Student name:

....

Class: .... Date: ....

3:01 Addition and subtraction in algebra

Like terms in algebra have exactly the same pronumerals, or combinations of pronumerals. They only differ in the number(s) in front of them. Like terms can be simplified when adding or subtracting.

$5x + 2x = 7x$
$14x^2 - x^2 = 13x^2$
$(3x + 4y + 5) + (8x - y - 7) = 11x + 3y - 2$

(*Note:* $11x + 3y - 2$ cannot be simplified further—all the terms are unlike.)

1 Simplify these expressions.

a $9x + 2x$ ______________________

b $6p - p$ ______________________

c $3c + 4c - 2c$ ______________________

d $4mn - 5mn$ ______________________

4

2 Here are six pairs of expressions. There are three pairs of like terms and three pairs of unlike terms.

$5x, 2$ $3pq, 2pq$ $6x, -4x$
$3x^2, 3x^5$ ab^2, a^2b $-2xy, 3xy$

Sort them into the correct columns in this table:

Like terms	Unlike terms

6

3 Here are three different expressions, and the result when someone has attempted to simplify them.

a $6x^3 - 2x^2 = 4x$ ______

b $21pq - 17pq = 4$ ______

c $15x - 7x = 8x$ ______

Which of A, B or C is true for **a**, **b**, **c**?

A: The answer is correct.

B: It is not possible to simplify this expression.

C: This expression can be simplified, but the answer is wrong.

3

4 Simplify these expressions by adding/subtracting like terms. Two cannot be simplified further—for these write 'does not simplify'.

a $5x + 7x - 2$ ______________________

b $3x + 8y - x - 4y$ ______________________

c $3x + 2$ ______________________

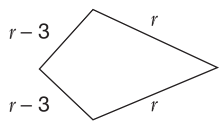
d $4x + x - 5x$ ______________________

e $x^2 - 4x - 4$ ______________________

f $4x - 1 + 8x - 6$ ______________________

6

5 a What name is given to this kind of quadrilateral?



____

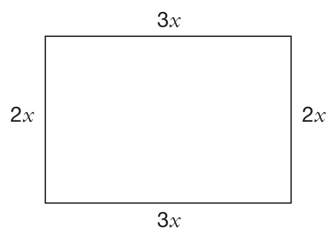
b Write an expression (in its simplest form) for its perimeter.

____

2

6 Write an expression for the perimeter of each of these shapes.

a



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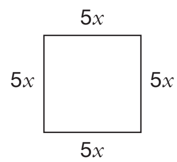

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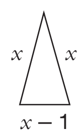

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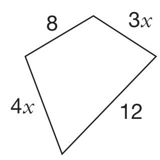


4

7 My friend took $5x$ books and $2y$ magazines on holiday. I took $3x$ books and y magazines. Write an expression (in its simplest possible form) for the number of books and magazines we took altogether.

____

1

Result /26

Student name: ....

Class: .... Date: ....

3:02 Multiplication in algebra

We don't use a multiplication sign in algebra—for example we write $8 \times x$ as $8x$.

Numbers are always written before letters when multiplication is implied.

Letters in products are written in alphabetical order.

$4 \times 3x = 12x$
$5x \times x = 5x^2$
$6qr \times 4pr = 24pqr^2$

1 Simplify these expressions.

a $6 \times x$ ______ b $x \times 4$ ______
c $1 \times x$ ______ d $y \times (-5)$ ______
e $2x \times 3y$ ______ f $d \times c$ ______
g $e \times 3c$ ______ h $5 \times 2x$ ______
i $3x \times 6$ ______ j $10x \times 4y$ ______
k $6p \times 1$ ______ l $q \times p$ ______
m $7p \times 2q$ ______ n $6x \times 5r$ ______
o $10 \times 4w$ ______ p $6v \times 5$ ______

16

2 Simplify these expressions.

a $(-4x) \times 5y$ ______
b $6y \times (-3x)$ ______
c $(-x) \times 2y$ ______
d $(-2p) \times (-5q)$ ______
e $x \times (-4y)$ ______
f $20y \times (-x)$ ______
g $4p \times (-2q) \times r$ ______
h $5d \times c \times (-2e)$ ______
i $y \times (-x) \times (-z)$ ______
j $(-2) \times (-6a) \times 2b$ ______
k $4x \times (-3) \times (-y)$ ______
l $(-2b) \times (-c) \times (10d)$ ______
m $20 \times (-3x) \times (5y)$ ______

