1

Linear relationships

Maths saves the planet!

Name: Zoe Ryan
Job: Forest Carbon Specialist
Qualifications: Bachelor of Forest Science

Developing countries with tropical rainforests have had a high rate of deforestation in the past. However, a change in attitude over the past few years and a growing interest in the environment is halting the devastation. Zoe Ryan is a Forest Carbon Specialist. Her job is to measure the amount of carbon stored in a forest to calculate the amount of greenhouse gases that would be emitted if they were cut down.

About a third of Zoe's time is spent abroad, travelling to tropical regions in countries such as Borneo, Papua New Guinea, Peru, Ecuador and Mozambique. She has to travel on canoes through croc-infested waters to get to some of the forests. 'It's really wild; you wouldn't find these places in a Lonely Planet travel guide … there's always something totally unexpected, you just have to let go and go with it.'

In the forest, Zoe takes measurements of the trees and uses sampling in order to measure the carbon stock in the forest. After the quantity of avoided emissions is calculated, it is then audited and verified by an independent third party. Carbon credits are then sold in the carbon market to companies who want, or need, to reduce their carbon emissions debit. Deforestation contributes about 15% of the world's greenhouse gas emissions. Getting companies to buy carbon credits means that the forest stays protected.

Knowing how important it is to manage the forest sustainably, it is crucial for Zoe to be accurate with her maths. 'I can see that the more samples we put in, the lower our error so the more carbon we're able to sell because our precision is greater. I feel like I'm genuinely doing something that's good for society, and it's also an adventure.'

Why learn this?

A variety of jobs use linear relationships and systems of linear equations to represent, analyse and solve a variety of problems, and to manipulate numbers to find quantities. Linear equations are used extensively in all branches of science and technology, including computer science, biology, chemistry, physics, anthropology and all the engineering professions. Other professionals such as economists, bankers, architects and photographers also use linear equations to understand information and to predict future results.

After completing this chapter you will be able to:

- solve linear equations
- find the gradient of a line joining two points or from an equation
- sketch linear graphs
- find the equations of parallel and perpendicular lines
- find the solutions for linear inequalities
- solve simultaneous linear equations.

1 Recall

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 If $P = 2t - 7$, find P when:

(a) $t = 0$ **(b)** $t = 5$ **(c)** $t = -4$ **(d)** $t = 2$

2 Solve the following linear equations.

(a) $2a - 3 = 11$ **(b)** $\frac{b}{4} + 7 = 13$ **(c)** $3(2c - 5) = 45$ **(d)** $\frac{2d + 6}{5} = 4$

3 Plot these coordinate points on a Cartesian plane.

(a) $A(4, 3)$ **(b)** $B(0, 2)$ **(c)** $C(-1, 2)$ **(d)** $D(-2, -3)$

4 Copy and complete the table of values below for each of the following equations, then plot each graph for $-2 \le x \le 2$ on a Cartesian plane.

x	-2	-1	0	1	2
y					

(a) $y = x + 3$ **(b)** $y = x - 2$ **(c)** $y = 2 - x$ **(d)** $y = 3x$

5 Sketch the graphs of:

(a) $y = 3$ **(b)** $x = 2$

6 Write **(i)** the gradient and **(ii)** the y-intercept for the graphs of the following equations.

(a) $y = 3x + 5$ **(b)** $y = 3 - 2x$ **(c)** $2x + 3y = 4$

7 Find the gradient of the line joining the points with coordinates:

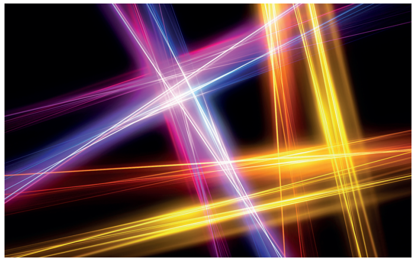
(a) (1, 3) and (4, -2) **(b)** (-3, -6) and (2, -1)

8 **(a)** Sketch the graphs of $x = 1$ and $y = 3$ on the same set of axes.

(b) Find the point of intersection of the two lines.

Exploration Task

You can download this activity from the eBook or the Pearson Places website.



Straight lines

In this activity you will explore how to match equations with different kinds of straight lines.

