QUADRATIC EQUATIONS

2

Contents

Syllabus references (See pages x–xv for details.)

Number and Algebra

Selections from ***Equations*** [Stages 5.2, 5.3§]

- Solve simple quadratic equations using a range of strategies (ACMNA241)
- Solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

2:01 Solution using factors

PREP QUIZ 2:01

Factorise: **1** $x^2 - 3x$ **2** $x^2 + 7x$ **3** $x^2 + 3x + 2$ **4** $x^2 - 4x - 12$
5 $x^2 + x - 20$ **6** $x^2 - 8x + 7$

Solve for x: **7** $3x = 12$ **8** $7x = 0$ **9** $x - 4 = 0$ **10** $x + 6 = 0$

In a quadratic equation the highest power of the pronumeral is 2.

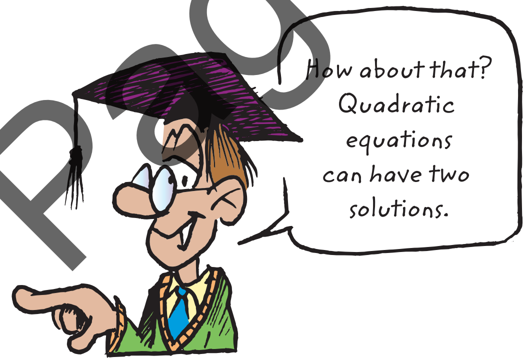
e.g. $x^2 = 9$ $\quad 5x^2 - 8 = 0$ $\quad x^2 - 6x = 0$ $\quad x^2 - 4x + 3 = 0$

Equations like the first two above can be solved directly but the second two require the expression to be factorised.

The term quadratic comes from *quadraticus*, which is the Latin word for square.

Equations of the form $ax^2 = c$

To solve $x^2 = 9$ we find the square root of both sides of the equation.
The square root of 9 is 3 or −3.



So if $x^2 = 9$, then $x = \pm 3$.
The equation has two solutions: $x = 3$ and $x = -3$

WORKED EXAMPLES

Solve these equations.

1 $x^2 - 16 = 0$ **2** $a^2 - 5 = 0$ **3** $3m^2 = 10$ **4** $k^2 + 4 = 0$

Solutions

1 $x^2 - 16 = 0$
$x^2 = 16$
$x = \pm\sqrt{16}$
$\therefore x = \pm 4$

2 $a^2 - 5 = 0$
$a^2 = 5$
$\therefore a = \pm\sqrt{5}$
(as decimal approximations
$a = 2{\cdot}236$ or $-2{\cdot}236$)

3 $3m^2 = 10$
$$m^2 = \frac{10}{3}$$
$$\therefore m = \pm\sqrt{\frac{10}{3}}$$
(as decimal approximations
$m = 1{\cdot}826$ or $-1{\cdot}826$)

4 $k^2 + 4 = 0$
$k^2 = -4$

The square of a real number is positive.
So this equation has no real solutions!

Equations of the form $ax^2 + bx + c = 0$

> A quadratic equation is an equation of the 'second degree'.

To solve $x^2 - 4x + 3 = 0$ we need to factorise the algebraic expression and then use the Null Factor Law.

> If $p \times q = 0$, then at least one of p and q must be zero.

$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

So either $x - 3 = 0$ or $x - 1 = 0$

$\therefore x = 3$ or $x = 1$

Substituting these values into the original equation will show that they are both solutions.

WORKED EXAMPLE 1

Solve the quadratic equations.

a If $(x - 1)(x + 7) = 0$

then either

$x - 1 = 0$ or $x + 7 = 0$

$\therefore x = 1$ or $x = -7$

b If $2x(x + 3) = 0$

then either

$2x = 0$ or $x + 3 = 0$

$\therefore x = 0$ or $x = -3$

c If $(2x - 1)(3x + 5) = 0$

then either

$2x - 1 = 0$ or $3x + 5 = 0$

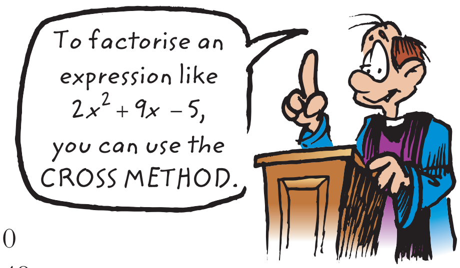
$2x = 1$ or $3x = -5$

$\therefore x = \frac{1}{2}$ or $x = -\frac{5}{3}$

> A quadratic equation can have two solutions.

WORKED EXAMPLE 2

To solve these equations, they are factorised first.



a $x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x + 3 = 0$ or $x + 1 = 0$

$\therefore x = -3$ or $x = -1$

b $x^2 - 49 = 0$

$(x - 7)(x + 7) = 0$

$x - 7 = 0$ or $x + 7 = 0$

$\therefore x = 7$ or $x = -7$

or $x^2 - 49 = 0$

$x^2 = 49$

$\therefore x = 7$ or -7

i.e. $x = \pm 7$

c $2x^2 + 9x - 5 = 0$

$(2x - 1)(x + 5) = 0$

$2x - 1 = 0$ or $x + 5 = 0$

$\therefore x = \frac{1}{2}$ or $x = -5$

WORKED EXAMPLE 3

Before these equations are solved, all the terms are gathered to one side of the equation and equated to zero.

a
$$\begin{aligned} x^2 + x &= 12 \\ x^2 + x - 12 &= 0 \\ (x + 4)(x - 3) &= 0 \\ x + 4 = 0 &\text{ or } x - 3 = 0 \\ \therefore x = -4 &\text{ or } \quad x = 3 \end{aligned}$$

b
$$\begin{aligned} 5x^2 &= 2x \\ 5x^2 - 2x &= 0 \\ x(5x - 2) &= 0 \\ x = 0 &\text{ or } 5x - 2 = 0 \\ \therefore x = 0 &\text{ or } \quad x = \tfrac{2}{5} \end{aligned}$$

c
$$\begin{aligned} 6x^2 &= 5x + 6 \\ 6x^2 - 5x - 6 &= 0 \\ (3x + 2)(2x - 3) &= 0 \\ 3x + 2 = 0 &\text{ or } 2x - 3 = 0 \\ \therefore x = -\tfrac{2}{3} &\text{ or } \quad x = \tfrac{3}{2} \end{aligned}$$

> To solve a quadratic equation:
> - gather all the terms to one side of the equation
> - factorise
> - solve the two resulting simple equations.

You can always check your solutions by substitution. For Worked Example 3a:

Substituting $x = -4$

$x^2 + x = 12$

$$\begin{aligned} \text{LHS} &= (-4)^2 + (-4) \\ &= 16 - 4 \\ &= 12 \\ &= \text{RHS} \end{aligned}$$

Substituting $x = 3$

$x^2 + x = 12$

$$\begin{aligned} \text{LHS} &= (3)^2 + (3) \\ &= 9 + 3 \\ &= 12 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Both $x = -4$ and $x = 3$ are solutions.

> LHS is left-hand side.
> RHS is right-hand side.

Exercise 2:01

Foundation worksheet 2:01
Quadratic equations

1 Solve these equations.

a $x^2 = 25$ **b** $x^2 = 81$ **c** $x^2 = 1$ **d** $x^2 = 100$

e $x^2 - 36 = 0$ **f** $x^2 - 64 = 0$ **g** $x^2 - 121 = 0$ **h** $x^2 - 400 = 0$

2 Solve these equations, leaving your answers in square root form.

a $x^2 = 6$ **b** $n^2 = 3$ **c** $y^2 - 10 = 0$ **d** $k^2 - 15 = 0$

e $4x^2 = 8$ **f** $5p^2 = 3$ **g** $8q^2 - 10 = 0$ **h** $7z^2 - 5 = 0$

3 Solve these equations, evaluating your answers correct to three decimal places.

a $x^2 = 2$ **b** $2a^2 = 7$ **c** $c^2 - 12 = 0$ **d** $8g^2 - 3 = 0$

4 **a** Solve the equation $x^2 + 9 = 0$. **b** Give a reason for your answer in **a**.

