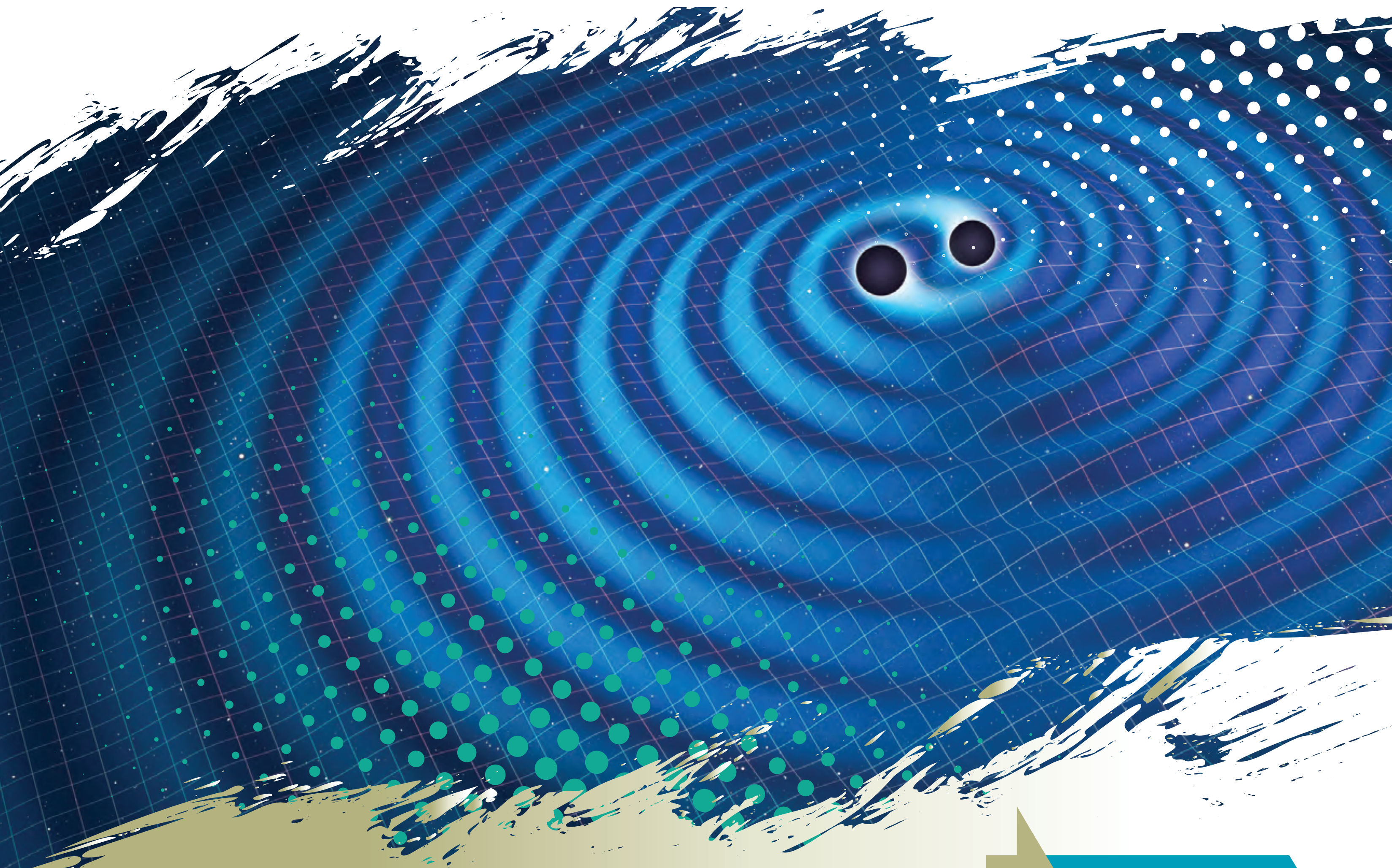
PEARSON PHYSICS QUEENSLAND

UNITS 3 & 4

2nd EDITION



Student Book

QCE 2025 Physics SYLLABUS

CHAPTER 03

Inclined planes and circular motion

This chapter explores the forces on objects resting or moving on a surface that is at an angle to the horizontal, also known as inclined planes, as well as objects experiencing circular motion. These forces will be analysed using free-body diagrams and formulae that were introduced in *Pearson Physics 11 Queensland*.

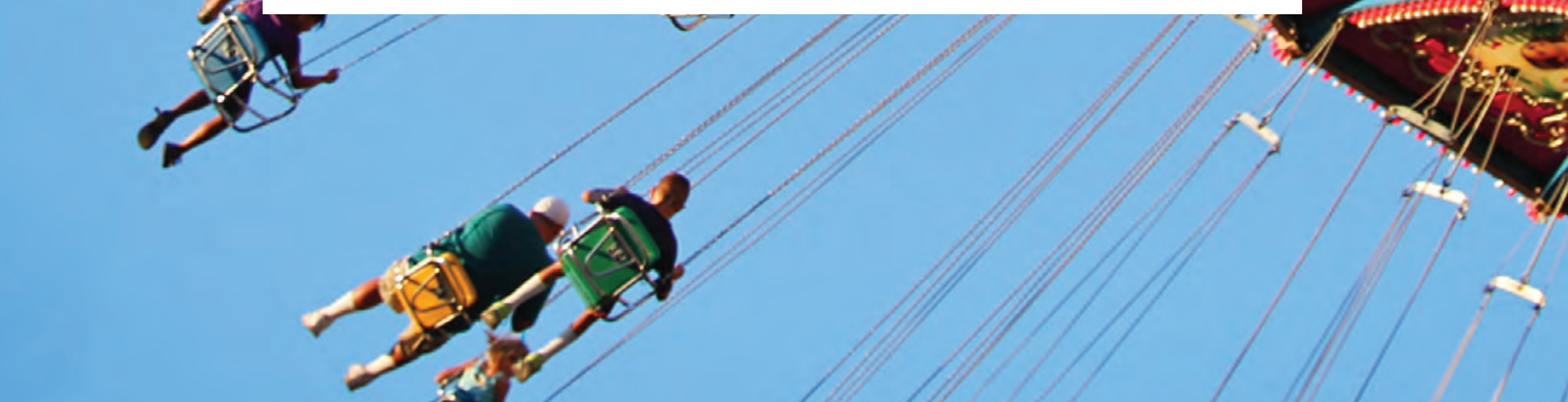
Inclined planes are used to change the magnitude or direction of a force. They are experienced in many aspects of everyday life from walking up a hill to building skyscrapers. Circular motion explores the idea that if a force is applied perpendicularly (at right angles) to the motion of an object, then the object will change direction. If this force is *continuously* applied at right angles to the motion of an object, then the object will *continue* to change direction, i.e. it will move in uniform circular motion.

Syllabus subject matter

Topic 1 • Gravity and motion

INCLINED PLANES AND CIRCULAR MOTION

- solve problems involving force due to gravity (weight) and mass using $F_g = mg$ **3.1**
- describe the concept of normal force **3.1**
- describe the forces acting on an object on an inclined plane (e.g. force due to gravity, normal force, tension, frictional force and applied force) through the use of free-body diagrams **3.1**
- determine the net force acting on an object on an inclined plane using vector analysis **3.1**
- describe the concept of uniform circular motion **3.2**
- describe the concepts of average speed and period **3.2**
- solve problems involving objects undergoing uniform circular motion at a constant speed using $v = \frac{2\pi r}{T}$ and $a_c = \frac{v^2}{r}$ **3.2**, **3.3**
- describe the concepts of centripetal acceleration and centripetal force **3.3**
- solve problems involving forces acting on objects in uniform circular motion using $F_c = F_{net} = \frac{mv^2}{r}$ **3.3**



3.1 Inclined planes

BY THE END OF THIS MODULE, YOU SHOULD BE ABLE TO:

- identify the forces acting on an object moving or resting on a surface at an angle to the horizontal, including the force due to gravity, friction and normal forces
- draw free-body diagrams to show the forces acting on an object on an inclined plane
- use vectors to analyse the net force acting on an object on an inclined plane.

FORCE DUE TO GRAVITY

In *Pearson Physics 11 Queensland* it was shown that a **force** is a push or a pull that can do one or more of the following to any object with mass:

- A force can change an object's velocity by changing its speed.
- A force can change an object's velocity by changing its direction of motion.
- A force can change the shape of an object.

The force **due to gravity** (also referred to as **weight** or simply **gravitational force**) is the force on an object due to its mass being in the presence of a gravitational field, such as Earth's gravitational field. The mass of an object is the same everywhere in the universe, but its gravitational force will change depending on the strength of the gravitational field the object is exposed to (you will learn more about this in Chapter 4. Any object with mass that is free to move in a gravitational field will experience an **acceleration due to gravity**, g.

> A force is a measure of the push or pull on an object.
>
> It is measured in newtons (N).
>
> It is a vector and so requires a magnitude and a direction to describe it fully.

The second law of motion can be used to calculate the gravitational force, F_g, of any mass in any gravitational field with an acceleration due to gravity g:

$$F_g = mg$$

g will vary depending on the location, but for objects on the surface of Earth, g is approximately $9.8\,\text{m}\,\text{s}^{-2}$ straight down towards the centre of Earth.

