Integers and indices

1

How low can you go? Humans have explored much of the land areas of our planet, but have seen very little of the vast depths of Earth's oceans.

The highest point on the Earth's surface is the summit of Mt Everest, a height of 8848 m above sea level (+8848 m). The lowest point on Earth is the bottom of the Mariana Trench, in the Pacific Ocean. It is believed to reach at least 10 994 m below sea level (-10 994 m). In 1960, naval lieutenant Don Walsh and engineer Jacques Piccard made the first expedition to the bottom of the trench. They reached a depth of -10 916 m. When they resurfaced, the windows of their submersible were cracked from the pressure of the water. Because of the dangers of high pressures, much of the Earth's oceans have only been explored to a depth of -300 m. Only a few submersibles can go beyond -3000 m, and even military submarines usually travel no deeper than -500 m. Below these depths, an amazing underwater world is waiting to be explored.

Forum

Have you ever been in negative temperatures? If you have, where were you?

If you were able to pick up Mt Everest and place it in the Mariana Trench, it would be completely covered by water. How much water would there be between the top of Mt Everest and sea level?

Why learn this?

Positive numbers are only half the story! You need to be able to work with numbers that are less than zero—the negative side of the number line. Negative numbers are used to show actions that are opposite to positives, such as a move backwards instead of forwards, or a fall in price instead of a rise.

Indices (also called powers or exponents) provide a convenient way to write and work with very large numbers. Having rules for working with indices helps make calculations more efficient.

After completing this chapter you will be able to:

- use directed numbers (integers) in everyday situations
- do the four operations (+, −, ×, ÷) on directed numbers
- use a number line to help with directed number calculations
- apply the order of operations rules to directed number calculations
- understand and use index notation
- work efficiently with index notation by applying the appropriate rules.

Recall 1

Prepare for this chapter by trying the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 To answer the following, it may help to draw a diagram.

(a) At 7 am on a winter morning, the temperature was 4 °C. At midday, the temperature was 13 °C. By midnight it had dropped to -2 °C.

(i) By how many degrees did the temperature increase, from 7 am to midday?

(ii) By how many degrees did the temperature decrease, from midday to midnight?

(iii) What was the difference between the temperature at 7 am and the temperature at midnight?

(b) A city building has 5 storeys above the ground floor and 2 basement levels below the ground floor.

(i) Jade parked her car in the 2nd basement level, got in the lift and moved up 5 floors. At which floor did she get out?

(ii) Erin got in the lift on the 4th floor. She went down 5 floors, then up 2 floors. At which floor did she get out?

2 Write the value of the following. **(a)** 2^3 **(b)** 3^4 **(c)** 5^2 **(d)** 10^6 **(e)** $\sqrt{36}$ **(f)** $\sqrt{49}$ **(g)** $\sqrt[3]{64}$ **(h)** $\sqrt[3]{8}$

3 Imagine that you are standing at -4 on a large number line. Which number would you land on if you walked:

(a) 7 places to the right **(b)** 5 places to the left?

4 Calculate: **(a)** 2×8 **(b)** 7×9 **(c)** 38×6 **(d)** 15×19 **(e)** $18 \div 3$ **(f)** $54 \div 9$ **(g)** $324 \div 3$ **(h)** $616 \div 4$

5 Calculate: **(a)** $(8 \div 4) + (35 \div 5)$ **(b)** $6 \times 8 + 2 \times 3$ **(c)** $50 - 5 \times 2 + 3$

6 Use a factor tree to find the prime factors of the following, then write each number as a product of its prime factors in index form.

(a) 24 **(b)** 36 **(c)** 120

7 Find the value of the following. **(a)** $4^2 + 3^2$ **(b)** $2^2 + 5^2$ **(c)** $8^2 - 6^2$ **(d)** $9^2 - 3^2$ **(e)** $5^2 \times 2^4$ **(f)** $3^3 \times 5 \times 7$ **(g)** $9^2 \div 3^2$ **(h)** $2^3 \times 7 \div 2^2$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Comparing powers

In this activity, you will explore what can make numbers raised to powers greater or smaller.

