CHAPTER 02

Newtonian theories of motion

An understanding of forces and fields has allowed humans to land on the Moon and explore the outer reaches of the solar system. Satellites in orbit around the Earth have changed the way people live. These advances have been achieved using Newton's laws of motion, which were published in the seventeenth century. Newton suggested that it should be possible to put satellites in orbit around the Earth almost 300 years before it became technically possible. While relativistic corrections introduced by Einstein are important in a limited number of contexts, Newton's description of gravitation and the laws governing motion are accurate enough for most practical purposes.

In this chapter Newton's laws will be used to analyse motion when two or more forces act on a body and how projectiles travel in the Earth's gravitational field. The chapter also covers how forces keep objects travelling in a circular path.

Key knowledge

- investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions **2.1**
- investigate and analyse theoretically and practically the uniform circular motion of an object moving in a horizontal plane: ($F_{net} = \frac{mv^2}{r}$), including:
  - a vehicle moving around a circular road **2.2**
  - a vehicle moving around a banked track **2.3**
  - an object on the end of a string **2.2**
- investigate and apply theoretically Newton's second law to circular motion in a vertical plane (forces at the highest and lowest positions only) **2.4**
- investigate and analyse theoretically and practically the motion of projectiles near Earth's surface, including a qualitative description of the effects of air resistance **2.5, 2.6**

REVISION

Equations of motion

The equations of motion can be used in situations where there is a constant acceleration a (in $\mathrm{m\,s^{-2}}$). These equations allow you to model the motion of objects and predict values for the initial velocity u (in $\mathrm{m\,s^{-1}}$), final velocity v (in $\mathrm{m\,s^{-1}}$), displacement s (in m) and time t (in s). A direction convention should also be followed when using the equations of motion.

The equations of motion for uniform acceleration are:

$$v = u + at$$

$$s = \tfrac{1}{2}(u + v)t$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

These equations will be used in this chapter in addition to a number of new equations.

2.1 Newton's laws of motion

On 14 July 2015, NASA's New Horizons spacecraft (Figure 2.1.1) sped past Pluto and sent back images to Earth that appeared on news broadcasts across the world. The principles of physics on which this mission depended were published by Isaac Newton in 1687 in a set of laws that radically challenged the understanding of his time.

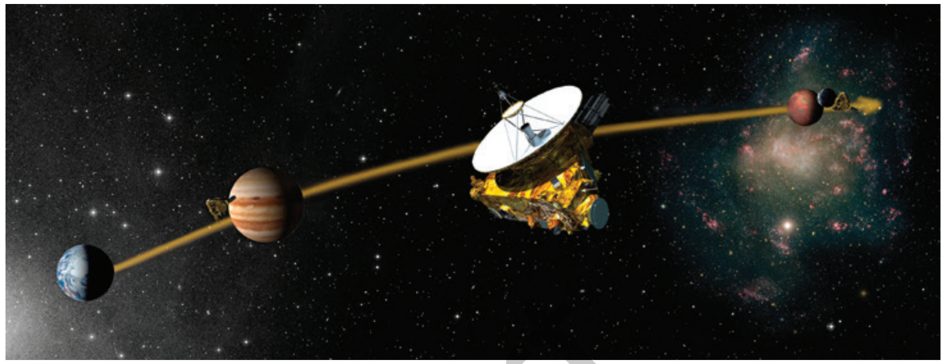


FIGURE 2.1.1 An artist's impression of New Horizons flying past Jupiter on its way to Pluto

Newton's laws are, in fact, only an approximation and have been superseded by Einstein's relativistic theories. In situations involving extremely high speeds (greater than 10% of the speed of light) or strong gravitational fields, Newton's laws are imprecise, and Einstein's theories must be used instead. However, Newton's laws are not obsolete. In most cases, Newton's laws remain invaluable for describing the motion of objects as diverse as planets and ping-pong balls.

NEWTON'S THREE LAWS OF MOTION

Newton's three laws of motion describe how forces affect the motion of bodies. The first law describes what happens to a body when there is no net force on it. The second law explains motion when there is an unbalanced force acting on a body. The third law states that all forces act in action–reaction pairs (that is, for every action there is an equal but opposite reaction).

Newton's first law

Newton's first law states that every object continues to be at rest, or continues with constant velocity, unless it experiences an unbalanced force. This is also called the law of inertia. An object that is moving at constant velocity will keep moving. This is seldom observed in everyday life due to the presence of forces such as friction and **air resistance** which eventually slow the motion of the object. To maintain constant motion, frictional forces must be balanced with some other force. For example, an object can keep moving at a constant velocity if it is driven by a motor.

An object that is stationary will remain stationary while the forces acting on it are balanced. For example, an object will fall due to the force of gravity, but it will remain at rest when this force is balanced by the **normal force** applied by a table on which the object comes to rest. A normal force is one that exists between a surface and an object, and it always act at right angles to the surface.

Newton's second law

Newton's second law states that the acceleration of a body experiencing an unbalanced force is directly proportional to the net force acting on it and inversely proportional to the mass of the body, i.e. $a = \frac{F_{net}}{m}$. This is commonly written as follows.

$F_{net} = ma$
where F_{net} is the net or resultant force acting (N)
m is the mass of the object (kg)
a is the acceleration of the object ($m\,s^{-2}$)

In other words, an object will accelerate at a greater rate when the force acting on it is increased. Heavy objects are harder to accelerate than lighter ones, so the rate of acceleration decreases as mass increases.

Newton's third law

Newton's third law states that when one body exerts a force on another body (an action force), the second body exerts an equal force on the first body but in the opposite direction (a reaction force):

$$F_{\text{on A by B}} = -F_{\text{on B by A}}$$

To simplify the notation, this text will use the following convention:

$$F_{AB} = F_{\text{on A by B}}$$

In this convention, the first subscript always indicates the body experiencing the force.

The forces in an action–reaction pair:

- are the same **magnitude**
- act in opposite directions and
- are exerted on two different objects.

It is important to note that action–reaction pairs can never be added together. This is because they act on different bodies. This is explained in Figure 2.1.2. In Figure 2.1.2 (b) the pair of forces shown, F_g and F_N, are not an action–reaction pair because both forces act on the same object (the basketball).

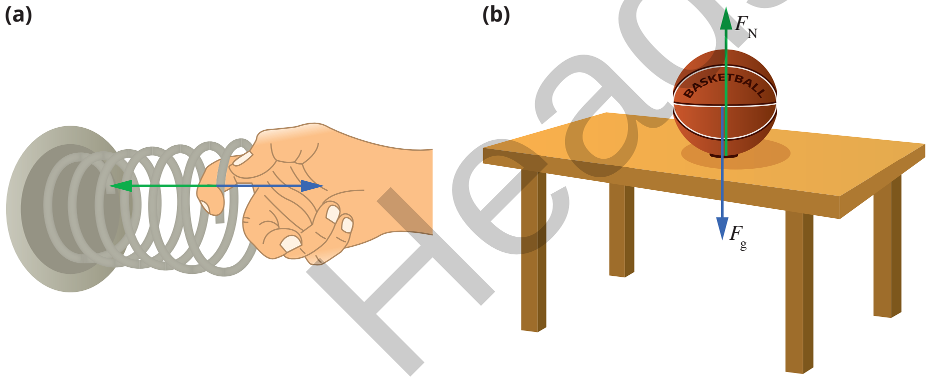


FIGURE 2.1.2 (a) An action–reaction pair: the hand pulls on the spring and the spring pulls back on the hand with an equal and opposite force. Figure (b) does not show an action-reaction pair. This is because the force due to gravity and the normal force both act on the same object, the basketball.

While the force is the same size on both objects, the resulting acceleration may not be. That is because the rate of acceleration depends on the mass of the objects concerned (from Newton's second law). Sometimes, when the objects have very different masses, the effect of one force in an action–reaction pair is much more noticeable. For example, if you stub your toe on a large heavy rock, the force exerted on your toe by the rock causes your foot to decelerate significantly. The equal and opposite force exerted by your toe on the rock does not cause any significant acceleration of the rock. This is because of its much greater mass.

$F_{\text{on A by B}} = -F_{\text{on B by A}}$

PHYSICSFILE

Tethered spacewalks

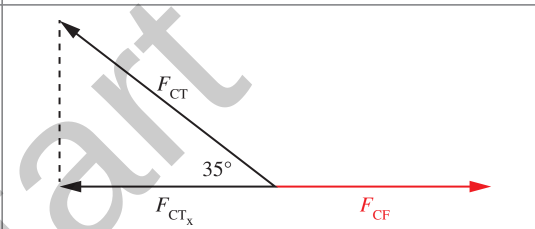
When stationed on the International Space Station (ISS), astronauts are often required to conduct spacewalks, that is, they need to complete tasks outside their spacecraft. During spacewalks, astronauts are tethered (i.e. attached) to their spacecraft. If they weren't, they would float off into space (remember Newton's first law of motion!). All the astronaut's tools are attached to their spacesuits, otherwise they too would float off into space. If an astronaut were to become accidentally untethered, it could be a disaster. Without a surface to push against, the astronaut would float off into space and be unable to return to the spacecraft. As a safety precaution, every astronaut is fitted with a small jet pack they can use to manoeuvre themselves back to their spacecraft. The jet pack propels the astronaut forward when it is fired backwards (remember Newton's third law of motion).

Worked example 2.1.1

APPLICATION OF NEWTON'S FIRST AND THIRD LAWS

A toddler drags a 4.5 kg cart of blocks across a floor at a constant speed of 0.75 m s^{-1}. It is being dragged by a handle which is at an angle of 35° above the horizontal. The force of friction between the cart and the floor is 5.0 N.

a Calculate the net force on the cart.	
Thinking	**Working**
The cart has constant speed, i.e. no acceleration. According to Newton's first law, the net force acting on the cart must be zero.	$F_{net} = 0\,N$

b Calculate the force that the toddler exerts on the cart.	
Thinking	**Working**
Draw a force diagram.	
If the net force is zero then the horizontal forces must be balanced. Therefore the horizontal component of the force on the cart by the toddler, F_{CT_x}, is equal to the magnitude of the frictional force, F_{CF}.	$F_{CT_x} = F_{CT} \cos 35° = F_{CF}$ $F_{CT} \cos 35° = 5.0N$ $F_{CT} = \dfrac{5.0}{\cos 35°} = 6.1N$ 35° above the horiztonal

c Determine the force that the cart exerts on the toddler.	
Thinking	**Working**
According to Newton's third law, the force on the cart by the toddler is equal and opposite to the force on the toddler by the cart. $F_{CT} = -F_{TC}$	Since the force on the cart is at an angle of 35° above the horizontal, the force of the cart on the toddler is 6.1 N at an angle of 35° below the horizontal.

Worked example: Try yourself 2.1.1

APPLICATION OF NEWTON'S FIRST AND THIRD LAWS

The toddler adds extra blocks to the cart and drags it across the floor more slowly. The 5.5 kg cart travels at a constant speed of 0.65 m s^{-1}. The force of friction between the cart and the floor is 5.2 N and the handle is now at an angle of 30° above the horizontal.

a Calculate the net force on the cart.

b Calculate the force that the toddler exerts on the cart.

c Determine the force that the cart exerts on the toddler.

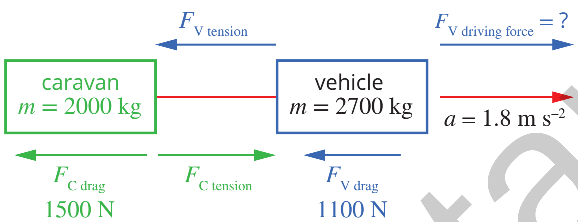
Applying Newton's first or second laws

When solving motion problems, a key strategy is to determine whether Newton's first law or second law should be applied. In the following examples, the objects in the questions are accelerating. Hence the second law should be used and the net force is proportional to the acceleration. In problems involving connected bodies, both the whole system and each component of the system have the same acceleration.

Worked example 2.1.2

APPLICATION OF NEWTON'S LAWS

A vehicle towing a caravan accelerates at 1.8 $m\,s^{-2}$ in order to overtake the car in front. The vehicle's mass is 2700 kg and the caravan's mass is 2000 kg. The drag force on the vehicle is 1100 N and the drag force on the caravan is 1500 N.

a Calculate the driving force of the engine.	
Thinking	**Working**
Draw a sketch showing all forces acting.	
Since there is an acceleration, Newton's second law can be applied to the whole system. Note that the caravan and vehicle are joined by a coupling and so the tension forces are not included at this stage. Consider the system as a whole.	$F_{\text{system}} = m_{\text{system}}a$ $F_{\text{V driving force}} - F_{\text{V drag}} - F_{\text{C drag}} = (m_V + m_C)a$ $F_{\text{V driving force}} - 1100 - 1500 = (2700 + 2000) \times 1.8$ $F_{\text{V driving force}} = 1.1 \times 10^4$ N in the direction of motion

b Calculate the magnitude of the tension in the coupling.	
Thinking	**Working**
Consider only one part of the system, for example the caravan, once again applying Newton's second law.	$F_{\text{C net}} = m_C a$ $F_{\text{C tension}} - F_{\text{C drag}} = m_C a$ $F_{\text{C tension}} = 2000 \times 1.8 + 1500$ $= 5.1 \times 10^3$ N

Worked example: Try yourself 2.1.2

APPLICATION OF NEWTON'S LAWS

A vehicle towing a trailer accelerates at 2.8 $m\,s^{-2}$ in order to overtake a car in front. The vehicle's mass is 2700 kg and the trailer's mass is 600 kg. The drag force on the vehicle is 1100 N and the drag force on the trailer is 500 N.

a Calculate the driving force of the engine.

b Calculate the magnitude of the tension in the coupling.

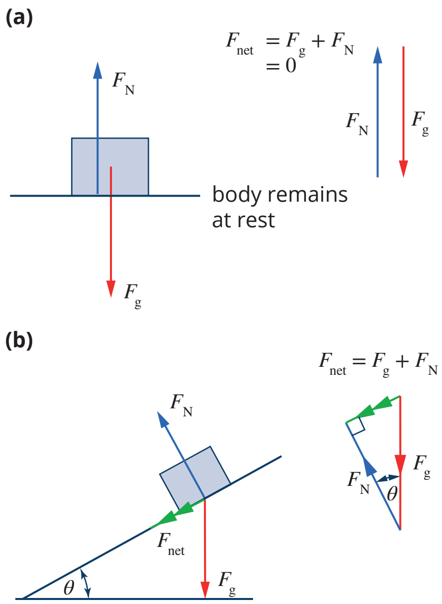


FIGURE 2.1.3 (a) A block on a level surface experiences a net force of zero, as F_N and F_g balance each other. (b) With the block on an incline, $F_N = F_g \cos \theta$, and the net force is given by $F_{net} = F_g + F_N$ added as vectors.

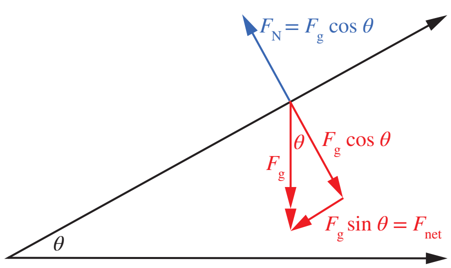


FIGURE 2.1.4 For a block on an incline, the force due to gravity can be resolved into a force perpendicular to the surface and a force parallel to the surface.

THE NORMAL FORCE AND INCLINED PLANES

One reaction force deserves a special mention. When an object exerts a force on a surface, the surface exerts a force back on the object that is at right angles (i.e. normal) to the surface. For example, the block in Figure 2.1.3(a) exerts a force on the level surface and the surface exerts a normal force back on the block. The force the block exerts on the surface is equal in size to the force due to gravity, F_g. Thus F_g is balanced by F_N, as shown in the figure. As there is no net force on the block, the object remains stationary.

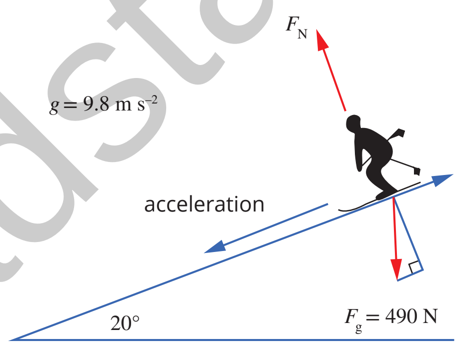
Consider an **inclined plane** (Figure 2.1.3(b)). The normal force is still at right angles to the surface. However, as the surface is not horizontal, F_N will be at an angle to F_g. There is a net force down the slope and the block accelerates, as predicted by Newton's second law.

Another way of viewing the forces along an inclined plane is to resolve the vector of the force due to gravity, F_g, into two components: one perpendicular to the slope and one parallel to the slope (Figure 2.1.4). The component perpendicular to the surface is balanced by the normal force F_N. The component of the force due to gravity that is parallel to the slope is the force that actually causes the acceleration.