> Force due to gravity is the force that acts on an object due to its mass in a gravitational field. It is given by:
>
> $$F_g = mg$$

THE NORMAL FORCE

The **normal force** is a very important force that acts on an object that is in contact with a surface. If we are at rest on a horizontal surface, and no other forces are acting, our gravitational force and the normal force acting on us are the same magnitude. These two forces will not be the same magnitude, however, if other forces act or the directions of the forces change, such as when flying in an aeroplane, or moving in an elevator.

The normal force, F_N, is always perpendicular to the surface in contact with the object (Figure 3.1.1). Remember that, although the gravitational force and the normal force may be equal in magnitude and opposite in direction, they are not an action-reaction pair defined by the third law of motion, as both forces are acting on the box from different sources.

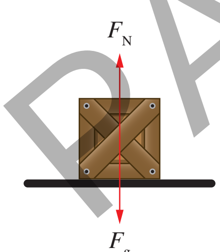


FIGURE 3.1.1 The normal force, F_N, is the force that pushes a mass upwards and is perpendicular to the surface in contact with the object.

In the example shown in Figure 3.1.1, the forces acting on the crate are:

- force due to gravity, F_g, and
- normal force, F_N.

The crate is resting on the ground so the net force must be zero. Using the second law of motion:

$$F_{net} = \text{vector sum of all forces acting on the crate}$$

So,

$$F_{net} = F_N \text{ (as a vector)} + F_g \text{ (as a vector)}$$

> The normal force is a force that acts on an object when the object is in contact with a surface.
>
> The normal force always acts perpendicular to the surface.

The gravitational force and normal force are in a straight line because the surface the crate rests on is exactly horizontal. This means we can set the upwards force as positive and the downwards force as negative. So, the net force becomes:

$$F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$$

$$\text{But } F_{\text{net}} = 0, \text{ so}$$

$$0 = F_{\text{N}} + F_{\text{g}}$$

$$\therefore F_{\text{N}} = -F_{\text{g}}$$

This shows that the normal force is the same size as the gravitational force but opposite in direction.

FREE-BODY DIAGRAMS

A diagram showing all of the forces acting on a single object is called a **free-body diagram**. Free-body diagrams are used to simplify the process of analysing the forces, and hence the resultant motion of a single object, by only showing forces that act directly on that particular object.

In this chapter, forces are drawn concurrently (i.e. from the same point on the object) and should show which force acts in which direction. Forces can be drawn to scale, but this is often not practical and can introduce the possibility of measuring error. Forces have not necessarily been drawn to scale in this book, but the length of the arrows are roughly relative to the magnitude of the force. Sometimes the net force is shown alongside the diagram to indicate the direction of acceleration if the object is not in equilibrium.

When a falling apple experienced **drag**, the free-body diagram could be as shown in Figure 3.1.2.

In this example the magnitude of the drag equals the magnitude of the gravitational force (the lengths of the two vectors are the same), so there is no net force and the apple falls with constant velocity.

An example of a free-body diagram of a remote-control car is shown to the left (Figure 3.1.3). The diagram shows four forces (gravitational force, F_{g}, normal force, F_{N}, **frictional force**, F_{f}, and **applied force**, F_{A}) acting on the car. The gravitational force and normal force are equal and opposite, so do not cause any acceleration up or down. There is a net force to the left, meaning the applied force is greater than the frictional force.

Drag and friction are forces that oppose the motion of the object. Drag acts to slow the motion of an object as it travels through air or a liquid. Frictional forces act to resist motion when two surfaces are in contact.

In a free-body diagram only forces that directly affect the object are shown. These add up to give the net force on the object.

The net force, in turn, then determines the acceleration of the object using the second law of motion, $F = ma$.

It is usually clear from the context whether something is a vector. Nonetheless, vectors can be represented using vector notation. In this book vectors are represented using italics, e.g. *F*, as are other variables and physical quantities. You might see a different type of vector notation in books and journals. This may include bold italics, such as ***F***, or a tilde or arrow above the letter such as $\tilde{F}$ or $\vec{F}$.

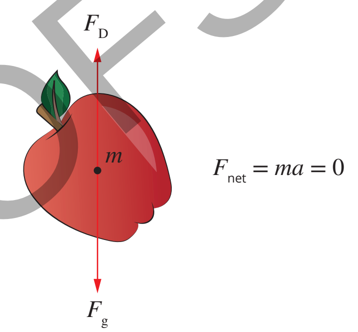


FIGURE 3.1.2 The free-body diagram of a falling apple with drag. In this example the drag is equal in magnitude to the gravitational force so there is no net force and the apple falls with constant velocity.

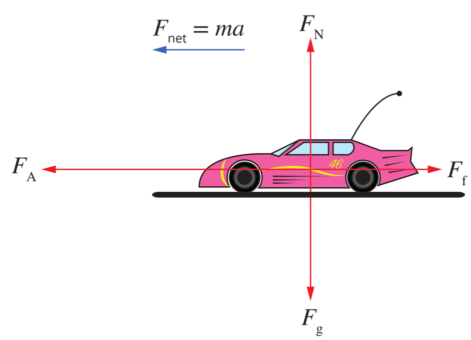
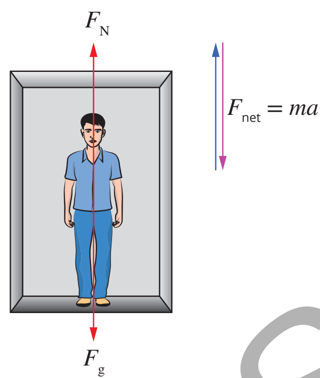


FIGURE 3.1.3 This free-body diagram of a remote-controlled car shows four forces acting (force due to gravity, normal force, friction and applied force) to give a net force to the left.

Worked example 3.1.1

FREE-BODY DIAGRAMS

A 78.0 kg student is standing in a lift on the fifth floor of a building. The lift accelerates downwards at 0.540 m s^{-2}.
a Draw a free-body diagram showing the forces that act on the student as the lift accelerates downwards.

Thinking	Working
There are two forces acting on the student: gravitational force and normal force. They are drawn parallel to each other, with the net force acting downwards.	

b Calculate the normal force the student experiences as the lift accelerates downwards.

Thinking	Working
There are two values given and these are mass and acceleration.	$m = 78.0\,kg$ $a = 0.540\,m\,s^{-2}$ downwards
Use the second law of motion to calculate the net force. As acceleration is downwards, F_g is greater than F_N.	$F_{net} = F_g - F_N$
Calculate the gravitational force.	$F_g = mg$ $= 78.0 \times 9.8$ $= 764.4\,N$
Substitute the known vales to find F_N.	$ma = F_g - F_N$ $78.0 \times 0.540 = 764.4 - F_N$ $42.12 = 764.4 - F_N$ $-F_N = 42.12 - 764.4$ $F_N = 722\,N$
Write the normal force as a vector with magnitude and direction.	$F_N = 722\,N$ upwards

➤ Try yourself 3.1.1

FREE-BODY DIAGRAMS

A 66.0 kg student is standing in a lift on the ground floor of a building. The student presses the button marked ‘2’ to go up to the second floor. The lift accelerates upwards at $0.820\,m\,s^{-2}$ for a few seconds before it then moves at a constant velocity.

a Draw a free-body diagram showing the forces that act on the student as the lift accelerates upwards.

b Calculate the normal force the student experiences as the lift accelerates upwards.

INCLINED PLANES

If an object such as a crate is on the ground, but the ground itself is at an angle to the horizontal, the gravitational force will still act straight down, and the normal force still acts perpendicular to the surface.

In this case the gravitational force and normal force are no longer parallel, and the magnitude of the normal force is no longer equal to the magnitude of the gravitational force. The magnitude of the gravitational force still has a negative value as its direction is straight down, but the normal force is no longer straight up.

In this situation, the crate can either be at rest, moving with a constant velocity, or accelerating along an **inclined plane**. All the laws of motion still apply; that is, the net force acting on the crate is equal to the mass of the crate multiplied by the acceleration of the crate. If the net force acting on the crate is zero, then the crate will be stationary or moving at a constant velocity.

As the angle that the inclined plane makes with the horizontal increases, the normal force decreases. This is because there is a smaller and smaller force pushing perpendicular to the surface when the plane is at an angle to the horizontal. The normal force is zero when the plane is perpendicular to the horizontal (Figure 3.1.4).

> The normal force is still perpendicular to the surface even if the surface is not horizontal.
>
> The force due to gravity is still straight down even if the surface is not horizontal.

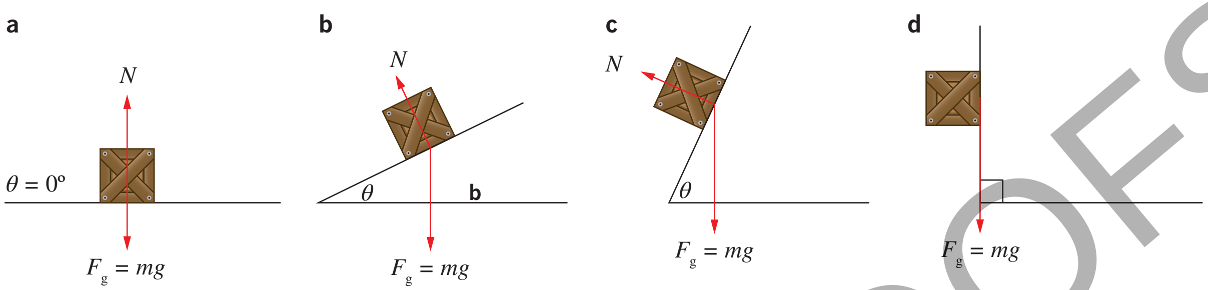


FIGURE 3.1.4 As the angle to the horizontal increases from 0° in (a), the magnitude and direction of the normal force changes, as in (b) and (c), until it is zero when the inclined plane is 90° to the horizontal. In all cases, the normal force is always perpendicular to the inclined plane.