Curriculum links

Australian CURRICULUM
VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Number and Algebra

Patterns and algebra

ACMNA234
Substitute values into formulas to determine an unknown

- solving simple equations arising from formulas

VCMNA333
Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term

- solving simple equations arising from formulas
- re-arranging expressions to make a specified variable the subject such as calculating the radius of a sphere to produce a given volume

Linear and non-linear relationships

ACMNA235/VCMNA335
Solve problems involving linear equations, including those derived from formulas

- representing word problems with simple linear equations and solving them to answer questions

ACMNA236/VCMNA336
Solve linear inequalities and graph their solutions on a number line

- representing word problems with simple linear inequalities and solving them to answer questions

ACMNA237/VCMNA337
Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology

- associating the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs

ACMNA238
Solve problems involving parallel and perpendicular lines

- solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel
- solving problems using the fact that the product of the gradients of perpendicular lines is -1 and conversely that if the product of the gradient of two lines is -1 then they are perpendicular

VCMNA338
Solve problems involving gradients of parallel and perpendicular lines

- solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel
- solving problems using the fact that the product of the gradients of perpendicular lines is -1 and conversely that if the product of the gradient of two lines is -1 then they are perpendicular

ACMNA240/VCMNA340
Solve linear equations involving simple algebraic fractions

- solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution
- representing word problems, including those involving fractions, as equations and solving them to answer the question

VCMNA342
Solve equations using systematic guess-check-and-refine with digital technology

- refining intervals on graphs and/or in tables of values to determine with increasing accuracy when the values of two functions are approximately equal

10A VCMNA364
Solve simultaneous equations using systematic guess-check-and-refine with digital technology contexts

- using graphs to determine a convergent set of intervals which contain a point of intersection of the graphs of two functions
- using cobweb diagram to solve simultaneous equations numerically

General capabilities

- Numeracy
- Literacy
- Critical and creative thinking
- Personal and social capability
- Information and communication technology (ICT) capability

Big ideas

- Solving linear equations follows the same set of steps, regardless of the form in which the equation is expressed.
- Linear equations produce straight line graphs. Sufficient information for a straight line graph to be drawn is the gradient and the y-intercept or the x- and y-intercepts (provided they are different points).
- Linear inequalities are solved using the same process as linear equations.
- Simultaneous linear equations may be solved using a graphical method or using algebraic methods of elimination and/or substitution.

Resources

Recall Worksheets

- R1.1: Substitution into a formula
- R1.2: Solving linear equations
- R1.3: Plotting points on the Cartesian plane
- R1.4: Plotting a linear equation using a table
- R1.5: Plotting graphs parallel to the axes
- R1.6: Gradient-intercept form of an equation
- R1.7: Gradient of a line joining two points
- R1.8: Finding points of intersection

Exploration Task

- Straight lines

Lightbook Starter

- Before you begin 1

notes:

Answers

Recall 1

1 (a) $P = 2t - 7$
$= 2(0) - 7$
$= 0 - 7$
$= -7$

(b) $P = 2t - 7$
$= 2(5) - 7$
$= 10 - 7$
$= 3$

(c) $P = 2t - 7$
$= 2(-4) - 7$
$= -8 - 7$
$= -15$

(d) $P = 2t - 7$
$= 2(2) - 7$
$= 2(2) - 7$
$= -3$

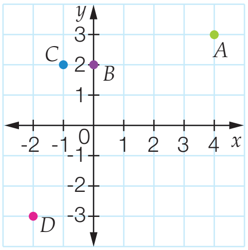
2 (a) $2a - 3 = 11$
$2a = 14$
$a = 7$

(b) $\frac{b}{4} + 7 = 13$
$\frac{b}{4} = 6$
$b = 24$

(c) $3(2c - 5) = 45$
$(2c - 5) = 15$
$2c = 20$
$c = 10$

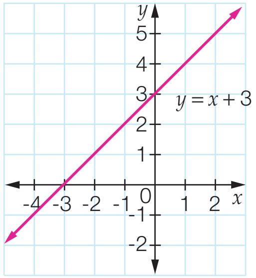
(d) $\frac{2d + 6}{5} = 4$
$2d + 6 = 20$
$2d = 14$
$d = 7$

3 (a)–(d)



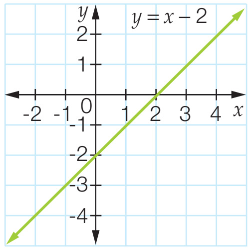
4 (a)

x	-2	-1	0	1	2
y	1	2	3	4	5



(b)