13

3 Simplify these expressions.

a $3 \times y^2$ ______ b $6x^2 \times (-2)$ ______
c $pr \times q^2$ ______ d $5x^2 \times y$ ______
e $(-8) \times 2r^2$ ______ f $e^2 \times df$ ______

6

4 Write the results of these as mathematical expressions.

a multiply p by q ______
b multiply x by 6 ______
c multiply $3y$ by $6x$ ______
d multiply x by twice y ______
e multiply $-p$ by $5r$ ______

5

5 Write these products in their simplest form.

a $p \times p$ ______
b $3g \times 5g$ ______
c $2x \times xy$ ______
d $\frac{1}{2}p \times 6p$ ______
e $(-x) \times 2xy$ ______
f $(-3c) \times (-7bc)$ ______

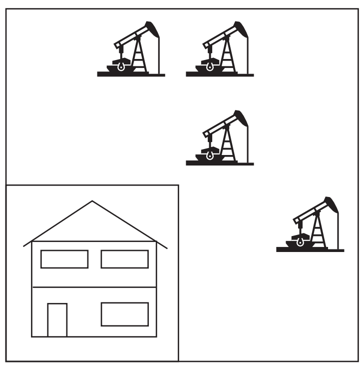
6

FUN SPOT

A farmer owns a small square farm, and has four sons and one daughter. There are four oil wells on the farm, and a large house in one corner.

In the farmer's will the house is left to the daughter, and each son is left an oil well. The four pieces of land with an oil well are the same shape and size.

Show how the farm should be divided in this way.



2

Result /48

3 ALGEBRA 2

Student name: ....

Class: .... **Date:** ....

3:03 Division in algebra

When representing a division in algebra we write the expression as a fraction.

Example:

$2x$ divided by 3 is written as $\frac{2x}{3}$.

Algebraic fractions can be simplified by dividing the top and bottom by common factors.

$\frac{4x}{12x} = \frac{1}{3}$ $\quad$ $\frac{10x}{15xy} = \frac{2}{3y}$

1 Write these expressions in fraction form.

a $6x \div y$ ______

b $3 \div x$ ______

c $x \div 4$ ______

d $x \div 4y$ ______

e $4x \div 5$ ______

5

2 Write the results of these as mathematical expressions.

a x divided by y ______

b $4p$ divided by $5q$ ______

c 1 divided by $2y$ ______

d x divided by 12 ______

e $6x^2$ divided by $5y$ ______

5

3 Write these quotients in fraction form.

a $p \div -3$ ______

b $(-4) \div x$ ______

c $(-3) \div (-5y)$ ______

d $(-1) \div 4y^2$ ______

4

4 Reduce these fractions to their lowest terms.

a $\frac{4y}{8}$ ______

b $\frac{12p}{18}$ ______

c $\frac{6m}{8m}$ ______

d $\frac{16pq}{12pq}$ ______

4

5 Simplify these algebraic fractions.

a $\frac{3x}{13x}$ ______

b $\frac{20x}{36y}$ ______

c $\frac{15pq}{40q}$ ______

d $\frac{8pqr}{2pr}$ ______

4

6 Simplify these divisions.

a $36y \div 18$ ______

b $4x^2 \div 2$ ______

c $6 \div 30g$ ______

d $48pq \div 20qr$ ______

4

7 Simplify these algebraic fractions to their lowest terms.

a $\frac{8p}{-4}$ ______

b $\frac{-16q}{12}$ ______

c $\frac{-30y}{-12}$ ______

d $\frac{15x}{-12}$ ______

e $\frac{-5x}{-40}$ ______

f $\frac{18f}{-63}$ ______

6

Result /32

Student name: ....

Class: .... Date: ....