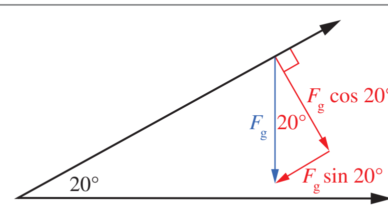
Worked example 2.1.3

INCLINED PLANES

A skier of mass 50 kg is skiing down an icy slope that is inclined at 20° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is $9.8\,\text{m}\,\text{s}^{-2}$.



a Determine the components of the force due to gravity on the skier perpendicular to the slope and parallel to the slope.

Thinking	Working
Draw a sketch and include the values provided.	
Resolve the force due to gravity into the component perpendicular to the slope.	The perpendicular component is: $F_\perp = F_g \cos 20°$ $= 490 \cos 20°$ $= 460$ $= 4.6 \times 10^2$ N

Resolve the force due to gravity into the component parallel to the slope.	The parallel component is: $F_{\parallel} = F_g \sin 20°$ $= 490 \sin 20°$ $= 168$ $= 1.7 \times 10^2$ N

b Determine the normal force that acts on the skier.	
Thinking	**Working**
The normal force is equal in magnitude to the perpendicular component of the force due to gravity.	$F_N = 4.6 \times 10^2$ N

c Calculate the acceleration of the skier down the slope.	
Thinking	**Working**
Apply Newton's second law. The net force along the plane is the component of the force due to gravity parallel to the slope.	$a = \frac{F_{net}}{m}$ $= \frac{168}{50}$ $= 3.4\,\text{m s}^{-2}$ down the slope

Worked example: Try yourself 2.1.3

INCLINED PLANES

A skier of mass 85 kg travels down the same icy slope inclined at 20° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is $9.8\,\text{m s}^{-2}$.

a	Determine the components of the force due to gravity on the skier perpendicular to the slope and parallel to the slope.
b	Determine the normal force that acts on the skier.
c	Calculate the acceleration of the skier down the slope.

Aside from rounding differences, the acceleration calculated in the Worked example and Try yourself questions above are equal. This is because acceleration is independent of the mass of the object (if we ignore friction forces). Mathematically, the relationship can be written as follows.

$$a = \frac{F_{net}}{m} = \frac{mg \sin\theta}{m} = g \sin\theta$$

where a is the acceleration of the object (m s^{-2})

g is the acceleration due to gravity (m s^{-2})

θ is the angle of the inclined plane from the horizontal

STRATEGIES FOR SOLVING FORCE AND MOTION PROBLEMS

Where forces on a body are given, Newton's laws can be applied. Two questions should be asked:

1 Is the object stationary or travelling at constant velocity? In these cases $F_{net} = 0$.
2 Is the object accelerating? In this case, $F_{net} = ma$.

When dealing with connected bodies, consider the whole system first, and then consider the separate parts of the system.

For coplanar forces that are not aligned (for example, on an inclined plane), resolve forces into their components.

Newton's second law can be used to find the acceleration of an object. It can then be used with the other equations of motion to find such quantities as displacement and final velocity.

2.1 Review

SUMMARY

- Newton's first law states that every object continues to be at rest, or continues with constant velocity, unless it experiences an unbalanced force. This is also called the law of inertia.
- Newton's second law states that the acceleration of a body experiencing an unbalanced force is directly proportional to the net force on the body and inversely proportional to the mass of the body: $F_{net} = ma$.
- Newton's third law states that when one body exerts a force on another body (an action force), the second body exerts an equal force on the first body but in the opposite direction (a reaction force): $F_{AB} = -F_{BA}$.
- The forces in an action–reaction pair are of the same magnitude, act in opposite directions and are exerted on two different objects.
- A normal force, F_N, acts between an object and a surface at right angles to the surface.
  - On a horizontal surface, $F_N = F_g$ and the object is stationary.
  - On an inclined surface, F_N is equal and opposite to the component of the force due to gravity acting perpendicular to the plane: $F_N = F_g \cos \theta$
- The net force (F_{net}) acting on an object on a plane inclined at an angle θ is $F_g \sin \theta$ (assuming that friction is negligible).

KEY QUESTIONS

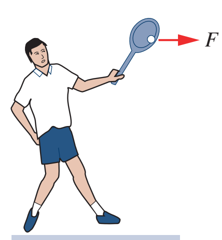
Knowledge and understanding

1 Phil is standing inside a tram when it starts off suddenly. Lisa, who is sitting down, comments that Phil was thrown backwards as the tram started moving. Is this a correct statement? Explain your answer in terms of Newton's laws.

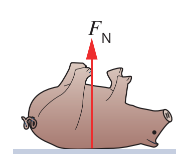
2 Consider an object of mass 5.3 kg sliding across a frictionless surface. What force is required to accelerate it at a rate of 2.2 m s–2?

3 On each of the following force diagrams, draw the reaction force that is the partner of the action force that is shown. For each force you draw, state what the force is acting on and what is providing the force.

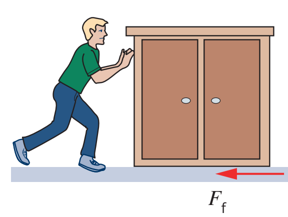
(a)



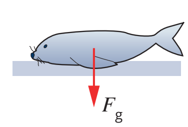
(b)



(c)



(d)



4 A table-tennis ball of mass 10 g is falling towards the ground at a constant speed of $8.2\,\text{m}\,\text{s}^{-1}$. Calculate the magnitude and direction of the force due to air resistance acting on the ball.

5 Ishtar is riding a motorised scooter along a level path. The combined mass of Ishtar and her scooter is 80.0 kg. The frictional and drag forces that are acting total 45.0 N. Determine the magnitude of the driving force provided by the motor under the following conditions.

- **a** Ishtar is moving at a constant speed of $10\,\text{m}\,\text{s}^{-1}$.
- **b** Ishtar is accelerating at $1.50\,\text{m}\,\text{s}^{-2}$.

6 A cyclist and his bike have a combined mass of 80 kg. When starting off from traffic lights, the cyclist accelerates uniformly and reaches a speed of $7.5\,\text{m}\,\text{s}^{-1}$ in 5.0 s.

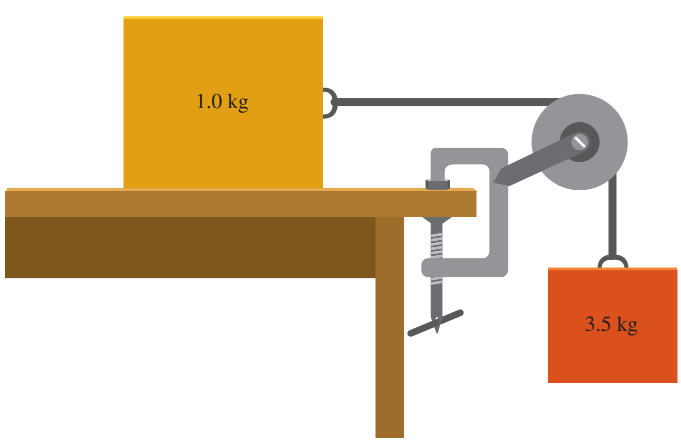
- **a** What is the acceleration during this time?
- **b** Calculate the driving force being provided by the cyclist's legs as he starts off. Assume that drag forces are negligible during this time.
- **c** The cyclist now rides at a constant speed of $15\,\text{m}\,\text{s}^{-1}$. If the force being provided by his legs is now 60 N, determine the magnitude of the drag forces that are acting.

7 During preseason football training, Matt was required to run dragging behind him a bag of sand of mass 50 kg. The bag was attached to a rope which made an angle of 25° to the horizontal. When Matt ran with a constant speed of $4.0\,\text{m}\,\text{s}^{-1}$, a frictional force of 60 N was acting on the bag.

- **a** What was the net force acting on the bag?
- **b** Calculate the size of the tension force that was acting in the rope.
- **c** What was the magnitude of the force the rope exerted on Matt as he ran?

Analysis

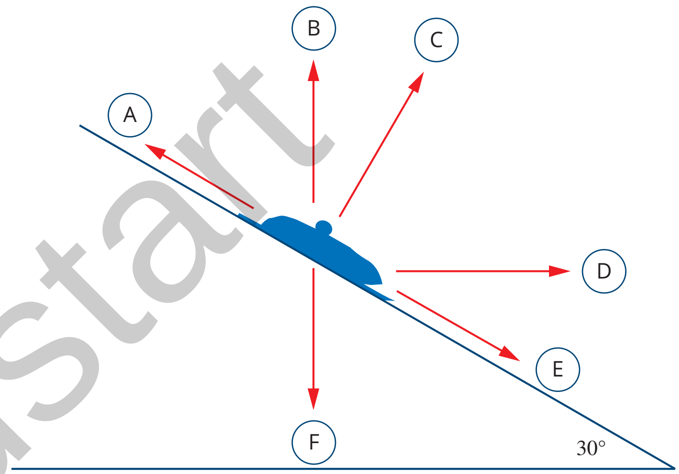
8 A block on a table is accelerated by a falling mass, as shown below. Calculate the acceleration of the blocks and the tension in the cord if the block on the table experiences a frictional force of 2.0 N as it slides along.



9 A 950 kg car is used to tow along a small trailer of mass 100 kg. The car and trailer have an acceleration of $0.800\,\text{m}\,\text{s}^{-2}$. The resistive forces acting on the car total 500 N. An additional 500 N of resistive forces act on the trailer.

- **a** Calculate the driving force required by the car's engine.
- **b** What tension exists in the tow rope between the car and trailer?

10 Kirsty is riding in a bobsled that is sliding down a snow-covered hill with a slope of 30° to the horizontal. The total mass of the sled and Kirsty is 100 kg. Initially the brakes are on and the sled moves down the hill with a constant velocity.



- **a** Which one of the arrows (A–F) best represents the direction of the frictional force acting on the sled?
- **b** Which one of the arrows (A–F) best represents the direction of the normal force acting on the sled?
- **c** Calculate the net frictional force acting on the sled.
- **d** Kirsty releases the brakes and the sled accelerates. What is the magnitude of her initial acceleration?
- **e** Kirsty returns to the top of the hill. A friend now joins her in the bobsled taking the total mass to 140 kg. The bobsled takes off down the same slope and with the brakes off (thus friction can be ignored). How will the extra mass affect the acceleration of the bobsled?

2.2 Circular motion in a horizontal plane

Circular motion is common throughout the universe. Children on a fairground ride (Figure 2.2.1) move in a circular path, and so do those in a car as it travels around a roundabout. In athletics, hammer throwers swing the hammer in a circular path before releasing it. On a much larger scale, the planets orbit the Sun in paths that are approximately circular. On an even grander scale, stars can travel in circular paths around the centres of their galaxies. This section explains the nature of circular motion in a horizontal plane and applies Newton's first and second laws to problems involving circular motion.

FIGURE 2.2.1 The people on this ride are travelling in a circular path.

UNIFORM CIRCULAR MOTION

In Figure 2.2.2, an athlete in a hammer-throw event swings a hammer—which is usually a steel ball—in a horizontal circle with a constant speed of $25\,\text{m}\,\text{s}^{-1}$. Although its speed is constant, its velocity is continually changing. This means that it is accelerating.

Remember that velocity is a vector. Since the direction of the hammer is changing, so too is its velocity, even though its speed is not changing. The velocity of the hammer at any instant is **tangential** (i.e. at a tangent) to its path. At one instant, the hammer is travelling at $25\,\text{m}\,\text{s}^{-1}$ north. An instant later it is travelling at $25\,\text{m}\,\text{s}^{-1}$ west, and then $25\,\text{m}\,\text{s}^{-1}$ south, and so on.

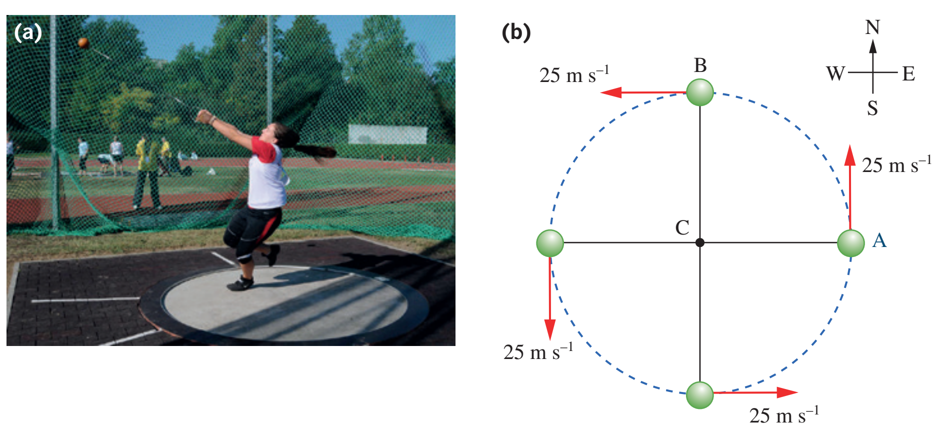


FIGURE 2.2.2 The velocity of the hammer at any instant is tangential to its path and is continually changing even though it has constant speed. Because its velocity is changing, the hammer is accelerating.

PERIOD AND FREQUENCY

Imagine that an object is moving in a circular path of radius r metres with a constant speed of v and takes T seconds to complete one revolution. The time taken to travel once around a circle is called the **period**, T, of the motion. The number of rotations each second is the **frequency**, f.

$$f = \frac{1}{T} \text{ and } T = \frac{1}{F}$$

where f is the frequency (Hz)
T is the period (s)

SPEED

An object that travels in a circle will travel a distance equal to the circumference of the circle, $2\pi r$, with each revolution (Figure 2.2.3). Given that the time for each revolution is the period, T, the average speed of the object is:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{T}$$

The average speed of an object moving in a circular path is:

$$v = \frac{2\pi r}{T}$$

where v is the speed of the object (m s^{-1})
r is the radius of the circle (m)
T is the period of motion (s)

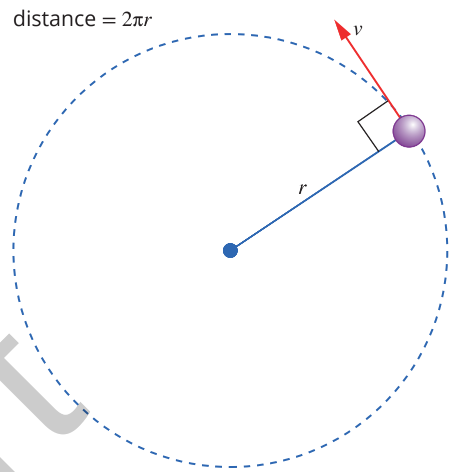


FIGURE 2.2.3 The average speed of an object moving in a circular path is given by the distance travelled in one revolution (the circumference, $2\pi r$) divided by the time taken (the period, T).

PHYSICSFILE

Wind generators

The wind generators in the figure below are part of a wind farm at Macarthur in south-west Victoria. The wind farm has 140 turbines and the towers are 85 m high. Each blade is 55 m long and can rotate at a maximum rate of 20 revolutions per minute. Although the blades are moving in a vertical, not horizontal, plane, their motion can be described using the same equation for circular motion given here. From the information given, you should be able to calculate that the tip of each blade is travelling at around 400 km h^{-1}.

The tips of these wind-generator blades travel in a circular path and can reach speeds of approximately 400 km h^{-1}.

Worked example 2.2.1

CALCULATING SPEED

A wind turbine has blades 55.0 m in length that rotate at a frequency of 20 revolutions per minute. At what speed do the tips of the blades travel? Express your answer in $km\,h^{-1}$.

Thinking	Working
Calculate the period, T. Remember to express frequency in the correct units. Alternatively, recognise that 20 revolutions in 60 seconds means that each revolution takes 3 seconds.	20 revolutions per minute $= \frac{20}{60} = 0.333\,Hz$ $T = \frac{1}{f}$ $= \frac{1}{0.333} = 3.0\,s$
Substitute r and T into the appropriate formula for speed and solve for v.	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 55.0}{3}$ $= 115.2\ m\,s^{-1}$ $= 1.15 \times 10^2\ m\,s^{-1}$
Convert $m\,s^{-1}$ into $km\,h^{-1}$ by multiplying by 3.6.	$115.2 \times 3.6 = 4.2 \times 10^2\,km\,h^{-1}$

Worked Example: Try yourself 2.2.1

CALCULATING SPEED

A water wheel has blades 2.0 m in length that rotate at a frequency of 10 revolutions per minute. At what speed do the tips of the blades travel? Express your answer in $km\,h^{-1}$.

CENTRIPETAL ACCELERATION

Since the velocity of an object travelling in a horizontal circle is changing, it is accelerating even though its speed is not changing. The object is continually deviating inwards from a straight-line direction and so has an acceleration towards the centre. This acceleration is known as **centripetal acceleration**. (The word 'centripetal' means to move towards a centre.)

In Figure 2.2.4, the velocity vector of an object travelling in a circular path is shown with an arrow labelled v, at a tangent to the circular path. The centripetal acceleration, a, is towards the centre of the circular path.

However, as Figure 2.2.5 shows, even though the object is accelerating towards the centre of the circle, it never gets any closer to the centre.

