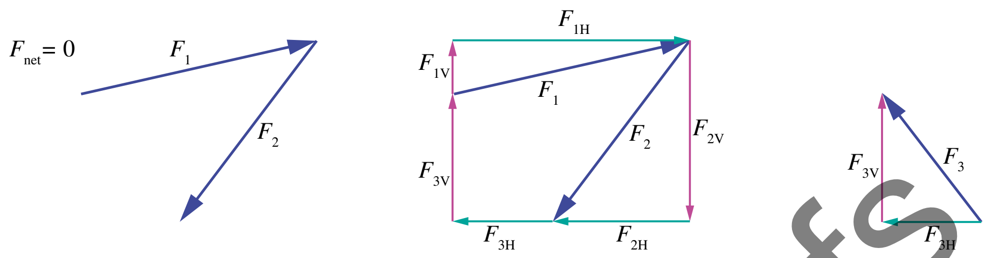


FIGURE 2.2.7 A method for finding F_3 that acts to make an equilibrium of forces with F_1 and F_2.

The guiding principle behind this method is that for the forces to be in equilibrium, the sum of the *x*-axis (horizontal or left–right) forces must equal zero, and the sum of the *y*-axis (vertical or up–down) forces must also equal zero.

$\Sigma Fx_{\text{-axis}} = 0$ or $\Sigma F_{\text{vertical}} = 0$ or $\Sigma F_{\text{left–right}} = 0$

$\Sigma Fy_{\text{-axis}} = 0$ $\quad$ $\Sigma F_{\text{horizontal}} = 0$ $\quad$ $\Sigma F_{\text{up–down}} = 0$

To draw a vector diagram (Figure 2.2.8):

- identify all of the forces that apply (e.g. tension, gravitational force)
- draw the vectors for each force separately and mark all known angles
- reposition the vectors so they are head to tail, adding all of the forces together.

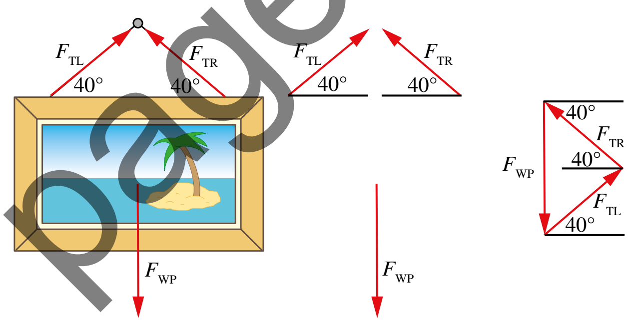
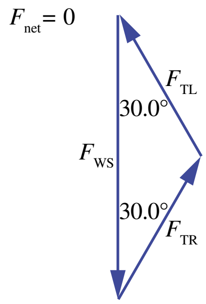
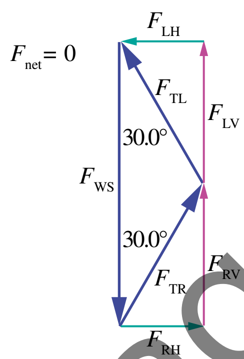
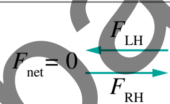
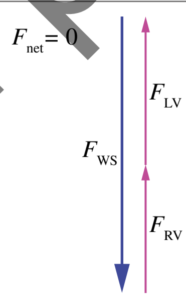


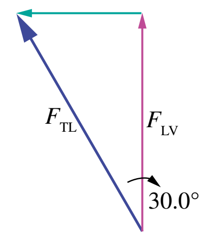
FIGURE 2.2.8 A vector diagram is drawn by identifying the vectors and then adding the vectors together, head to tail.

Worked example 2.2.2

CALCULATING TRANSLATIONAL EQUILIBRIUM IN TWO DIMENSIONS

An advertising sign is hung by two cables to the ceiling of a shop. The sign has a mass of 45.0 kg and the cables are at an angle of 30.0° to the vertical as shown in the image below. If the mass of the cables is ignored, calculate the tension in each cable when the sign is suspended. Use $g = 9.80\,\text{N kg}^{-1}$ when answering this question.

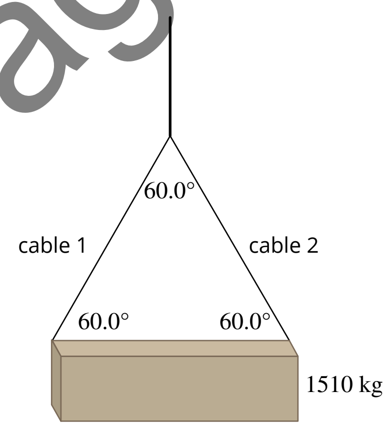
Thinking	Working
Construct a vector diagram adding all of the forces together.	 F_{WS} is the weight of the sign, F_{TL} is the tension of the left cable and F_{TR} is the tension of the right cable.
Apply left and right components and up and down components.	
In the horizontal dimension, F_{LH} is in equilibrium with F_{RH}.	
In the vertical dimension, F_{WS} is in equilibrium with F_{LV} and F_{RV}.	
Apply the equation for translational equilibrium in one dimension. F_{RV} and F_{LV} are equal in magnitude and therefore each is half of F_{WS}.	$\Sigma F_{up-down} = 0$ $F_{RV} = F_{LV}$
Expand the equation to include each of the vertical forces acting on the sign.	$F_{WS} + F_{RV} + F_{LV} = 0$ $m_S g + F_{RV} + F_{RV} = 0$
Substitute the data into the equation and solve for the unknown.	$(45.0)(-9.80) + 2F_{RV} = 0$ $-441 + 2F_{RV} = 0$ $F_{RV} = \frac{441}{2}$ $F_{RV} = F_{LV} = 220.5\,\text{N}$

Draw the right triangle with one of the vertical components of the tension and the angle.	
Use trigonometry to solve for the tension in one of the cables, which will equal the tension in the other cable. Present your answer to the appropriate number of significant figures.	$\cos\theta = \frac{F_{LV}}{F_{TL}}$ $F_{TL} = \frac{F_{LV}}{\cos\theta}$ $F_{TL} = \frac{(220.5)}{\cos(30.0°)}$ $F_{TL} = 254.61$ $F_{TL} = F_{TR} = 255\,N$

Worked example: Try yourself 2.2.2

CALCULATING TRANSLATIONAL EQUILIBRIUM IN TWO DIMENSIONS

A concrete beam of mass 1510 kg is being lifted by cables labelled 1 and 2, as shown in the diagram. The beam is moving upwards with a constant velocity of 2.00 m s^{-1}. Calculate the tension in cable 1 and cable 2. Ignore the mass of the cables and use $g = 9.80\,N\,kg^{-1}$ when answering this question.