5 Find the two solutions for each equation. Check your answers by substitution.

a $x(x-5)=0$ b $x(x+7)=0$ c $2x(x+1)=0$
d $5a(a-2)=0$ e $4q(q+5)=0$ f $6p(p-7)=0$
g $(x-2)(x-1)=0$ h $(x-7)(x-3)=0$ i $(a-5)(a-2)=0$
j $(y+3)(y+4)=0$ k $(t+3)(t+2)=0$ l $(x+9)(x+5)=0$
m $(a-6)(a+6)=0$ n $(y+8)(y-7)=0$ o $(n+1)(n-1)=0$
p $(a+1)(2a-1)=0$ q $(3x+2)(x-5)=0$ r $2x(3x-1)=0$
s $(4x-1)(2x+1)=0$ t $(3a-4)(2a-1)=0$ u $(6y-5)(4y+3)=0$
v $6x(5x-3)=0$ w $(9y+1)(7y+2)=0$ x $(5x-1)(5x+1)=0$

6 After factorising the left-hand side of each equation, solve the following.

a $x^2+3x=0$ b $m^2-5m=0$ c $y^2+2y=0$
d $6x^2+12x=0$ e $9n^2-3n=0$ f $4x^2+8x=0$
g $x^2-4=0$ h $a^2-49=0$ i $y^2-36=0$
j $a^2-1=0$ k $n^2-100=0$ l $m^2-64=0$
m $x^2+3x+2=0$ n $a^2-5a+6=0$ o $y^2+12y+35=0$
p $a^2-10a+21=0$ q $x^2-10x+16=0$ r $m^2-11m+24=0$
s $h^2+h-20=0$ t $x^2+2x-35=0$ u $a^2-4a-45=0$
v $x^2+x-56=0$ w $y^2-8y+7=0$ x $a^2+9a-10=0$

7 a Solve the equation $x^2-4x+4=0$.
b How many solutions are there for this quadratic equation?
c When will a quadratic equation have only one solution?

Check answers by substitution.

8 Solve:

a $x^2+4x+4=0$ b $m^2-6m+9=0$ c $z^2+2z+1=0$
d $k^2-8k+16=0$ e $a^2-10a+25=0$ f $y^2+20y+100=0$

9 Factorise and solve the following.

a $2x^2+x-1=0$ b $3x^2+7x+2=0$
c $3x^2+17x+10=0$ d $2x^2-11x+12=0$
e $2x^2-x-10=0$ f $2x^2-11x-21=0$
g $4x^2+21x+5=0$ h $4x^2-19x-5=0$
i $4x^2-21x+5=0$ j $5x^2+16x+3=0$
k $2x^2+13x-24=0$ l $7x^2+48x-7=0$
m $4x^2-4x-3=0$ n $6x^2-x-1=0$
o $9x^2+9x+2=0$ p $10x^2+9x+2=0$
q $12x^2-7x+1=0$ r $10x^2-13x+4=0$

10 Gather all the terms to one side of the equation and then solve.

a $x^2=3x$ b $m^2=8m$ c $t^2=-5t$
d $x^2=5x-4$ e $a^2=2a+15$ f $y^2=3y-2$
g $m^2=9m-18$ h $n^2=7n+18$ i $h^2=4h+32$
j $x^2+x=2$ k $y^2+2y=3$ l $x^2-7x=-10$
m $y^2+3y=18$ n $t^2+3t=28$ o $y^2+2y=15$
p $2x^2+x=1$ q $2x^2-x=15$ r $4m^2-5m=6$
s $3x^2=13x-14$ t $5p^2=17p-6$ u $2x^2=11x-5$

2:02 Solution by completing the square

This method depends upon completing an algebraic expression to form a perfect square, that is, an expression of the form $(x + a)^2$ or $(x - a)^2$.

Reminder:

$(x + a)^2 = x^2 + 2ax + a^2$

- The middle term in the expansion is $2ax$.
- The coefficient of x in the middle term is $2a$.

So, the value of a can be found by halving the coefficient of the x term.

This is also true for: $(x - a)^2 = x^2 - 2ax + a^2$

WORKED EXAMPLE 1

What must be added to the following to make perfect squares?

a $x^2 + 8x$

b $x^2 - 5x$

Solutions

a $x^2 + 8x + \ldots$

Half of 8 is 4, so the perfect square is:

$$\begin{aligned} x^2 + 8x + 4^2 &= (x + 4)^2 \\ &= x^2 + 8x + 16 \end{aligned}$$

So 16 must be added to make a perfect square.

b $x^2 - 5x + \ldots$

Half of -5 is $-\frac{5}{2}$, so the perfect square is:

$$\begin{aligned} x^2 - 5x + \left(-\tfrac{5}{2}\right)^2 &= \left(x - \tfrac{5}{2}\right)^2 \\ &= x^2 - 5x + \tfrac{25}{4} \end{aligned}$$

So $\frac{25}{4}$ must be added to make a perfect square.

Now, to solve a quadratic equation using this technique follow these steps.

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ x^2 + 4x &= 21 \\ x^2 + 4x + 2^2 &= 21 + 2^2 \\ \therefore (x + 2)^2 &= 25 \\ x + 2 &= \pm\sqrt{25} \\ x &= -2 \pm 5 \\ \therefore x &= 3 \text{ or } -7 \end{aligned}$$

Move the constant to the RHS.

Add (half of x coefficient)^2 to both sides.

Note that $x^2 + 4x - 21 = 0$ could have been solved using the Null Factor Law as in the previous exercise.

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ (x - 3)(x + 7) &= 0 \\ \therefore x = 3 &\text{ or } x = -7 \end{aligned}$$

In this case, this is an easier and quicker way to find the solution. The method of completing the square, however, can determine the solution of quadratic equations that cannot be factorised. This can be seen in the following examples.

WORKED EXAMPLE 2

Solve:

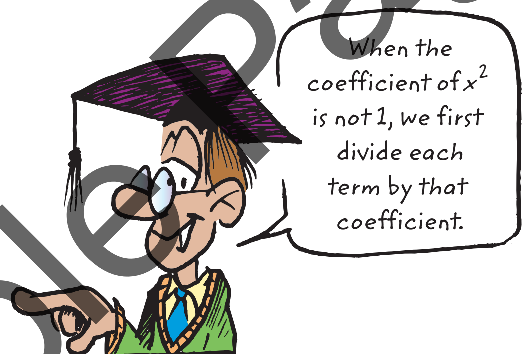
a $x^2 + 6x + 1 = 0$ **b** $x^2 - 3x - 5 = 0$ **c** $3x^2 - 4x - 1 = 0$

Solutions

a
$$\begin{aligned} x^2 + 6x + 1 &= 0 \\ x^2 + 6x &= -1 \\ x^2 + 6x + 3^2 &= -1 + 3^2 \\ (x+3)^2 &= 8 \\ x + 3 &= \pm\sqrt{8} \\ \therefore\ x &= -3 \pm \sqrt{8} \\ x &= -3 + \sqrt{8} \text{ or } -3 - \sqrt{8} \\ (x &\doteqdot -0{\cdot}17 \text{ or } -5{\cdot}83) \end{aligned}$$

Note that the solution involves a square root, i.e. the solution is irrational. Using your calculator, approximations may be found.