The centripetal acceleration, a, of an object moving in a circular path of radius r with a velocity v can be found from the relationship:

$$a = \frac{v^2}{r}$$

A substitution can be made in this equation for the speed of the object, which was found earlier to be:

$$v = \frac{2\pi r}{T}$$

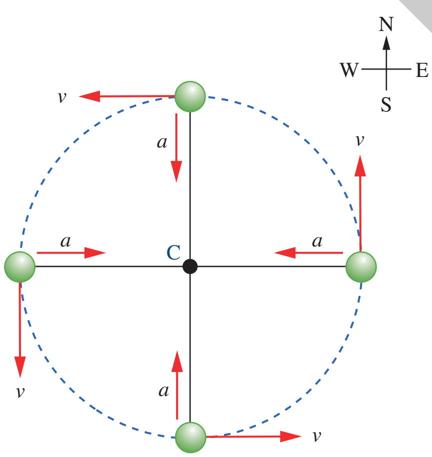


FIGURE 2.2.4 A body moving in a circular path has an acceleration towards the centre of the circle. This is known as centripetal acceleration.

Thus:

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \left(\frac{2\pi r}{T}\right)^2 \times \frac{1}{r} \\ &= \frac{4\pi^2 r}{T^2} \end{aligned}$$

Centripetal acceleration is always directed towards the centre of the circular path and is given by:

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

where a is the centripetal acceleration ($\mathrm{m\,s^{-2}}$)
v is the speed ($\mathrm{m\,s^{-1}}$)
r is the radius of the circle (m)
T is the period of motion (s)

FORCES THAT CAUSE CIRCULAR MOTION

As with all forms of motion, an analysis of the forces that are acting is needed to understand why circular motion occurs. In the hammer throw described earlier in this section, the hammer is continually accelerating. It follows from Newton's second law that there must be a net unbalanced force continuously acting on it. The net unbalanced force that gives the ball its acceleration towards the centre of the circle is known as a **centripetal force**.

In every case of circular motion, a real force is necessary to provide the centripetal force. The force acts in the same direction as the acceleration, that is, towards the centre of the circle. This centripetal force can be provided in a number of ways. For the hammer in Figure 2.2.5(a), the centripetal force is the tension force in the cable. Three other examples of centripetal force are shown in Figure 2.2.5.

Consider the consequences if the unbalanced force ceases to act. In the example of the hammer thrower, if the tension in the cable became zero—as happens when the thrower releases the hammer—there is no longer a force causing the hammer to change direction. The result is that the hammer moves in a straight line tangential to its circular path, as would be expected from Newton's first law.

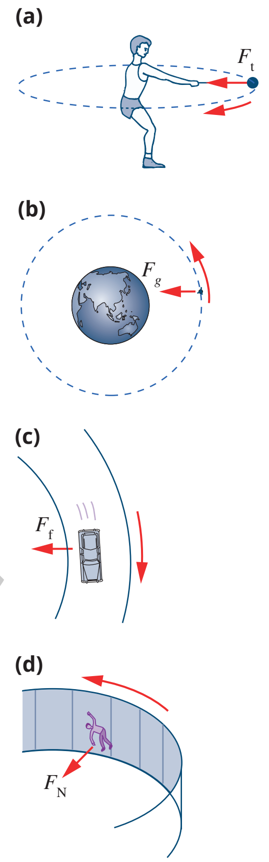


FIGURE 2.2.5 (a) In a hammer throw, tension in the cable provides the centripetal force. (b) For planets and satellites, the gravitational attraction towards the central body provides the centripetal force. (c) For a car on a curved road, the friction between the tyres and the road provides the centripetal force. (d) For a person in a Gravitron ride, it is the normal force from the wall that provides the centripetal force.

Centripetal force is given by:

$$F_{\text{net}} = ma = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$$

where F_{net} is the net or centripetal force on the object (N)
m is the mass (kg)
a is the acceleration ($\mathrm{m\,s^{-2}}$)
v is the speed ($\mathrm{m\,s^{-1}}$)
r is the radius of the circle (m)
T is the period of motion (s)

Worked example 2.2.2

CENTRIPETAL FORCES

An athlete in a hammer-throw event is swinging a metal ball of mass 7.0 kg in a horizontal circular path of radius 60 m. The ball is moving at 20.0 m s^{-1}.

a Calculate the magnitude of the acceleration of the ball.	
Thinking	**Working**
As the object is moving in a circular path, the centripetal acceleration is towards the centre of the circle. To find the magnitude of this acceleration, write down the variables that are given.	$v = 20.0\ \text{m s}^{-1}$ $r = 1.60\ \text{m}$ $a = ?$
Select the equation for centripetal acceleration that fits the values you have, and substitute the values.	$a = \dfrac{v^2}{r}$ $= \dfrac{20.0^2}{1.60}$ $= 250\ \text{m s}^{-2}$
Only the magnitude is required, so no direction is needed in the answer.	The acceleration of the ball is 250 m s^{-2}.

b Calculate the magnitude of the tensile force (tension) acting in the wire used to swing the ball.	
Thinking	**Working**
Identify the unbalanced force that is causing the object to move in a circular path. Write down the information that you are given.	$m = 7.0\ \text{kg}$ $a = 250\ \text{m s}^{-2}$ $F_{net} = ?$
Select the appropriate equation for centripetal force and substitute the variables you have.	$F_{net} = ma$ $= 7.0 \times 250$ $= 1750$ $= 1.8 \times 10^3\ \text{N}$
State the magnitude only as no direction is required.	The force of tension in the wire is the unbalanced force that is causing the ball to accelerate. Tensile force $F_t = 1.8 \times 10^3\ \text{N}$

Worked example: Try yourself 2.2.2

CENTRIPETAL FORCES

An athlete in a hammer-throw event is swinging a ball of mass 7.0 kg in a horizontal circular path of radius 20 m. The ball is moving at 25.0 m s^{-1}.

a Calculate the magnitude of the acceleration of the ball.

b Calculate the magnitude of the tensile force (tension) acting in the wire.

CASE STUDY

The Gravitron

When a car turns sharply to the left, the passengers in the car seem to sway to the right. Many mistakenly think that a force to the right is acting. In fact, the passengers are simply maintaining their motion in the original direction of the car, as described by Newton's first law, that is, they are experiencing inertia. If the passengers are (unwisely) not wearing seatbelts, they may be squashed against the right-hand door as the car turns. This will exert a large force to the left on them, which causes them to move to the left.

People moving rapidly in circular paths might also mistakenly think that there is an outward force acting on them. In order for the physics to be explained, it is necessary to use different frames of reference. For example, riders on the Gravitron (also known as the Vortex or Rotor), like those in Figure 2.2.6, will feel a force pushing them into the wall. This outwards force is commonly known as a centrifugal force. (centrifugal means 'centre-fleeing'.) This force does not actually exist in their frame of reference. The riders think that it does because they are in a rotating frame of reference. From outside the Gravitron, it is evident that there is an inwards force (the normal force) that is holding them in a circular path. If the walls disintegrated and this normal force ceased to act, they would not fly outwards, but move at a tangent to their circle.

A Gravitron can rotate at 24 rpm with a radius of 7 m. The centripetal acceleration can be over 40 m s^{-2}. This is caused by a very large centripetal force from the wall i.e. the normal force, F_N. In the vertical direction, F_g is balanced by an upwards frictional force, F_f, so the riders experience no vertical motion even if the floor then drops away. It is important to remember that there is no force acting outwards. In fact, as you can see in Figure 2.2.7, the forces are unbalanced and the net force is equal in size and direction to the normal force towards the centre of the circle.

FIGURE 2.2.6 There is a large inwards force from the wall (a normal force) that causes these Gravitron riders to travel in a circular path.

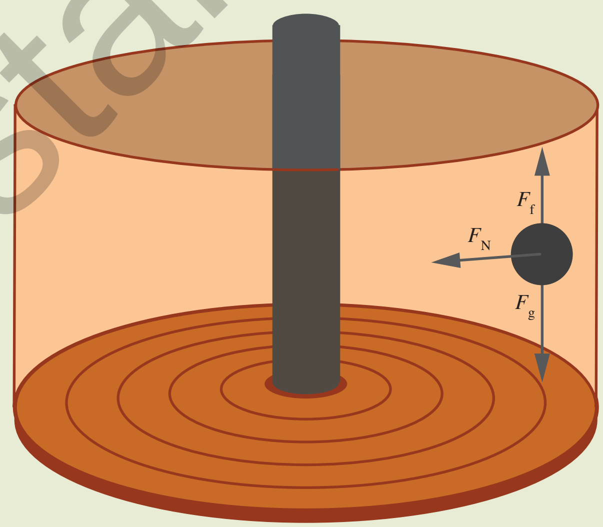


FIGURE 2.2.7 There are three forces acting on the rider in a Gravitron. Vertically, the forces are balanced and so no motion occurs in this direction. The remaining force, F_N, provides the net (centripetal) force to the centre of the ride.

BALL ON A STRING

You may have played totem tennis. This is a game where a ball is attached to a pole by a string and can travel in a horizontal circle, although the string itself is not horizontal (Figure 2.2.8).

If the ball at the end of the string is swinging slowly, the string swings down at an angle closer to the pole. If the ball was swung faster, the string would become closer to being horizontal. In fact, it is not possible for the string to be absolutely horizontal, although as the speed increases, the closer to horizontal it becomes. This system is known as a conical pendulum.

If the angle of the conical pendulum is known, trigonometry can be used to find the radius of the circle and the forces involved.

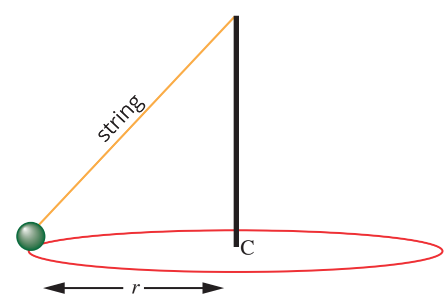
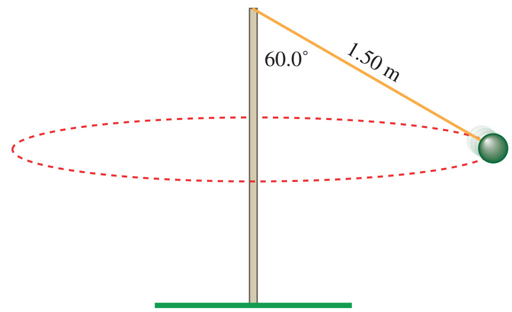


FIGURE 2.2.8 This ball is travelling in a horizontal circular path of radius r. The centre of its circular motion is at C.

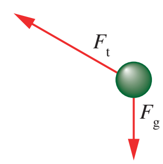
Worked example 2.2.3

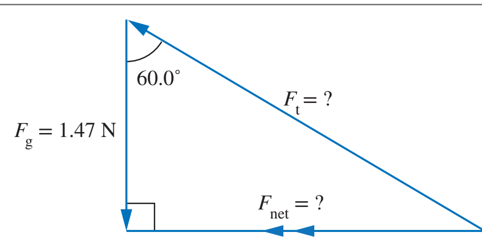
OBJECT ROTATING ON THE END OF A STRING

During a game of totem tennis, a ball of mass 150 g is swinging freely in a horizontal circular path. The cord is 1.50 m long and at an angle of 60.0° to the vertical.



a Calculate the radius of the ball's circular path.	
Thinking	**Working**
The radius of the circular path and the pole form a right angle. Use trigonometry to find the radius.	$r = 1.50 \sin 60.0° = 1.30$ m

b Draw and label the forces that are acting on the ball at the instant shown in the diagram.	
Thinking	**Working**
There are two forces acting: the tension in the cord, F_t, and the force due to gravity, F_g. These forces are unbalanced.	

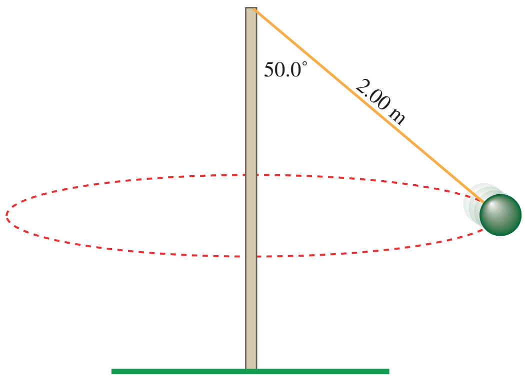
c Determine the net force that is acting on the ball at this time.	
Thinking	**Working**
First calculate the force due to gravity, F_g.	$F_g = mg$ $= 0.150 \times 9.8$ $= 1.47$ N
The ball has an acceleration that is towards the centre of its circular path. This is horizontal and acts towards the left at this instant. The net force will also be acting in this direction. A force triangle and trigonometry can be used to determine the net force.	 $F_{net} = 1.47 \tan 60.0°$ $= 2.55$ N towards the centre of the circular path

d Calculate the size of the tensile force in the cord.	
Thinking	**Working**
Use trigonometry to find F_t.	$F_t = \dfrac{1.47}{\cos 60.0°}$ $= 2.94\,\text{N}$

Worked example: Try yourself 2.2.3

OBJECT ROTATING ON THE END OF A STRING

During a game of totem tennis, a ball of mass 200 g is swinging freely in a horizontal circular path. The cord is 2.00 m long and at an angle of 50.0° to the vertical.



a	Calculate the radius of the ball's circular path.

b	Draw and label the forces that are acting on the ball at the instant shown in the diagram.

c	Determine the net force that is acting on the ball at this time.

d	Calculate the size of the tensile force in the cord.

2.2 Review

SUMMARY

- Frequency, f, is the number of revolutions each second and is measured in hertz (Hz).
- Period, T, is the time for one revolution and is measured in seconds.
- The relationship between T and f is:

$$f = \frac{1}{T} \text{ and } T = \frac{1}{f}$$

- An object moving with a uniform speed in a circular path of radius r and with period T has an average speed given by:

$$v = \frac{2\pi r}{T}$$

- The velocity of an object moving with a constant speed in a circular path is continuously changing. The velocity vector is always directed at a tangent to the circular path.
- An object moving in a circular path with a constant speed has an acceleration due to its circular motion. This acceleration is directed towards the centre of the circular path and is called centripetal acceleration, a, where:

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

- Centripetal acceleration is a consequence of a centripetal force acting to make an object move in a circular path.
- A centripetal force is directed towards the centre of the circle and its magnitude can be calculated using Newton's second law:

$$F_{net} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$$

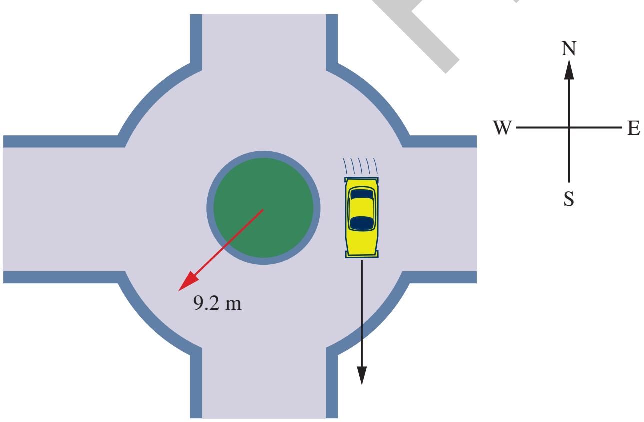
- A centripetal force is always supplied by a real force the nature of which depends on the situation. The real force is commonly friction, gravitation or the tension in a string or cable.

KEY QUESTIONS

Knowledge and understanding

The following information relates to questions 1–5.

A car of mass 1200 kg is travelling in a roundabout in a circular path of radius 9.2 m. The car moves with a constant speed of 8.0 m s^{-1}. The direction of the car is clockwise around the roundabout when viewed from above.



1 Which two of the following statements correctly describe the motion of the car as it travels around the roundabout?

A It has a constant speed.
B It has a constant velocity.
C It has zero acceleration.
D It has an acceleration that is directed towards the centre of the roundabout.

2 When the car is in the position shown in the diagram, what is the:

a speed of the car
b velocity of the car
c magnitude and direction of the acceleration of the car?

3 Calculate the magnitude and direction of the net force acting on the car at the position shown.

4 When the car has travelled halfway around the roundabout, what is the:

a velocity of the car at this point
b direction of its acceleration at this point?

5 If the driver of the car kept speeding up, what would eventually happen to the car as it travelled around the roundabout? Explain your answer.

6 An ice skater of mass 75 kg is skating in a horizontal circle of radius 2.5 m at a constant speed of $1.5\,m\,s^{-1}$.

a Determine the magnitude of the skater's acceleration.

b Are the forces acting on the skater balanced or unbalanced? Explain your answer.

c Calculate the magnitude of the centripetal force acting on the skater.