In drawing free-body diagrams of objects on inclined planes, all forces are written according to a set of axes that are either parallel (the *x*-direction) or perpendicular (the *y*-direction) to the inclined plane. In defining the axes this way, only the gravitational force will need to be separated into components, as the normal force will always be perpendicular to the plane and any frictional forces will always act parallel to the plane (since the object's motion will either be up or down the incline).

To find the value of the normal force, the gravitational force must be separated into components:

- F_{gy}, the component of F_g perpendicular to the plane (which will be equal in magnitude and opposite in direction to the normal force)
- F_{gx}, the component of F_g parallel to the plane.

These components are shown as a right-angled triangle in Figure 3.1.5 and with the full system of forces in Figure 3.1.6.

Note that the angle formed between the gravitational force and the component of the gravitational force perpendicular to the plane is the same as the angle the inclined plane makes with the horizontal.

The components of the gravitational force can be calculated using trigonometry:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{F_{gy}}{F_g}$$

or

$$F_{gy} = F_g \cos\theta$$

Using the definition of gravitational force as $F_g = mg$, we can now write F_{gy} as:

$$F_{gy} = mg\cos\theta$$

and as this vector is parallel to the normal force, it must be equal in magnitude to the normal force:

$$F_N = F_{gy} = mg\cos\theta$$

The component of gravitational force acting parallel to the plane, F_{gx}, is found using sine of θ in a similar way to give:

$$F_{gx} = F_g \sin\theta$$

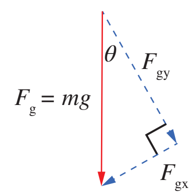


FIGURE 3.1.5 The components of gravitational force parallel (F_{gx}) and perpendicular (F_{gy}) to the inclined plane are shown in purple. These vectors make a right-angled triangle, with the gravitational force (F_g) as the hypotenuse.

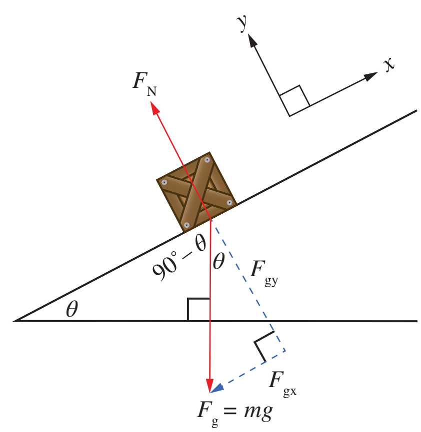


FIGURE 3.1.6 The angle formed between the gravitational force and the component of the gravitational force perpendicular to the plane is the same as the angle the inclined plane makes with the horizontal.

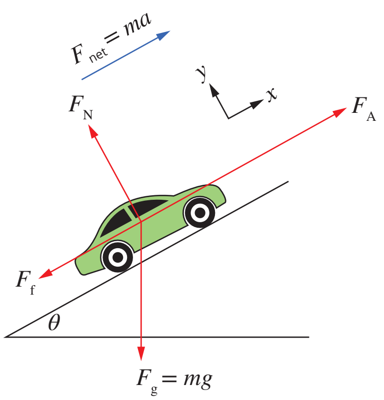


FIGURE 3.1.7 A full free-body diagram showing all the typical forces acting on an object that is accelerating up an inclined plane. All forces, except the gravitational force, act directly along either the *x*- or *y*-axes shown in the top right corner.

The normal force, F_N, and the component of gravitational force *perpendicular* to the plane, F_{gy}, are the only forces that act perpendicular to the plane and are balanced. Since they are equal, the net force will not be perpendicular to the inclined plane. The component of the gravitational force *parallel* to the plane, F_{gx}, is not balanced so, unless there is another force acting parallel to the plane, the crate will accelerate down the plane.

Typically, there are at least two other forces acting on an object on an inclined plane and they are the applied force (if the object has an engine or can supply a force that helps the object to move in a forward direction) and friction (which always opposes the motion of the object). These are added to the inclined plane force diagram to show the forces acting on a car accelerating up the plane (Figure 3.1.7).

The applied force and friction do not need to have their components resolved because they are already parallel to the inclined plane, that is, along the *x*-axis. In all situations, unless otherwise stated, the direction of acceleration (if any), will only be up or down the incline, not at an angle into the plane or out of the plane.

Brief descriptions of the common forces that can act on an object are shown in Table 3.1.1.

TABLE 3.1.1 Summary of the common forces that act on an object

Type of force	Symbol	Description	Where it acts
applied force	F_A	the force given to an object by an external push or pull, such as an engine	in the same direction as the push or pull
gravitational force	F_g	the force on an object due to the presence of a gravitational field	always straight down on the surface of Earth towards the centre of Earth
normal force	F_N or N	the force on an object that is due to the contact between the object and the surface it is touching. The surface pushes back on the object due to the third law of motion.	always perpendicular to the surface the object sits on
frictional force	F_f	the force on an object that opposes the motion of the object	always opposite to the direction of motion
tension	F_T or T	the force that acts along a string or wire when it is pulled tightly by forces at either end. It acts equally at all points in the string or wire.	along the string or wire, but away from the object
drag	F_D	air resistance, or the retarding force on an object due to its motion through a gas or liquid	always opposite to the direction of motion

Strategies for solving inclined plane problems

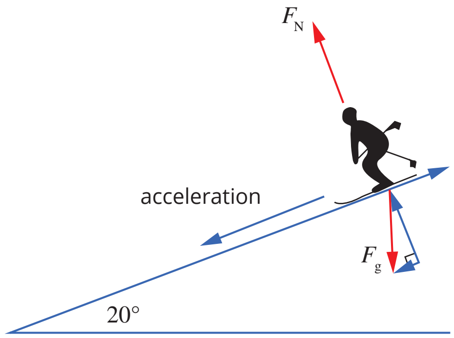
The following summarises strategies for solving inclined plane problems.

- List the information you are given.
- Draw a free-body diagram.
- Label each force acting on the mass (see Table 3.1.1).
- If the mass is accelerating, then there will be a net force in the direction of this acceleration.
- If the mass is not accelerating, then there will be zero net force.
- Remember that the net or resultant force is equal to the sum of all forces acting on the mass.
- Gravitational force is typically the only force that requires its components to be resolved perpendicular and parallel to the incline. Use trigonometry to calculate these components.
- A question might ask for the final velocity, time or displacement of the mass moving along the inclined plane. Refer to *Pearson Physics 11 Queensland* for the equations of motion with uniform acceleration to calculate these values.

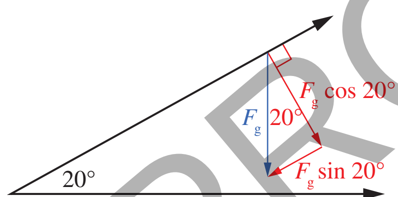
Worked example 3.1.2

INCLINED PLANES I

A skier of mass 50.0 kg is skiing freely down an icy slope that is inclined at 20.0° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is 9.8 m s^{-2}.



a Determine the components of the gravitational force of the skier perpendicular to the slope and parallel to the slope.

Thinking	Working
The components of the gravitational force perpendicular and parallel to the slope are given by trigonometric equations.	
Calculate the gravitational force of the skier.	$F_g = mg$ $= 50.0 \times 9.8$ $= 490$ N down
F_{gx} is the component of the gravitational force parallel to the slope.	$F_{gx} = F_g \sin 20.0°$ $= 490 \sin 20.0°$ $F_{gx} = 167.6 = 168$ N down (parallel to the slope)
F_{gy} is the component of the gravitational force perpendicular to the slope.	$F_{gy} = F_g \cos 20.0°$ $= 490 \cos 20.0°$ $F_{gy} = 460$ N down (perpendicular to the slope)

b Determine the normal force acting on the skier.

Thinking	Working
The normal force is equal in magnitude but opposite in direction to F_{gy}.	$F_N = -F_{gy} = 460$ N up (perpendicular to the slope)

c Calculate the acceleration of the skier down the slope.

Thinking	Working
The net force is down the slope, so this will be equal to the sum of all forces acting parallel to the slope. The acceleration is then equal to the net force divided by the mass of the skier.	$a = \frac{F_{net}}{m}$ $a = \frac{F_{net}}{m} = \frac{F_{gx}}{m} = \frac{167.6}{50.0} = 3.35$ $a = 3.35$ m s^{-2} down the slope

➤ Try yourself 3.1.2

INCLINED PLANES I

A skier of mass 85.0 kg is skiing freely down an icy slope that is inclined at 20.0° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is $9.8\,\text{m s}^{-2}$.

a Determine the components of gravitational force of the skier perpendicular to the slope and parallel to the slope.

b Determine the normal force acting on the skier.

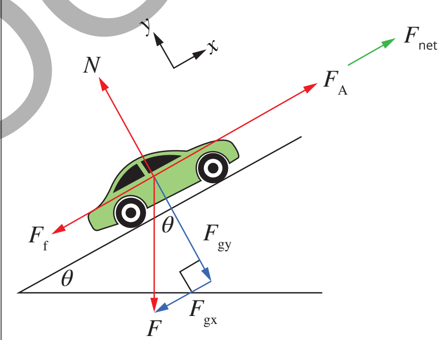
c Calculate the acceleration of the skier down the slope.