b
$$\begin{aligned} x^2 - 3x - 5 &= 0 \\ x^2 - 3x &= 5 \\ x^2 - 3x + \left(-\tfrac{3}{2}\right)^2 &= 5 + \left(-\tfrac{3}{2}\right)^2 \\ \left(x - \tfrac{3}{2}\right)^2 &= 7\tfrac{1}{4} \\ x - \tfrac{3}{2} &= \pm\sqrt{7\tfrac{1}{4}} \\ \therefore\ x &= \frac{3}{2} \pm \frac{\sqrt{29}}{2} \\ x &= \frac{3 + \sqrt{29}}{2} \text{ or } \frac{3 - \sqrt{29}}{2} \\ (x &\doteqdot 4{\cdot}19 \text{ or } -1{\cdot}19) \end{aligned}$$



c
$$\begin{aligned} 3x^2 - 4x - 1 &= 0 \\ x^2 - \tfrac{4}{3}x - \tfrac{1}{3} &= 0 \\ x^2 - \tfrac{4}{3}x &= \tfrac{1}{3} \\ x^2 - \tfrac{4}{3}x + \left(-\tfrac{2}{3}\right)^2 &= \tfrac{1}{3} + \left(-\tfrac{2}{3}\right)^2 \\ \left(x - \tfrac{2}{3}\right)^2 &= \tfrac{7}{9} \\ x - \tfrac{2}{3} &= \pm\frac{\sqrt{7}}{3} \\ \therefore\ x &= \frac{2}{3} \pm \frac{\sqrt{7}}{3} \\ x &= \frac{2 + \sqrt{7}}{3} \text{ or } \frac{2 - \sqrt{7}}{3} \\ (x &\doteqdot 1{\cdot}55 \text{ or } -0{\cdot}22) \end{aligned}$$

You can use the following fact to check your answers.
For the equation $ax^2 + bx + c = 0$
the two solutions must add to $-\frac{b}{a}$.
In Worked Example 2a, $(-0{\cdot}17) + (-5{\cdot}83) = -6$ or $\frac{-6}{1}$
In Worked Example 2b, $1{\cdot}55 + (-0{\cdot}22) = 1{\cdot}33\ \left[\doteqdot \frac{4}{3}\right]$

Exercise 2:02

1 What number must be inserted into each expression to complete the square?

a $x^2 + 6x + \ldots^2 = (x + \ldots)^2$
b $x^2 + 8x + \ldots^2 = (x + \ldots)^2$
c $x^2 - 2x + \ldots^2 = (x - \ldots)^2$
d $x^2 - 4x + \ldots^2 = (x - \ldots)^2$
e $x^2 + 3x + \ldots^2 = (x + \ldots)^2$
f $x^2 - 7x + \ldots^2 = (x - \ldots)^2$
g $x^2 + 11x + \ldots^2 = (x + \ldots)^2$
h $x^2 - x + \ldots^2 = (x - \ldots)^2$
i $x^2 + \frac{5x}{2} + \ldots^2 = (x + \ldots)^2$
j $x^2 - \frac{2x}{3} + \ldots^2 = (x - \ldots)^2$

2 Solve the following equations, leaving your answers in surd form.

a $(x - 2)^2 = 3$
b $(x + 1)^2 = 2$
c $(x + 5)^2 = 5$
d $(x - 1)^2 = 10$
e $(x - 3)^2 = 7$
f $(x + 2)^2 = 11$
g $(x + 3)^2 = 8$
h $(x + 10)^2 = 12$
i $(x - 3)^2 = 18$
j $(x + \frac{1}{2})^2 = 5$
k $(x - \frac{2}{3})^2 = 3$
l $(x + 1\frac{1}{2})^2 = 12$
m $(x - 1)^2 = 2\frac{1}{2}$
n $(x + 3)^2 = 4\frac{1}{2}$
o $(x - \frac{1}{3})^2 = \frac{5}{9}$

3 Solve the following equations by completing the square. Also find approximations for your answers, correct to two decimal places.

a $x^2 + 2x - 1 = 0$
b $x^2 - 2x - 5 = 0$
c $x^2 - 4x - 8 = 0$
d $x^2 + 6x - 8 = 0$
e $x^2 - 6x + 2 = 0$
f $x^2 + 4x + 1 = 0$
g $x^2 + 10x = 5$
h $x^2 + 2x = 4$
i $x^2 - 12x = 1$
j $x^2 + 5x + 2 = 0$
k $x^2 + 7x - 3 = 0$
l $x^2 + x - 3 = 0$
m $x^2 + 9x + 3 = 0$
n $x^2 + 3x - 5 = 0$
o $x^2 - 11x + 5 = 0$
p $x^2 - x = 3$
q $x^2 + 3x = 2$
r $x^2 - 5x = 1$
s $2x^2 - 4x - 1 = 0$
t $2x^2 + 3x - 4 = 0$
u $2x^2 - 8x + 1 = 0$
v $3x^2 + 2x - 3 = 0$
w $5x^2 - 4x - 3 = 0$
x $4x^2 - x - 2 = 0$

2:03 The quadratic formula

As we have seen in the previous section, a quadratic equation is one involving a squared term. In fact, any quadratic equation can be represented by the **general form** of a quadratic equation:

$$ax^2 + bx + c = 0$$

where a, b, c are all integers, and $a \neq 0$.

If any quadratic equation is arranged in this form, a formula using the values of a, b and c can be used to find the solutions.

The quadratic formula for $ax^2 + bx + c = 0$ is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof of the quadratic formula:

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

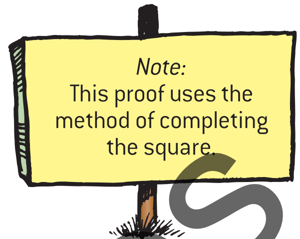
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



WORKED EXAMPLE 1

Solve the following by using the quadratic formula.

a $2x^2 + 9x + 4 = 0$

b $x^2 + 5x + 1 = 0$

Solutions

a For $2x^2 + 9x + 4 = 0$,
$a = 2, b = 9, c = 4$.
Substituting into the formula gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times 4}}{2 \times 2}$$

$$= \frac{-9 \pm \sqrt{81 - 32}}{4}$$

$$= \frac{-9 \pm \sqrt{49}}{4}$$

$$= \frac{-9 \pm 7}{4}$$

$$= -\tfrac{2}{4} \text{ or } -\tfrac{16}{4}$$

$$\therefore x = -\tfrac{1}{2} \text{ or } -4$$

b For $x^2 + 5x + 1 = 0$,
$a = 1, b = 5, c = 1$.
Substituting into the formula gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-5 \pm \sqrt{25 - 4}}{2}$$

$$= \frac{-5 \pm \sqrt{21}}{2}$$

There is no rational equivalent to $\sqrt{21}$ so the answer may be left as:

$$x = \frac{-5 + \sqrt{21}}{2} \text{ or } \frac{-5 - \sqrt{21}}{2}$$

Approximations for these answers may be found using a calculator.
$x \doteqdot -0{\cdot}21$ or $-4{\cdot}79$ (2 dec. pl.)

WORKED EXAMPLE 2

Solve the following by using the quadratic formula.

a $3x^2 = 2x + 2$ **b** $2x^2 + 2x + 7 = 0$

Solutions

a The equation $3x^2 = 2x + 2$ must first be written in the form $ax^2 + bx + c = 0$:

$3x^2 - 2x - 2 = 0$

So $a = 3$, $b = -2$, $c = -2$.

Substituting these values gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times (-2)}}{2 \times 3}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{6}$$

$$= \frac{2 \pm \sqrt{28}}{6}$$

$$\therefore\ x = \frac{2 + \sqrt{28}}{6} \text{ or } \frac{2 - \sqrt{28}}{6}$$

$x \doteqdot 1{\cdot}22$ or $-0{\cdot}55$ (2 dec. pl.)

b For $2x^2 + 2x + 7 = 0$,

$a = 2$, $b = 2$, $c = 7$.

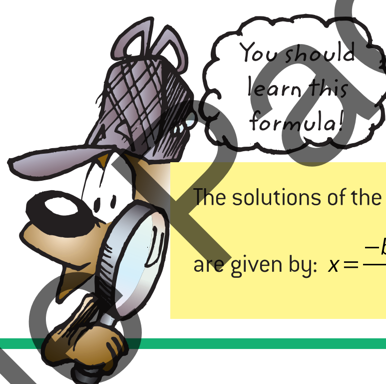
Substituting these values gives:

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 2 \times 7}}{2 \times 2}$$

$$= \frac{-2 \pm \sqrt{-52}}{4}$$

But $\sqrt{-52}$ is not real!