7 A 1.5 kg ball is made to swing in a horizontal circle of radius 1.2 m at 2.5 revolutions per second.

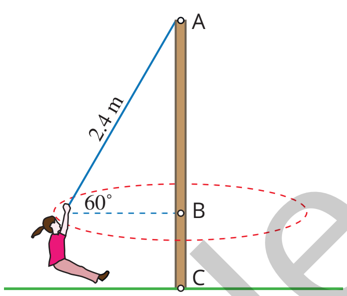
a What is the period of rotation of the ball?

b What is the orbital speed of the ball?

c What is the magnitude of the acceleration of the ball?

d What is the magnitude of the net force acting on the ball?

8 A child of mass 30.0 kg is playing on a maypole swing in a playground. The rope is 2.40 m long and at an angle of 60.0° to the horizontal as she swings freely in a circular path. In answering the following questions, ignore the mass of the rope in your calculations.



a Calculate the radius of the child's circular path.

b Identify the forces that are acting on her as she swings freely.

c What is the direction of her acceleration when she is at the position shown in the diagram?

d Calculate the net force acting on the girl.

e What is her speed as she swings?

Analysis

9 A car with a mass of 1500 kg is travelling around a circular curve of radius 30 m at a constant speed of $25\,m\,s^{-1}$.

a Calculate the centripetal force required for it to round the curve.

b What provides the centripetal force?

c Why do the passengers in the car slide towards the outside of the car when the car rounds the curve?

d If there is any ice or oil on the road, the friction between the car tyres and the road is reduced. If the car is travelling too fast for the turn, what path will the car take?

2.3 Circular motion on banked tracks

FIGURE 2.3.1 Australia's Paige Greco, a Paralympic cyclist who won a gold medal at the 2020 Tokyo Paralympics in the 3000 m Individual Pursuit C1–3

The previous section considered relatively simple situations involving uniform circular motion in a horizontal plane. However, there are more complex situations involving circular motion. On many road bends, the road is not horizontal, but at a small angle to the horizontal. This enables vehicles to maintain their speed without skidding. A similar situation is at a cycling velodrome (Figure 2.3.1). The velodrome at the Darebin International Sports Centre in Thornbury has banked or inclined corners that peak at 43°. This enables cyclists to travel at much higher speeds than if the track were flat. This section examines the principles underlying banked cornering and applies Newton's laws to problems involving circular motion on banked tracks.

BANKED TRACKS

Cars and bikes can travel much faster around corners when the road or track surface is inclined or banked at an angle to the horizontal. Banked tracks are used at cycling velodromes and certain motor sport events, such as NASCAR races. Road engineers design roads to be banked in places where there are sharp corners, such as exit ramps from freeways.

When cars travel in circular paths on horizontal roads, they are relying on the force of friction between the tyres and the road to provide the sideways force that keeps the car in its circular path.

Consider a car travelling clockwise around a horizontal roundabout at a constant speed, v (Figure 2.3.2). The car has an acceleration towards C (the centre of the circle) and so the net force is also sideways on the car towards C. The vertical forces (gravity and the normal force) are balanced (Figure 2.3.3). The only horizontal force is the sideways force that the road exerts on the car tyres. This is a force of friction, F_f. It is unbalanced and so must equal the net force, F_{net}.

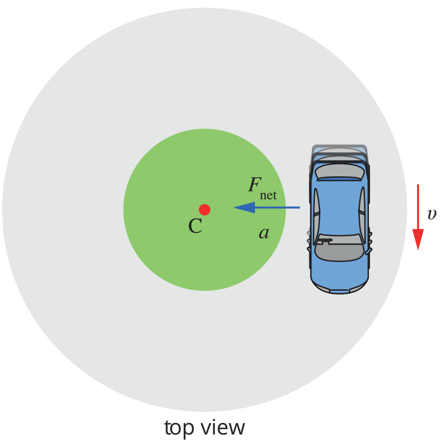


FIGURE 2.3.2 A car travelling in a circular path on a horizontal track

If the car drove over an icy patch, there would be no friction and the car would not be able to turn. It would skid in a straight line at a tangent to the circular path.

Creating a **banked track** by angling the road reduces the need for a sideways frictional force and allows cars to travel faster without skidding off the road and away from the circular path. Consider the same car travelling around a circular, banked road at constant speed, v (Figure 2.3.4). It is possible for the car to travel at a speed so that there is no need for a sideways frictional force. This is called the **design speed** and it is dependent on the angle, θ at which the road is banked. At this speed, the car exhibits no tendency to drift higher or lower on the road.

The car still has an acceleration towards the centre of the circle, C, and so there must be an unbalanced force in this direction. Due to the banking, there are now only two forces acting on the car: its force due to gravity, F_g, and the normal force, F_N, from the road. As can be seen in Figure 2.3.4(b), these forces are unbalanced. They add together to give a net force that is horizontal and directed towards C (Figure 2.3.4(c)).

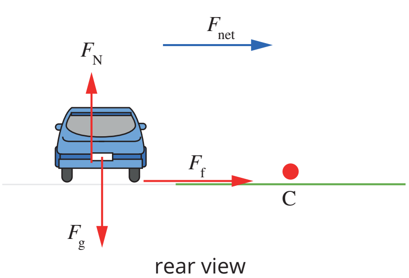


FIGURE 2.3.3 The vertical forces are in balance. It is friction between the tyres and the road that enables the car to turn.

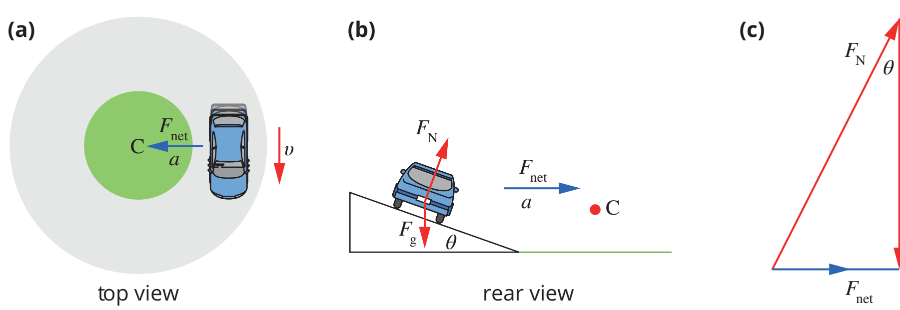


FIGURE 2.3.4 (a) A car is travelling in a circular path on a banked road. (b) The acceleration and net force are towards the centre of the path, C. The banked road means that the normal force (F_N) has an inwards component. This is what enables the car to turn the corner. (c) Vector addition gives the net force (F_{net}) acting horizontally towards the centre.

Banking angle

From Figure 2.3.4(c), it can be seen that the banking angle, θ, at the design speed of a road or track can be found by trigonometry:

$$\tan\theta = \frac{F_{net}}{F_g}$$

where F_{net} is the force acting towards the centre of the circle (N)

F_g is the force due to gravity on the object (N).

Substituting $F_{net} = \frac{mv^2}{r}$ and $F_g = mg$ into the equation and simplifying gives the following equation.

$$\tan\theta = \frac{v^2}{rg}$$

$$\therefore \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

where v is the speed of the vehicle ($m\,s^{-1}$)

r is the radius of the track (m)

θ is the banking angle (degrees)

g is the acceleration due to gravity ($9.8\,m\,s^{-2}$ near the surface of the Earth)

Thus if the banking angle is known, trigonometry can be used to calculate the design speed. Rearranging $\tan\theta = \frac{v^2}{rg}$ gives the following equation for the design speed, v.

$$v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$

Note that the normal force on an object will be larger when it travels on a banked track than when it travels on a flat track. For example, the cyclist in Figure 2.3.5 would feel a larger force acting from the road when she is on a banked track than when she is cycling on a flat track.

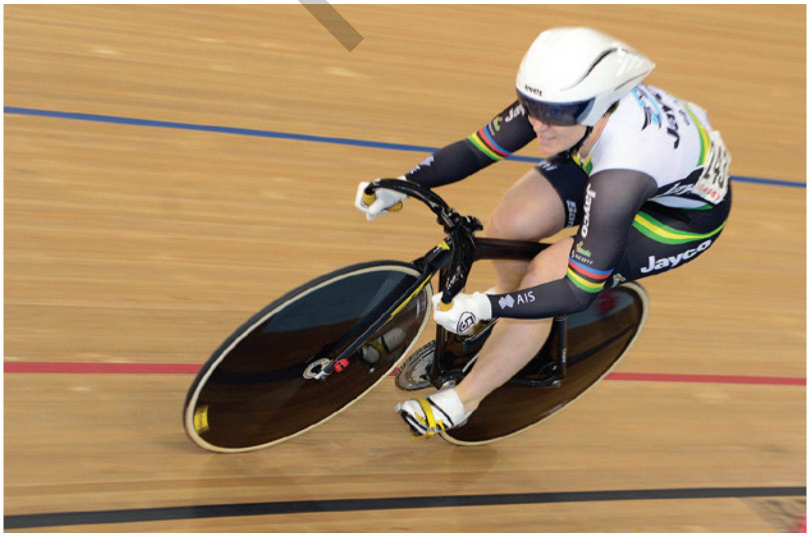


FIGURE 2.3.5 Australian cyclist Anna Meares on a banked velodrome track is cornering at speeds far higher than she could on a flat track. Cyclists on a velodrome will have a greater normal force from the track than cyclists on a flat track.

PHYSICSFILE

Inclined planes vs banked tracks

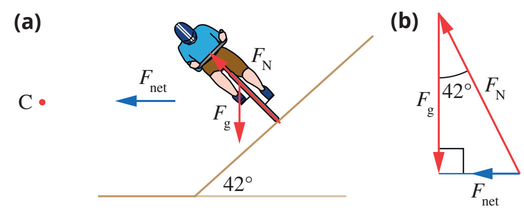
It is easy to confuse problems involving inclined planes with those involving banked tracks. Inclined-plane problems involve static objects overcoming the coefficient of static friction to slide down the plane. Banked-track problems involve an object moving in uniform circular motion on an inclined plane.

For banked-track problems, it is helpful to consider the vector components of the normal force, F_N, when completing calculations. For inclined-plane problems, components of the force due to gravity, F_g, are used.

Worked example 2.3.1

BANKED TRACKS

A curved section of track on an Olympic velodrome has a radius of 50 m and is banked at an angle of 42° to the horizontal. A cyclist of mass 75 kg is riding on this section of track at the design speed. Assume that g is 9.8 m s^{-1}.

a Calculate the net force acting on the cyclist at this instant.	
Thinking	**Working**
Draw a force diagram and include all forces acting on the cyclist. These forces are the force due to gravity and the normal force from the track, and these are unbalanced. The net force is horizontal and towards the centre of the circular track, as shown in diagram (a) and the force triangle of diagram (b).	
Calculate the force due to gravity, F_g.	$F_g = mg$ $= 75 \times 9.8$ $= 735$ N
Use the force triangle and trigonometry to calculate the net force, F_{net}.	$\tan\theta = \frac{F_{net}}{F_g}$ $\tan 42° = \frac{F_{net}}{735}$ $F_{net} = 0.90 \times 735$ $= 662$ N
As force is a vector, a direction is needed in the answer.	The net force is 6.6×10^2 N horizontally towards the centre of the circle.

b Calculate the design speed for this section of the track.	
Thinking	**Working**
List the relevant values.	$g = 9.8$ m s^{-2} $r = 50$ m $\theta = 42°$ $v = ?$
Use the design speed formula.	$v = \sqrt{rg\tan\theta}$ $= \sqrt{50 \times 9.8 \times \tan 42°}$ $= 21$ m s^{-1}

Worked example: Try yourself 2.3.1

BANKED TRACKS

A curved section of track on an Olympic velodrome has radius of 40 m and is banked at an angle of 37° to the horizontal. A cyclist of mass 80 kg is riding on this section of track at the design speed. Assume that g is 9.8 m s^{-1}.

a Calculate the net force acting on the cyclist at this instant.

b Calculate the design speed for this section of the track.

Worked example 2.3.2

FINDING THE BANKING ANGLE

The curved portion of a highway needs to be banked to prevent cars from skidding off it. Assume that the banked track of the highway is designed for a top vehicle speed of 80 km h^{-1} (i.e. the maximum speed limit for this portion of the highway is 80 km h^{-1}). The banked track portion of the highway has a radius of 500 m.

What is the value of the banking angle, θ, such that the forces acting on a car keep it on the highway without the need for friction? Assume g is 9.8 m s^{-2}.

Thinking	Working
Recall the formula for finding the banking angle.	$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$
Convert the design speed from km h^{-1} to m s^{-1}.	$v = \frac{80 \text{ km h}^{-1}}{3.6}$ $= 22.2 \text{ m s}^{-1}$
Calculate the angle.	$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ $= \tan^{-1}\left(\frac{22.2^2}{500 \times 9.8}\right)$ $= 5.7°$

Worked example: Try yourself 2.3.2

FINDING THE BANKING ANGLE

The curved portion of a highway needs to be banked to prevent cars from skidding off it. Assume that the banked track of the highway is designed for a top vehicle speed of 110 km h^{-1}. The banked track portion of the highway has a radius of 750 m.

What is the value of the banking angle, θ, such that the forces keep the car on the highway without the need for friction? Assume that g is 9.8 m s^{-1}.

PHYSICSFILE

Wall of Death

In some amusement parks around the world, there is a ride known menacingly as the Wall of Death (see the figure below). It consists of a cylindrical enclosure with vertical walls. People on bicycles and motorbikes ride into the enclosure and around the vertical walls, so the angle of banking is 90°! The riders need to keep moving and are depending on friction to hold them up. By travelling fast, the centripetal force (the normal force from the wall) is large and this increases the size of the grip (i.e. friction) between the wall and tyres. If the rider slammed on the brakes and stopped, they would simply plummet to the ground.

For a rider to successfully conquer the Wall of Death, they need to travel fast and there must be a good grip between the tyres and the track. The rider is relying on friction to maintain their motion along the wall.

2.3 Review

SUMMARY

- A banked track is one where the track is inclined at an angle to the horizontal. This enables vehicles to travel at higher speeds when cornering than when travelling around a flat curved path.
- Banking a track eliminates the need for a sideways frictional force to make a turn. When the speed and angle are such that there is no sideways frictional force, the speed is known as the design speed.
- The forces acting on a vehicle travelling at the design speed on a banked track are gravity and the normal force from the track. These forces are unbalanced and add to give a net force directed towards the centre of the circular motion.
- At the design speed, the banking angle of the track is given by:

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

- For a given banking angle and curve radius, the design speed is given by:

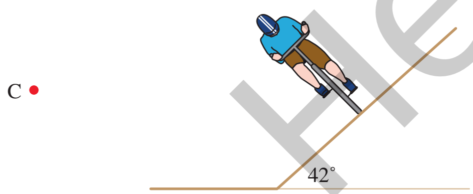
$$v = \sqrt{rg\tan\theta}$$

KEY QUESTIONS

Knowledge and understanding

1 A cyclist is riding along a circular section of a velodrome where the radius is 30 m and the track is inclined at 30° to the horizontal. The cyclist is riding at the design speed and maintains a constant speed. Describe the direction of the acceleration on the cyclist.

2 Copy the following diagram and then draw on it the normal force, the force due to gravity and the net force acting on the bicycle. Label each force.



3 The net force acting on an 80 kg bike racing around a banked track is 780 N. What is the banked angle of the track, given that the bike is racing at the design speed?

4 A cycling velodrome has a turn that is banked at 25° to the horizontal. The radius of the track at this point is 35 m.

- **a** Determine the speed (in km h^{-1}) at which a cyclist of mass 75 kg would experience no sideways force as they ride on this section of the track.
- **b** Calculate the size of the normal force acting on the cyclist.
- **c** How would this compare with the normal force if they were riding on a flat track?

5 A car-racing track is banked so that when the cars corner at 55 m s^{-1}, they experience no sideways frictional forces. The track is circular with a radius of 275 m. Calculate the angle to the horizontal at which the track is banked.

6 A curved portion of a highway with a speed limit of 90 km h^{-1} needs to be banked to prevent cars from skidding off it. The curved portion has a radius of 450 m. What is the value of the banking angle that will keep a car travelling at the speed limit on that portion of the highway without the need of friction?

Analysis

7 An architect is designing a velodrome and the original plans have semi-circular sections of radius 15 m and a banking angle of 30°. The architect is asked to make changes to the plans that will increase the design speed for the velodrome. What two design elements could the architect change in order to meet this requirement?

2.4 Circular motion in a vertical plane

Just as a body moving with constant speed in a horizontal circular path has an acceleration that is directed towards the centre of the circle, so does a body moving in a vertical circular path.

If you have been on a rollercoaster ride you will have travelled over humps and down dips at high speeds and, at times, in circular arcs. Some rides even have 360° circular tracks that are entirely vertical (Figure 2.4.1). During these rides, your body may experience forces that you find unpleasant.

When you travel on a rollercoaster, you can experience quite strong forces pushing you down into the seat as you fly through the dips. Then, as you travel over the humps, you tend to lift off your seat. These forces will be discussed in this section. As in the previous sections, Newton's laws are used to solve problems involving this type of circular motion.

FIGURE 2.4.1 This rollercoaster has a circular path in a vertical plane.