Worked example 3.1.3

INCLINED PLANES II

An 850.0 kg car is accelerating up a road with a slope of 12.0° at $0.25\,\text{m s}^{-2}$. Calculate the force the engine provides to the car if the frictional forces on the car add to 500.0 N.

Take g as $9.8\,\text{m s}^{-2}$.

Thinking	Working
Draw a diagram of the car on the inclined plane showing all the forces acting on it. For calculations, take forces down the slope as negative and forces up the slope as positive.	
Calculate the component of the gravitational force acting parallel to the slope, F_{gx}.	$F_{gx} = F_g \sin 12.0°$ $= 850.0 \times 9.8 \times \sin 12.0°$ $F_{gx} = 1732$ N down the slope
Use the second law of motion to determine all forces acting parallel to the slope.	$F_{net} = ma = F_A - F_f - F_{gy}$
Substitute the values for m, a, F_f and F_{gx}.	$F_{net} = ma = F_A - F_f - F_{gy}$ $= 850.0 \times 0.25 = F_A - 500.0 - 1732$ $F_A = 212.5 + 1732 + 500.0$ $= 2444\,\text{N} = 2.4 \times 10^3\,\text{N}$
Write the applied force as a vector with magnitude and direction.	F_A, the applied force, is 2.4×10^3 N up the slope.

➤ Try yourself 3.1.3

INCLINED PLANES II

Calculate the acceleration of the car if the same car turns around and is now accelerating down the slope with the same frictional force and applied forces acting.

Take g as $9.8\,\text{m s}^{-2}$.

3.1 Review

SUMMARY

- The force due to gravity, F_g, of an object acts straight down in all situations.
- A normal force, F_N, acts between an object and a surface, at right angles to the surface.
- The component of the gravitational force on an inclined plane acting parallel or perpendicular to the plane can be determined using trigonometry.
- On a horizontal surface, $F_N = -F_g$ and the object is not accelerating if there are no other forces present.
- On an inclined plane, F_N is equal and opposite to the component of the gravitational force acting perpendicular to the plane.
- A free-body diagram shows all the forces acting on a single object.
- Vectors are used to analyse the net force acting on an object on an inclined plane.

KEY QUESTIONS

Describe

1 Name the unit of gravitational force.

2 Identify the direction in which the gravitational force of an object acts.

3 Identify the direction in which the normal force acts.

4 Identify the relationship between the magnitude of the normal force on an inclined plane as the angle to the horizontal increases.

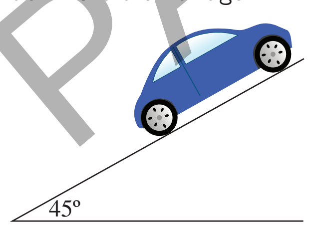
Apply

5 Explain why an object must be in contact with a surface for a normal force to exist.

6 Contrast how the normal force for objects on an inclined plane differs from that on horizontal surfaces.

7 Describe how the gravitational force balances with the normal force on inclined planes.

8 In 2009 in Wisconsin, USA, a car stalled and became stuck on a drawbridge while the drawbridge rose to an angle of 45° (see below). The frictional force between the car's tyres and the road stopped it from rolling down the drawbridge.



Describe how the car's gravitational force, normal force and frictional force acting on the tyres of the car changed as the drawbridge rose. (Apparently the driver, uninjured, waited until the bridge came back down and drove off as if nothing had happened.)

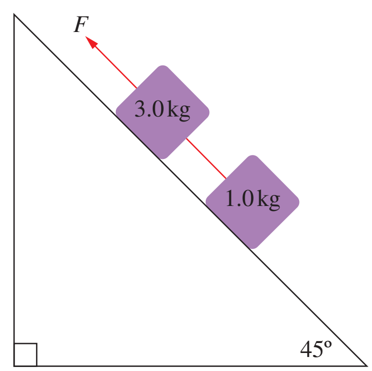
Analyse

9 Calculate the normal force acting on a skateboarder with a combined mass of 65.0 kg on a 25.0° incline.

10 Calculate the net force acting on a snowboarder with a combined mass of 88.0 kg travelling down a frictionless slope 12.5° to the horizontal.

11 Determine the force the engine must be applying to a 35 000 kg fully loaded bus being driven at a constant velocity up a 5.1° slope, if frictional forces add to 1600 N. Draw a free-body diagram to support your understanding.

12 Two masses, 1.0 kg and 3.0 kg, are tied together with a light rope and pulled up a 45° frictionless incline with a force *F*, as shown below.
Determine the tension in the rope and the value of *F* if the acceleration of the system is 2.0 m s^{-2} up the incline.



Remember that tension is a force directed along a rope or chain that is away from the object the rope or chain is supporting. Draw free-body diagrams on each block showing the directions of each force.

3.2 Circular motion

BY THE END OF THIS MODULE, YOU SHOULD BE ABLE TO:

- describe and solve problems using average speed, average velocity, period and frequency in motion around a circle
- describe uniform circular motion as the motion of an object moving around a circle with a constant speed
- differentiate between average speed and average velocity
- solve problems involving the average speed of objects undergoing uniform circular motion.

Whenever an object moves with constant speed in a circle, it is changing three quantities:

- direction of its motion
- angle from its starting position
- distance from its starting point.

When an object is undergoing **uniform circular motion**, as seen in the hammer throw in the Figure 3.2.1, the direction of the object's motion is constantly changing. This means that while the *speed* is constant, the *velocity* is also constantly changing. Recall from *Pearson Physics 11 Queensland* that:

- The **average speed** of an object is scalar and is a measure of distance over time ($m\,s^{-1}$).
- The **average velocity** of an object is a vector and is a measure of displacement over time ($m\,s^{-1}$).
- The **period** is the total time taken for a full revolution (s).
- The **frequency** is the number of revolutions per second (s^{-1} or Hz).

For an object travelling in a circle, the circumference is calculated using $2\pi r$.

Recall that period (T) and frequency (f) are related by the rule:

$$f = \frac{1}{T}$$

Some examples of objects that have different average speeds of rotation but the same period are shown in Figures 3.2.2 and 3.2.3.

The average speed of an object undergoing uniform circular motion can be calculated using:

$$v = \frac{2\pi r}{T}$$

where:

v is the average speed around the circle ($m\,s^{-1}$)

r is the radius of the circular path in (m)

T is the period of motion of the object around the circular path (s).

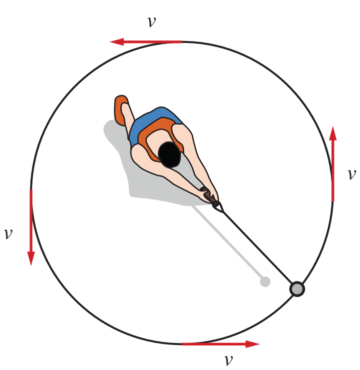


FIGURE 3.2.1 When an object undergoes uniform circular motion, the velocity is constantly changing.

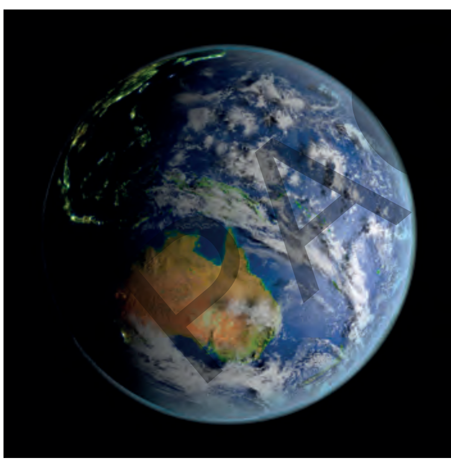


FIGURE 3.2.2 Earth rotates once every 23 hours, 56 minutes and 4.1 seconds, which is a period of 86 164.1 s and a frequency of $1.160\,58 \times 10^{-5}$ Hz. At the equator, Earth rotates at about 465 $m\,s^{-1}$.

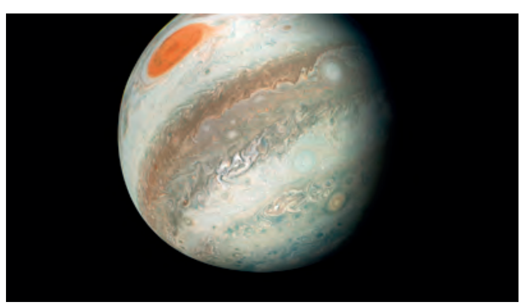


FIGURE 3.2.3 Jupiter rotates once every 9.925 hours, which is a frequency of 2.799×10^{-5} Hz. At the equator, Jupiter rotates at about 13 $km\,s^{-1}$.

Units of circular motion

Note that in problems of circular motion, the rotational rate of the object is sometimes given in revolutions per minute (rpm) rather than $m\,s^{-1}$. Before any values are substituted into the formulas, the values of rotation must first be converted into $m\,s^{-1}$. Some common rotational units and their conversion factors are given in Table 3.2.1.