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0

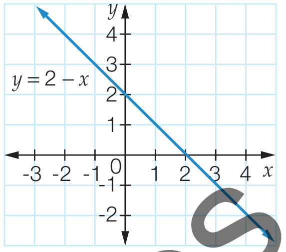


notes:

Answers

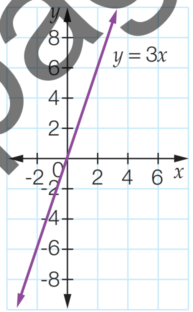
4 (c)

x	-2	-1	0	1	2
y	4	3	2	1	0

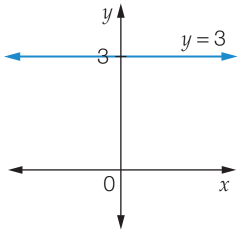


(d)

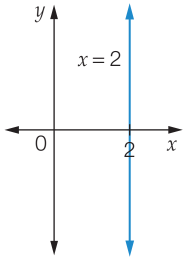
x	-2	-1	0	1	2
y	-6	-3	0	3	6



5 (a)



(b)



6 (a) (i) 3 (ii) (0, 5)

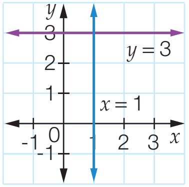
(b) (i) -2 (ii) (0, 3)

(c) (i) $-\frac{2}{3}$ (ii) $(0, \frac{4}{3})$

7 (a) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{4 - 1} = -\frac{5}{3}$

(b) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - -6}{2 - -3} = \frac{5}{5} = 1$

8 (a)



(b) (1, 3)

1.1

Linear equations

A **linear relationship** exists between two variables if the graph of the relationship is a **linear graph** (a straight line). In a linear relationship, a uniform change in one variable, the **independent variable**, produces a uniform change in the other, the **dependent variable**. The variables x and y are commonly used because they represent the axes of the Cartesian plane on which linear relationships are graphed.

A linear relationship is described by a **linear equation**. If you know the value of one of the variables, you can solve the equation to find the corresponding value of the other variable. You can solve a linear equation using inverse operations. Linear equations have only one solution.

Worked example 1 W.E. 1

Solve each of the following equations to find the value of the unknown.

(a) $5x - 6 = 34$ **(b)** $\frac{3a}{2} + 4 = -1$

Thinking	Working
(a) 1 Use inverse operations in the opposite order to find the value of the unknown. (Here, add 6 to both sides and then divide both sides by 5.)	(a) $5x - 6 = 34$ $5x = 40$ $x = 8$
2 Check your answer by substituting the value of the pronumeral into the equation.	$\text{LHS} = 5x - 6$ $= 5 \times 8 - 6$ $= 34$ $= \text{RHS}$
(b) 1 Use inverse operations in the opposite order to find the value of the unknown. (Here, subtract 4 from both sides, multiply both sides by 2 and then divide both sides by 3.)	(b) $\frac{3a}{2} + 4 = -1$ $\frac{3a}{2} = -5$ $3a = -10$ $a = \frac{-10}{3}$ $= -3\frac{1}{3}$
2 Check your answer by substituting the value of the pronumeral into the equation.	$\text{LHS} = \frac{3a}{2} + 4$ $= \frac{3 \times -3\frac{1}{3}}{2} + 4$ $= \frac{3 \times -\frac{10}{3}}{2} + 4$ $= -\frac{10}{2} + 4$ $= -5 + 4$ $= -1$ $= \text{RHS}$

Equations where the unknown appears more than once

To solve equations of the type $5a - 3 = 2a + 6$ you must first add or subtract a **term** containing the pronumeral on both sides of the equation, so that the terms with pronumerals appear on one side only (usually the LHS). If the equations involve brackets, you can expand the brackets using the distributive law $a(b + c) = ab + ac$.

Worked example 2 W.E. 2

Solve each of the following equations to find the value of the unknown.