MOVING IN VERTICAL CIRCLES

A body moving with constant speed in a horizontal circular path has an acceleration that is directed towards the centre of the circle. The same applies for vertical circular paths. However, circular motion in a vertical plane in real life is often more complex, as it does not usually involve constant speeds.

An example is illustrated in Figure 2.4.2(a). The speed of the skateboarder practising in a half-pipe will increase on the way down as gravitational potential energy is converted into kinetic energy. This means that the skater will experience linear acceleration, a_l, as well as centripetal acceleration, a_c. The resultant acceleration is not directed towards the centre of the circular path.

At the bottom of the half-pipe, the skateboarder will be neither slowing down nor speeding up, so the acceleration is entirely centripetal at this point (Figure 2.4.2(b)). The same applies at the very top of a circular path. For this reason, motion at these points is easier to analyse.

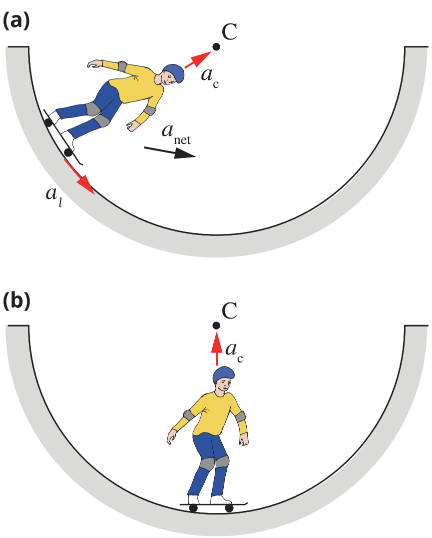


FIGURE 2.4.2 (a) When coming down the sides of a half-pipe, the skateboarder speeds up, and so has both a linear and a centripetal acceleration. The net acceleration, a_{net}, is not towards the centre, C. (b) At the lowest point, the velocity of the skateboarder is momentarily constant, so there is no linear acceleration. The acceleration is supplied completely by the centripetal acceleration, a_c, which is acting towards C.

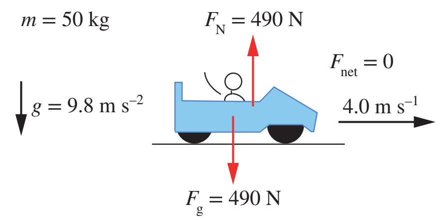


FIGURE 2.4.3 The vertical forces are in balance in this situation, i.e. $F_N = F_g$.

Uniform horizontal motion

Theme park rides make you appreciate that the forces you experience throughout a ride can vary greatly. First, consider the case of a person in a rollercoaster cart, like that shown in Figure 2.4.3, travelling horizontally at $4.0\,m\,s^{-1}$. If the person's mass is 50 kg and the gravitational field strength is $9.8\,m\,s^{-2}$, the forces acting on the person can be easily calculated. These forces are the gravitational force, F_g, and the normal force, F_N, from the seat.

The person is moving in a straight line with a constant speed, so there are no unbalanced forces acting. The force due to gravity balances the normal force from the seat. The normal force is therefore 490 N up, which is what usually acts upwards on the person when moving horizontally. Hence they would feel the same as their usual force due to gravity.

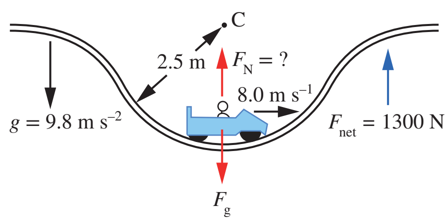


FIGURE 2.4.4 The person has a centripetal acceleration that is directed upwards towards the centre of the circle, and so the net force is also upwards. In this case, the magnitude of the normal force, F_N, is greater than the force due to gravity, F_g, and produces a situation where the rider feels heavier than usual.

Circular motion: travelling through dips

Now consider the forces that act on the person as the cart reaches the bottom of a circular dip. Suppose that the dip has a radius of 2.5 m and the cart is moving at $8.0\,m\,s^{-1}$ (Figure 2.4.4).

The person will have a centripetal acceleration due to the circular path. This centripetal acceleration is directed towards the centre, C, of the circular path—in this case, vertically upwards. The person's centripetal acceleration, a, is:

$$
\begin{aligned}
a &= \frac{v^2}{r} \\
&= \frac{8.0^2}{2.5} \\
&= 26\,\text{m s}^{-2} \text{ upwards towards C}
\end{aligned}
$$

The net centripetal force acting on the person is given by:

$$
\begin{aligned}
F_{net} &= ma \\
&= 50 \times 26 \\
&= 1300 \text{ N upwards}
\end{aligned}
$$

The normal force, F_N, and the force due to gravity, F_g, are no longer in balance. They add together to give an upwards force of 1300 N. This indicates that the normal force must be greater than the force due to gravity by 1300 N. In other words, the normal force is 490 N + 1300 N = 1790 N up. This is more than three times greater than the normal force of 490 N that usually acts on a person of mass 50 kg. That is the reason why, when in a ride, you feel the seat pushing up against you much more strongly at this point and you feel much heavier than usual.

Circular motion: travelling over humps

Now consider the situation as the cart moves over the top of a hump of radius 2.5 m with a lower speed of $2.0\,m\,s^{-1}$ (Figure 2.4.5).

The person now has a centripetal acceleration that is directed vertically downwards towards the centre of the circle, C. Therefore the net force acting at this point is directed vertically downwards. The centripetal acceleration is:

$$
\begin{aligned}
a &= \frac{v^2}{r} \\
&= \frac{2.0^2}{2.5} \\
&= 1.6\,\text{m s}^{-2} \text{ downwards towards C}
\end{aligned}
$$

The net centripetal force is:

$$
\begin{aligned}
F_{net} &= ma \\
&= 50 \times 1.6 \\
&= 80\,\text{N downwards}
\end{aligned}
$$

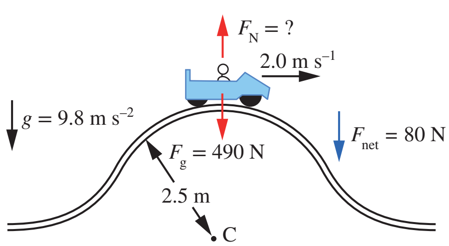


FIGURE 2.4.5 The centripetal acceleration is downwards towards the centre of the circle, and so the net force is also in that direction. At this point, the magnitude of the normal force, F_N, is less than the force on the person due to gravity, F_g.

As in the dip, the force due to gravity and the normal force are not in balance. They add to give a net force of 80 N down. The force due to gravity, F_g, must therefore be 80 N greater than the normal force, F_N. This tells us that the normal force is 490 N − 80 N = 410 N up. This explains why you feel lighter when travelling over a hump.

Circular motion: travelling through loops

You might have been on a rollercoaster like the one in Figure 2.4.6 where you were upside down at times during the ride. The speed of these rides and the radius of their circular path is what prevents riders from falling out. In theory, the safety harness worn by a rider is not needed to hold them in their seats.

FIGURE 2.4.6 The thrill seekers on this rollercoaster ride don't fall out when upside down because the centripetal acceleration of their cart is greater than $9.8\,\text{m}\,\text{s}^{-2}$ down.

The reason people don't fall out of the rollercoaster is that their centripetal acceleration is greater than the acceleration due to gravity ($9.8\,\text{m}\,\text{s}^{-2}$). To illustrate this, try the following activity. Extend a hand palm up, place an eraser on your palm then turn your hand over and move it rapidly towards the floor. You should find that it is possible to keep the eraser in contact with your hand as you move your hand down. The eraser is under your hand but it is not falling out of your hand. Your hand must be, for a short time, moving downwards with an acceleration in excess of $9.8\,\text{m}\,\text{s}^{-2}$ and continually exerting a normal force on the eraser. If your hand had an acceleration less than $9.8\,\text{m}\,\text{s}^{-2}$, the eraser would fall away from your hand to the floor.

A similar principle holds with rollercoaster rides. The people on the ride don't fall out at the top because the motion of the rollercoaster gives them a centripetal acceleration that is greater than $9.8\,\text{m}\,\text{s}^{-2}$ down. The engineers who designed the ride would have ensured that the rollercoaster can move with sufficient speed and in a circle of an appropriate radius so that this happens.

To explore this further, consider a rollercoaster ride of radius 15 m in a vertical circle (Figure 2.4.7).

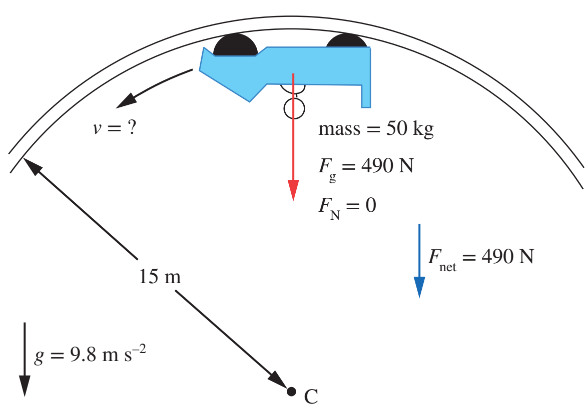


FIGURE 2.4.7 A rollercoaster cart travelling upside down through a loop. At the critical point where the cart just stays in contact with the track, the normal force can be considered to be zero.

PHYSICSFILE

Fighter pilots

A fighter pilot in a vertical loop manoeuvre can safely experience centripetal accelerations of up to around 9 g, or 88 m s^{-2}. In a loop where the g-force is greater than this, the pilot may pass out. The centripetal acceleration of the plane will push her into her seat and make the blood flow away from her head. The resulting reduction of blood in the brain may cause her to experience visual disturbance (a 'grey out') or even lose consciousness (a 'black out'). This type of force is a called a positive g-force. Fighter pilots wear g-suits which pressurise the legs and limit the blood flowing to them. This helps them to maintain consciousness.

On the other hand, if the pilot's head is on the outside of the loop, the centripetal acceleration will pull the pilot onto their harness and the additional blood flow to the head can make the whites of the eyes turn red. The excess blood flow in the head may cause 'red out'. This type of force is called a negative g-force.

It is possible to calculate the speed that would ensure that a rider does not fall out. At the critical speed (i.e. the minimum speed), the normal force, F_N, on the person will be zero. In other words, the seat will exert no force on them at this speed. The critical speed is independent of the mass of the person. Assuming that $g = 9.8\,\text{m s}^{-2}$, the centripetal force, F_{net}, is:

$F_{net} = F_g + F_N$ but $F_N = 0$, so:

$F_{net} = F_g$

Therefore:

$$\frac{mv^2}{r} = mg$$

$$v^2 = \frac{mgr}{m}$$

$$= gr$$

$$v = \sqrt{gr}$$

$$= \sqrt{9.8 \times 15}$$

$$= 12\,\text{m s}^{-1}$$

This speed is approximately equal to 43 km h^{-1} and is the minimum speed needed to prevent riders—whatever their mass—from falling out of their cart in a loop of that particular radius. In practice, the rollercoaster would move with a speed much greater than this to ensure that there was a significant force between the riders and their seats, rather than zero normal force as calculated for the critical speed. Corkscrew rollercoasters can travel at up to 110 km h^{-1} and the riders can experience accelerations of up to 50 m s^{-2} (or 5 g). So safety harnesses are really only needed when the speed is below the critical value. Their primary function is to prevent people from moving around while on the ride.

How the normal force varies during the ride

It is interesting to compare the normal force that acts on the 50 kg rollercoaster rider in the three situations explored, that is, when they are travelling horizontally with uniform motion, when they are at the bottom of a dip and when they are at the top of a loop.

- The normal force when travelling horizontally is 490 N upwards.
- At the bottom of a dip, the normal force is 1790 N upwards. In other words, in the dip the seat pushes into the rider with a greater force than usual. As the rider experiences a normal force of 1790 N, they feel much heavier than normal. If the rider had been sitting on weighing scales at this time, the scales would have shown a higher-than-usual reading.
- At the top of a hump, the normal force is 410 N upwards. In other words, over the hump the seat pushes into the rider with a smaller force than usual. As the rider experiences a normal force of 410 N, this gives them the sensation of feeling lighter.

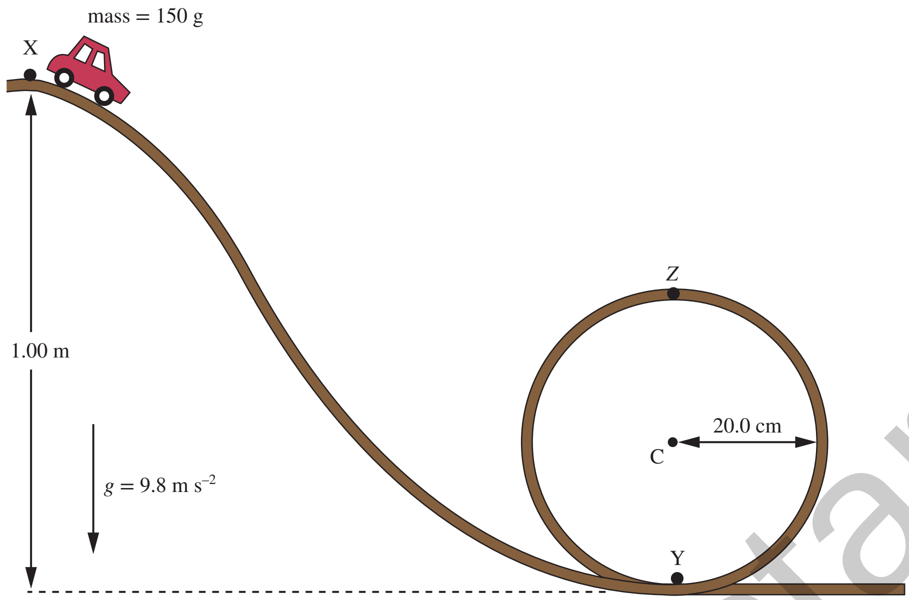
The force on the rider due to gravity has not changed throughout the ride: F_g remains at 490 N. It is the normal force acting on them that varies. It is the normal force that makes the rider feel heavier and lighter as they travel through the dips and humps respectively.

Worked example 2.4.1

VERTICAL CIRCULAR MOTION

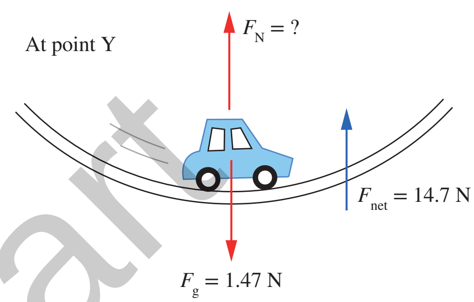
A student arranges a toy car track with a vertical loop of radius 20.0 cm.

A toy car of mass 150 g is released from rest at a height of 1.00 m (point X). The car rolls down the track and travels inside the loop. Assume that g is $9.8\,m\,s^{-2}$ and ignore friction.



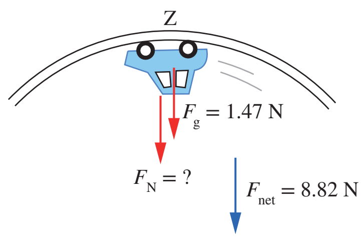
a Calculate the speed of the car as it reaches point Y at the bottom of the loop.	
Thinking	**Working**
Note all the variables given to you in the question.	At X: $m = 150\,g = 0.150\,kg$ $\Delta h = 1.00\,m$ $v = 0$ $g = 9.8\,m\,s^{-2}$
Approach the problem by considering that energy is conserved during the car's motion. Calculate the total mechanical energy first. Note that the initial speed is zero, so E_k at X is zero.	The total mechanical energy, E_m, at X is: $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $= 0 + (0.150 \times 9.8 \times 1.00)$ $= 1.47\,J$
Use conservation of energy ($E_m = E_k + E_g$) to determine the velocity at point Y. As the car rolls down the track, it loses its gravitational potential energy and gains kinetic energy. At the bottom of the loop (Y), the car has zero potential energy.	At Y: $E_m = 1.47\,J$ $\Delta h = 0$ $E_g = 0$ $E_m = E_k + E_g$ $E_m = \frac{1}{2}mv^2 + mg\Delta h$ $1.47 = \frac{1}{2} \times 0.150v^2 + 0$ $v^2 = \frac{1.47}{0.0750}$ $v = \sqrt{19.6}$ $= 4.4\,m\,s^{-1}$

b Calculate the normal force from the track at point Y.

Thinking	Working
To solve for F_N, start by working out the net, or centripetal, force. At Y, the car has a centripetal acceleration towards *C* (i.e., upwards), so the net centripetal force must also be vertically upwards at this point.	$F_{net} = \frac{mv^2}{r}$ $= \frac{0.150 \times 4.43^2}{0.200}$ $= 14.7\,\text{N}$ up
Calculate the force due to gravity, F_g, and add it to a force diagram.	$F_g = mg$ $= 0.150 \times 9.8$ $= 1.47\,\text{N}$ down 
Work out the normal force using vectors. Note up as positive and down as negative in your calculations. The forces acting are unbalanced, as the car has a centripetal acceleration upwards (towards C). The upwards (normal) force must be larger than the downwards force.	$F_{net} = F_g + F_N$ $+14.7 = -1.47 + F_N$ $F_N = +14.7 + 1.47$ $= 16\,\text{N}$ up Note that the force the track exerts on the car is much greater (by about ten times) than the force due to gravity. If the car were travelling horizontally on a flat surface, the normal force would be just 1.47 N up.

c What is the speed of the car as it reaches point Z?