TABLE 3.2.1 Common units for circular motion

Unit	Symbol	Description	Is it an SI unit?	Conversion to SI unit
$m\,s^{-1}$	v	metres per second	yes	not applicable
$km\,h^{-1}$	v	kilometres per hour	no	divide by 3.6 to convert to $m\,s^{-1}$
rpm	rpm	revolutions per minute	no	divide by 60 to convert to frequency (Hz)
rps	rps	revolutions per second	no	same as frequency (Hz)
period	T	time for one revolution	yes	not applicable
frequency	f	how many revolutions per second	yes	not applicable

Revolutions per minute are very commonly used to describe the rotational speeds of engines and motors. Since 2014, Formula 1 cars (Figure 3.2.4) can only use engines that are limited to 15 000 rpm. This corresponds to the piston moving inside the cylinder at 250 Hz and up to 75 $km\,h^{-1}$.

FIGURE 3.2.4 Formula 1 cars have engines that rotate at a maximum of 15 000 rpm.

Worked example 3.2.1

UNITS OF CIRCULAR MOTION

The Windy Hill Wind Farm outside of Ravenshoe on the Atherton Tablelands in Queensland produces 12 MW of electricity. There are 20 turbines on the site, and each has blades 20.0 m long that rotate at a maximum rate of 38 revolutions per minute.

a Determine the period of the motion of the blade.

Thinking	Working
Calculate the frequency by dividing the revolutions per minute by 60.	Revolutions per second = $\frac{38}{60}$ = 0.633 rps f = 0.633 Hz
Calculate the period by taking the reciprocal of frequency.	$f = \frac{1}{T}$ $0.633 = \frac{1}{T}$ $T = \frac{1}{0.633} = 1.58 = 1.6\,s$

b Calculate how fast the tip of one blade is moving in $m\,s^{-1}$ and in $km\,h^{-1}$.

Thinking	Working
Calculate the circumference of the blade's motion.	$C = 2\pi r$ $= 2 \times \pi \times 20.0$ $= 125.7\,m$
The average speed is the circumference divided by the period.	$v = \frac{125.7}{1.58}$ $= 79.6\,m\,s^{-1}$
To find the average speed in $km\,h^{-1}$ multiply by 3.6.	$v = 79.6 \times 3.6 = 287\,km\,h^{-1}$

➤ Try yourself 3.2.1

UNITS OF CIRCULAR MOTION

A typical internal hard disc drive (HDD) for a desktop computer can rotate up to 7200 revolutions per minute.

a Determine the period of the HDD.

b Calculate how fast, in $m\,s^{-1}$, a point on the edge of the HDD rotates if the diameter of the HDD is 8.89 cm.

Worked example 3.2.2

AVERAGE SPEED AND PERIOD AROUND A CIRCLE

A hammer is thrown in a cage by an Olympic athlete who rotates 3.05 times each second. The total length of the athlete's arms and hammer is 2.1 m.

a Determine the period of the hammer's motion.

Thinking	Working
Calculate the period by taking the reciprocal of the frequency.	$f = \frac{1}{T}$ $3.05 = \frac{1}{T}$ $T = \frac{1}{3.05}$ $= 0.328\,s$

b Calculate the average speed of the hammer.

Thinking	Working
The average speed of the hammer is the circumference divided by the period.	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 2.1}{0.328}$ $= 40\,m\,s^{-1}$

c Calculate the average velocity of the hammer.

Thinking	Working
The average velocity of the hammer is the displacement divided by the period.	$v = \frac{s}{T}$ $= \frac{0}{0.328}$ $= 0\,m\,s^{-1}$

➤ Try yourself 3.2.2

AVERAGE SPEED AND PERIOD AROUND A CIRCLE

A hammer is being thrown in the cage by another athlete who rotates 3.60 times each second. The total length of the athlete's arms and hammer is 1.95 m.

a Calculate the period of the hammer's motion.

b Calculate the average speed of the hammer.

c Calculate the average velocity of the hammer.

3.2 Review

SUMMARY

- Uniform circular motion is the motion of an object moving around a circle with a constant speed.
- The time taken for an object to complete one revolution around a circle is called the period, T, and is measured in seconds.
- The number of complete rotations per second is called the frequency, f. Frequency is measured in hertz (Hz) and is related to period by:

$$f = \frac{1}{T}$$

- For an object moving in a circle of radius r, the average speed, v, is the circumference of the circle divided by the period:

$$v = \frac{2\pi r}{T}$$

- For an object moving in a circle, the average velocity is zero if the object starts and finishes at the same point.

KEY QUESTIONS

Describe

1 State the SI units of period, frequency, average speed and average velocity.

2 Identify the relationship between frequency and period as the period increases.

3 Describe what period measures in circular motion.

4 Describe the conditions required for 'uniform' circular motion.

Apply

5 Determine the frequency of an object rotating at 1200 rpm.

6 Contrast average speed and average velocity for an object completing one full rotation in uniform circular motion.

7 Contrast the period and frequency of an object undergoing uniform circular motion.

8 Explain how you could be changing velocity even if you are moving at a constant speed.

Analyse

9 Calculate the average speed of a cyclist who takes 44.9 s to complete one circuit of a circular track with a radius of 50 m.

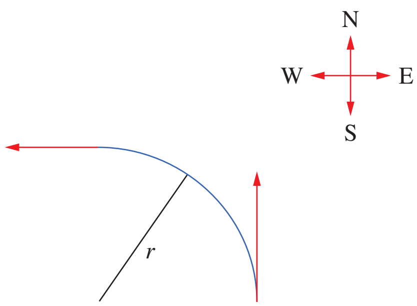
10 Calculate the length of a fan blade if the edge of the blade has an average speed of 28.3 m s^{-1} with a period of 40 ms.

11 The Hubble Space Telescope makes an approximately circular orbit of Earth at an average altitude of 540 km above Earth's surface, and a period of 95.47 minutes. The radius of Earth is 6400 km.

a Calculate the average speed of the telescope.

b Calculate the average velocity of the telescope.

12 David takes a ride along a circularly curved track that has a radius of 1250 m as shown below. He starts the trip riding exactly due north and finishes facing due west. He travels at an average speed of 12.3 km h^{-1}. Calculate the average velocity of his trip.



3.3 Centripetal force

BY THE END OF THIS MODULE, YOU SHOULD BE ABLE TO:

- recall that the velocity of an object moving in a circle is always perpendicular to the radius of the circle, i.e. at a tangent to the circle
- understand that motion around a circle involves a force that is always perpendicular to the velocity of the object
- describe centripetal acceleration as the acceleration that keeps an object moving in uniform circular motion
- describe centripetal force as the *net* force an object experiences that keeps it moving in uniform circular motion
- understand that centripetal acceleration and centripetal force are always directed towards the centre of the circle the object is moving around
- solve problems involving centripetal acceleration and centripetal force.

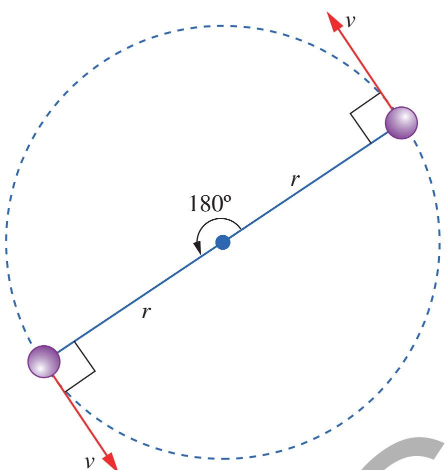


FIGURE 3.3.1 The velocity vector of an object in circular motion always points at a tangent to its direction of motion.

If the motion of an object moving in a circle were to be frozen, as in Figure 3.3.1, we would find that the velocity vector is always pointing in a direction at a tangent to its motion around the circular path, and at right angles to a line drawn to the centre of the path. The direction of the velocity shows where the object would move if the force causing it to move in a circle were to disappear.

This direction is continuously changing so that exactly one half of a rotation later the velocity vector is pointing in the opposite direction.

CENTRIPETAL ACCELERATION

Because the direction of an object in uniform circular motion is changing, the object's *velocity* is changing, but its speed is constant. So, even if an object's speed is constant, it is still accelerating because its *direction* is changing.

This acceleration, a_c, is known as **centripetal acceleration** and is the resulting, or net, acceleration of a body that is moving in a circle of radius r, with a velocity v.

The centripetal acceleration of an object moving with uniform circular motion is given by:

$$a_c = \frac{v^2}{r}$$

where:

a is the centripetal acceleration of the object ($\mathrm{m\,s^{-2}}$)

v is average speed of the object ($\mathrm{m\,s^{-1}}$)

r is the radius of the circular path in (m)

As it is a vector, centripetal acceleration requires a direction. A vector that is at right angles to a tangent must point towards the centre of the circle; hence, the direction of centripetal acceleration is directed towards the centre of the circle.

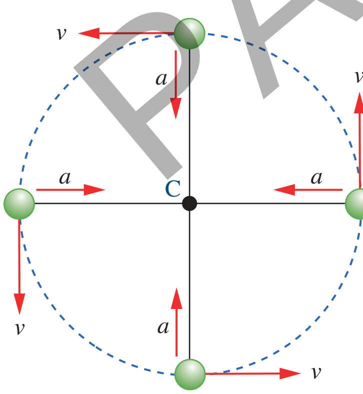


FIGURE 3.3.2 The centripetal acceleration of an object moving with uniform circular motion is always directed towards the centre of the circle, and at right angles to the object's velocity.

> The magnitude of centripetal acceleration is $a_c = \frac{v^2}{r}$. It is directed towards the centre of the circle that the object is moving around.

No matter where the object is, if it is moving in a circle then it will have a velocity vector at a tangent to the radius and a centripetal acceleration vector pointing towards the centre of the circle as shown in Figure 3.3.2. These vectors are always at right angles to each other.