So $2x^2 + 2x + 7 = 0$ has no real solutions.



The solutions of the equation $ax^2 + bx + c = 0$ are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 2:03

P Foundation worksheet 2:03
The quadratic formula

1 Use the quadratic formula to solve the following equations. All have rational answers.

a $x^2 + 5x + 6 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 10x + 9 = 0$
d $x^2 - 3x - 10 = 0$ **e** $x^2 - 2x - 15 = 0$ **f** $x^2 + 4x - 12 = 0$
g $x^2 - 9x + 14 = 0$ **h** $x^2 - 8x + 12 = 0$ **i** $x^2 - 6x + 5 = 0$
j $3x^2 + 7x + 2 = 0$ **k** $2x^2 + 11x + 5 = 0$ **l** $4x^2 + 11x + 6 = 0$
m $2x^2 - 5x - 3 = 0$ **n** $5x^2 - 9x - 2 = 0$ **o** $3x^2 - 5x + 2 = 0$
p $6x^2 + 7x + 2 = 0$ **q** $6x^2 + 7x - 3 = 0$ **r** $8x^2 - 14x + 3 = 0$

2 Solve the following, leaving your answers in surd form. (*Remember:* A surd is an expression involving a square root.)

a $x^2 + 4x + 2 = 0$ **b** $x^2 + 3x + 1 = 0$ **c** $x^2 + 5x + 3 = 0$
d $x^2 + x - 1 = 0$ **e** $x^2 + 2x - 2 = 0$ **f** $x^2 + 4x - 1 = 0$
g $x^2 - 2x - 1 = 0$ **h** $x^2 - 7x + 2 = 0$ **i** $x^2 - 6x + 3 = 0$
j $x^2 - 10x - 9 = 0$ **k** $x^2 - 8x + 3 = 0$ **l** $x^2 - 5x + 7 = 0$
m $2x^2 + 6x + 1 = 0$ **n** $2x^2 + 3x - 1 = 0$ **o** $2x^2 - 7x + 4 = 0$
p $3x^2 + 10x + 2 = 0$ **q** $3x^2 - 9x + 2 = 0$ **r** $5x^2 + 4x - 2 = 0$
s $4x^2 - x + 1 = 0$ **t** $3x^2 - 3x - 1 = 0$ **u** $4x^2 - 3x - 2 = 0$
v $2x^2 + 11x - 5 = 0$ **w** $2x^2 - 9x + 8 = 0$ **x** $5x^2 + 2x - 1 = 0$

3 Use the formula to solve the following. Give answers as decimal approximations correct to two decimal places.

a $x^2 - 4x + 1 = 0$ b $x^2 - 6x + 3 = 0$ c $x^2 + 8x - 5 = 0$
d $x^2 + 9x + 1 = 0$ e $x^2 + 2x - 5 = 0$ f $x^2 + 3x - 1 = 0$
g $x^2 + 2 = 0$ h $x^2 - 7x = 2$ i $x^2 = 6x - 11$
j $2x^2 + x - 2 = 0$ k $2x^2 - 5x - 2 = 0$ l $3x^2 + 9x + 5 = 0$
m $2x^2 = 7x - 2$ n $5x^2 - 3x = 4$ o $6x^2 = x + 3$

INVESTIGATION 2:03 HOW MANY SOLUTIONS?

Consider these three quadratic equations:

a $x^2 + 6x + 5 = 0$ b $x^2 + 6x + 9 = 0$ c $x^2 + 6x + 12 = 0$

Use the quadratic formula to solve each equation.

Solutions

a
$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 5}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{16}}{2}$$
$$= \frac{-6 \pm 4}{2}$$
$$\therefore\ x = -1 \text{ or } -5$$

b
$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{0}}{2}$$
$$= \frac{-6}{2}$$
$$\therefore\ x = -3$$

c
$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 12}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{-12}}{2}$$

$\sqrt{-12}$ has no real solution

$\therefore\ x$ has no real solution

It appears that a quadratic equation may have two, one or no real solutions. The 'key' is the part of the formula under the square root sign.

The number of real solutions is determined by $b^2 - 4ac$.

If $b^2 - 4ac$ is:

- *positive* then the equation will have *two* real solutions
- *zero* then the equation will have *one* real solution
- *negative* then the equation will have *no* real solution.

Exercises

By evaluating $b^2 - 4ac$ for each equation, determine how many real solutions it will have.

$b^2 - 4ac$ is called the *discriminant*.

1 $x^2 + 4x + 3 = 0$ 2 $x^2 + 4x + 4 = 0$ 3 $x^2 + 4x + 5 = 0$
4 $x^2 - x - 2 = 0$ 5 $x^2 - x = 0$ 6 $x^2 - x + 2 = 0$
7 $4x^2 - 12x + 9 = 0$ 8 $4x^2 - 12x + 7 = 0$ 9 $4x^2 - 12x + 11 = 0$
10 $5x^2 - x + 7 = 0$ 11 $5x^2 - x - 7 = 0$ 12 $9x^2 + 6x + 1 = 0$

2:04 Choosing the best method

PREP QUIZ 2:04

Factorise:

1 $5x^2 - 10x$ 2 $x^2 - 5x - 14$ 3 $x^2 - 81$ 4 $x^2 + 5x + 6$

Solve:

5 $(x - 2)(x + 7) = 0$ 6 $(2x - 3)(3x + 1) = 0$ 7 $x^2 - 16 = 0$

8 $3x^2 - 12x = 0$ 9 $x^2 - 3x + 2 = 0$

10 Write the formula for the solutions of the equation $ax^2 + bx + c = 0$.

Some quadratic equations may appear in a different form from those seen so far, but they can always be simplified to the general form $ax^2 + bx + c = 0$. They may then be solved by factorising or by using the quadratic formula.

WORKED EXAMPLES

Solve the following equations.

1 $x^2 - 2x + 1 = 3x + 6$ 2 $x(x - 5) = 6$ 3 $x = \dfrac{5x - 6}{x}$

Solutions

1 In this example, all the terms must be gathered to the LHS.

$$x^2 - 2x + 1 = 3x + 6$$
$$-3x - 6 = -3x - 6$$
$$x^2 - 5x - 5 = 0$$

This cannot be factorised, so the quadratic formula is used.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{5 \pm \sqrt{25 + 20}}{2}$$
$$= \frac{5 \pm \sqrt{45}}{2}$$
$$\therefore\ x = \frac{5 + \sqrt{45}}{2} \text{ or } \frac{5 - \sqrt{45}}{2}$$

2 Expand and gather the terms to the LHS.

$$x(x - 5) = 6$$
$$x^2 - 5x = 6$$
$$x^2 - 5x - 6 = 0$$

Factorising gives:

$$(x - 6)(x + 1) = 0$$
$$\therefore\ x = 6 \text{ or } -1$$

3 $x = \dfrac{5x - 6}{x}$

Multiplying both sides by x gives:

$$x^2 = 5x - 6$$
$$x^2 - 5x + 6 = 0$$

Factorising gives:

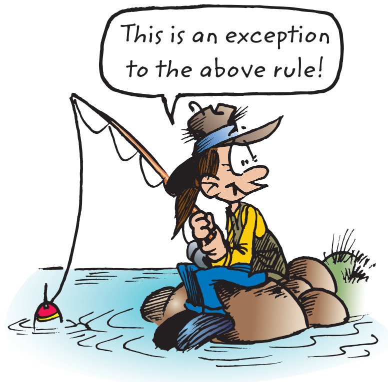
$$(x - 2)(x - 3) = 0$$
$$\therefore\ x = 2 \text{ or } 3$$

When solving a quadratic equation:

Step 1: Express the equation in general form:
$ax^2 + bx + c = 0$

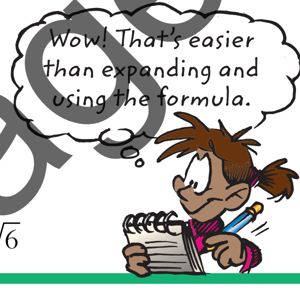
Step 2: Factorise, if you can, and solve it.
or
use the formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

WORKED EXAMPLE



$(a + 7)^2 = 6$

For equations like this, where one side is a perfect square, it is easier to follow the final steps in the method of completing the square.