Thinking	Working
Calculate the velocity from the values you have, using $E_m = E_k + E_g$.	At Z: $m = 0.150\,\text{kg}$ $\Delta h = 2 \times 0.200 = 0.400\,\text{m}$ Mechanical energy is conserved, so $E_m = 1.47\,\text{J}$ (from part **a**). At Z: $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.47 = (0.5 \times 0.150 \times v^2) + (0.150 \times 9.8 \times 0.400)$ $1.47 = 0.075 \times v^2 + 0.588$ $v^2 = 11.76$ $v = 3.4\,\text{m}\,\text{s}^{-1}$

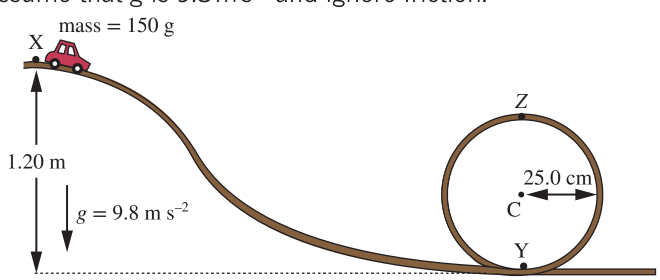
d What is the normal force acting on the car at point Z?	
Thinking	**Working**
To find F_N, start by working out the net, or centripetal, force. At Z, the car has a centripetal acceleration towards C (i.e., downwards), so the net centripetal force must also be vertically downwards at this point.	$F_{net} = \frac{mv^2}{r}$ $= \frac{0.150 \times 3.43^2}{0.200}$ $= 8.82\text{ N down}$
Work out the normal force using vectors. Note up as positive and down as negative in your calculations.	 $F_{net} = F_g + F_N$ $-8.82 = -1.47 + F_N$ $F_N = -8.82 + 1.47$ $= -7.35$ $= 7.4\text{ N down}$ Note that there is still strong contact between the car and the track—as given by the normal force—but that it is only about half the size compared to when the car was at the bottom of the track. If the car had progressively lower speeds, the normal force at Z would decrease and eventually drop to zero. At this point, the car would lose contact with the track, fall off the track and its acceleration would be equal to *g*.

Worked example: Try yourself 2.4.1

VERTICAL CIRCULAR MOTION

A student arranges a toy car track with a vertical loop of radius 25.0 cm, as shown.

A toy car of mass 150 g is released from rest at a height of 1.20 m (point X). The car rolls down the track and travels around the loop. In answering the following questions, assume that *g* is 9.8 m s^{-2} and ignore friction.



a Calculate the speed of the car as it reaches point Y the bottom of the loop.

b Calculate the normal force from the track at point Y.

c What is the speed of the car as it reaches point Z?

d What is the normal force acting on the car at point Z?

2.4 Review

SUMMARY

- The gravitational force, F_g, and normal force, F_N, must be considered when analysing the motion of an object moving in a vertical circle.
- If the normal force is greater than the gravitational force ($F_N > F_g$) the passenger or rider will feel heavier than they really are.
- If the normal force is less than the gravitational force ($F_N < F_g$) the passenger or rider will feel lighter than they really are.
- In vertical circular motion, the gravitational force always acts vertically downwards regardless of the position of the rider or passenger around the circle, the net force always acts towards the centre of the circle, and the normal force always acts between the seat and the passenger or rider.
- The normal force and the gravitational force are added together as vectors in a force diagram to give the resultant as the net force.
- At the point where a moving object falls from its circular path, the normal force is zero. The object will be moving with a centripetal acceleration equal to that due to gravity (9.8 m s^{-2} down).
- Problems relating to motion in vertical circles can often be solved by noting that energy is conserved at all points in the motion:

$$E_m = E_k + E_g = \frac{1}{2}mv^2 + mg\Delta h$$

KEY QUESTIONS

Knowledge and understanding

In answering the following questions, assume that $g = 9.8$ m s^{-2} and ignore the effects of air resistance.

1 A yo-yo is swung with a constant speed in a vertical circle.

a Describe the magnitude of the acceleration of the yo-yo along its path.

b At which point in the circular path is there the most tension in the string?

c At which point in the circular path is there the least tension in the string?

d At which point is the string most likely to break?

e If the yo-yo has a mass of 100 g and the radius of the circle is 1.25 m, find the minimum speed that the yo-yo must have at the top of the circle so that the string does not slacken.

2 A car of mass 800 kg encounters a speed hump of radius 10 m. The car drives over the hump at a constant speed of 14.4 km h^{-1}.

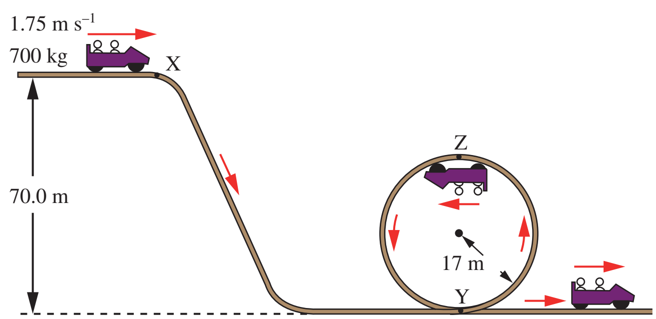
a Name all the vertical forces acting on the car when it is at the top of the hump.

b Calculate the resultant force acting on the car when it is at the top of the hump.

c After travelling over the hump, the driver remarked to a passenger that she felt lighter as the car moved over the top of the hump. Is this possible? Explain your answer.

d What is the maximum speed (in km h^{-1}) that this car can have at the top of the hump and still have its wheels in contact with the road?

3 A student is designing an amusement park ride that includes a loop-the-loop, in which a cart descends a steep incline at point X, enters a circular loop at point Y, and makes one complete revolution of the loop. The cart has a mass of 700 kg, and it carries passengers at a speed of 1.75 m s^{-1} before it begins its descent from point X, which is 70 m higher than the bottom of the loop.



a Calculate the speed of the cart at point Y.

b What is the speed of the cart at point Z?

c Calculate the normal force acting on the cart at point Z.

d What is the minimum speed that the cart can have at point Z and still stay in contact with the track?

4 A stunt pilot appearing at an air show decides to perform a vertical loop so that she is upside down at the top of the loop. During the stunt she maintains a constant speed of $35.0\,\mathrm{m\,s^{-1}}$ and the loop has a radius of 100 m.

Calculate the normal force acting on the 80.0 kg pilot when she is at the top of the loop.

5 A skateboarder of mass 72 kg is practising on a half-pipe of radius 3.0 m. At the lowest point of the half-pipe, the speed of the skater is $7.0\,\mathrm{m\,s^{-1}}$.

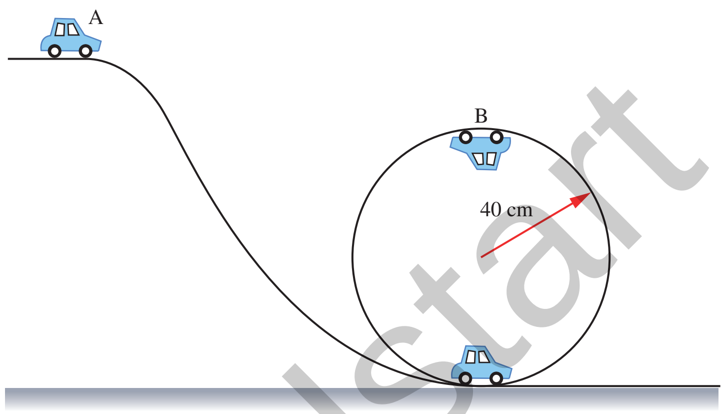
a What is the acceleration of the skater at this point? Specify both the magnitude and direction.

b Calculate the size of the normal force acting on the skater at this point.

Analysis

6 The maximum value of acceleration that the human body can safely tolerate for a short time is nine times that due to gravity. Calculate the maximum speed with which a pilot could safely pull out of a circular dive of radius 400 m.

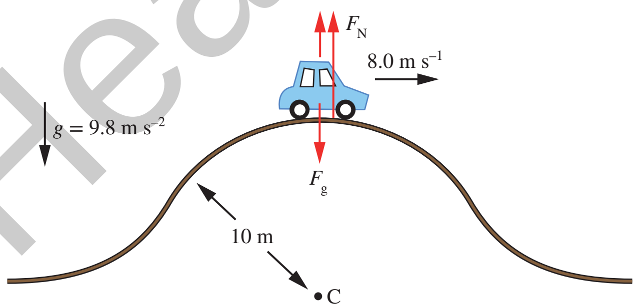
7 A student rolls a toy car of mass 50 g along a smooth track in the shape of a loop-the-loop. They try to give the car a launch speed at point A so that the car just maintains contact with the track as it passes through point B.



a Determine the acceleration of the toy car as it passes point B.

b How fast is the toy car travelling at point B?

8 A car of mass 1500 kg slows to travel over an old stone bridge of radius 10 m. The car's speed at the top of the bridge is $8.0\,\mathrm{m\,s^{-1}}$.



a Calculate the magnitude and direction of the resultant force acting on the car when it is at the top of the bridge.

b Calculate the magnitude and direction of the normal force acting on the car when it is at the top of the bridge.

c What is the maximum speed that the car can have at the top of the bridge and still have its wheels in contact with the road?

2.5 Projectiles launched horizontally

FIGURE 2.5.1 A multi-flash photograph of a golf ball that has been bounced on a hard surface. The ball moves in a series of parabolic paths.

A **projectile** is any object that is thrown or projected into the air and is moving freely, that is, it has no power source (such as a rocket engine or propeller) driving it. A netball as it is passed, a cricket ball that is hit for six and an aerial skier flying through the air are examples of projectiles. People have long argued about the path that projectiles follow, with some thinking that they were circular or had straight sections. It is now known that if projectiles are not launched vertically, and if air resistance is ignored, they move in smooth parabolic paths (Figure 2.5.1). This section considers projectiles that are launched horizontally and shows how Newton's laws can be used to solve problems involving projectile motion.

PROJECTILE MOTION

It is a very common misconception that when a projectile travels forwards through the air, it has a forwards force acting on it. This is incorrect. There may have been some forwards force acting as the projectile was launched, but once the projectile is released, this forwards force is no longer acting.

In fact, if air resistance is ignored, the only force acting on a projectile during its flight is the force due to gravity, F_g. This force is constant and always directed vertically downwards. This causes the projectile to continually deviate from a straight-line path and follow a parabolic path (Figure 2.5.2).

Projectile motion is quite complex compared to straight-line motion. It must be analysed by considering the different components—horizontal and vertical—of the actual motion. The vertical and horizontal components are independent of each other and must be treated separately.

FIGURE 2.5.2 The motorcycle and rider travel in a parabolic path as they fly through the air.

Given that the only force acting on a projectile is the force due to gravity, F_g, it follows that the projectile must have a vertical acceleration of $9.8\,\text{m s}^{-2}$ downwards throughout its motion.

In the vertical direction, a projectile accelerates due to the force of gravity, that is, at a rate of $9.8\,\text{m s}^{-2}$ downwards.

In the horizontal direction, a projectile has a uniform velocity. This is because there are no forces acting on it in this direction (if air resistance is ignored). Thus the horizontal acceleration is zero.

PROJECTILES LAUNCHED HORIZONTALLY

Projectiles can be launched at any angle. The launch velocity needs to be resolved into vertical and horizontal components and trigonometry used to solve most problems involving projectile motion. For projectiles launched horizontally, calculating the vector components of the launch velocity is straightforward: the initial vertical velocity is zero (although it increases during the flight) and the horizontal velocity is constant (equal to its launch velocity). This can be verified using trigonometric ratios and a launch angle of 0°.

Tips for solving projectile motion problems

1. Construct a diagram showing the projectile's motion. Write down the information supplied for the horizontal and vertical components.
2. In the horizontal direction, the velocity, v, is constant, and the only formula needed to calculate it is $v_{av} = \frac{s}{t}$.
3. In the vertical direction, the projectile is moving with a constant acceleration ($9.8\,\text{m s}^{-2}$ downwards), so the equations of motion for uniform acceleration can be used. These include:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

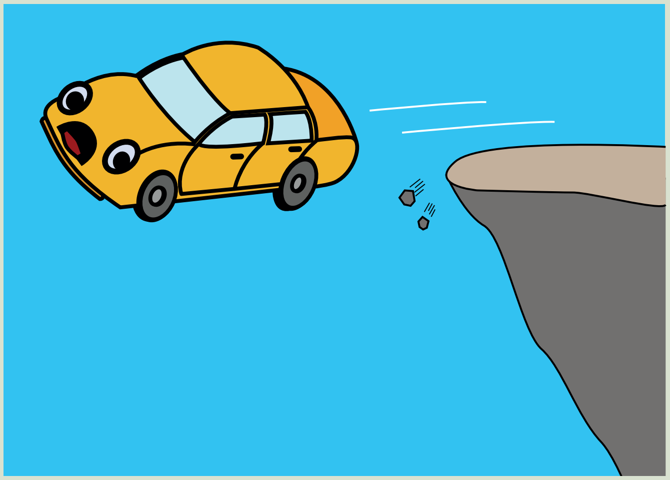
4. In the vertical direction, it is important to clearly specify whether up or down is the positive or negative direction. Either choice will work just as effectively. However, the same convention needs to be used consistently throughout each problem.
5. If a projectile is launched horizontally, its horizontal velocity throughout the flight is the same as its initial velocity.
6. Pythagoras's theorem can be used to determine the actual speed of the projectile at any point.
7. If the velocity of the projectile is required, it is necessary to provide a direction with respect to the horizontal plane as well as its speed.

PHYSICSFILE

Cartoon physics

It is easy to get the wrong idea about projectile motion from watching cartoon characters running or driving off cliffs.

In many cartoons, the character leaves the cliff and travels horizontally outwards, stopping in mid-air (see figure at right). Once they realise where they are, they immediately fall vertically downwards. Clearly, this is not what happens in reality! The character should start falling in a smooth parabolic arc as soon as they leave the cliff-top.

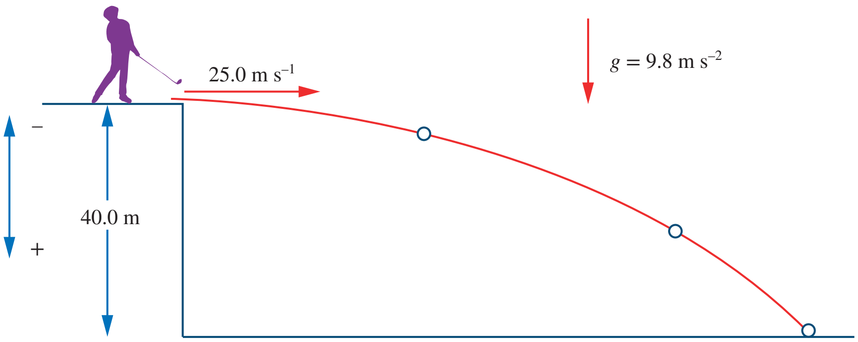


Many misconceptions can arise from what is shown in cartoons. In real life, this car would start falling as soon as it leaves the cliff top and travel in a parabolic arc.

Worked example 2.5.1

PROJECTILE LAUNCHED HORIZONTALLY

A golf ball of mass 150 g is hit horizontally with a speed of 25.0 m s^{-1} from the top of a cliff 40.0 m high. In answering the following questions, assume that $g = 9.8$ m s^{-2} and ignore air resistance.



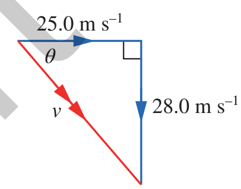
a Calculate the time the ball takes to land.

Thinking	Working
Let the downwards direction be positive. Write down the information relevant to the vertical component of the motion. Note that the instant the ball is hit, it is travelling only horizontally, so its initial vertical velocity is zero.	Down is positive. Vertically: $u = 0$ m s^{-1} $s = 40.0$ m $a = 9.8$ m s^{-2} $t = ?$
In the vertical direction, the ball has constant acceleration, so use an equation for uniform acceleration. Select the equation that best fits the information you have.	$s = ut + \frac{1}{2}at^2$
Substitute values, rearrange the equation and solve for t.	$40.0 = 0 + \frac{1}{2} \times 9.8t^2$ $t = \sqrt{\frac{40.0}{4.90}}$ $= 2.86$ $= 2.9$ s

b Calculate the distance the ball travels from the base of the cliff, i.e. the range of the ball.