3:04A Simplifying algebraic fractions: Addition and subtraction

To add algebraic fractions use the same method as adding numerical fractions.

$$\frac{x}{12}+\frac{5x}{12}=\frac{x+5x}{12}$$
$$=\frac{6x}{12}$$
$$=\frac{x}{2}\text{ (simplifying)}$$

Write fractions with a common denominator before adding or subtracting.

$$\frac{x}{4}-\frac{x}{6}=\frac{3x}{12}-\frac{2x}{12}$$
$$=\frac{3x-2x}{12}$$
$$=\frac{x}{12}$$

1 Add or subtract these fractions. Simplify your answer if possible.

a $\frac{3x}{7}+\frac{x}{7}$ ______

b $\frac{x}{12}+\frac{7x}{12}$ ______

c $\frac{4x}{15}-\frac{x}{15}$ ______

d $\frac{17y}{40}-\frac{9y}{40}$ ______

e $\frac{x}{20}+\frac{9x}{20}$ ______

f $\frac{11x}{12}-\frac{7x}{12}$ ______

6

2 Write each of these expressions as a single fraction.

a $\frac{x}{5}+\frac{x}{3}$ ______

b $\frac{2x}{5}+\frac{x}{2}$ ______

c $\frac{5x}{6}-\frac{x}{12}$ ______

d $\frac{5x}{8}-\frac{13x}{24}$ ______

e $\frac{x}{2}-\frac{x}{4}$ ______

f $\frac{2x}{3}-\frac{5x}{12}$ ______

g $\frac{4x}{5}+\frac{x}{10}$ ______

7

FUN SPOT THE INNKEEPER'S CONUNDRUM

Ten weary, footsore travellers,
All in a woeful plight,
Sought shelter at a wayside inn
One dark and stormy night.

'Nine rooms, no more,' the landlord said
'Have I to offer you.
To each of eight a single bed,
But the ninth must serve for two.'

A din arose. The troubled host
Could only scratch his head,
For of those tired men no two
Would occupy one bed.

The puzzled host was soon at ease—
He was a clever man—
And so to please his guests devised
This most ingenious plan.

In room marked A two men were placed,
The third was lodged in B,
The fourth to C was then assigned,
The fifth retired to D.

In E the sixth he tucked away,
In F the seventh man,
The eighth and ninth in G and H,
And then to A he ran.

Wherein the host, as I have said,
Had laid two travellers by;
Then taking one—the tenth and last—
He lodged him safe in I.

Nine single rooms—a room for each—
Were made to serve for ten;
And this it is that puzzles me
And many wiser men.

Explain this paradox.

______
______
______
______
______
______
______
______

3

Result /16

3 ALGEBRA 3

Student name: ....

Class: .... Date: ....

3:04B Simplifying algebraic fractions: Multiplication and division

Multiply algebraic fractions by multiplying the numerators and multiplying the denominators. Simplify your answer if possible.

$$\frac{x}{15} \times \frac{5}{3x} = \frac{5x}{45x} = \frac{1}{9}$$

Divide fractions by multiplying the first fraction by the *reciprocal* of the second fraction.

$$\frac{4p}{15q} \div \frac{3p}{5} = \frac{4p}{15q} \times \frac{5}{3p} = \frac{20p}{45pq} = \frac{4}{9q}$$