(a) $5a - 3 = 2a + 6$ **(b)** $2a - 5 = 6a + 1$ **(c)** $4(k + 1) = 3(6 - k)$

Thinking	Working
(a) 1 Add or subtract a term so that the terms with pronumerals appear on one side of the equation only. (Here, subtract $2a$ from both sides.)	(a) $5a - 3 = 2a + 6$ $3a - 3 = 6$
2 Use inverse operations in the opposite order to find the value of the unknown. (Here, add 3 to both sides and then divide both sides by 3.)	$3a = 9$ $a = 3$
3 Check your answer by substituting the value of the pronumeral into each side of the equation.	$\text{LHS} = 5a - 3$ $\text{RHS} = 2a + 6$ $= 5 \times 3 - 3$ $= 2 \times 3 + 6$ $= 15 - 3$ $= 6 + 6$ $= 12$ $= 12$ $\text{LHS} = \text{RHS}$
(b) 1 If the coefficient of the pronumeral term on the RHS is greater than the coefficient of the pronumeral on the LHS, then swap the sides of the equation. (Here, 6 > 2 so swap sides.)	(b) $2a - 5 = 6a + 1$ $6a + 1 = 2a - 5$
2 Add or subtract a term so that the terms with pronumerals appear on one side of the equation only.	$4a + 1 = -5$
3 Use inverse operations in the opposite order to find the value of the unknown.	$4a = -6$ $a = -\frac{6}{4}$
4 Simplify your solution if necessary.	$= -\frac{3}{2}$
5 Check your answer by substituting the value of the pronumeral into each side of the equation.	$\text{LHS} = 2a - 5$ $\text{RHS} = 6a + 1$ $= 2 \times -\frac{3}{2} - 5$ $= 6 \times -\frac{3}{2} + 1$ $= -3 - 5$ $= -9 + 1$ $= -8$ $= -8$ $\text{LHS} = \text{RHS}$

1.1

Resources

eWorked examples

- Solving equations
- Solving equations with a pronumeral on both sides
- Solving equations involving algebraic fractions
- Worded equation problems

Lightbook Starter

- Check-in 1.1

Appendices

- Appendix 1A: Which step first?

Recap

Question	Answer
1 In a right-angled triangle, find the length of the hypotenuse if the other two side lengths are 5 cm and 12 cm.	$h^2 = 5^2 + 12^2$ $h = 13$ cm (5, 12, 13 triad)
2 If $a = 1$, $b = 2$, $c = -3$, $d = -4$, find the value of the following. (a) $(a - b)^2 + (c - d)^2$ (b) $(a - c)^2 + (b - d)^2$ (c) $\frac{c - d}{b - a}$	(a) $(1 - 2)^2 + (-3 - -4)^2 = (-1)^2 + (1)^2$ $= 1 + 1$ $= 2$ (b) $(1 - -3)^2 + (2 - -4)^2 = (4)^2 + (6)^2$ $= 16 + 36$ $= 52$ (c) $\frac{-3 - -4}{2 - 1} = \frac{1}{1}$ $= 1$
3 Solve: (a) $5x = 25$ (b) $x - 7 = 8$	(a) $5x = 25$ $x = 5$ (b) $x - 7 = 8$ $x = 15$
4 Complete a table of values for each of the following linear relationships. $\begin{array}{cccccc} x & -2 & -1 & 0 & 1 & 2 \\ y & & & & & \end{array}$ (a) $y = 2x - 1$ (b) $2x + y = 4$	(a) $\begin{array}{cccccc} x & -2 & -1 & 0 & 1 & 2 \\ y & -5 & -3 & -1 & 1 & 3 \end{array}$ (b) $\begin{array}{cccccc} x & -2 & -1 & 0 & 1 & 2 \\ y & 8 & 6 & 4 & 2 & 0 \end{array}$
5 Evaluate the following. (a) $\frac{2}{3} + \frac{4}{5}$ (b) $\frac{5}{9} - \frac{3}{8}$	(a) $\frac{2}{3} + \frac{4}{5} = \frac{2 \times 5}{15} + \frac{4 \times 3}{15}$ $= \frac{10}{15} + \frac{12}{15}$ $= \frac{22}{15}$ $= 1\frac{7}{15}$ (b) $\frac{5}{9} - \frac{3}{8} = \frac{5 \times 8}{72} - \frac{3 \times 9}{72}$ $= \frac{40}{72} - \frac{27}{72}$ $= \frac{13}{72}$

notes:

(c) **1** Expand brackets.	(c) $4(k+1)=3(6-k)$ $4k+4=18-3k$
2 Add or subtract a term so that the terms with pronumerals appear on one side of the equation only. (Here, add 3*k* to both sides.)	$7k+4=18$
3 Use inverse operations in the opposite order to find the value of the unknown. (Here, subtract 4 from both sides and then divide both sides by 7.)	$7k=14$ $k=2$
4 Check your answer by substituting the value of the pronumeral into both sides of the equation.	$\text{LHS}=4(k+1)$ $=4(2+1)$ $=4\times 3$ $=12$ $\text{RHS}=3(6-k)$ $=3(6-2)$ $=3\times 4$ $=12$ $\text{LHS}=\text{RHS}$

To solve an equation containing fractions, find the lowest common multiple (LCM) of the denominators, called the lowest common denominator (LCD). You then multiply both sides of the equation by the LCD. This simplifies the equation by eliminating the fractions. The equation can now be solved as shown previously.