2.2 Review

SUMMARY

- Translational equilibrium in one dimension can be represented mathematically as $\Sigma F = 0$.
- Translational equilibrium in two dimensions can be represented mathematically as $\Sigma Fx = 0$ and $\Sigma Fy = 0$.
- In two dimensions, an equilibrium of forces can be represented in a closed vector diagram.
- By calculating x and y components of the forces in equilibrium, the forces in equilibrium in each dimension can be found.

KEY QUESTIONS

1 Select the correct statement explaining when translational equilibrium occurs.

A A net force acts through the centre of mass and causes no acceleration.

B A net force acts through the centre of mass and causes an acceleration.

C No net force acts through the centre of mass resulting in an acceleration.

D No net force acts through the centre of mass and so there is no acceleration.

2 An object is in translational equilibrium. Which of the following statements applies?

A The object must be stationary.

B The object must be moving at constant velocity.

C The object must be experiencing translational acceleration.

D The object must not be experiencing translational acceleration.

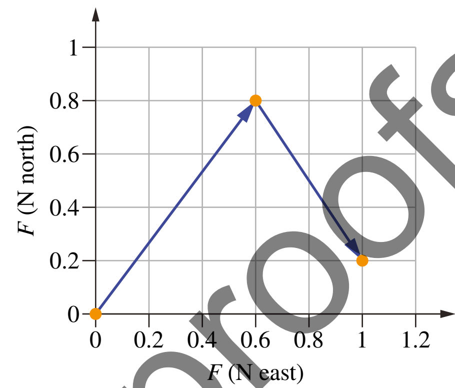
3 A string supports a birdfeeder that has a mass of 355 g. If the birdfeeder is in translational equilibrium, calculate the tension in the string supporting the birdfeeder.

4 Calculate the mass of a pendulum bob that hangs stationary from a chain when the tension in the chain is 7.50 N.

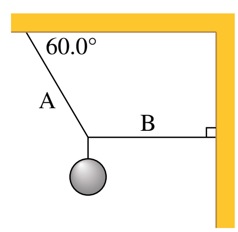
5 A chef wants to make sure that the wire cables attached to each end of a kitchen rail will be able to hold the weight of the rail, two tools and a frying pan that must hang from it. The mass of the rail is 3.05 kg, the mass of each tool is 345 g and the mass of the frying pan is 2.25 kg. Calculate the tension force in each wire cable if the load is spread evenly along the rail.

6 Two window cleaners working on the windows of a high-rise building work on a platform suspended by four cables. Tom has a mass of 79.0 kg, Jack has a mass of 68.0 kg and the mass of the platform is 225 kg. The platform is moving down the side of the building at a constant speed and all cables provide the same tension. Calculate the tension in one of the cables.

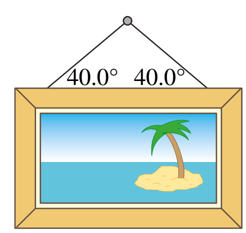
7 A shopping trolley is pushed in different directions by three children. The force vectors of two of the children are shown in the diagram. Draw the force vector from the third child that would cause the shopping trolley to be in translational equilibrium.



8 A bowling alley wants to promote its business by suspending a 112 kg fibreglass bowling ball on a frame outside its alley. The bowling ball is supported by two steel cables capable of withstanding up to 1550 N of tension before breaking. Cable A is at an angle of 60.0° to the horizontal frame member and cable B is perpendicular to the vertical frame member. Assuming the mass of the cables is negligible, calculate the tension in cable A and in cable B.



9 A picture is hung on a wall as shown in the diagram below. If the hanging wire is capable of supporting a maximum force of 40.0 N, what is the maximum mass of the picture that can be supported before the wire snaps?



10 A street performer stands on a rope tied between two posts. When the performer is standing at the centre, the rope makes an angle of 10.0° to the horizontal. Assuming the mass of the rope is negligible and that the mass of the performer is 75.0 kg, calculate the tension in the rope. Give your answer to three significant figures.

2.3 Static equilibrium

FIGURE 2.3.1 When a cyclist doesn't need to pedal, they can stand on both pedals with equal force. This causes an equal torque in the clockwise and anticlockwise directions. The pedals are in rotational equilibrium.

Objects are in translational equilibrium when the sum of the forces acting through their centre of mass equals zero. However, not all forces act through the centre of mass of an object; some forces may act in ways that generate torque or moment on an object. In order for an object to be in rotational equilibrium, the sum of the moments in a clockwise direction must balance the sum of the moments in an anticlockwise direction (Figure 2.3.1). This relationship is called the **principle of moments**.

By combining the conditions for translational equilibrium and rotational equilibrium, an object can be made to be in a state of static equilibrium. Static equilibrium will be explored in more detail in this section.

ROTATIONAL EQUILIBRIUM

A **rotational equilibrium** of torque or moments occurs about a **reference point** when the sum of all the torques acting on an object in the clockwise direction is equal to the sum of all the torques acting in the anticlockwise direction.

When analysing a situation involving more than one torque acting on an object, the principle of moments can be said to apply when rotational equilibrium exists, or the sum of the torques about a reference point is equal to zero. That is:

$\Sigma\tau = 0$

This is also represented as:

$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$

Figure 2.3.2 illustrates the masts of a boat that are in rotational equilibrium around their base of support.

FIGURE 2.3.2 To keep each mast in rotational equilibrium relative to its base, the stainless-steel cables (stays) attached to it must provide opposing torques on the mast.

When the sum of the torques is not balanced, however, the object will experience an unbalanced torque and will rotate about the reference point.