Solution

$(a + 7)^2 = 6$
$\therefore\ a + 7 = \pm\sqrt{6}$
$a = -7 \pm \sqrt{6}$
$a = -7 + \sqrt{6}$ or $-7 - \sqrt{6}$

Exercise 2:04

1 Solve the following quadratic equations. Give answers to two decimal places if necessary.

a $x^2 + 7x + 6 = 0$ b $x^2 - 8x + 12 = 0$ c $x^2 + 5x - 24 = 0$
d $x^2 - 3x + 1 = 0$ e $x^2 + 3x - 3 = 0$ f $x^2 + 4x + 2 = 0$
g $x^2 + 8x = 0$ h $x^2 - 10x = 0$ i $5x^2 - 10x = 0$
j $x^2 - 81 = 0$ k $x^2 - 121 = 0$ l $4x^2 - 9 = 0$
m $2x^2 + 4x + 1 = 0$ n $3x^2 - x - 1 = 0$ o $2x^2 - 5x + 1 = 0$
p $2x^2 + 6x + 4 = 0$ q $3x^2 + 15x + 18 = 0$ r $2x^2 - 6x - 8 = 0$
s $x^2 = 6x + 27$ t $x^2 = 13x - 36$ u $2x^2 - 5x = 12$
v $25 = 10x - x^2$ w $36 = 13x - x^2$ x $2 = 9x - 5x^2$

2 Rearrange each equation below into the form $ax^2 + bx + c = 0$, and solve.

a $x^2 + 9x = 2x - 12$ b $x^2 + 20 = 8x + 5$ c $x^2 - 4x + 10 = 2x + 2$
d $3x^2 + 5x = 2x^2 - 6$ e $4x^2 + 5x = 3x^2 - 2x$ f $x^2 + 3x - 10 = 3x - 1$
g $x^2 + 5x = 3x + 1$ h $x^2 + 7 = 5 - 4x$ i $2x + 1 = x^2 + x$
j $x(x + 5) = 6$ k $x(x - 7) = 18$ l $x^2 = 4(x + 8)$
m $(m - 1)^2 = 4$ n $(x + 3)^2 = 9$ o $(x + 5)^2 = 11$
p $(2a + 1)^2 = 16$ q $(5y - 3)^2 = 7$ r $(6n - 7)^2 = 3$
s $x = \dfrac{2x + 15}{x}$ t $x = \dfrac{3x + 28}{x}$ u $1 = \dfrac{2 - x^2}{x}$
v $x = \dfrac{5x - 3}{x}$ w $\dfrac{3(x + 1)}{x} = x$ x $2(x + 2) = \dfrac{1}{x}$

FUN SPOT 2:04 WHAT IS AN ITALIAN REFEREE?

Work out the answer to each question and write the letter for that part in the box that is above the correct answer.

Solve:

A $(x + 3)(x - 5) = 0$

M $(x + 1)(x - 1) = 0$

R $(2x - 1)(x - 7) = 0$

R $(3x + 5)(2x - 3) = 0$

Solve by factorising:

M $x^2 - 5x + 6 = 0$

A $x^2 - x - 6 = 0$

E $x^2 - 5x - 6 = 0$

I $x^2 + x - 6 = 0$

Solve:

N $(x - 2)^2 = 5$

P $(x + 1)^2 = 2$

O $(x + 7)^2 = 9$

U $(x + 3)^2 = 3$

$-2, 3$	$-\frac{1}{2}, 7$	$-4, -10$	$2, 3$	$-3, 5$	$2 \pm \sqrt{5}$	$-3 \pm \sqrt{3}$	$-1, 1$	$-1 \pm \sqrt{2}$	$-3, 2$	$-\frac{5}{3}, \frac{3}{2}$	$-1, 6$

2:05 Problems involving quadratic equations

PREP QUIZ 2:05

From the list of numbers, 1, 2, $3\frac{1}{2}$, 5·2, 9, 10, write the numbers that are:

1 integers **2** odd **3** square

Write the next two consecutive integers after:

4 8 **5** n

Write the next two consecutive even numbers after:

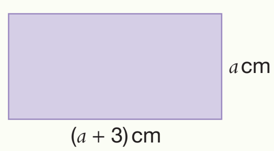
6 10 **7** x (x is even)

Write expressions for:

8 a number that is 3 less than x

9 the area of this rectangle

10 the perimeter of this rectangle.



When solving a problem or applying a given formula, a quadratic equation may be involved. Consider the following examples.

WORKED EXAMPLES

1 The product of two consecutive positive even numbers is 48. Find the numbers. (Hint: If the first number is x, then the next even number is $x + 2$).

2 The length of a rectangle is 5 cm longer than its breadth. If the area of the rectangle is 84 cm^2, find the length of the rectangle.

3 A projectile is fired vertically upwards and its height h, in metres, after t seconds is given by the formula:
$h = 40t - 8t^2$
Find the time taken by the projectile to first reach a height of 48 m.

Solutions

1 The problem gives the equation:
$x(x + 2) = 48$

Solving this gives:

$$x^2 + 2x = 48$$
$$x^2 + 2x - 48 = 0$$
$$(x + 8)(x - 6) = 0$$
$$\therefore x = -8 \text{ or } 6$$

The numbers are positive, so $x = 6$.
∴ The two consecutive numbers are 6 and 8.

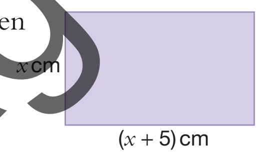
3 $h = 40t - 8t^2$
To find the time t when the height $h = 48$ metres, substitute into the formula:
$48 = 40t - 8t^2$

Gathering all the terms on the LHS:

$$8t^2 - 40t + 48 = 0$$
$$8(t^2 - 5t + 6) = 0$$
$$8(t - 2)(t - 3) = 0$$
$$\therefore t = 2 \text{ or } 3$$

∴ The projectile was at a height of 48 m at 2 seconds and 3 seconds. So it took 2 seconds to first reach a height of 48 m.

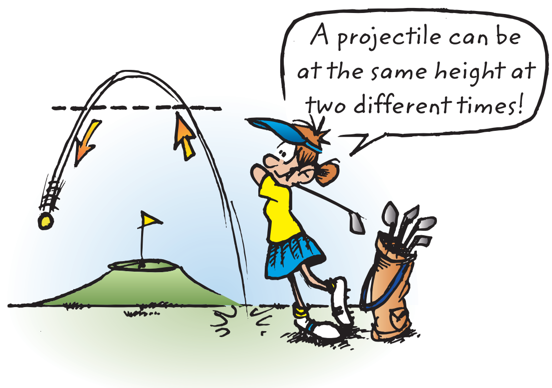
2 If the breadth is x, then the length is $x + 5$.



Since the area of a rectangle is equal to length times breadth, then:

$$x(x + 5) = 84$$
$$x^2 + 5x = 84$$
$$x^2 + 5x - 84 = 0$$
$$(x + 12)(x - 7) = 0$$
$$\therefore x = -12 \text{ or } 7$$

Now, since the dimensions must be positive, the breadth must be 7 cm.
∴ The length is 12 cm.

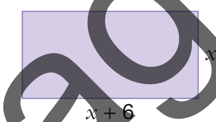


Exercise 2:05

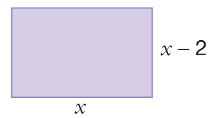
1 Find the two consecutive positive integers:
- a if their product is 20
- b if their product is 90
- c if they are even and their product is 120
- d if they are odd and their product is 63.

2
- a The sum of a positive integer and its square is 90. Find the number.
- b The sum of a positive integer and its square is 132. Find the number.
- c The difference between a positive integer and its square is 56. Find the number.
- d The square of a number is equal to 5 times the number. What are the two possible numbers?
- e When a number is subtracted from its square, the result is 42. Find the two possible numbers.