Worked example 3

W.E. 3

Solve each of the following equations to find the value of the unknown.

(a) $\frac{2x+1}{3}=\frac{x-3}{5}$ **(b)** $\frac{x}{3}-2=\frac{x-1}{4}$

Method 1: Multiplying by the LCD

Thinking	Working
(a) **1** Find the LCD of all fractions in the equation.	(a) $\text{LCD}=15$
2 Multiply every term in the equation by the LCD. Insert brackets if required.	$\frac{2x+1}{3}=\frac{x-3}{5}$ $\frac{15(2x+1)}{3}=\frac{15(x-3)}{5}$
3 Divide the denominator of each term into the LCD.	$5(2x+1)=3(x-3)$
4 Expand any brackets.	$10x+5=3x-9$
5 Add or subtract a term so that the terms with the pronumeral is on one side of the equation only. (Here, subtract 3*x* from both sides.)	$7x+5=-9$
6 Use inverse operations to find the value of the unknown.	$7x=-14$ $x=-2$
7 Check the value of your solution by substituting the value of the pronumeral into both sides of the original equation.	$\text{LHS}=\frac{2\times -2+1}{3}$ $=\frac{-3}{3}$ $=-1$ $\text{RHS}=\frac{-2-3}{5}$ $=\frac{-5}{5}$ $=-1$ $\text{LHS}=\text{RHS}$
(b) **1** Find the LCD of all fractions in the equation.	(b) $\text{LCD}=12$ $\frac{x}{3}-2=\frac{x-1}{4}$
2 Multiply every term in the equation by the LCD. Insert brackets if required.	$\frac{12x}{3}-12\times 2=\frac{12(x-1)}{4}$
3 Divide the denominator of each term into the LCD.	$4x-24=3(x-1)$
4 Expand any brackets.	$4x-24=3x-3$
5 Add or subtract a term so that the terms with the pronumerals are is on one side of the equation only. (Here, subtract 3*x* from both sides.)	$x-24=-3$
6 Use inverse operations to find the value of the unknown.	$x=21$
7 Check the value of your solution by substituting the value of the pronumeral into both sides of the original equation.	$\text{LHS}=\frac{21}{3}-2$ $=7-2$ $=5$ $\text{RHS}=\frac{21-1}{4}$ $=\frac{20}{4}$ $=5$ $\text{LHS}=\text{RHS}$

Alternatively, you can write all fractions with a common denominator (the LCD) and then multiply both sides of the equation by the LCD.

Method 2: Writing all terms as fractions using the LCD

Thinking	Working
(a) **1** Find the LCD of all fractions in the equation.	(a) $\text{LCD}=15$
2 Write all terms in the equation as fractions using the LCD as the denominator. Use brackets if required. (Here, $\frac{1}{3}=\frac{5}{15}$ and $\frac{1}{5}=\frac{3}{15}$.)	$\frac{2x+1}{3}=\frac{x-3}{5}$ $\frac{5(2x+1)}{15}=\frac{3(x-3)}{15}$
3 Multiply every term on both sides of the equation by the denominator (LCD) to eliminate the fraction.	$5(2x+1)=3(x-3)$
4 Expand any brackets.	$10x+5=3x-9$

Suggested examples

1 **Solve each of the following equations.**

(a) $3x - 1 = 8$

(b) $3(3x - 1) = 8$

(c) $\frac{3(3a - 1)}{2} = -6$

(d) $\frac{2c}{3} - 5 = -1$

Answers:

(a)
$$3x - 1 = 8$$
$$3x - 1 + 1 = 8 + 1$$
$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

(b)
$$3(3x - 1) = 8$$
$$9x - 3 = 8$$
$$9x - 3 + 3 = 8 + 3$$
$$9x = 11$$
$$\frac{9x}{9} = \frac{11}{9}$$
$$x = 1\tfrac{2}{9}$$