CONDITIONS FOR STATIC EQUILIBRIUM

When a body or a system is not accelerating or rotating, it must be in both translational and rotational equilibrium. This situation satisfies the conditions for **static equilibrium**, which can be represented by:

$\Sigma F = 0$ and $\Sigma\tau = 0$

This can also be shown as:

$\Sigma Fx = 0$

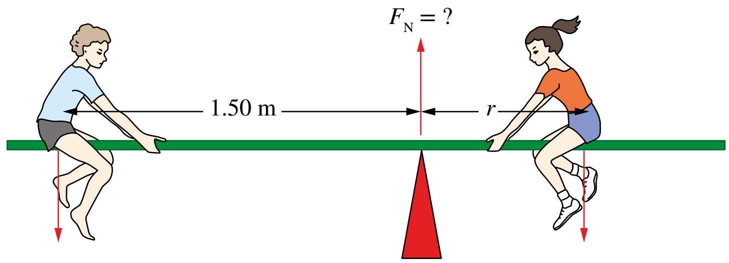
$\Sigma Fy = 0$

$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$

Worked example 2.3.1

CALCULATING STATIC EQUILIBRIUM

Two young children make a seesaw with a long plank. The boy sits on the seesaw 1.50 m from the pivot. The masses of the boy and the girl are 20.0 kg and 30.0 kg respectively. Assume that the mass of the plank is negligible. Use $g = 9.80\ \text{N kg}^{-1}$ in your calculations where required.



a Calculate the force applied to the plank by the pivot point when the children are sitting on the seesaw.

Thinking	Working
Identify the variables involved and state them with directions in their standard form.	$m_g = 30.0\ \text{kg}$ $m_b = 20.0\ \text{kg}$ $g = 9.80\ \text{N kg}^{-1}$ down
Apply a sign convention to the vector force data.	$g = -9.80\ \text{N kg}^{-1}$
Identify the object that is in translational equilibrium. This is the object on which all the forces are acting.	The object experiencing translational equilibrium is the plank.
Apply the equation for translational equilibrium in one dimension.	$\Sigma F_y = 0$
Expand the equation to include each of the forces acting on the plank.	$F_{Wg} + F_{Wb} + F_P = 0$ $m_g g + m_b g + F_P = 0$
Substitute the data into the equation and solve for the unknown.	$(30.0)(-9.80) + (20.0)(-9.80) + F_P = 0$ $(-294) + (-196) + F_P = 0$ $(-490) + F_P = 0$ $F_P = +490$
State the answer to the appropriate number of significant figures and with the appropriate direction.	$F_P = 4.90 \times 10^2\ \text{N}$ up

b Calculate where the girl has to sit in order to balance the boy.

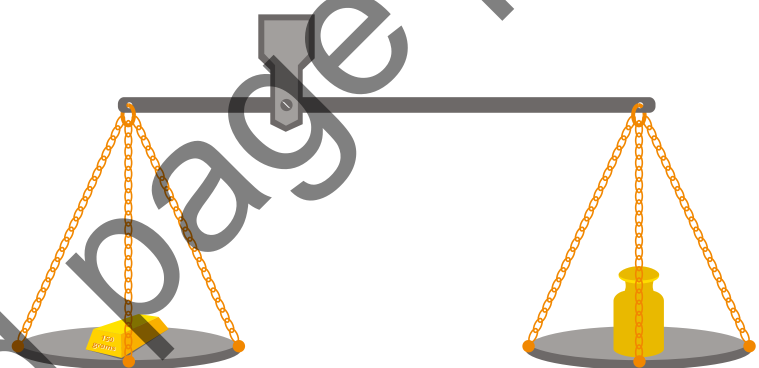
Thinking	Working
Identify the variables involved and state them in their standard form.	$m_g = 30.0\ \text{kg}$ $m_b = 20.0\ \text{kg}$ $r_{\perp b} = 1.50\ \text{m}$ $g = 9.80\ \text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the seesaw.
Identify the reference point about which the torques will be calculated.	The reference point is the pivot of the seesaw.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the girl on the seesaw provides the clockwise torque. The force of the boy on the seesaw provides the anticlockwise torque.

Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the seesaw.	$F_{Wg}r_{\perp g} = F_{Wb}r_{\perp b}$
Substitute the data into the equation and solve for the unknown. State the answer to the appropriate number of significant figures.	$F_{W}r_{\perp g} = F_{Wb}r_{\perp b}$ $m_{g}gr_{\perp g} = m_{b}gr_{\perp b}$ $(30.0)(9.80)r_{\perp g} = (20.0)(9.80)(1.50)$ $r_{\perp g} = \dfrac{(20.0)(9.80)(1.50)}{(30.0)(9.80)}$ $r_{\perp g} = 1.00\text{ m}$

Worked example: Try yourself 2.3.1

CALCULATING STATIC EQUILIBRIUM

A set of scales (with one longer arm) is used to measure the mass of gold when it is larger than the standard set of masses. A lump of gold weighing 185 g is placed on the short arm, which is 10.0 cm long, and the standard masses are placed on the long arm. Use $g = 9.80\text{ N kg}^{-1}$ in your calculations where required. Give your answers to three significant figures.



a Calculate the force applied to the scale's arm due to the pivot point if a standard mass of 50.0 g exactly balances the gold.

b Calculate the length the long arm should have in order to balance the gold.

In Worked example 2.3.1, the seesaw is in equilibrium because all the forces and torques are balanced. In solving the problem, it seemed obvious to choose the pivot as the reference point around which the torques are determined. But because the seesaw plank is in equilibrium, any point could have been chosen as the reference point. For example, you could take the reference point to be where the girl is sitting (Figure 2.3.3), meaning that the torques acting on the plank would be due to the boy and to the seesaw pivot point. The torque due to the girl becomes zero, as the lever arm distance for her will be zero. The solutions to the questions in the Worked example will still be the same for this reference point.