Thinking	Working
Write down the information relevant to the horizontal component of the motion. As the ball is hit horizontally, the initial speed gives the horizontal component of the velocity throughout the flight.	Horizontally: $u = 25.0$ m s^{-1} $t = 2.86$ s from part **a** $s = ?$
Select the equation that best fits the information you have.	As the horizontal speed is constant (i.e. $u = v$), you can use $v_{av} = \frac{s}{t}$.
Substitute values, rearrange the equation and solve for s.	$25.0 = \frac{s}{2.86}$ $s = 25.0 \times 2.86$ $= 72$ m Note that the mass of the ball does not affect its motion, as is the case with all objects in projectile motion or in free fall.

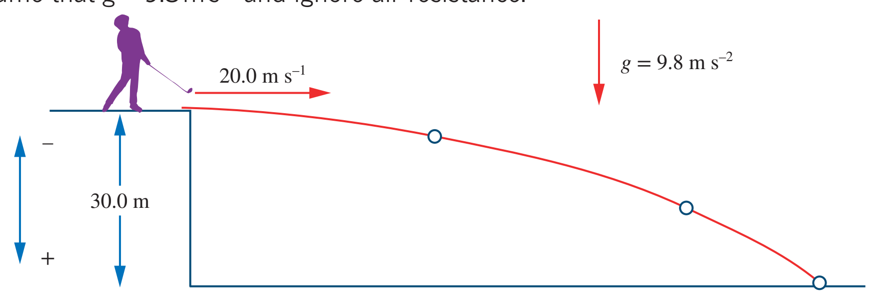
c Calculate the velocity of the ball as it lands.

Thinking	Working
Find the horizontal and vertical components of the ball's speed as it lands. Write down the information relevant to both the vertical and horizontal components.	Horizontally: $u = v = 25.0\,\text{m}\,\text{s}^{-1}$ Vertically, with downwards as positive: $u = 0$ $a = 9.8\,\text{m}\,\text{s}^{-2}$ $s = 40.0\,\text{m}$ $t = 2.86\,\text{s}$ $v = ?$
To find the final vertical speed, use the equation for uniform acceleration that best fits the information you have.	$v = u + at$
Substitute values and solve for the variable you are looking for, in this case v.	Vertically: $v = 0 + 9.8 \times 2.86$ $= 28\,\text{m}\,\text{s}^{-1}$ down
Add the components as vectors.	
Use Pythagoras's theorem to work out the actual speed, v, of the ball.	$v = \sqrt{v_h^2 + v_v^2}$ $= \sqrt{25.0^2 + 28.0^2}$ $= \sqrt{1409}$ $= 38\,\text{m}\,\text{s}^{-1}$
Use trigonometry to find the angle, θ.	$\theta = \tan^{-1}\left(\frac{28.0}{25.0}\right)$ $= 48.2°$
Specify the velocity with a magnitude and a direction relative to the horizontal. Express the answer to 2 significant figures.	The final velocity of the ball is $38\,\text{m}\,\text{s}^{-1}$ at 48° below the horizontal.

Worked example: Try yourself 2.5.1

PROJECTILE LAUNCHED HORIZONTALLY

A golf ball of mass 100 g is hit horizontally with a speed of $20.0\,\text{m}\,\text{s}^{-1}$ from the top of a 30.0 m high cliff. In answering the following questions, assume that $g = 9.8\,\text{m}\,\text{s}^{-2}$ and ignore air resistance.



a Calculate the time the ball takes to land.

b Calculate the distance the ball travels from the base of the cliff, i.e. the range of the ball.

c Calculate the velocity of the ball as it lands.

PHYSICSFILE

Aerodynamic design

In the track-and-field event of javelin, the aerodynamic shape of the javelin once used proved to be too successful. The javelin had been progressively streamlined to reduce the drag force so that the athletes could throw it further. This was not a problem until the 1980s—the javelin could now be thrown so far that runners competing in nearby track events were endangered. The design of the javelin was changed again. It was made more snub-nosed to increase drag and reduce the distance it could be thrown (see figure below). In 1983, the world record was 104.8 m. In 1986, with the modified design, the world record dropped to 85.7 m.

Australian Kelsey-Lee Barber winning a bronze medal in the women's javelin final at the 2020 Tokyo Olympics. Note the javelin's snub-nosed end.

THE EFFECTS OF AIR RESISTANCE

The interaction between a projectile and the air can have a significant effect on its motion particularly if the projectile has a large surface area and a relatively low mass. If you try throwing an inflated party balloon, it will not travel very far compared to throwing a marble at the same speed.

The size of the air resistance (i.e. the drag force) that acts on an object as it moves depends on s factors as:

- the speed of the object: the faster an object moves, the greater the drag force becomes.
- the cross-sectional area of the object in its direction of motion: a greater area means greater drag
- the aerodynamic shape of the object: a more streamlined shape experiences less drag
- the density of the air: higher air density means greater drag.

When a pilot drops a supply parcel from a plane, the drag force from the air acts in the opposite direction to the parcel's velocity. If the parcel were dropped on the Moon, where there is no air and hence no air resistance, the parcel would continue its horizontal motion and would remain directly below the plane as it fell.

Figure 2.5.3 shows a supply parcel being dropped from a plane moving at a constant velocity. If air resistance is ignored, the parcel falls in the parabolic arc shown by the darker blue curved line in diagram (a). It would continue moving horizontally at the same rate as the plane, that is, as the parcel falls it would stay directly beneath the plane until it hits the ground. The effect of air resistance is shown by the light-blue curved path. Air resistance (i.e. drag) is a retarding force and it acts in a direction that is opposite to the motion of the projectile. Air resistance makes the parcel fall more slowly and over a shorter path. If air resistance is taken into account, there are now two forces acting, as shown in diagram (b): the force due to gravity, F_g, and air resistance, F_a. Therefore the resultant force, F_{net}, that acts on the projectile is not vertically down and nor is its acceleration.

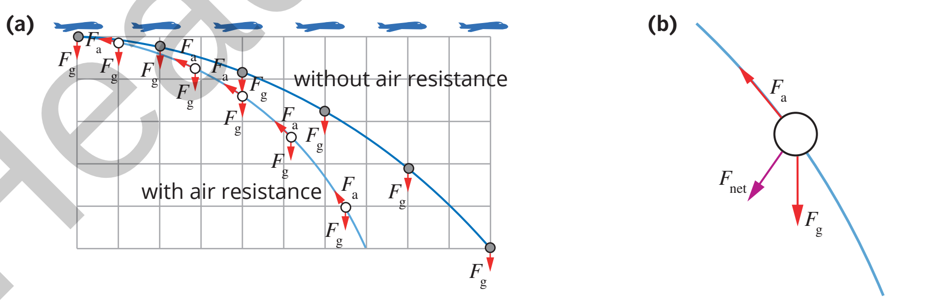


FIGURE 2.5.3 (a) The paths of a supply parcel dropped from a plane with and without air resistance. (b) When air resistance is acting, the net force on the parcel is not vertically down.

2.5 Review

OA ✓✓

SUMMARY

- If air resistance is ignored, the only force acting on a projectile is the force due to gravity, F_g. This results in the projectile having a vertical acceleration of $9.8\,\mathrm{m\,s^{-2}}$ downwards during its flight.
- Projectiles move in parabolic paths that can be analysed by considering the horizontal and vertical components of their motion.
- The following equations of motion for uniform acceleration can be used to determine the vertical component of the motion:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

- The horizontal velocity of a projectile remains constant throughout its flight if air resistance is ignored. Therefore the following equation for average velocity can be used for the horizontal component of its motion:

$$v_{av} = \frac{s}{t}$$

- Pythagoras's theorem can be used with the vertical and horizontal components of velocity to work out the speed, v, of the projectile.
- If the velocity of the projectile is required, trigonometry can be used to find the angle to the horizontal at which the projectile is travelling.

KEY QUESTIONS

Knowledge and understanding

For the following questions, assume that the acceleration due to gravity is $9.8\,\mathrm{m\,s^{-2}}$ and ignore the effects of air resistance unless otherwise stated.

1 A skateboard travelling at $5.5\,\mathrm{m\,s^{-1}}$ rolls off a horizontal bench that is 1.7 m high.

- **a** How long does the board take to hit the ground?
- **b** How far does the board land from the base of the bench?
- **c** What is the magnitude and direction of the board's acceleration just before it lands?

2 Two identical tennis balls, X and Y, are hit horizontally from a point 2.5 m above the ground with different initial speeds: ball X has an initial speed of $7.5\,\mathrm{m\,s^{-1}}$ and ball Y has an initial speed of $12\,\mathrm{m\,s^{-1}}$.

- **a** Calculate the time it takes for ball X to strike the ground.
- **b** Calculate the time it takes for ball Y to strike the ground.
- **c** How much further than ball X does ball Y travel in the horizontal direction before bouncing?

3 An archer stands on top of a platform that is 45 m high and fires an arrow horizontally at $70\,\mathrm{m\,s^{-1}}$.

- **a** What is the speed of the arrow as it reaches the ground?
- **b** At what angle relative to the horizontal is the arrow travelling as it reaches the ground?

4 A bowling ball of mass 9.5 kg travelling at $6.5\,\mathrm{m\,s^{-1}}$ rolls off a horizontal table 1.0 m high.

- **a** Calculate the ball's horizontal velocity just as it strikes the floor.
- **b** What is the vertical velocity of the ball as it strikes the floor?
- **c** Calculate the velocity of the ball as it strikes the floor.
- **d** What time interval has elapsed between the ball leaving the table and striking the floor?
- **e** Calculate the horizontal distance travelled by the ball as it falls.
- **f** Draw a diagram showing the forces acting on the ball as it falls towards the floor.

Analysis

5 A golf ball of mass 175 g is hit horizontally from the top of a cliff 75.0 m high. The golf ball lands 100 m from the base of the cliff. Calculate the horizontal speed at which the golf ball left the cliff top.

2.6 Projectiles launched obliquely

The previous section looked at projectiles that were launched horizontally. Another common situation is when projectiles are launched obliquely (i.e. at an angle) by being thrown forwards and upwards at the same time. An example of an oblique launch is shooting for a goal in basketball (Figure 2.6.1). Once the ball is released, the only forces acting on it are gravity (pulling it down to Earth) and air resistance (which slightly retards the ball's motion).

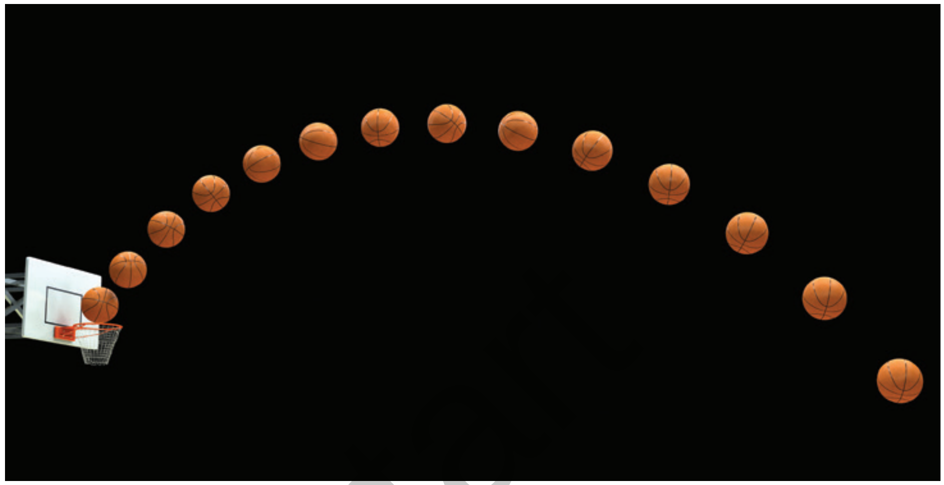


FIGURE 2.6.1 A basketball thrown up towards the ring travels in a smooth parabolic path.

PROJECTILES LAUNCHED AT AN ANGLE

If drag forces are ignored, the only force acting on a projectile that is launched at an angle to the horizontal is gravity, F_g. This is the same as with projectiles launched horizontally.

Gravity acts vertically downwards and so it has no effect on the projectile's horizontal motion. The vertical and horizontal components of the motion are independent of each other and once again must be treated separately.

In the vertical direction, a projectile accelerates due to the force of gravity, that is, at a rate of $9.8\,\text{m}\,\text{s}^{-2}$ downwards. Thus the vertical component of the projectile's velocity decreases as the projectile rises. It is momentarily zero at the top of the flight and then it increases again as the projectile descends.

In the horizontal direction, the projectile has a uniform velocity since there are no forces acting in this direction (if air resistance is ignored).

Tips for problems involving projectile motion

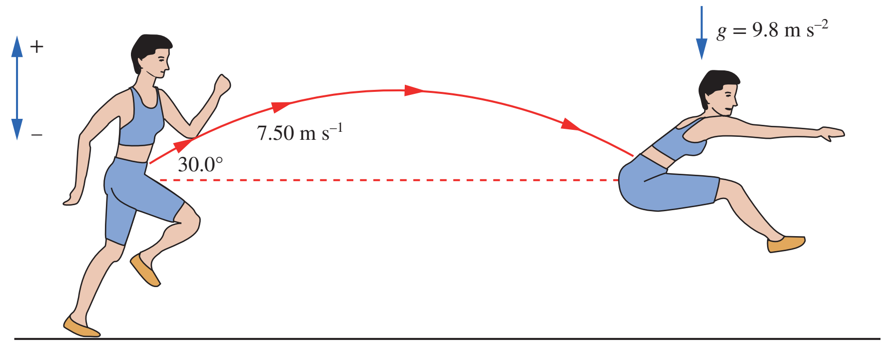
General rules for solving problems involving projectile motion were given in the previous section—see page 95 for a reminder.

If a projectile is launched at an angle to the horizontal, trigonometry can be used to find the components of the initial horizontal and vertical velocity. Pythagoras's theorem can then be used to determine the actual velocity of the projectile at any point as well as its direction with respect to the horizontal plane. Worked example 2.6.1 demonstrates how this is done.

Worked example 2.6.1

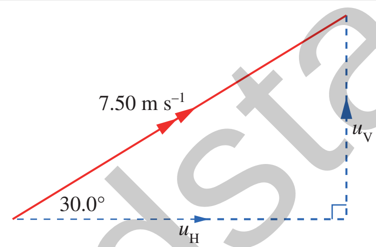
LAUNCH A PROJECTILE AT AN ANGLE

A 65 kg athlete in a long-jump event leaps with a velocity of 7.50 m s^{-1} at an angle of 30.0° to the horizontal.



In answering the following questions, treat the athlete as a point mass, ignore air resistance and use $g = 9.8$ m s^{-2}.

a What is the athlete's velocity at the highest point in the jump?

Thinking	Working
First find the horizontal and vertical components of the initial speed.	 Using trigonometry: $u_H = 7.50 \cos 30.0°$ $= 6.50$ m s^{-1} $u_V = 7.50 \sin 30.0°$ $= 3.75$ m s^{-1}
Projectiles that are launched obliquely move only horizontally at the highest point. The vertical component of the velocity at this point is zero. Thus the actual velocity is given by the horizontal component of the velocity.	At maximum height, $v = 6.50$ m s^{-1} horizontally to the right.

b What is the maximum height gained by the athlete's centre of mass during the jump?

Thinking	Working
To find the maximum height you must work with the vertical component of the velocity. Recall that the vertical component of velocity at the highest point is zero.	Vertically, taking up as positive: $u = 3.75$ m s^{-1} $a = -9.8$ m s^{-2} $v = 0$ $s = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v^2 = u^2 + 2as$ $0 = 3.75^2 + 2 \times -9.8 \times s$
Rearrange the equation and solve for s.	$s = \dfrac{3.75^2}{19.6}$ $= 0.72$ m

PHYSICSFILE

Motorcycle jumping

A motorcycle jumping is an example of projectile motion at an angle. The distance the motorcycle can jump is a function of its approach velocity to the ramp, the ramp angle, and the mass of the motorcycle and rider. Other factors also play a significant role, such as frictional forces, air resistance, wind speed, etc.

Robert 'Robbie' Maddison is an Australian motorbike stunt performer. He has broken several world records for the distance jumped and has successfully jumped distances over 105 m and made a perfect landing. Some of his feats include jumping the Tower Bridge in London, with a backflip, while the drawbridge was open.

A motorcycle jumping on a dirt track is an example of a projectile launched at an angle

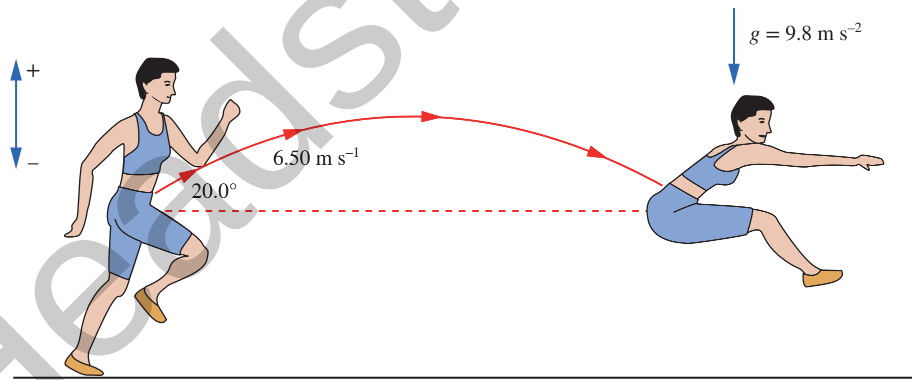
c Assuming a return to the original height, what is the total time the athlete is in the air?