(c)
$$\frac{3(3a - 1)}{2} = -6$$
$$3(3a - 1) = -6 \times 2$$
$$9a - 3 = -12$$
$$9a - 3 + 3 = -12 + 3$$
$$9a = -9$$
$$\frac{9a}{9} = \frac{-9}{9}$$
$$a = -1$$

(d)
$$\frac{2c}{3} - 5 = -1$$
$$\frac{2c}{3} - 5 + 5 = -1 + 5$$
$$\frac{2c}{3} = 4$$
$$\frac{2c}{3} \times \frac{3}{1} = 4 \times 3$$
$$2c = 12$$
$$c = 6$$

2 **Solve each of the following equations.**

(a) $3d + 5 = 4d - 2$

(b) $5(3f + 5) = 3(4f - 2)$

Answers:

(a)
$$3d + 5 = 4d - 2$$
$$3d = 4d - 7$$
$$-d = -7$$
$$d = 7$$

(b)
$$5(3f + 5) = 3(4f - 2)$$
$$15f + 25 = 12f - 6$$
$$15f = 12f - 31$$
$$3f = -31$$
$$f = -\frac{31}{3}$$
$$= -10\tfrac{1}{3}$$

3 **Solve each of the following equations.**

(a) $\frac{3x - 1}{4} = \frac{x - 3}{2}$

(b) $\frac{3m - 1}{4} = \frac{2m - 3}{3} + 2$

Answers:

(a)
$$\frac{3x - 1}{4} = \frac{x - 3}{2}$$
$$\frac{4(3x - 1)}{4} = \frac{4(x - 3)}{2}$$
$$3x - 1 = 2(x - 3)$$
$$3x - 1 = 2x - 6$$
$$3x = 2x - 5$$
$$x = -5$$

(b)
$$\frac{3m - 1}{4} = \frac{2m - 3}{3} + 2$$
$$\frac{12(3m - 1)}{4} = \frac{12(2m - 3)}{3} + 2 \times 12$$
$$3(3m - 1) = 4(2m - 3) + 24$$
$$9m - 3 = 8m - 12 + 24$$
$$9m - 3 = 8m + 12$$
$$9m = 8m + 15$$
$$m = 15$$

4 **Write an equation for each of the following and then solve it to find the unknown numbers.**

(a) **Find three consecutive even numbers whose sum is 510.**

(b) **Uncle Luij is 3 times as old as his niece Tess. His nephew Jack is 3 years younger than Tess. If their ages add to 82, how old is Uncle Luij?**

Answers:

(a) Let the first even number be n, where n is an integer (whole number).

Three consecutive even numbers: $n, n + 2, n + 4$

Equation:
$$n + (n + 2) + (n + 4) = 510$$
$$3n + 6 = 510$$
$$3n = 504$$
$$\frac{3n}{3} = \frac{504}{3}$$
$$n = 168$$

The three consecutive even numbers are 168, 170 and 172.

(b) Let t = Tess's age.

Luij: $3t$

Jack: $t - 3$
$$t + 3t + (t - 3) = 82$$
$$5t - 3 = 82$$
$$5t = 85$$
$$t = 17$$
$$3t = 51$$

Uncle Luij is 51 years.

Teaching strategies

Equations with fractions

When solving equations involving fractions, remove the fractions by multiplying by the lowest common denominator. In this way, all of the fractions will have a denominator of 1.

For example, to solve $\frac{3x - 1}{5} = \frac{x - 3}{2}$, multiply both sides by 10. Students need to be competent with this technique before being shown the short cut of 'cross-multiplying' to obtain $2(3x - 1) = 5(x - 3)$. Multiplying by the lowest common denominator avoids the common error in equations such as $\frac{3m - 1}{4} = \frac{2m - 3}{3} + 2$ when students cross-multiply the fractions and forget to multiply the (+ 2) term by 12.

Equations with the pronumeral on both sides

Students often try to divide by a negative number. Remind them to multiply by -1 to remove the negative. Or, to avoid the opportunity for mistake, students could remove the pronumeral from the side with the smaller coefficient of the pronumeral. Students tend to remove the pronumeral from the right-hand side of the equation. (This is still a legitimate procedure, but students then need to be confident removing the negative.)

$2m + 6 = 5m - 3$	$2 < 5$ so subtract $2m$ from both sides
$4 - 2m = 5m - 3$	$-2 < 5$ so add $2m$ to both sides
$4 - 2m = 3 - 5m$	$-5 < -2$ so add $5m$ to both sides