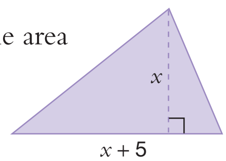
3
- a Find the dimensions of this rectangle if the length is 6 cm longer than the breadth and its area is 40 cm^2.



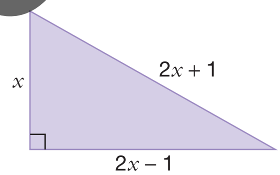
- b The breadth of a rectangular room is 2 m shorter than its length. If the area of the room is 255 m^2, find the dimensions of the room.



- c The base of a triangle is 5 cm longer than its height. If the area of the triangle is 7 cm^2, find the length of the base.



- d A right-angled triangle is drawn so that the hypotenuse is twice the shortest side plus 1 cm, and the other side is twice the shortest side less 1 cm. Find the length of the hypotenuse.



4
- a Michelle threw a ball vertically upwards. Its height h metres after time t seconds is given by the formula:

  $h = 8t - t^2$

  Find after what time the ball was first at a height of 12 m.
- b The sum, S, of the first n positive integers is given by the formula:

  $$S = \frac{n}{2}(n + 1)$$

  Find the number of positive integers needed to give a total of 78.
- c For the formula $s = ut + \frac{1}{2}at^2$, find the values of t if:
  - i $s = 18, \quad u = 7, \quad a = 2$
  - ii $s = 6, \quad u = 11, \quad a = 4$
  - iii $s = 7, \quad u = 1, \quad a = 6$

5 An n-sided polygon has $\frac{1}{2}n(n-3)$ diagonals. How many sides has a figure if it has 90 diagonals?

6 Jenny is y^2 years old and her daughter Allyson is y years old. If Jenny lives to the age of $13y$, Allyson will be y^2 years old. How old is Allyson now? (*Note:* the difference in ages must remain constant.)

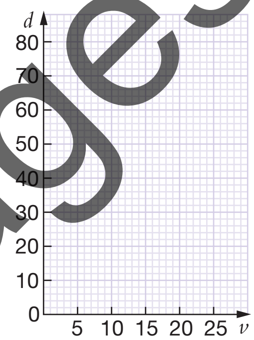
7 Kylie bought an item for \$$x$ and sold it for \$10.56. If Kylie incurred a loss of x per cent, find x.

8 A relationship that is used to approximate car stopping distances (d) in ideal road and weather conditions is: $d = t_r v + kv^2$ where t_r is the driver's reaction time, v is the velocity and k is a constant.

a Stirling's reaction time was measured to be 0·8 seconds. The distance it took him to stop while travelling at 20 m/s (72 km/h) was 51 m. Substitute this information into the formula to find the value of k.

b If, for these particular conditions, Stirling's breaking distance is given by $d = 0{\cdot}8v + 0{\cdot}0875v^2$ complete the table below, finding d correct to the nearest metre in each case.

v (m/s)	0	5	10	15	20	25
d (m)						



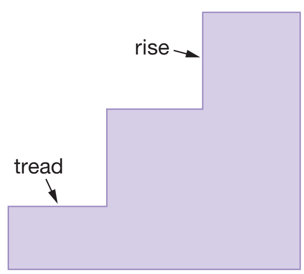
c Graph d against v using the number plane shown on the right. What kind of curve is produced?

d Use your graph to find the velocity (in m/s) that would produce a stopping distance of 40 m. Check your accuracy by solving the equation $40 = 0{\cdot}8v + 0{\cdot}0875v^2$ using the formula

$$v = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

e What factors would determine the safe car separation distance in traffic?

9 The rise and tread of a staircase have been connected using the formula $r = \frac{1}{2}(24 - t)$, or $r = \frac{66}{t}$ where r and t are measured in inches (1 inch is about 2·54 cm).



a If the tread should not be less than 9 inches, what can be said about the rise?

b Graph both functions on the same set of axes and compare the information they provide.

c What are the points of intersection of the two graphs? Check the accuracy of your graphs by solving the two simultaneous equations:

$$r = \frac{1}{2}(24 - t) \qquad 1$$

$$r = \frac{66}{t} \qquad 2$$

(Hint: Substitute 2 into 1 and solve the resulting quadratic equation.)

d Convert the formulas 1 and 2 to centimetres rather than inches.

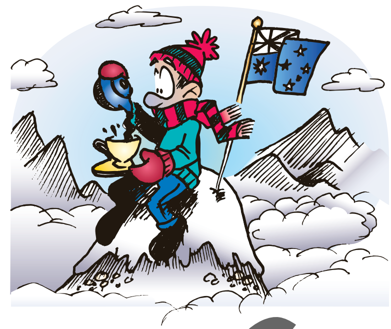
e Do the measurements of staircases you have experienced fit these formulas? Investigate other methods used by builders to determine r and t.

INVESTIGATION 2:05 TEMPERATURE AND ALTITUDE

The following formula has been used to give the boiling point of water at various heights above or below sea-level:

$h = 520(212 - T) + (212 - T)^2$

where height, h, is in feet and the temperature, T, is in degrees Fahrenheit (°F).



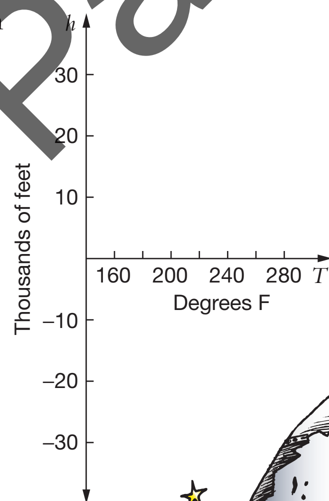
1 Show that $h = (212 - T)(732 - T)$.

2 At what height above sea-level does water boil at:
 a 200°F
 b 250°F?

3 Plot a graph of h against T. (Use values of T from 160°F to 280°F. Use values of h from −30 000 feet to 30 000 feet.)

T (°F)	160	180	200	220	240	260	280
h (feet)	29 744						−30 736

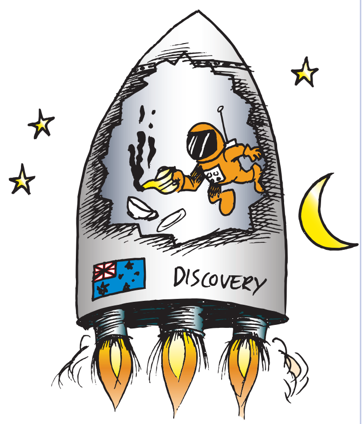
4 Use your graph to find the temperature at which water boils, to the nearest degree:
 a at Flinders Peak (1155 feet above sea-level)
 b atop Mt Everest (29 000 feet above sea-level)
 c on the shore of the Dead Sea (1300 feet below sea level).



5 Check your answers to Question 4 by substituting each height into the formula and solving the resulting quadratic equation.

6 Change the units on the axes of your graph so that they are in degrees Celsius (°C) and thousands of metres. To do this, use the formula $C = \frac{5}{9}(F - 32)$ and the approximation, 1 foot = 0·305 metres.

7 Discuss:
- Over what temperature range is this formula useful or valid?
- Find the height at which the International Space Station orbits. Can the formula be used there? Is the station pressurised to some equivalent height above sea-level?



2:06 Equations reducible to quadratics

PREP QUIZ 2:06

Solve these equations:

1 $2a(a-3)=0$
2 $(2a+1)(a-5)=0$
3 $(3a-4)(5a+2)=0$
4 $a^2-5a+6=0$
5 $a^2+14a+49=0$
6 $a^2-a-12=0$
7 $3a^2+7a+2=0$
8 $16a^2-24a+9=0$

Solve for x:

9 $x^2-16=0$
10 $x^4-16=0$

Some equations that appear more difficult to factorise may be changed into a quadratic equation by substituting an appropriate pronumeral.