1 Multiply these fractions and write the answer in its simplest form.

a $\frac{x}{4} \times \frac{x}{3}$ ____________________

b $\frac{x}{8} \times \frac{4}{5}$ ____________________

c $\frac{3}{5} \times \frac{x}{6}$ ____________________

d $\frac{7}{x} \times \frac{x}{3}$ ____________________

e $\frac{3x}{8} \times \frac{2}{15x}$ ____________________

5

2 Multiply and write the answer in its simplest form.

a $\frac{4}{x} \times \frac{x}{8}$ ____________________

b $\frac{18}{x} \times \frac{x}{24}$ ____________________

c $\frac{2x}{5} \times \frac{3}{4}$ ____________________

d $\frac{2x}{3y} \times \frac{y}{4x}$ ____________________

e $\frac{x}{12} \times \frac{2}{3x}$ ____________________

f $\frac{3c}{5d} \times \frac{10}{c}$ ____________________

6

3 Divide these fractions and write the answer in its simplest form.

a $\frac{x}{12} \div \frac{x}{3}$ ____________________

b $\frac{7x}{4} \div \frac{x}{2}$ ____________________

c $\frac{3x}{8} \div \frac{6}{5x}$ ____________________

3

4 Divide and simplify if possible.

a $\frac{2x}{3} \div \frac{4x}{5}$ ____________________

b $\frac{2x}{5} \div \frac{x}{10}$ ____________________

c $\frac{3}{5x} \div \frac{12}{x}$ ____________________

d $\frac{4x}{y} \div \frac{2x}{3y}$ ____________________

4

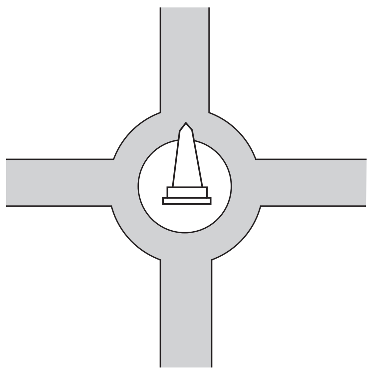
FUN SPOT

'There are only four hotels in this town,' the visitor was informed, 'one on each road. The town's four roads meet at right angles at the War Memorial. This road is the Pirie Highway.'

'To reach the Criterion from the Endeavour you must turn left. To reach the Miners' Arms from the Standard you have to turn right.'

The visitor entered three of the hotels; he arrived at the War Memorial three times during this search, turning left the first time, going straight across the second, and turning right the third time. He spent the night at the Criterion.

Which hotel stands on Pirie Highway?



2

Result /20

3 ALGEBRA 3

Student name: ....

Class: .... Date: ....

3:05 Grouping symbols

Grouping symbols indicate that the expression inside should be evaluated first. However, often in algebra we need to write an expression without grouping symbols. The process of 'expanding' an expression or removing grouping symbols is called the **distributive law**.

Each term in the grouping symbols is multiplied by the term outside.

Example:
Expand:

a $-3(2x-4)$

b $x(x+5)$

Answer:

a $-3(2x-4) = -3 \times 2x - -3 \times 4$
$= -6x + 12$

b $x(x+5) = x^2 + 5x$

The first example is written in full. However, usually the multiplication does not need to be shown.

1 Complete the gaps using the distributive law.

a $4(x+6) = 4 \times$ ______ $+ 4 \times$ ______
$=$ ______ $+$ ______

b $-3(8x+1) = -3 \times$ ______ $+ -3 \times$ ______
$=$ ______ $-$ ______

c $-10(5x-2) =$ ______ $\times 5x -$ ______ $\times 2$
$=$ ______ $+$ ______

3

2 Expand these expressions.

a $3(p+q)$ ______
b $4(x+2)$ ______
c $-4(x+3)$ ______
d $2(3x-1)$ ______
e $-3(2x-9)$ ______
f $5(2x-3y)$ ______
g $10(3-x)$ ______
h $-7(2x+3)$ ______
i $4(6-3p)$ ______
j $-5(3x-2y)$ ______
k $8(x-2y)$ ______
l $20(7-3d)$ ______
m $-2(8+p)$ ______
n $-4(2c-3d)$ ______

14

3 Remove the grouping symbols.

a $x(2+x)$ ______

b $3(1-2x)$ ______

c $x(3x+4)$ ______

d $3x(2x+7)$ ______

e $x(2x-9)$ ______

f $x(2-x)$ ______

g $2p(3p+q)$ ______

h $6m(3n-5m)$ ______

8

4 Remove the grouping symbols.

a $-(x+2)$ ______

b $-(7-x)$ ______

c $-(x^2-2x)$ ______

d $-4x(2+3x)$ ______

e $-x(x+1)$ ______

5

5 Expand and simplify.

a $6\left(\frac{y}{3}+2\right)$ ______

b $10\left(\frac{2p}{5}-3\right)$ ______

c $6\left(\frac{3x}{2}+\frac{x}{3}\right)$ ______

d $8\left(\frac{3y}{4}-\frac{y}{2}\right)$ ______

e $24\left(\frac{3x}{8}+\frac{1}{6}\right)$ ______

f $18\left(\frac{5x}{6}-\frac{2y}{3}\right)$ ______

6

FUN SPOT — TRUTH AND LIES

Here are five numbered statements. How many of them are true?

1 Exactly one of these statements is false.
2 Exactly two of these statements are false.
3 Exactly three of these statements are false.
4 Exactly four of these statements are false.
5 Exactly five of these statements are false.

______

2

Result /38

3 ALGEBRA 4

Student name: ....

Class: .... Date: ....