CENTRIPETAL FORCE

If the sum of all forces that act on an object cause it to move in a circle, then the second law of motion can be written using centripetal acceleration instead of linear acceleration.

This force is called the **centripetal force**, F_c. It is the resulting force that causes an object of mass m to move in a circle of radius r, with a velocity v.

The centripetal force, which is also the net force, acting on an object moving with uniform circular motion is given by:

$$F_c = F_{net} = \frac{mv^2}{r}$$

where:

F_c is the centripetal force acting on the object (N)

m is the mass of the object (kg)

v is average speed of the object ($m\,s^{-1}$)

r is the radius of the circular path in (m)

Like centripetal acceleration, the centripetal force is directed towards the centre of the circle the object is moving around. If the force acts on an object at right angles to its uniform motion (i.e. constant speed), then the object will be continually changing direction, hence moving in a circle.

It is important to remember that the centripetal force is the resulting, or net, force that causes the circular motion of the object. There must always be a non-zero net force acting on an object for it to move in a circle.

The magnitude of centripetal force is $F_c = F_{net} = \frac{mv^2}{r}$, and this is directed towards the centre of the circle that the object is moving around, i.e. in the same direction as the centripetal acceleration.

Examples of centripetal force in action

As a car turns a left corner, the tyres exert a frictional force on the road towards the right of the car's motion. By the third law of motion, the road must also exert a frictional force on the tyres in the opposite direction (the left) to the force exerted by the tyres on the road, causing the tyres and car to move to the left.

If the force on the road from the tyres is larger than the friction between the tyres and the road, then the tyres will lose grip and the centripetal force (i.e. net force) from the tyres disappears. As there is no net force on the tyres, there will be no force on the car and the car will continue to move at a constant velocity in a straight line rather than turning the corner (Figure 3.3.3).

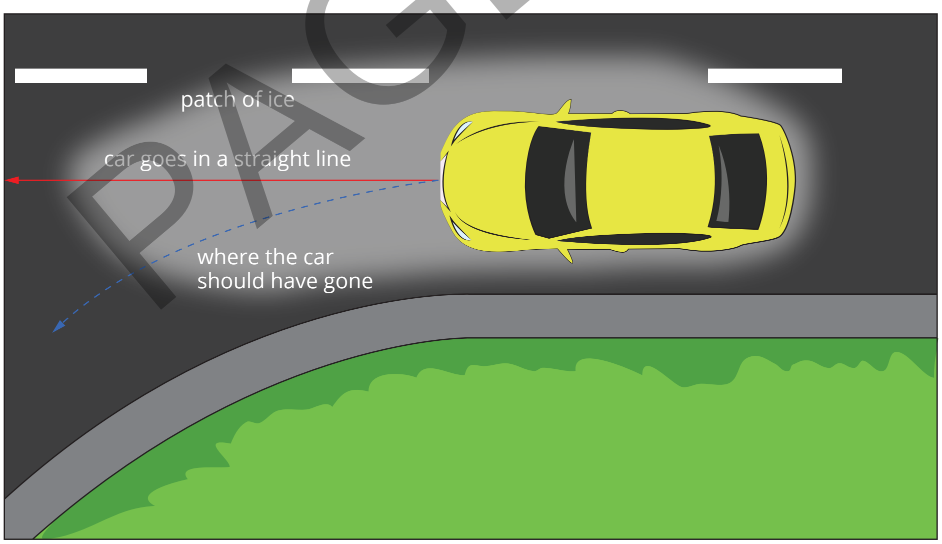


FIGURE 3.3.3 A car will skid off the road when trying to turn a corner when there is insufficient frictional force between the road and the tyres. In this case there is no centripetal force, thus the car continues to move in a straight line.

When turning a corner in a car, a rolling object may experience a 'force' pushing it outwards. This is commonly, but incorrectly, referred to as 'centrifugal force'. This force does not exist (Figure 3.3.4).

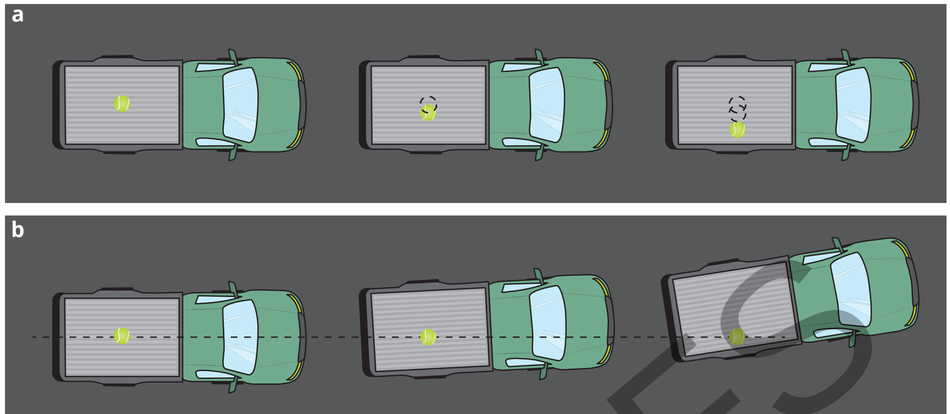


FIGURE 3.3.4 Two different points of view of a ball in the tray of a utility truck moving at a constant speed but turning left, as viewed (a) from above from the utility's point of view and (b) from the ball's point of view.

Figure 3.3.4a shows the motion of a ball in the tray of a utility truck that is turning left, from the point of view of the utility. The ball appears to violate the laws of motion, displaying a sideways force (the apparent 'centrifugal force') that is not the result of the interaction of a force with any other object. As nothing is causing this force, it does not exist. So, what is causing the apparent motion of the ball?

Figure 3.3.4b shows the motion of the ball from its own point of view, or from the point of view of an outside observer watching the utility turn the corner. In this point of view the ball obeys the first law of motion, i.e. it will move in a straight line with the same speed until an unbalanced force acts upon it. No forces are being applied to the ball, so it continues moving in a straight line, shown with the dashed line in Figure 3.3.4b. It is the truck that is being forced to move left via friction with the road, and the ball is simply hit by the right side of the utility because the utility is turning into the path of the ball. Hence, there is no need for a centrifugal force to explain the ball's motion.

Worked example 3.3.1

CENTRIPETAL FORCE AND ACCELERATION

The Moon has a mass of 7.3477×10^{22} kg and takes 27.322 days to orbit Earth at an average distance of 384 399 km.	
a Calculate the average speed of the Moon in its orbit around Earth.	
Thinking	**Working**
Convert the period and radius to SI units.	$T = 27.322$ days $= 27.322 \times 24 \times 3600$ s $= 2360620.8$ s $r = 384399$ km $= 384399 \times 10^3$ m $= 3.84399 \times 10^8$ m
The average speed is the circumference of the circular orbit divided by the period.	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 3.84399 \times 10^8}{2360620.8}$ $= 1023.14 = 1.0231 \times 10^3\ \text{m s}^{-1}$

b Calculate the centripetal acceleration of the Moon.	
Thinking	**Working**
Substitute the calculated value for v into the formula for centripetal acceleration.	$a_c = \frac{v^2}{r}$ $= \frac{1023.14^2}{3.84399 \times 10^8}$ $= 0.00272325 = 2.7233 \times 10\text{-}3\,\text{m}\,\text{s}^{-2}$ towards Earth
c Calculate the centripetal force acting on the Moon.	
Thinking	**Working**
Recall the equation for the centripetal force, substitute the values and solve. Note that this could also be calculated using $F_c = ma_c$.	$F_c = \frac{mv^2}{r}$ $= \frac{7.3477 \times 10^{22} \times 1023.14^2}{3.84399 \times 10^8}$ $= 2.0010 \times 10^{20}$ N towards Earth

➤ Try yourself 3.3.1

CENTRIPETAL FORCE AND ACCELERATION

Earth has a mass of 5.97×10^{24} kg and takes exactly 1.0 year to orbit the Sun at an average distance of 150 000 000 km.

a Calculate the average speed of Earth in its orbit around the Sun.

b Calculate the centripetal acceleration of Earth.

c Calculate the centripetal force acting on Earth.

CIRCULAR MOTION IN A HORIZONTAL PLANE

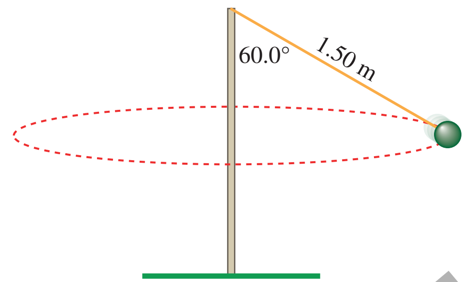
The interaction between the gravitational force, the normal force and the centripetal force on an object moving in circular motion depends on whether the object is rotating about a horizontal or a vertical plane. At every point in a horizontal plane, the centripetal force will act perpendicular to the gravitational and normal forces (Figure 3.2.X). The speed of the object will be constant, so the centripetal force will be constant.

When an object is twirled on the end of a string in a horizontal circle, it will always hang below the pivot point. If the object were to trace a horizontal circle at the same height as the pivot point, the string would be perpendicular to the direction of the object's gravitational force, and there would be no component of tension in the string in the vertical direction to balance the gravitational force of the object. Therefore, the object traces a circle lower than the pivot point and has both vertical and horizontal components. The vertical component balances the gravitational force of the object, and the horizontal component provides the centripetal force to keep the object moving in a circle. The tension in the string is therefore greater than the centripetal force keeping the object moving in a circle.