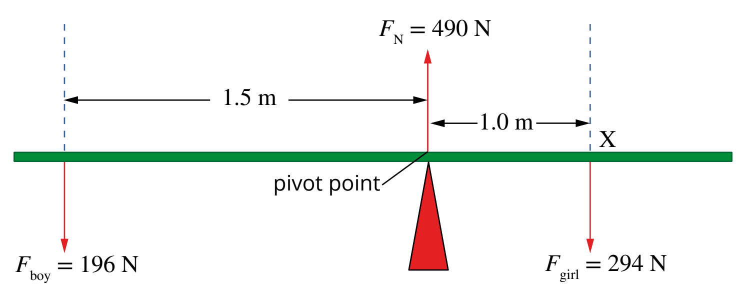


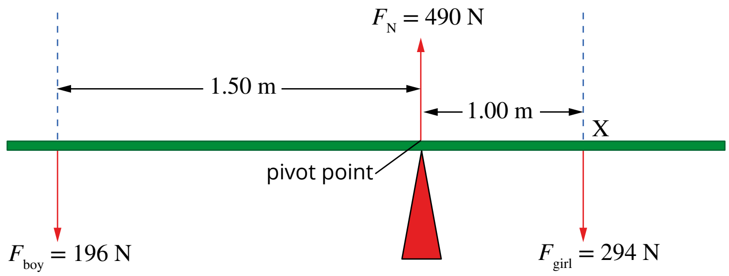
FIGURE 2.3.3 A force diagram for the seesaw problem from Worked example 2.3.1, showing that the point at which the girl sits (labelled X) has been chosen as the reference point.

Thus, the boy will create an anticlockwise torque around the girl, while the normal force at the pivot for the seesaw creates an equal clockwise torque around her. In the following Worked example, it can be verified that the seesaw is in rotational equilibrium by calculating the torques around the position of the girl. The plank will be in rotational equilibrium if the clockwise torque equals the anticlockwise torque.

Worked example 2.3.2

CALCULATING STATIC EQUILIBRIUM USING A DIFFERENT REFERENCE POINT

Verify that the seesaw plank in the image below is in rotational equilibrium about the reference point X, where the girl is sitting. The weights of the boy and girl are 196 N and 294 N respectively, and the force of the pivot on the plank is 490 N upwards. Assume that the mass of the plank is negligible.



Here, the point at which the girl is sitting (labelled X) has been chosen as the reference point. The force due to the boy acts downwards on the plank and the force due to the pivot point acts upwards on the plank.

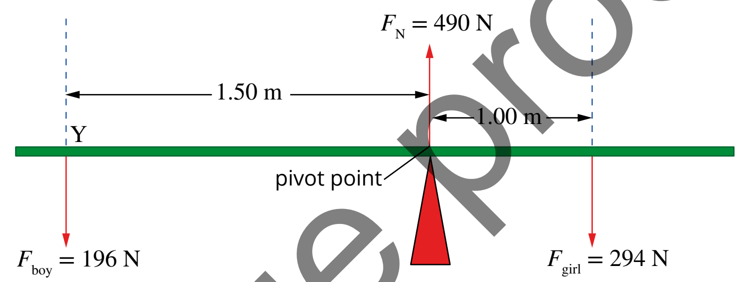
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$F_P = 490\,N$ $F_g = 294\,N$ $F_b = 196\,N$ $r_{\perp b} = 1.00 + 1.50 = 2.50\,m$ $r_{\perp P} = 1.00\,m$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the seesaw.
Identify the reference point about which the torques will be calculated.	The reference point is the position of the girl at X.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the pivot on the plank provides the clockwise torque. The force of the boy on the plank provides the anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the seesaw. Note that the girl's torque is not included here as, being the reference point, her torque is zero.	$F_P r_{\perp P} = F_b r_{\perp b}$
Substitute the data into the equation and solve.	$F_P r_{\perp P} = F_b r_{\perp b}$ $(490)(1.00) = (196)(2.50)$ $490 = 490$

Identify the magnitude of the clockwise torque compared to the magnitude of the anticlockwise torque.	Around reference point X (the position of the girl), the clockwise torque due to the pivot point on the plank is equal to the anticlockwise torque due to the boy on the plank. So, the plank is in rotational equilibrium.

Worked example: Try yourself 2.3.2

CALCULATING STATIC EQUILIBRIUM USING A DIFFERENT REFERENCE POINT

Verify that the seesaw plank in the figure below is also in rotational equilibrium about the reference point Y, where the boy is sitting. The weights of the boy and girl are 196 N and 294 N respectively, and the force of the pivot on the plank is 490 N upwards. Assume that the plank's mass is negligible.



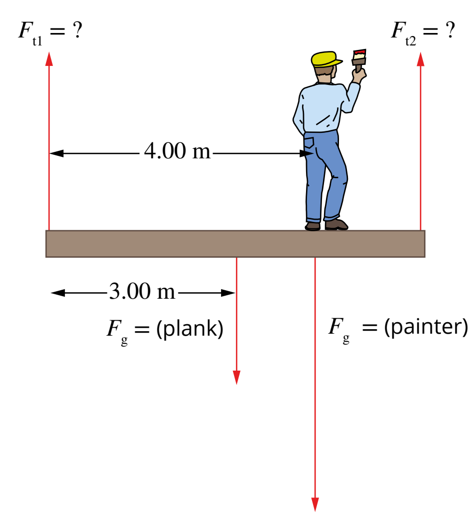
STATIC EQUILIBRIUM WITH TWO UNKNOWN FORCES

The seesaw problem is relatively straightforward since in each situation there is only one unknown force. If there are two unknown forces, the reference point can be chosen to coincide with one of the forces. Using this strategy means that the force acting at the reference point contributes no torque (since $r = 0$). The remaining unknown force can then be found using the relationship $\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$, as shown in Worked example 2.3.3.

Worked example 2.3.3

CALCULATING STATIC EQUILIBRIUM WITH TWO UNKNOWNS

A 70.0 kg painter stands 4.00 m from the end of a 6.00 m long plank that is supported by a rope at each end. The plank has a mass of 20.0 kg. Determine the tension on the left-hand rope (F_{t1}). Use $g = 9.80\ \text{N kg}^{-1}$ in your calculations where required.