Thinking	Working
As the motion is symmetrical, the time required to complete the motion will be double the time taken to reach the maximum height. First, the time it takes to reach the maximum height must be found.	Vertically, taking up as positive: $u = 3.75\,\text{m s}^{-1}$ $a = -9.8\,\text{m s}^{-2}$ $v = 0$ $t = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v = u + at$ $0 = 3.75 - 9.8t$
Rearrange the formula and solve for t, the time needed to reach the maximum height.	$t = \dfrac{3.75}{9.8}$ $= 0.38\,\text{s}$
The time to complete the jump is double the time it takes to reach the maximum height.	Total time $= 2 \times 0.383$ $= 0.77\,\text{s}$

Worked example: Try yourself 2.6.1

LAUNCH A PROJECTILE AT AN ANGLE

A 50 kg athlete in a long-jump event leaps with a velocity of $6.50\,\text{m s}^{-1}$ at 20.0° to the horizontal.



In answering the following questions, treat the athlete as a point mass, ignore air resistance and use $g = 9.8\,\text{m s}^{-2}$.

a What is the athlete's velocity at the highest point in the jump?

b What is the maximum height gained by the athlete's centre of mass during the jump?

c Assuming a return to the original height, what is the total time the athlete is in the air?

CASE STUDY ANALYSIS

The physics of shot putting

In shot-put competitions there is an advantage in being tall. It means that the release height of the shot will be higher than that of a competitor who is not as tall. It also means the distance travelled by the shot will be greater.

At the 2020 Tokyo Olympic Games the men's event was won by Ryan Crouser of the United State of America, with a distance of 23.30 m. Crouser is 201 cm tall. The gold medal for women was won by Gong Lijiao of China (175 cm tall), with a distance of 20.58 m.

When a projectile is launched at an angle to the horizontal, the theoretical launch angle that gives the maximum range is 45°. This applies only where a projectile lands at the same height as it was launched. A projectile could land at a point lower than its launch height. For example, shot putters launch their shot from above the ground. The theoretical launch angle for maximum range in this case is approximately 43°, depending on the actual release height. In reality, however, shot putters never release at this angle. This is because the speed at which they can launch the shot decreases as the angle gets further from the horizontal. Figure 2.6.2 shows the relationship between launch speed and launch angle.

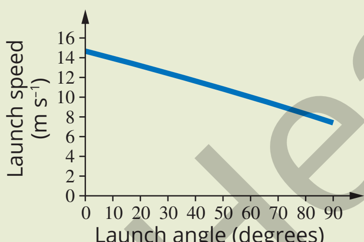


FIGURE 2.6.2 A graph showing how launch speed is greatest with a horizontal launch and decreases as the launch angle increases

The decrease in launch speed with an increase in launch angle is caused by two factors:

- When throwing with a high launch angle, the shot putter must expend a greater effort during the delivery phase to overcome the force of gravity. This reduces the launch speed.
- The structure of the shoulder and arm favours the production of putting force in the horizontal direction more than in the vertical direction.

The optimum launch angle for an athlete is obtained by combining the speed–angle relation for the athlete with the equation for the range of a projectile in free flight. For these reasons, the optimum launch angle for shot putters is actually around 34°.

Analysis

In a shot-put event a 3.0 kg shot is launched with an initial velocity of 7.5 $m\ s^{-1}$ from a height of 1.6 m and at an angle of 45° to the horizontal.

1. What is the initial horizontal speed of the shot?
2. What is the initial vertical speed of the shot?
3. How long does it take the shot to reach its maximum height?
4. What is the maximum height from the ground reached by the shot?
5. What is the speed of the shot when it reaches its maximum height?
6. What distance does the shot put travel?

2.6 Review

SUMMARY

- Projectiles move in parabolic paths that can be analysed by considering the horizontal and vertical components of their motion.
- If air resistance is ignored, the only force acting on a projectile is the force due to gravity, F_g. This results in the projectile having a vertical acceleration of 9.8 m s^{-2} downwards during its flight.
- The equations for uniform acceleration can be used to determine the vertical component. These equations are:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

- If air resistance is ignored, the horizontal velocity of a projectile remains constant throughout its flight. The appropriate equation is:

$$v_{av} = \frac{s}{t}$$

- For objects initially launched at an angle to the horizontal, it is useful to calculate the initial horizontal and vertical velocities using trigonometry.
- At its highest point, the projectile is moving horizontally. Its velocity at this point is given by the horizontal component of its launch velocity. The vertical component of the velocity is zero at this point.

KEY QUESTIONS

Knowledge and understanding

In answering the following questions, assume that the acceleration due to gravity is 9.8 m s^{-2} and ignore the effects of air resistance unless otherwise stated.

1 A javelin thrower launches her javelin at 35° above the horizontal. Describe how the horizontal velocity of the javelin changes during its flight.

2 James and Ollie are using a garden hose to water some raised veggie beds that are approximately waist height. The hose is quite short, and the water does not reach the farthest plants. James thinks if they hold the hose horizontally, all of the velocity of the water will be in the horizontal direction, so the water will reach the furthest. Ollie thinks the water will reach furthest if they hold the hose at 45°. Who is correct, and why?

3 A rugby player kicks for a goal by taking a place kick with the ball at rest on the ground. The ball is kicked at 40° to the horizontal at 25 m s^{-1}. At its highest point, what is the speed of the ball?

4 A basketballer shoots for a goal by launching the ball at 25 m s^{-1} at 30° to the horizontal.

a Calculate the initial horizontal speed of the ball.

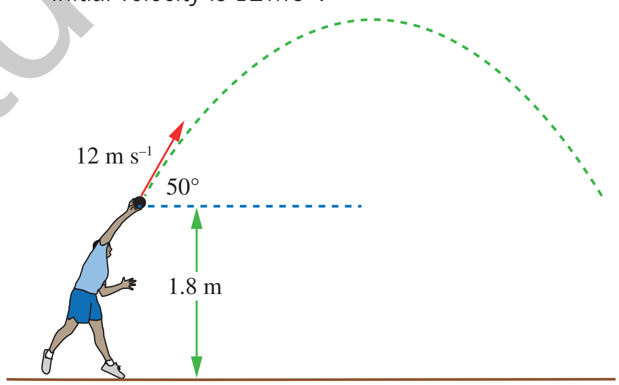
b What is the initial vertical speed of the ball?

c What is the magnitude and direction of the acceleration of the ball when it is at its maximum height?

d What is the speed of the ball when it is at its maximum height?

Analysis

5 In a shot-put event a 3.5 kg shot is launched from a height of 1.8 m at an angle of 50° to the horizontal. Its initial velocity is 12 m s^{-1}.



a What is the initial horizontal speed of the shot?

b What is the initial vertical speed of the shot?

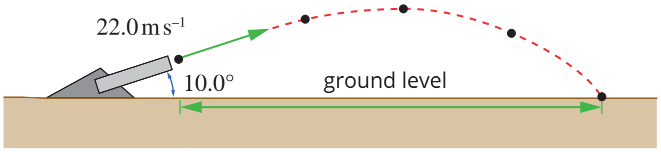
c How long does it take the shot to reach its maximum height?

d What is the maximum height from the ground reached by the shot?

e What is the speed of the shot when it reaches its maximum height?

f What distance does the shot-put travel?

6 A tennis player is using a tennis ball machine to practise her forehand. She sets the machine to launch tennis balls with an initial velocity of 22.0 m s^{-1} at an angle of 10.0° above the horizontal. The balls are launched from approximately the same height as her racquet. Give your answers to 3 significant figures.



a Calculate the horizontal component of the velocity of the ball:

i initially

ii after 0.25 s

iii after 0.50 s

b Calculate the vertical component of the velocity of the ball:

i initially

ii after 0.25 s

iii after 0.50 s

c What is the speed of the tennis ball after 0.50 s?

d What is the speed of the ball as it hits her racquet?

e What horizontal distance does the ball travel before it hits her racquet—that is, what is its range?

f Describe what effect air resistance has on the ball.

Chapter review

02

KEY TERMS

air resistance
banked track
centripetal acceleration
centripetal force
design speed
frequency
inclined plane
magnitude
normal force
period
projectile
tangential

REVIEW QUESTIONS

Knowledge and understanding

1 A bowling ball is rolling down a smooth straight ramp. Describe the speed and acceleration of the ball.

2 A cyclist is riding at high speed around a steeply banked section of a velodrome. Choose the option below that best describes the magnitude of the normal force acting on the cyclist.

A zero
B greater than the force of gravity on the cyclist
C equal to the force of gravity on the cyclist
D less than the force of gravity on the cyclist

3 A bowling ball is rolling down a smooth track that is inclined at 45° to the horizontal.

a What is the magnitude of the acceleration of the ball?
b How does the magnitude of the normal force acting on the ball compare to its force due to gravity?

4 Two students are discussing a demonstration in which a bucket half filled with water was swung at high speed in a vertical circle. No water spilled out of the bucket as it passed the overhead position. Dave states that at the top of the circle, a centrifugal force acts outwards against the force due to gravity and so holds the water in the bucket. Emma states that inertia keeps the water in the bucket and the normal force from the bucket keeps the water travelling in a circular path. Who is correct, and why?

5 Two blocks are connected by a string that passes over a smooth pulley. One of the blocks is placed on a frictionless table and the other is free to move up or down. The block on the table has a mass of 5.0 kg and the connected block has a mass of 10.0 kg.

a At what rate do the blocks accelerate?
b What is the magnitude of the tension in the string?

6 A skier of mass 90 kg is skiing down an icy slope that is inclined at 15° to the horizontal. In answering the following questions, assume that friction is negligible and that the acceleration due to gravity is 9.8 m s^{-2}.

a Determine the components of the force on the skier due to gravity perpendicular to the slope and parallel to the slope.
b Determine the normal force that acts on the skier.
c Calculate the acceleration of the skier down the slope.

7 A locomotive of mass 7500 kg is pulling two carriages. Carriage A has a mass of 10 000 kg and carriage B has a mass of 5000 kg. The locomotive accelerates at 2.0 m s^{-2}. The drag force on carriage A is 2000 N, on carriage B it is 1000 N and on the locomotive it is 1500 N.

a Calculate the driving force of the locomotive engine.
b Calculate the magnitude of the tension in the coupling between the two carriages.

8 A 1000 kg car tows a 200 kg trailer along a level surface at an acceleration of 2.5 m s^{-2}. The frictional drag on the car is 800 N and the frictional drag on the trailer is 700 N. Calculate the driving force provided by the car engine to give this acceleration.

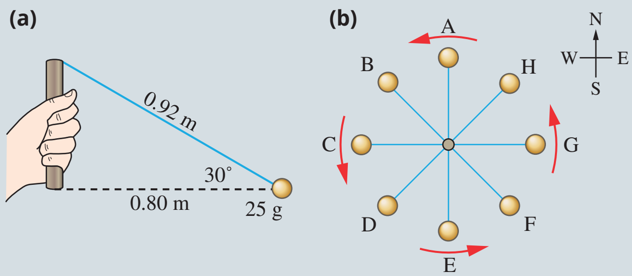
9 A marble is rolled from rest down a smooth slide that is 3.5 m long. The slide is inclined at an angle of 40° to the horizontal.

a Calculate the acceleration of the marble.
b What is the speed of the marble as it reaches the end of the slide?

10 Marshall has a mass of 57 kg and is riding his 3.0 kg skateboard down a 5.0 m long ramp that is inclined at an angle of 65° to the horizontal. Ignore friction when answering questions **a** to **d**.

a Calculate the magnitude of the normal force acting on Marshall and his skateboard.
b What is the acceleration of Marshall as he travels down the ramp?
c What is the net force acting on Marshall and his board when no friction acts?
d If Marshall's initial speed is zero at the top of the ramp, calculate his final speed as he reaches the bottom of the ramp.
e Marshall now stands halfway up the incline while holding his board in his hands. Friction now acts on him. Calculate the frictional force acting on Marshall while he is standing stationary on the ramp.

11 During a high-school physics experiment, a copper ball of mass 25.0 g is attached to a very light piece of steel wire 0.920 m long and whirled in a circle at 30.0° to the horizontal, as shown in diagram (a). The ball moves in a circular path of radius 0.800 m with a period of 1.36 s. The top view of the ball's motion is shown in diagram (b).



- **a** Calculate the orbital speed of the ball.
- **b** What is the centripetal acceleration of the ball?
- **c** What is the magnitude of the centripetal force acting on the ball?
- **d** Draw a diagram similar to diagram (a) showing all the forces acting on the ball.
- **e** What is the magnitude of the tension in the wire?

12 A toy car is travelling in a circular path of radius 15 m at a constant speed of 7.5 m s^{-1}.

- **a** What is the acceleration of the toy car?
- **b** What force is keeping the toy car moving in its circular path?

13 A cycling track has a turn that is banked at 40° to the horizontal. The radius of the track at this point is 30 m. Determine the speed at which a cyclist of mass 60 kg would experience no sideways force as they ride along this section of track.

14 The Ferris wheel at an amusement park has an arm of 10 m radius and its compartments move with a constant speed of 5.0 m s^{-1}.

- **a** Calculate the normal force that a 50 kg boy would experience from the seat when at the:
  - **i** top of the ride
  - **ii** bottom of the ride.
- **b** After getting off the ride, the boy remarks to a friend that he felt lighter than usual at the top of the ride. Which option explains why he might feel lighter at the top of the ride?
  - **A** He lost weight during the ride.
  - **B** The strength of the gravitational field was weaker at the top of the ride.
  - **C** The normal force there was larger than the gravitational force.
  - **D** The normal force there was smaller than the gravitational force.

15 A toy car is moving at 3.75 m s^{-1} as it rolls off a horizontal table. The car takes 1.5 s to reach the floor.

- **a** How far does the car land from the table?
- **b** What is the magnitude and direction of acceleration when the car is halfway to the floor?

16 A bowling ball of mass 8.75 kg travelling at 15.0 m s^{-1} rolls off a horizontal table that is 1.27 m high.

- **a** What is the horizontal speed of the ball as it strikes the floor?
- **b** What is the vertical speed of the ball as it strikes the floor?
- **c** Calculate the speed of the ball as it reaches the floor.

Application and analysis

17 In a tennis match, a tennis ball is hit from a height of 1.70 m with an initial velocity of 18.5 m s^{-1} at an angle of 46.0° to the horizontal. Ignore drag forces in answering the following questions.

- **a** What is the initial horizontal speed of the ball?
- **b** What is the initial vertical speed of the ball?
- **c** What is the maximum height that the ball reaches above the court surface?

18 An experiment examined the relationship between the distance an object travels and the time it takes to travel that distance as it falls from a height of 20 m. The following data show the distance the object has fallen and the time when the object was at that distance. For example, 1.212 seconds after the object was dropped it had fallen 7.2 metres.

Distance (m)	Time (s)
0	0.000
1	0.452
2.5	0.714
3	0.782
5	1.010
7.2	1.212
9.1	1.363
11.5	1.532
13.2	1.641

- **a** What is the rate at which the object drops?
- **b** Plot the time to fall (*s*) as a function of distance the object has fallen (*m*).
- **c** Predict the time taken to fall 15 m.

19 An experiment is conducted to examine the relationship between the horizontal distance an object travels and the vertical distance an object falls when launched at an angle of 25° from the horizontal, from a vertical height of 15 m. An electronic device collected the data: the horizontal distance travelled and the vertical height at each 0.1 s in time.

Horizontal distance travelled (m)	Time (s)	Height (m)
0.1	0.1	15
0.2	0.2	15.2
0.3	0.3	15.6
0.4	0.4	16.2
0.5	0.5	17
0.6	0.6	18
0.7	0.7	19.2
0.8	0.8	18
0.9	0.9	17
1	1	16.2
1.1	1.1	15.6
1.2	1.2	15.2
1.3	1.3	15
1.4	1.4	14.3
1.45	1.5	13.6
1.5	1.6	12.8
1.55	1.7	12.2
1.6	1.8	11.5
1.65	1.9	10.9
1.7	2	10.1
1.75	2.1	9.5
1.8	2.2	8.7
1.85	2.3	8.2
1.9	2.4	7.6
1.95	2.5	6.5
2	2.6	5.8
2.05	2.7	5.1
2.1	2.8	4.3
2.15	2.9	3.7
2.2	3	2.9
2.25	3.1	2

a Plot the horizontal distance travelled by the object as a function of time.

b Using the data, calculate the horizontal velocity of the object.

c Plot the horizontal velocity as a function of time. What observations can you make from the plot?

d Using the data, calculate the horizontal acceleration of the object.

e Plot the horizontal acceleration as a function of time. What observations can you make from the plot?

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