For example: $x^4-13x^2+36=0$
If we let $u=x^2$ and then substitute, the equation becomes: $u^2-13u+36=0$

This equation can now be solved by factorising:
$(u-9)(u-4)=0$
$\therefore\ u=9$ or $u=4$

But $u=x^2$ so $x^2=9$ or $x^2=4$
$\therefore\ x=\pm 3$ or $x=\pm 2$

So the equation $x^4-13x^2+36=0$ has four solutions, $x=-3, -2, 2$ and 3.

WORKED EXAMPLE 1

By making suitable substitutions, solve these equations.

a $3a^4-14a^2+8=0$
Let $u=a^2$
So equation becomes:
$3u^2-14u+8=0$
$(3u-2)(u-4)=0$
$\therefore\ u=\frac{2}{3}$ or 4
But $u=x^2$, so
$x^2=\frac{2}{3}$ or 4
$\therefore\ x=\pm\sqrt{\frac{2}{3}}$ or ± 2

b $9^x-28(3^x)+27=0$
Let $u=3^x$
So equation becomes:
$u^2-28u+27=0$
$(u-27)(u-1)=0$
$\therefore\ u=27$ or 1
But $u=3^x$, so
$3^x=27$ or 1
$3^3=27$ or $3^0=1$
$\therefore\ x=3$ or 0

Note: $9^x=(3^x)^2$

WORKED EXAMPLE 2

By making suitable substitutions, solve these equations.

a $m^6 - 7m^3 - 8 = 0$

Let $X = m^3$

So the equation becomes:

$X^2 - 7X - 8 = 0$

$(X - 8)(X + 1) = 0$

$\therefore\ X = 8 \text{ or } -1$

But $X = m^3$, so

$m^3 = 8 \text{ or } -1$

$\therefore\ m = 2 \text{ or } -1$

b $y^4 - 7y^2 - 18 = 0$

Let $A = y^2$

So the equation becomes:

$A^2 - 7A - 18 = 0$

$(A - 9)(A + 2) = 0$

$\therefore\ A = 9 \text{ or } -2$

But $A = y^2$, so

$y^2 = 9 \text{ or } -2$ ($y^2 \neq$ negative number)

$\therefore\ y = \pm 3$ are the only solutions

Exercise 2:06

1 Use the substitution given to reduce each equation to a quadratic equation and then solve.

- **a** $x^4 - 5x^2 + 4 = 0,\ u = x^2$
- **b** $a^4 - 10a^2 + 9 = 0,\ u = a^2$
- **c** $n^4 - 29n^2 + 100 = 0,\ u = n^2$
- **d** $k^4 - 8k^2 + 16 = 0,\ X = k^2$
- **e** $m^4 - 6m^2 + 8 = 0,\ A = m^2$
- **f** $z^4 - 14z^2 + 45 = 0,\ Y = z^2$

2 By using the substitution given, solve these equations.

- **a** $4x^4 - 5x^2 + 1 = 0,\ u = x^2$
- **b** $9a^4 - 37a^2 + 4 = 0,\ u = a^2$
- **c** $3n^4 - 28n^2 + 9 = 0,\ u = n^2$
- **d** $8k^4 - 22k^2 + 9 = 0,\ X = k^2$

3 Reduce each equation to a quadratic equation, using the substitution given, and then solve.

- **a** $y^6 - 9y^3 + 8 = 0,\ u = y^3$
- **b** $m^8 - 17m^4 + 16 = 0,\ u = m^4$
- **c** $x^6 - 1008x^3 + 8000 = 0,\ X = x^3$
- **d** $n^8 - 82n^4 + 81 = 0,\ A = n^4$

4 By using the substitution given, solve these *exponential* equations.

- **a** $4^x - 5(2^x) + 4 = 0,\ u = 2^x$ [*Note:* $4^x = (2^x)^2$]
- **b** $9^x - 4(3^x) + 3 = 0,\ u = 3^x$
- **c** $4^x - 12(2^x) + 32 = 0,\ X = 2^x$
- **d** $9^x - 12(3^x) + 27 = 0,\ A = 3^x$

5 By using the substitution given, solve these equations.

- **a** $x^4 + 5x^2 - 6 = 0,\ u = x^2$
- **b** $x^4 - 5x^2 + 6 = 0,\ u = x^2$
- **c** $x^4 - 5x^2 - 6 = 0,\ u = x^2$
- **d** $x^4 + 5x^2 + 6 = 0,\ u = x^2$
- **e** $x^4 - 7x^2 + 10 = 0,\ u = x^2$
- **f** $a^4 - 2a^2 - 15 = 0,\ u = a^2$

6 Substitute u for $(x - 1)^2$ to solve the equation: $(x - 1)^4 - 29(x - 1)^2 + 100 = 0$

7 Solve the equation: $(a + 2)^4 - 20(a + 2)^2 + 64 = 0$

8 Solve these equations using the substitution $X = x^2 - 2x$.

- **a** $(x^2 - 2x)^2 - 23(x^2 - 2x) + 120 = 0$
- **b** $(x^2 - 2x)^2 + 3(x^2 - 2x) - 40 = 0$

FUN SPOT 2:06 — DID YOU KNOW THAT 2 = 1?

Now that your algebra skills are more developed,
you can be let into the secret that 2 really is equal to 1.

Proof: Assume that $x = y$.

Multiply both sides by x:	$x^2 = xy$
Subtract y^2 from both sides:	$x^2 - y^2 = xy - y^2$
Factorise both sides:	$(x - y)(x + y) = y(x - y)$
Divide by $(x - y)$:	$x + y = y$

If we let $x = y = 1$, then $2 = 1$. QED

MATHS TERMS 2

coefficient
- the number that multiplies a pronumeral in an equation or algebraic expression,
  e.g. $3x^2 - x + 5 = 0$
  coefficient of x^2 is 3
  coefficient of x is -1

completing the square
- completing an algebraic expression to form a perfect square, i.e. $(x + a)^2$ or $(x - a)^2$,
  e.g. to complete the square for $x^2 + 6x$, the number 9 is added:
  $x^2 + 6x + 9 = (x + 3)^2$

factorise
- to write an expression as the product of its factors
- the reverse of expand

quadratic equation
- an equation in which the highest power of the unknown pronumeral is 2,
  e.g. $x^2 - 16 = 0$, $x^2 + 5x + 6 = 0$
- a quadratic equation may have two, one or no solutions

quadratic formula
- a formula that gives the solutions to equations of the form $ax^2 + bx + c = 0$
- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

DIAGNOSTIC TEST 2 — QUADRATIC EQUATIONS

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Solve these equations, giving the answers correct to three decimal places if necessary. — 2:01

a $x^2 = 49$ b $a^2 = 7$ c $m^2 - 16 = 0$ d $3y^2 - 5 = 0$

2 Solve these equations: — 2:01

a $(x + 7)(x - 3) = 0$ b $x(x - 5) = 0$
c $(2x - 1)(x + 1) = 0$ d $(3x + 2)(4x - 5) = 0$

3 Factorise and solve: — 2:01

a $x^2 + 5x = 0$ b $x^2 + 9x + 14 = 0$
c $x^2 - 49 = 0$ d $2x^2 + 5x - 3 = 0$

4 What number must be inserted to complete the square? — 2:02

a $x^2 + 6x + \ldots$ b $x^2 - 4x + \ldots$
c $x^2 + 3x + \ldots$ d $x^2 - x + \ldots$

5 Solve the following by completing the square. — 2:05

a $x^2 + 2x - 2 = 0$ b $x^2 - 6x + 1 = 0$
c $x^2 - 3x - 5 = 0$ d $2x^2 - 10x = 1$

6 Solve using the quadratic formula. (Leave answers in surd form.) — 2:03

a $x^2 + x - 3 = 0$ b $x^2 - 5x + 2 = 0$
c $2x^2 + 4x + 1 = 0$ d $3x^2 + 2x - 2 = 0$

7 Solve the following: — 2:04

a $x^2 - x + 1 = 4x + 7$ b $x(x - 5) = x - 9$
c $(x + 4)^2 = 6$ d $x = \dfrac{2x + 8}{x}$

8 Solve these equations using the substitutions given. — 2:06

a $x^4 - 20x^2 + 64 = 0,\ u = x^2$ b $a^4 - 11a^2 + 18 = 0,\ u = a^2$

One of these gear wheels has 28 teeth and the other has 29 teeth. How many revolutions of each wheel must be completed for the same two teeth to be in the same position next to each other?