3:06 Simplifying expressions with grouping symbols

Sometimes after expanding, an expression can be simplified by adding/subtracting like terms.

Example:
Expand and simplify $2x + 4(3 - x)$.

Answer:
$$2x + 4(3 - x) = 2x + 12 - 4x$$
$$= -2x + 12$$

Example:
Expand and simplify $2(3x - 2) - (4x + 7)$.

Answer:
$$2(3x - 2) - (4x - 7) = 6x - 4 - 4x + 7$$
$$= 2x + 3$$

1 Expand and simplify each of these expressions.

a $5(x + 2) + 8$

b $4x + 2(x + 3)$

c $6x - 2(x + 1)$

d $8x - 4(x - 4)$

e $15 - 2(x + 1)$

f $18x - 3(2x + 7)$

g $3x + 2(1 - 2x)$

h $2x - (x + 3)$

i $6 - (x - 2)$

j $5x - 3(x + 2)$

k $6p - 2(1 - p)$

11

2 Expand and simplify each of these expressions.

a $5(x + 3) + 2(x + 6)$

b $-2(3x - 1) + 3(x - 5)$

c $3(2x + 5) - 2(x + 1)$

3

3 Expand all grouping symbols, then simplify each expression.

a $3(2x - 7) - (x + 2)$

b $5 - x - (3x + 2)$

c $4x + 2 - 2(x + 7)$

d $4(x - 9) - (6 + x)$

e $3(x + 2) + 2(2x + 1)$

f $10(5x + 2) - 7(3x - 9)$

6

4 Expand all grouping symbols, then simplify each expression.

a $x(x - 3) + 8(x - 1)$

b $x(2 + 3x) - x(2x + 1)$

c $x(8x + 3) - 2(x + 2)$

d $4x(x - 2) + 5(2x + 1)$

e $x(2x - 3) - 4(3 - x)$

5

Result /25

Student name:

Class: Date:

3:07 Factorising using common factors

The process of factorising is the opposite of expanding. When factorising an expression, we seek to write it as a product using grouping symbols. We look for a common factor first.

Example:
$7x + 7y = 7(x + y)$

The common factor is 7. We write 7 outside brackets.

The common factor can be a number only, as shown in the above example, or a pronumeral such as x, or a combination of both.

Worked examples:

a $8x + 12 = 4 \times 2x + 4 \times 3$
$= 4(2x + 3)$

b $3x + x^2 = 3 \times x + x \times x$
$= x(3 + x)$

c $12x^2 + 6x = 6x(2x + 1)$

Note: Always factorise as far as possible, so take out the highest common factor (HCF). In the above example when factorising $8x + 12$ the answer $4(2x + 3)$ is more complete than $2(4x + 6)$.

1 Factorise the following by taking out the HCF.

a $4a + 4b$ ______
b $12x - 12y$ ______
c $pq + pr$ ______
d $3a + 6b$ ______
e $4x + 2y$ ______
f $2x + 4$ ______
g $4x - 16$ ______
h $6x + 6$ ______
i $4 + 8x$ ______
j $5x + 20y$ ______
k $12x + 6$ ______
l $8x - 4$ ______
m $3x + 24$ ______
n $8 - 12x$ ______
o $10x - 5$ ______

15

2 Factorise completely.

a $2pq + 8p$ ______
b $16mn + 4m$ ______
c $abc - ac$ ______
d $6x^2 + 4xy$ ______
e $18xy - 28y$ ______
f $6abd + 3ac$ ______

6

3 Factorise by taking out the negative common factor.

a $-4x + 8$ ______
b $-2y - 10$ ______
c $-6x - 4y$ ______
d $-x^2 + 8x$ ______
e $-x^2 - 3x$ ______

5

4 Factorise each of the following.

a $x^2 + 10x$ ______
b $x^2 - 20x$ ______
c $5x - x^2$ ______
d $x - 2x^2$ ______
e $xy + 3x^2$ ______
f $6x^2 - 36x$ ______
g $4x^2 + 12$ ______
h $12x^2 - 8x$ ______
i $6x^2 + 2x$ ______
j $4 + 2x^2$ ______
k $7x - 3x^2$ ______
l $3x^2 + 3y^2$ ______
m $15xy + 20x^2$ ______

13

Result /39