Thinking	Working
Identify the variables involved and state them in their standard form.	$m_{pl} = 20.0\,kg$ $m_{pa} = 70.0\,kg$ $r_{\perp F_{t1}-F_{t2}} = 6.00\,m$ $r_{\perp c-F_{t2}} = 3.00\,m$ $r_{\perp pa-F_{t2}} = 2.00\,m$ $g = 9.80\,N\,kg^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the plank.
Identify the reference point about which the torques will be calculated. Always choose a point at which one of the unknown forces acts in order to form an equation with just one unknown variable. This avoids the need to solve simultaneous equations.	The reference point is the point at which the rope providing the tension force F_{t2} is attached.
Decide which force causes the clockwise torques and which forces cause the anticlockwise torques around the chosen reference point.	The tension force of the left-hand rope on the plank provides the clockwise torque. The force of the painter on the plank provides an anticlockwise torque. The force of gravity on the plank provides another anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$
Expand the equation to include each of the torques acting on the plank. The torque of the right-hand rope is not included as it acts through the reference point.	$F_{t1}r_{\perp F_{t1}-F_{t2}} = F_{pl}r_{\perp c-F_{t2}} + F_{pa}r_{\perp pa-F_{t2}}$ $F_{t1}r_{\perp F_{t1}-F_{t2}} = m_{pl}gr_{\perp c-F_{t2}} + m_{pa}gr_{\perp pa-F_{t2}}$
Substitute the data into the equation and solve for the unknown force.	$F_{t1}(6.00) = (20.0)(9.80)(3.00) + (70)(9.80)(2.00)$ $F_{t1} = \dfrac{(196)(3.00) + (686)(2.00)}{(6.00)}$ $F_{t1} = \dfrac{(588) + (1372)}{(6.00)}$ $F_{t1} = 326.67$ $F_{t1} = 327\,N$

Work example: Try yourself 2.3.3

CALCULATING STATIC EQUILIBRIUM WITH TWO UNKNOWNS

For the painter on the plank scenario in Worked example 2.3.3, determine the tension on the right-hand rope (F_{t2}).

Another way to determine the second unknown force is to apply the conditions for translational equilibrium. You can check these values:

$\Sigma F = 0$: the sum of the two upward forces (tensions) $555\,N + 327\,N = 882\,N$.

This balances the sum of the two downward forces: $196\,N + 686\,N = 882\,N$.

OTHER STATIC EQUILIBRIUM SCENARIOS

The balanced seesaw and the supported plank are just two of many situations in which static equilibrium can occur. Other conditions in which static equilibrium exists include the following scenarios.

Cantilevers

A beam that extends beyond its support structure is called a **cantilever**. Cantilevers are common structural elements. For example, the diving board at the local pool is a cantilever.

A cantilever bridge might be used to span a river or valley (Figure 2.3.4). Pillars are built at regular intervals across a gap in order to support a number of beams. The cantilever beams are then joined at the centre of each span. The forces on the pillars are not affected by joining the beams; they remain the same as they would be if the beams were not connected. All the support for the cantilever is supplied by pillars.

Other structures involving cantilevers include shelving, awnings over the footpath outside shops and the wings of a plane.

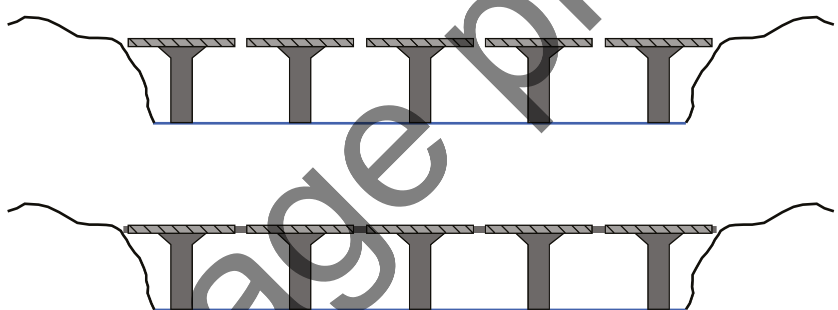


FIGURE 2.3.4 Each beam in the cantilever bridge is fully supported by the pillar below the beam. No added support is provided by connecting the beams.

PHYSICS IN ACTION

Cantilever ruler

- Is it possible to support a ruler with two fingers when one finger is not in the middle of the ruler? Try this experiment using a 1-metre ruler. A smaller ruler will do if that is all that is available.
- Place your index fingers underneath each end of the ruler to support it, as shown in Figure 2.3.5.
- Move your fingers towards each other to find the centre of mass of the ruler.
- Place your right hand at the 30 cm mark of the ruler. Now determine where you need to place your left hand if it must be placed at a point that is less than the 50 cm mark.
- You will find that, regardless of where you start and provided that the ruler is balanced, your fingers will come together at the centre of mass of the ruler. The downward force from the ruler on each finger will vary depending upon its position, and the friction between finger and ruler will vary as a result of the change in that downward force. The result is that both fingers will end up on either side of the centre of mass.

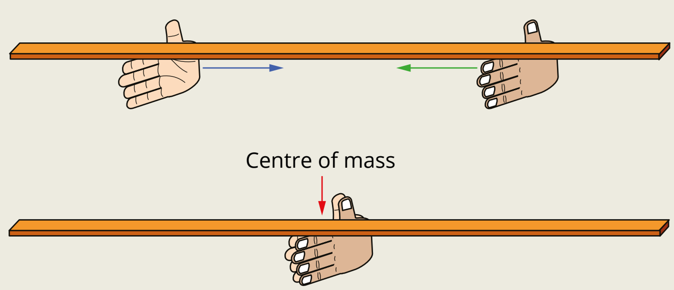


FIGURE 3.3.5 Finding the centre of mass of a ruler with your fingers.

When the centre of mass of a beam is not directly above a support, the force due to gravity acting on the centre of mass will create a torque that must be factored into the conditions of rotational equilibrium. In this case, two supports are usually required: one providing an upward force on the beam and the other providing a downward force on the beam.

Figure 2.3.6 shows a swimming pool diving board with one support applying an upward force on the board and the other support, the metal strap, applying a downward force on the board. A diving board must have a downward force on the fixed end of the board to create an opposing torque to the torque provided by the force due to gravity acting on the board.