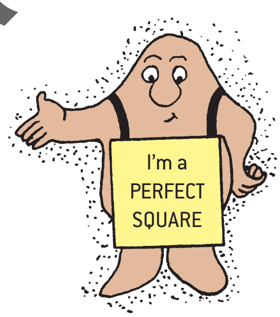
ASSIGNMENT 2A Chapter review

1 Solve the following quadratic equations using the method you feel is most appropriate.

a $x^2 + x - 30 = 0$
b $x(x - 7) = 0$
c $(x + 1)^2 = 9$
d $2x^2 + 7x - 15 = 0$
e $x^2 + 2x = 24$
f $(x + 2)(3x - 1) = 0$
g $x^2 - 14x + 49 = 0$
h $5x(2x - 3) = 0$
i $x^2 - 100 = 0$
j $x^2 - 5x - 14 = 0$
k $x^2 = 28 + 3x$
l $x^2 + 5x + 1 = 0$
m $(x - 3)^2 = 2$
n $10x^2 - 3x - 1 = 0$
o $x^2 - 20 = 0$
p $5x^2 + 3x = 0$
q $x^2 + 10x + 25 = 0$
r $x^2 + 2x - 4 = 0$
s $3x^2 - x - 2 = 0$
t $2x^2 + 5x + 1 = 0$
u $x(x + 5) = 24$
v $(x + 2)^2 = x + 2$
w $x = \dfrac{5x - 2}{x}$
x $(2x + 1)^2 = (x + 3)^2$

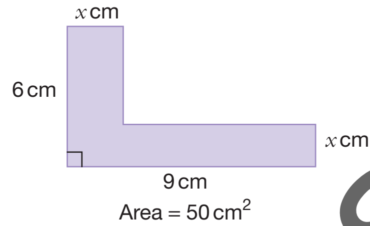
2 Solve by completing the square.

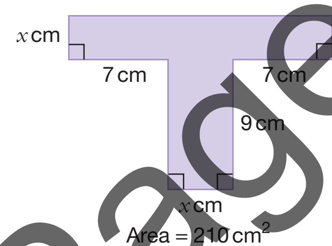
a $x^2 + 4x - 32 = 0$
b $x^2 - 3x - 40 = 0$
c $x^2 - 10x + 4 = 0$
d $2x^2 + 6x - 3 = 0$



3 Find three consecutive positive integers if the sum of their squares is 50.

4 Find x in the following figures.

a


b


5 If a rectangular field has an area of 0·28 ha and its length is 30 m more than its width, find the width of the field.

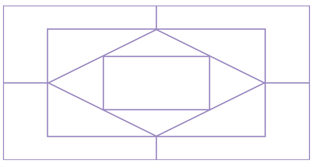
6 Solve each equation using the substitution given.

a $x^4 - 26x^2 + 25 = 0,\ u = x^2$
b $n^4 - 12n^2 + 27 = 0,\ u = n^2$
c $z^4 - 5z^2 - 12 = 0,\ u = z^2$

ASSIGNMENT 2B Working mathematically

1 An odd number between 301 and 370 has three different digits. If the sum of its digits is five times the hundreds digit, what is the digit?

2 What is the minimum number of colours needed to shade this diagram if no two adjacent regions may have the same colour?



3 Emma's passbook savings account allowed her to deposit or withdraw at any time. Her interest, which was 2·5% pa, was calculated on the minimum monthly balance and was paid twice yearly into her account. She could withdraw up to $500 in cash per day or any amount in the form of a cheque. Cheques for the payment of bills (third party cheques) were provided free of charge. She was able to start her account with as little as $1.

a What is the minimum balance required?

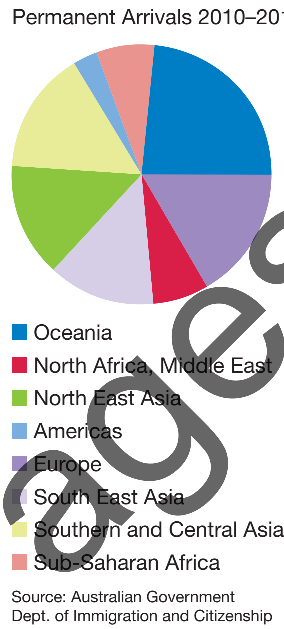
b On what amount is the interest calculated?

c Does the interest earned in one month automatically begin to earn interest during the next month?

4 Decrease $360 by 20% and then increase the result by 20%. What is the difference between $360 and your final answer?

5 50% more than what number is 25% less than 60% more than 10?

6 The immigration to Australia for 2010–2011 is shown by this pie chart.



a Which region provided:

i the most immigrants

ii the fewest immigrants?

b Measure the angle of each sector to determine the percentage of immigrants from:

i South East Asia

ii Europe

c If the total number of immigrants was 126 536, how many (to the nearest hundred) came from:

i the Americas

ii Oceania and Antarctica?

ASSIGNMENT 2C Cumulative revision

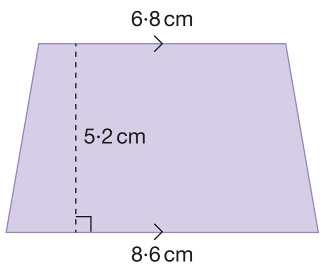
1 **a** Find 15% of \$125. 1:01D
b What percentage is \$25 of \$125?
c Decrease a price of \$125 by 30%.
d 15% discount of a price is equal to \$24. What is the full price?

2 Simplify the following expressions. 1:02
a $5x - 2y - x + y$ **b** $10ax \div 5a$ **c** $\dfrac{2a}{3} + \dfrac{3a}{5}$ **d** $\dfrac{2a}{3x} \div \dfrac{4a}{9y}$

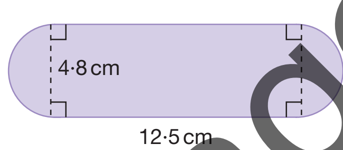
3 A card is drawn from a standard pack of 52 playing cards. 1:03
What is the probability the card is:
a red **b** a club **c** a Jack **d** the 7 of spades?

4 Find the area of each shape. 1:04

a



b

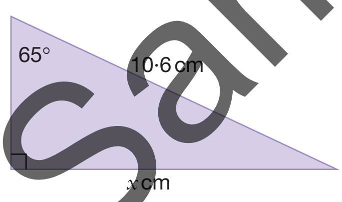


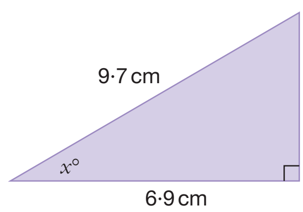
5 Evaluate: 1:05
a $5^2 \times 2^5$ **b** $4^5 \div 4^4$ **c** 4^{-2} **d** $(2^3)^{-1}$ **e** $8^{\frac{2}{3}}$

6 Simplify each expression: 1:06
a $3\sqrt{18}$ **b** $4\sqrt{8} - \sqrt{32}$ **c** $(2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} - 3\sqrt{2})$

7 Yvonne is paid an hourly rate of \$28.40 for a 36 hour week. The first 6 hours overtime are paid at time-and-a-half; after that extra hours worked are paid double-time. Find Yvonne's wage for a week in which she works 45 hours. 1:08

8 For these triangles, find: 1:12
a the value of x to one decimal place **b** the value of x to the nearest degree.





9 For the set of scores 3, 5, 4, 7, 5, 4, 8, 3, 4, find the: 1:13
a range **b** mode **c** median **d** mean **e** Q_1 **f** Q_3
g interquartile range.