Worked example 3.3.2

HORIZONTAL CENTRIPETAL FORCE AND ACCELERATION

During a game of totem tennis, a 150 g tennis ball is swinging freely in a horizontal circular path as shown below.

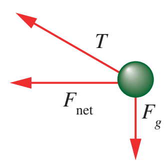


The cord is 1.50 m long and is at an angle of 60.0° to the vertical.

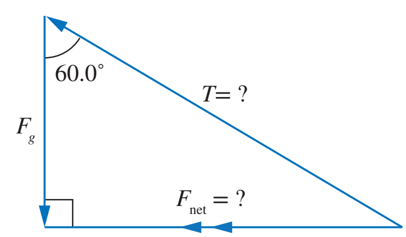
a Calculate the radius of the ball's circular path.

Thinking	Working
Use an appropriate trigonometric ratio to determine the radius of the ball's path.	$\sin 60.0° = \frac{r}{1.50}$ $r = 1.50 \sin 60.0°$ $= 1.30\text{ m}$

b Sketch a free-body diagram showing all the forces acting on the ball.

Thinking	Working
Forces that act on the ball are the tension in the string and the gravitational force of the ball. Centripetal force ($F_c = F_{net}$) is the sum of the tension and the gravitational force of the ball.	

c Determine the net force acting on the ball.

Thinking	Working
The net force on the ball is equal to the sum of the gravitational force and the tension in the string. This gives a vector acting directly to the left which is the centripetal force, since the ball is moving in a circle.	
Calculate the gravitational force first.	$F_g = mg = 0.150 \times 9.8$ $= 1.47\text{ N}$ straight down
Using trigonometry and the vertical and horizontal components of the tension, the net force can be calculated. Note that that centripetal force cannot be directly calculated using the formula for centripetal force because neither the speed nor period are known.	$\tan 60.0° = \frac{F_{net}}{F_g}$ $F_{net} = F_g \tan 60.0°$ $= 1.47 \tan 60.0°$ $= 2.55\text{ N}$ towards the pole

d Calculate the tension in the string.	
Thinking	**Working**
The tension in the string can also be found using the force triangle in part **c**, using either the vertical or horizontal component of the tension.	$\cos 60.0° = \frac{F_g}{T}$ $T = \frac{F_g}{\cos 60.0°} = \frac{1.47}{\cos 60.0°}$ $T = 2.94$ N along the cord towards the top of the pole. or $\sin 60.0° = \frac{F_{net}}{T}$ $T = \frac{F_{net}}{\sin 60.0°} = \frac{2.55}{\sin 60.0°}$ $T = 2.94$ N along the cord towards the top of the pole.

e Calculate the speed of the ball.	
Thinking	**Working**
The speed of the ball can be found using the formula for centripetal force.	$F_c = \frac{mv^2}{r} = 2.55$ $v = \sqrt{\frac{2.55 \times 1.30}{0.150}} = 4.70\ \text{m s}^{-1}$

➤ Try yourself 3.3.2

HORIZONTAL CENTRIPETAL FORCE AND ACCELERATION

The ball on the totem tennis pole is swapped with a slightly heavier ball with a mass of 175g. The same 1.50 m cord is used which is now swung 50.0° to the vertical.

a Calculate the radius of the ball's circular path.

b Sketch a free-body diagram showing all the forces acting on the ball.

c Determine the net force acting on the ball.

d Calculate the tension in the string.

e Calculate the speed of the ball.

3.3 Review

SUMMARY

- The direction of centripetal acceleration is always towards the centre of the circle the object is moving around.
- The centripetal force, F_c, is the *net* force an object of mass *m* experiences that keeps it moving in uniform circular motion with speed, *v*, period, *T*, and radius *r*:

$$F_c = \frac{mv^2}{r}$$

- Centripetal force is also always directed towards the centre of the circle the object is moving around.

Continued over page

3.3 Review *continued*

KEY QUESTIONS

Describe

1 State the direction of the velocity vector of an object that moves in a circle.

2 State the direction in which centripetal acceleration acts.

3 Describe the relationship that must exist between the net force and the velocity of a moving mass for uniform circular motion to result.

4 Recall the formula used to calculate centripetal acceleration using the velocity and radius of the circular path.

Apply

5 Explain why the centripetal force is not necessarily a force that acts directly on an object moving in a circle.

6 Identify at which point in their rotation hammer and discus throwers need to release their projectiles in order for the projectile to land on the arena.

7 A car is travelling over the top of a hill that models uniform circular motion.

Draw an arrow that represents the direction of the net force acting on the car. Explain your reasoning.

8 Explain why a tighter (smaller radius) turn results in more centripetal force for a car moving at the same speed.

Analyse

9 A student is swinging a cork on a string horizontally above their head. The cork experiences a tension force of 2.5 N from the string. The student keeps the length of string the same but doubles the speed of rotation. Determine the tension force from the string when the cork is spun at the new speed.

10 Calculate the magnitude of the centripetal acceleration of a person standing on the surface of Earth at the equator. Use the radius of Earth at the equator as 6378 km.

11 A car of mass 750 kg is travelling around a roundabout with a 5.0 m radius. They are driving with a speed of 4.0 m s^{-1}. Calculate the net force experienced by the car as it travels around the roundabout.

12 A cyclotron is a device used to accelerate subatomic particles to very high speeds for them to collide and create new particles. The cyclotron consists of an electromagnet and a circular tube for the particles to move along. Calculate the speed of the protons if the electromagnet exerts a force of 7.50×10^{-13} N on a beam of protons, causing them to move in a circular path of radius 1.20 m. The mass of a proton is 1.67×10^{-27} kg.

Chapter review

03

KEY TERMS

acceleration due to gravity
applied force
average speed
average velocity
centripetal acceleration
centripetal force
drag
force
force due to gravity
free-body diagram
frequency
frictional force
gravitational force
inclined plane
normal force
period
tension
uniform circular motion
weight

KEY QUESTIONS

Describe

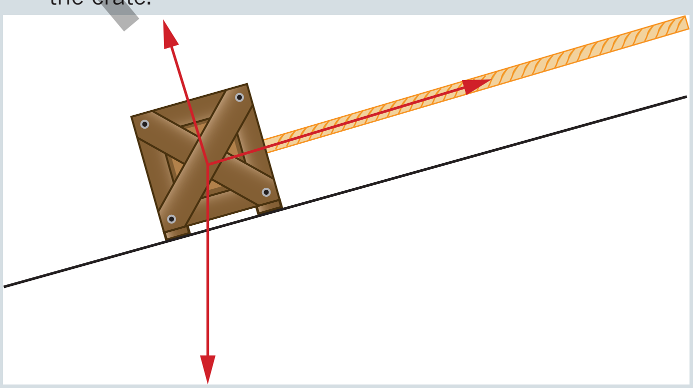
1 Identify the situation in which gravitational force is equal to the normal force.
 A in all situations
 B when the surface is at 0° to the horizontal
 C when the surface is at 45° to the horizontal
 D when the surface is at 90° to the horizontal

2 Identify the option that best describes the motion of a rubber ball rolling down a slope on which friction is negligible.
 A It moves with constant velocity
 B It moves with constant acceleration
 C It moves with increasing acceleration
 D It moves with decreasing acceleration

3 Identify the formula that correctly describes the magnitude of centripetal force.
 A $F_c = \frac{mr^2}{v}$
 B $F_c = \frac{mv^2}{2}$
 C $F_c = \frac{mv}{r}$
 D $F_c = \frac{mv^2}{r}$

4 State in which direction friction acts on an object moving down an incline plane.

5 A heavy crate is being pulled up a ramp by a strong rope. On the diagram below, label the tension force, the normal force and the gravitational force acting on the crate.



6 Describe the relationship between the normal force and gravitational forces acting on an object at rest on an incline plane.

7 State how the speed of an object changes as it moves in uniform circular motion.

8 State the unit used to measure period.

9 Identify what two values you need to calculate the average speed of an object in uniform circular motion.

10 Identify what provides the centripetal force needed to keep an object in an orbit around Earth.

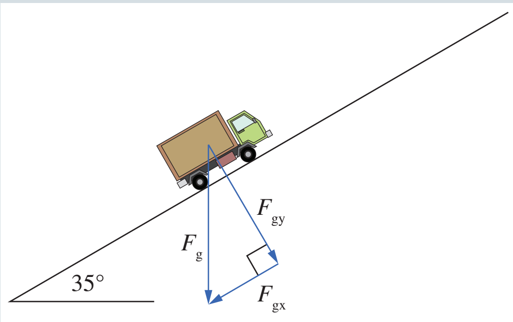
Apply

11 Identify which of the following forces must balance for an object to remain stationary on an inclined plane.
 A gravitational force and normal force
 B gravitational force and frictional force
 C frictional force and the component of gravitational force parallel to the plane
 D frictional force and the component of gravitational force perpendicular to the plane

12 Identify which of the following statements describes the forces acting on an object on a plane inclined at an angle θ, ($\theta > 0°$).
 A The normal force and the gravitational force cancel out
 B The normal force is equal in magnitude to the gravitational force
 C The normal force is always perpendicular to the surface
 D In the absence of friction, a component of the normal force causes the object to accelerate down the slope

13 A box is placed on a frictionless surface 10° to the horizontal. No external force is applied, but the box starts sliding down the slope. Identify the force that is causing the box to move.