Integers review

1.1

A **positive** (+) or a **negative** (-) sign is used to show the direction of a number. Some words associated with negative numbers are: down, loss, below, decrease, lose, withdrawal. Some words associated with positive numbers are: up, profit, above, increase, gain, deposit.

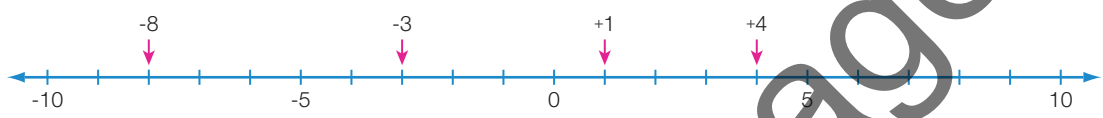
For example, a deposit of $50 into your bank account could be written as +50, and a withdrawal of $180 could be written as -180.

Date	Transaction	Balance
		$500
17 July	ATM Withdrawal -$180	$320
20 July	Birthday money from Mum +$50	$370

What is an integer?

The **integers** are all of the positive whole numbers 1, 2, 3, … , the negative whole numbers -1, -2, -3, … , and zero. (The use of '…' in mathematics shows that the sequence of numbers continues forever.) Positive and negative integers are also called *directed numbers*.

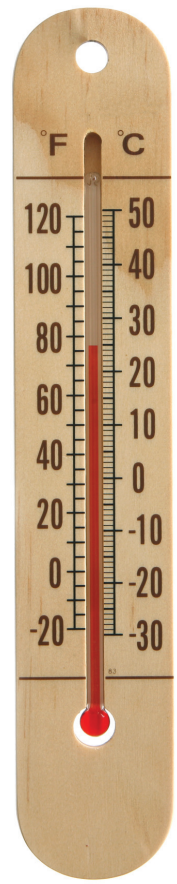
Integers can be represented on a number line:



The numbers on a number line get larger in value as you move from left to right.

If a number has no sign in front of it, assume it is positive. 4 = +4

From the number line above, you can write the statement +4 > -8. You could put this in words as 'positive 4 is greater than negative 8'. You could also write the statement -3 < +1, which is the same as saying 'negative three is less than positive one'. Number lines can also be written vertically, with the positive numbers above the negative ones. A thermometer is an example of a vertical number line.

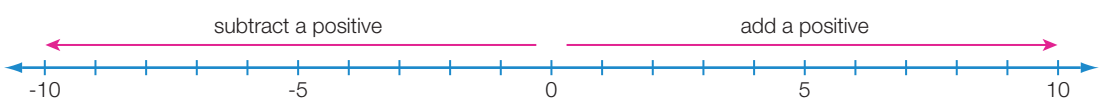


Adding and subtracting integers

Adding and subtracting positive integers is the straightforward addition you have been doing for years.

To *add a positive integer*, imagine moving that many places to the *right* on the number line (or upwards on a vertical number line), in the positive direction.

To *subtract a positive integer*, imagine moving that many places to the *left* on the number line (or downwards on a vertical number line), in the negative direction.



Adding a negative integer is the opposite of adding a positive one. It means you move in the negative direction, to the *left* on the number line (or downwards on a vertical number line). You can see that this is the same movement as subtracting a positive integer.

Subtracting a negative integer is the opposite of subtracting a positive one. It means you move in the positive direction, to the *right* on the number line (or upwards on a vertical number line).

Simplifying addition

When adding and subtracting integers, brackets are often placed around the second number and its sign, to separate it from the addition or subtraction symbol; for example, +5 + (+9) or +7 – (+3). However, as you can write a positive integer without the + sign in front, you can drop the positive signs in front of the numbers, remove the brackets, and simply write 5 + 9 or 7 – 3.

You can see from the previous number lines that subtracting a negative integer is the same as adding the positive integer. This means that you could write 8 – (-2) as 8 + 2.