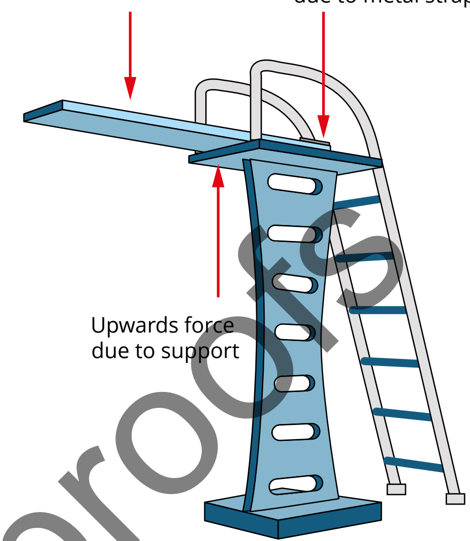
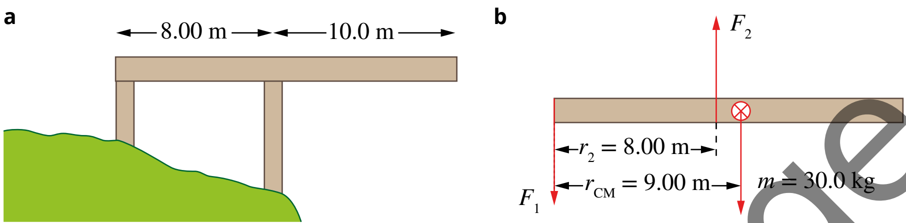


FIGURE 2.3.6 A diving board showing that a metal strap is needed to provide the downward force on the fixed end of the board.

Worked example 2.3.4

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER

A uniform cantilever beam 18.0 m long is used as a viewing platform. It extends 10.0 m beyond two supports that are 8.00 m apart. The beam has a mass of 30.0 kg. Determine the magnitude and direction of the force that the right-hand support must supply so that the beam is in static equilibrium.

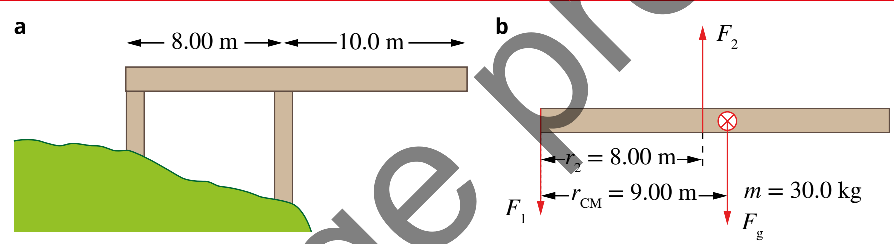


Thinking	Working
Identify the variables involved and state them in their standard form.	$m_b = 30.0$ kg $r_{\perp F_2-F_1} = 8.00$ m $r_{\perp c-F_1} = 9.00$ m $g = 9.80\ \text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object upon which all the torques are acting.	The object experiencing rotational equilibrium is the beam.
Identify the reference point about which the torques will be calculated. Choose the point on which the other unknown force acts to eliminate it as an unknown in the equation.	The reference point is the point at which the support providing the force F_1 is attached.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the right-hand support on the beam provides the anticlockwise torque. The force of gravity on the beam provides the clockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the beam. The torque of the left-hand support is not included, as it acts through the reference point.	$F_b r_{\perp c-F_1} = F_2 r_{\perp F_2-F_1}$

Substitute the data into the equation and solve for the unknown.	$(30.0)(9.80)(9.00) = F_2\,(8.00)$ $F_2 = \dfrac{(30.0)(9.80)(9.00)}{(8.00)}$ $F_2 = \dfrac{(2646)}{(8.00)}$ $F_2 = 330.75$
State the direction of the force acting on the object in equilibrium and present to the appropriate number of significant figures with the appropriate units.	The force of 331 N is upwards on the beam.

Worked example: Try yourself 2.3.4

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER



Determine the magnitude and direction of the force that the left-hand support must supply so that the beam is in static equilibrium (F_1).

You can check these values using $\Sigma F = 0$.

The sum of the upward forces (F_2) is 331 N.

This balances the sum of the two downward forces, $(F_g + F_1) = 294 + 36.8 \approx 331$ N.

Struts and ties

In addition to balancing the forces on their main beams and pillars, many structures are strengthened in a number of ways. For example, a cantilever may be supported by struts or ties, as shown in Figure 2.3.7. A strut is placed below the beam and is rigid, so it will be under compression. A tie, attached from above a beam, may be either rigid or flexible and will be under tension.

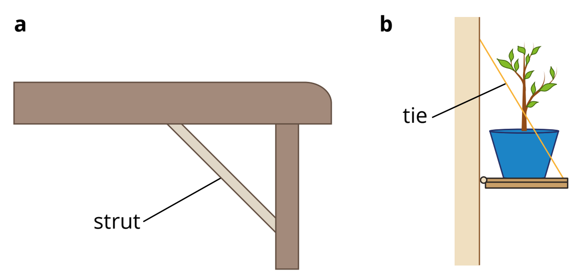
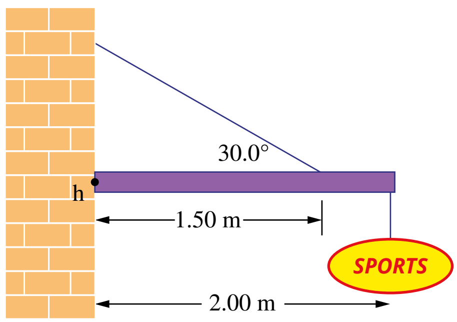


FIGURE 2.3.7 (a) A strut helps to support a cantilevered beam and is under compression. (b) A tie helps to support a fold-out shelf and is under tension.

Worked example 2.3.5

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER SUPPORTED BY A TIE

A sign of mass 10.0 kg is suspended from the end of a uniform 2.00 m long cantilevered beam. The other end of the beam is attached to the wall by a hinge labelled h. The beam has a mass of 25.0 kg and is further supported by a wire tie that makes an angle of 30.0° to the beam. The wire is attached to the beam at a point 1.50 m from the wall. Use $g = 9.80\,\text{N kg}^{-1}$ and ignore the mass of the wire for these calculations.



Calculate the tension (F_t) in the wire that is supporting the beam.