14 This is a partially drawn free-body diagram of a 2.58×10^3 kg broken-down truck resting on an inclined plane at an angle of 35° to the horizontal. Only the gravitational force and its components are shown.



- **a** Describe the net force, and hence the acceleration, of the truck
- **b** Complete the free-body diagram by drawing vectors representing the frictional force and normal force
- **c** Calculate the magnitudes of the components of the gravitational force parallel, F_{gx}, and perpendicular, F_{gy}, to the surface of the plane
- **d** Calculate the magnitude of the normal force acting on the truck

15 Explain why an object in uniform circular motion is accelerating even though its speed is constant.

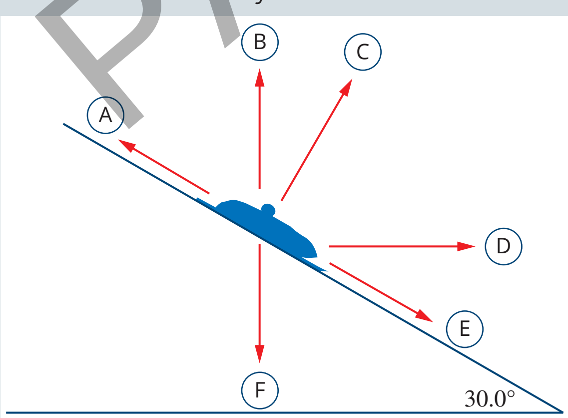
16 Identify the relationship between period and average speed of an object as the average speed increases.

17 Determine the average period of the seconds hand on a clock.

18 Explain why a hammer moves off in a straight line when the hammer thrower lets go.

Analyse

19 Penny is riding in a bobsled that is sliding down a snow-covered hill with a slope of 30.0°. The bobsled is frictionless in situations where brakes are not applied. The total mass of the sled and Penny is 102 kg. Initially the brakes are on and the sled moves down the hill with a constant velocity.



- **a** Determine which one of the arrows A–F best represents the direction of the frictional force acting on the sled
- **b** Determine which one of the arrows A–F best represents the direction of the normal force acting on the sled
- **c** Calculate the frictional force acting on the sled
- **d** Calculate the magnitude of Penny's acceleration when she releases the brakes and the sled accelerates
- **e** Determine how the acceleration of the bobsled will be affected when Penny rides the bobsled down the same slope with an extra passenger so that its total mass is 144 kg. Penny rides with the brakes off, so friction can be ignored again

20 Declan has a mass of 57 kg and he is riding his 3.0 kg skateboard down a 5.0 m high ramp that is inclined at an angle of 65° to the horizontal. Ignore friction when answering question parts **a**–**d**.

- **a** Calculate the magnitude of the normal force acting on Declan and his skateboard
- **b** Determine the net force acting on Declan and his board when no friction acts
- **c** Calculate Declan's acceleration as he travels down the ramp
- **d** Calculate his final speed as he reaches the bottom of the ramp, if Declan's initial speed is zero at the top of the ramp
- **e** Declan now stands halfway up the incline while holding the board in his hands. Friction now acts on Declan. Calculate the frictional force acting on Declan while he is standing stationary on the ramp

21 Angelina twirls a 7.00 kg hammer tied to the end of a 1.3 m rope in an approximately horizontal circle. The hammer moves at a rate of 1.0 revolutions per second.

- **a** Calculate the average speed of the hammer as it travels around in its path
- **b** Calculate the centripetal acceleration of the hammer
- **c** Calculate the tension in the rope

22 A turbocharger is a device that is used to increase the power and efficiency of a normal internal combustion engine. It works using a set of blades rotating at 250 000 rpm to draw air into the engine. The blades of a particular turbocharger are 6.0 cm in radius.

- **a** Calculate the frequency of the blades
- **b** Calculate the period of the blades
- **c** Calculate the average speed of the tip of the blades

23 The galaxy NGC 1365, in the constellation Fornax, shown below, is thought to have a supermassive black hole in its centre. This black hole has a mass two million times the mass of the Sun and is also the fastest spinning object ever measured, rotating at 85% of the speed of light.

If the radius of the black hole is 1.6 million km and the speed of light is 3.00×10^8 m s^{-1}:

a calculate the period of the black hole

b calculate the frequency of the black hole

c calculate the magnitude of the centripetal acceleration of the black hole

24 A passenger riding on a very fast rollercoaster feels weightless when travelling upside down through a series of loops. Describe the forces acting on the passenger for this to occur.

Interpret

25 The highest waterslide in the world is in Brazil. It is 49.0 m tall and is inclined at an angle of 60.0° to the horizontal. It is known that riders reach a speed of 91.0 km h^{-1} on this slide. Do not assume friction is negligible.

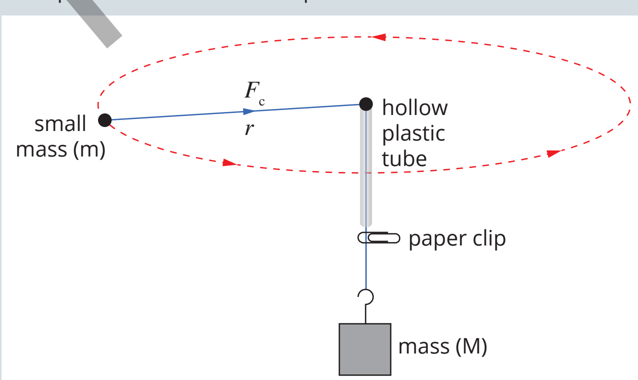
a Calculate the net force on a 70.0 kg teenager using the slide

b Calculate the magnitude of the average frictional force opposing the teenager's motion

c Determine the normal force acting on the teenager

26 Rob performed an experiment to determine the value of the acceleration due to Earth's gravity, *g*. A known, but variable mass, *m*, connected to a light string was threaded through a small tube. Connected to the other end of the string was a small, but constant mass, M.

The small mass, *M*, was then twirled around so that the radius, *r*, of the motion was kept constant. A paperclip was attached to the string just underneath the end of the tube so the radius of the circular motion could be kept constant. This set-up is shown below.



The mass underneath the tube, *m*, was varied and the period, *T*, of one swing was recorded in the table below. The radius of the string was kept constant at 30.0 cm, and the small mass being twirled was kept constant at 50.0 g. Rob recorded the values in the table below:

Mass underneath tube, *m* (kg)	**Period of one swing, *T* (s)**
0.050	1.11
0.100	0.82
0.150	0.66
0.200	0.55
0.250	0.49
0.300	0.45

a Show that the gravitational force, *mg*, of the mass underneath the tube can be written as:

$$mg = \frac{4\pi^2 Mr}{T^2}$$

b When plotting *T* against *m*, the data demonstrates a non-linear relationship. Determine how the data can be linearised

c Draw a graph of the new linearised data including a line of best fit

d Use the derived formula and the gradient of the line of best fit to determine acceleration due to gravity from this investigation

Data analysis

DATA SET 1

A student has conducted an experiment to measure acceleration due to gravity. This was completed by recording the average acceleration of ice blocks sliding down an inclined plane. The angle of the inclined plane was increased after each trial.

The experimental data was collected and is presented in Table 1. The data was processed and is presented in Figure 1.

TABLE 1 Average time of journey and average acceleration of ice blocks released down an incline plane of varying angles.

Angle (degrees)	Average time of ice block journey (s)	Average acceleration of the ice block ($m\,s^{-2}$)
5.00	4.30	0.21
10.00	2.00	0.95
15.00	1.30	2.30
20.00	1.20	2.60
25.00	1.00	3.80
30.00	0.90	4.69

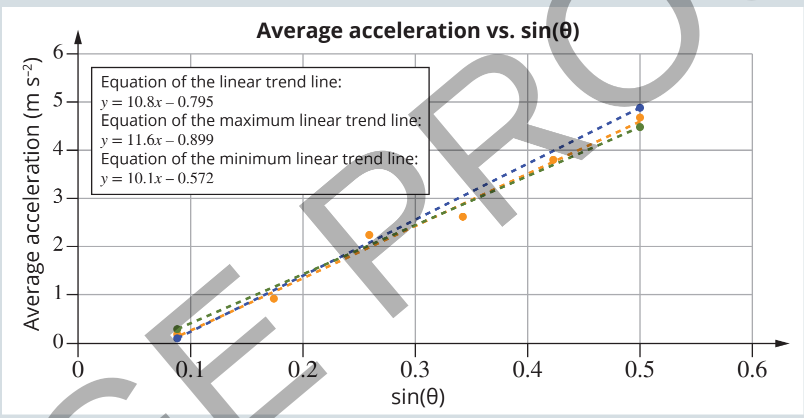


FIGURE 1 Graph of average acceleration against sine of the angle of inclination for ice blocks released down an incline plane.

Question 1 (3 marks)

Identify the mathematical relationship between the average acceleration of the ice blocks and the angle of inclination, including the uncertainty of the gradient and *y*-intercept.

Question 2 (1 mark)

Use the line of best fit to calculate the average acceleration of an ice block released on an inclined plane with an angle of 45°.

Question 3 (3 marks)

Draw a conclusion that quantifies the acceleration due to gravity for a frictionless surface, including the percentage uncertainty in the value you determine. Show your reasoning.

Question 4 (1 mark)

Calculate the percentage error for your calculated value compared to the theoretical value, $g = 9.8\,m\,s^{-2}$.