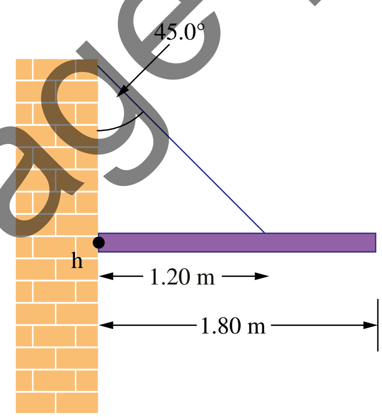
Thinking	Working
Identify the variables involved and state them in their standard form.	$m_b = 25.0\,\text{kg}$ $m_s = 10.0\,\text{kg}$ $r_{\perp\text{b–h}} = 1.00\,\text{m}$ $r_{\perp\text{s–h}} = 2.00\,\text{m}$ $r_{\perp\text{w–h}} = 1.50\,\text{m}$ $g = 9.80\,\text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the beam.
Identify the reference point about which the torques will be calculated. Choose the point at which the other unknown force acts to eliminate it from the equation.	The reference point is the hinge (h) at which the beam is connected to the wall. As the unknown reactionary force at the hinge passes through this reference point, it does not need to be considered.
Decide which force causes the anticlockwise torque and which forces cause the clockwise torques around the chosen reference point.	The force of the wire tie on the beam provides the anticlockwise torque. The force of gravity on the beam provides one clockwise torque. The force of gravity on the sign provides another clockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the beam.	$F_b r_{\perp\text{b–h}} + F_s r_{\perp\text{s–h}} = F_w r_{\perp\text{w–h}}$
Substitute the data into the equation to solve for the perpendicular distances from the force arm to the line of action of the force.	$r_{\perp\text{w–h}} = r_{\perp\text{w–h}} \sin(30°)$ $r_{\perp\text{w–h}} = 1.50 \times \sin(30°)$ $r_{\perp\text{w–h}} = 0.750\,\text{m}$

Substitute the data into the equation and solve for the unknown force. Present your answer with the correct units and to the correct number of significant figures.	$(25.0)(9.80)(1.00) + (10.0)(9.80)(2.00) = F_t\,(0.750)$ $F_t = \dfrac{(245)+(196)}{(0.750)}$ $F_t = \dfrac{(441)}{(0.750)}$ $F_t = 588.00$ $F_t = 588\,\text{N}$
By finding the perpendicular distance using the angle at which the tension force is acting, the tension force can be calculated directly by the equation. If the vertical component of the tension force was found, then the tension force could be calculated separately using the angle at which the force is acting.	

Worked example: Try yourself 2.3.5

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER SUPPORTED BY A TIE

A uniform 5.00 kg beam, 1.80 m long, extends from the side of a building and is supported by a wire tie that is attached to the beam 1.20 m from a hinge (h) at an angle of 45.0°.



Calculate the tension (F_t) in the wire that is supporting the beam.

2.3 Review

SUMMARY

- Static equilibrium occurs when an object experiences translational equilibrium and rotational equilibrium.
- Static equilibrium can be represented mathematically as $\Sigma F = 0$ and $\Sigma \tau = 0$.
- Rotational equilibrium is best represented mathematically as $\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$.
- In calculations of static equilibrium, the reference point is the point about which the torques act.
- In calculations of static equilibrium with two unknown forces, the reference point can be placed at the point at which one of the unknown forces acts. This eliminates any torque due to this force, as the distance from the force to the reference point is zero.

KEY QUESTIONS

1 Select the correct statement describing the requirements for an object to be in rotational equilibrium.

A A net torque acts about the reference point and rotation does not occur.

B A net torque acts about the reference point and rotation occurs.

C No net torque acts about the reference point and the rate of rotation does not increase.

D No net torque acts about the reference point and the rate of rotation increases.

2 An adult of mass 75.0 kg sits on a seesaw at a playground with a 25.0 kg child sitting 2.25 m from the pivot point on the other side from the adult. Calculate how far from the pivot the adult must sit for the seesaw to remain balanced and horizontal.

3 Select the statement that correctly describes an object in static equilibrium.

A The object experiences rotational equilibrium, but not translational equilibrium.

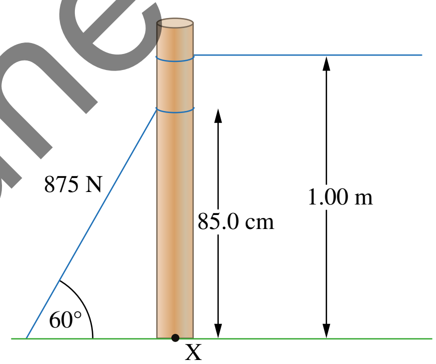
B The object experiences rotational equilibrium and translational equilibrium.

C The object experiences neither rotational equilibrium nor translational equilibrium.

D The object experiences translational equilibrium, but not rotational equilibrium.

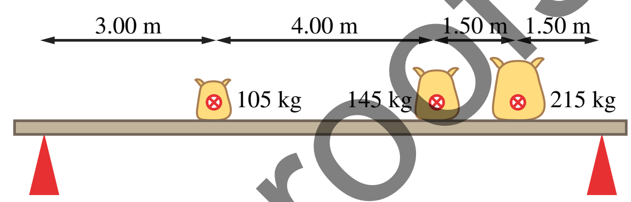
4 An adult of mass 90.0 kg sits on a seesaw at a playground with two 20.0 kg children. One child is sitting 1.50 m from the pivot point and the other is sitting 2.50 m from the pivot point, both on the other side from the adult. Calculate how far from the pivot the adult must sit for the seesaw to remain balanced and horizontal.

5 The end post of a vineyard trellis is held in position by a backstay that is under a tension of 875 N at an angle of 60.0° to the horizontal. The geometry of the situation is shown in the diagram below.



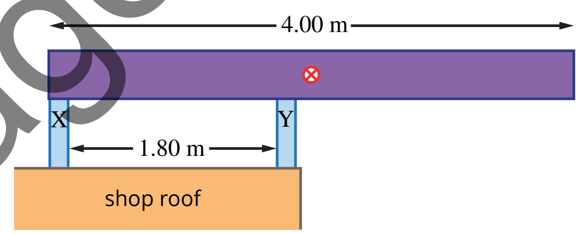
Using the base of the post, X, as the pivot point, determine the size of the tension in the vineyard trellis wire, F_t.

6 A makeshift shelf is used in a bakery to store sacks of flour. The shelf is constructed using a 10.0 m beam with a mass of 50.0 kg, with one support positioned at each end. The shelf holds sacks of mass 105 kg, 145 kg and 215 kg at the positions shown in the figure below. Calculate the forces on the beam due to the left and right supports.



The following information applies to questions 7–9.

A 4.00 m cantilever-type awning is constructed on the roof of a shop so that it shades the front. The awning has a mass of 925 kg and is supported by two supporting columns, X and Y, which produce forces on the awning of F_X and F_Y respectively.



7 Determine the force supplied by column Y on the awning (F_Y).

8 Determine the direction in which F_X acts on the awning.

9 Determine the force supplied by column X on the awning (F_X).

Chapter review

02

KEY TERMS

axis of rotation
base
cantilever
centre of gravity
centre of mass
component
force arm
line of action
neutral equilibrium
pivot point
principle of moments
reference point
rotational equilibrium
stable equilibrium
static equilibrium
torque
translational equilibrium
unstable equilibrium
vector diagram

Use $g = 9.80\,m\,s^{-2}$ to answer the following questions.

1 Which of the options best completes the following statement about torque?
'When a force acts such that the line of action of the force is directed through the pivot point of the object, …'
A a torque will result
B a torque will result only if the applied force is greater than the weight of the pivot point
C a torque will result only if the weight of the pivot point is greater than the applied force
D no torque will result

2 Tom is riding his skateboard and is in a state of translational equilibrium. Select the correct statement regarding Tom's motion.
A He must be decelerating.
B He must be maintaining a constant velocity.
C He must be experiencing a translational acceleration.
D He must be experiencing a rotational acceleration.

3 A cyclist is coasting down a road at a decreasing velocity while standing on the pedals. Which one or more of the following objects is in static equilibrium?
A the rear wheel
B the front wheel
C the front cog connected to the pedals
D the cyclist standing on the pedals

4 A child pulls down on a lever-type door handle with a force of 30.0 N. Given that the length of the handle is 12.0 cm, calculate the maximum possible torque acting on the handle's pivot.

5 A railway signalman is responsible for pushing levers that move train tracks, which switches a train from one line to another. Calculate the torque on the axle at the bottom of a 1.35 m lever if a 74.0 N force acts at an angle of 60.0° to the lever.

6 Define 'stability'. Describe the different types of stability and the factors that affect stability.

The following information applies to questions 7 and 8.
A woman whose car has a flat tyre has two wheel spanners in the boot of her car. One wheel spanner is 15.0 cm long, the other is 75.0 cm long.

7 In order to undo the wheel nuts with a minimum amount of effort, which wheel spanner should the woman select? Explain.

8 If the maximum force that the woman can apply is 45.0 N, determine the maximum torque that can be delivered to a wheel nut.

9 In the following situations, a torque is acting. In each case, identify the axis of rotation or pivot point about which the torque acts and estimate the length of the lever arm.
a A garden tap is turned on.
b A wheelbarrow is lifted by the handles.
c An object is picked up with a pair of tweezers.
d A screwdriver is used to lever open a tin of paint.

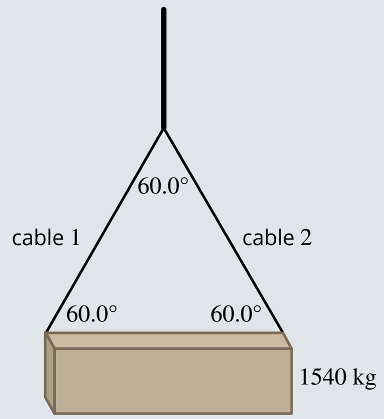
The following information applies to questions 10 and 11.
A crane with a horizontal lever arm is lifting a concrete wall of mass 2.50 tonnes. The load is 20.0 m from the axis of rotation.

10 Calculate the torque created by this load.

11 What stops the crane from toppling over as a result of this torque?

12 Which of the following are in translational equilibrium? More than one correct answer is possible.
A a stationary elevator
B an elevator going up with constant velocity
C an aeroplane during take-off
D a container ship sailing with constant velocity
E a car plummeting off a cliff

The following information applies to questions 13 and 14.
A concrete beam of mass 1540 kg is being lifted by steel cables, as shown in the diagram.

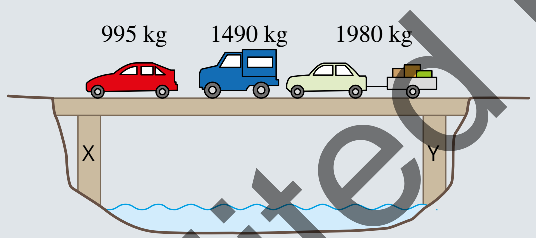


The beam is moving upwards with a constant velocity of 2.00 m s^{-1}. Ignore the mass of the cable.

13 Draw a force diagram showing all the forces acting on the beam.

14 Calculate the tension in cable 1 and cable 2. Show your answer in kN.

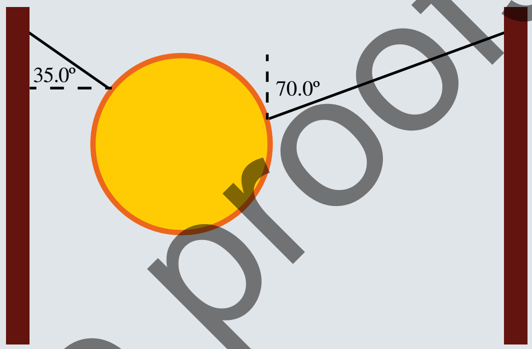
15 Three cars are crossing a beam bridge of mass 525 kg in a single line. At one instant, the left pillar (X) is providing a force of 2.00×10^4 N upwards. What is the size and direction of the force exerted by pillar Y at this instant?



16 A dining tabletop, of mass 40.0 kg, is supported by four legs. On the table there is 4.50 kg of food. If the table is stationary, and the legs all provide the same support, calculate the force each leg would be exerting on the table.

17 A climber of mass of 86.5 kg is hanging from a rope half-way up a rock wall. Calculate the tension force provided by the rope in order for the climber to be in translational equilibrium.

18 A 12.5 kg sign is hung between two poles by two cables of negligible mass, as shown in the diagram below. The sign is in translational equilibrium. Calculate the tension in the shorter left-hand cable and the longer right-hand cable.



19 A child's toy is to be suspended above her bed from a string attached to the ceiling. The toy is made from a 1.00 m long aluminium rod of mass 10.0 g, with a 145 g model of the Sun hanging at one end of the rod and a 22.5 g model of the Earth hanging at the other end. Calculate how far from the Sun, in centimetres, the ceiling attachment string needs to be tied to the bar, so that the whole toy is in static equilibrium.