You can also see that adding a negative integer is the same as subtracting the positive integer. This means you could write 4 + (-10) as 4 – 10.

+ (+) and – (-) can be replaced with + – (+) and + (-) can be replaced with –

You could also say:

When the two signs are the *same*, add. When the two signs are *different*, subtract.

Worked example 1 W.E. 1

Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) -4 + (+11) **(b)** -9 + (+3) **(c)** +2 – (+8) **(d)** -5 – (+11)

Thinking	Working
(a) 1 Simplify the addition by writing positive integers without their signs, and removing the brackets.	(a) -4 + (+11) = -4 + 11
2 Imagine (or draw) a number line. Start at the first number (-4), then move the number of places indicated by the second number (11) in the positive direction (to the right). Write your answer.	= 7
(b) 1 Simplify the addition by writing positive integers without their signs, and removing the brackets.	(b) -9 + (+3) = -9 + 3
2 Imagine (or draw) a number line. Start at the first number (-9), then move the number of places indicated by the second number (3) in the positive direction (to the right). Write your answer.	= -6

(c)	**1**	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	*(c)* +2 – (+8) = 2 – 8
	2	Imagine (or draw) a number line. Start at the first number (2), then move the number of places indicated by the second number (8) in the negative direction (to the left). Write your answer.	= -6
(d)	**1**	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	*(d)* -5 – (+11) = -5 – 11
	2	Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (11) in the negative direction (to the left). Write your answer.	= -16

Worked example 2

W.E. 2

Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) +9 + (-7) **(b)** -12 + (-1) **(c)** +4 – (-6) **(d)** -5 – (-7)

Thinking			Working
(a)	**1**	Remove the brackets and write a single subtraction symbol between the numbers. Write positive integers without their signs.	*(a)* +9 + (-7) = 9 – 7
	2	Complete this straightforward subtraction. Write your answer.	= 2
(b)	**1**	Remove the brackets and write a single subtraction symbol between the numbers. Write positive integers without their signs.	*(b)* -12 + (-1) = -12 – 1
	2	Imagine (or draw) a number line. Start at the first number (-12), then move the number of places indicated by the second number (1) in the negative direction (to the left). Write your answer.	= -13
(c)	**1**	Remove the brackets and write a single addition symbol between the numbers. Write positive integers without their signs.	*(c)* +4 – (-6) = 4 + 6
	2	Complete this straightforward addition.	= 10

(d) 1 Remove the brackets and write a single addition symbol between the numbers. Write positive integers without their signs.

(d) -5 – (-7)
= -5 + 7

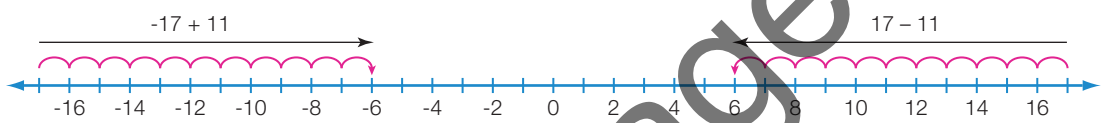
2 Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (7) in the positive direction (to the right). Write your answer.

= 2

Using number line symmetry

The number line is symmetrical about zero. A positive number is the same distance from zero as its negative opposite; for example, negative 6 is the same distance from zero as positive 6. This can be useful when adding integers, especially large ones.

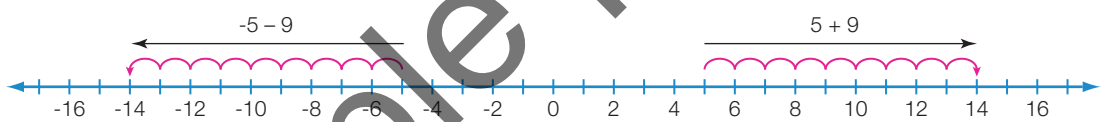
For example, if you show -17 + 11 on a number line, it is the mirror image of showing 17 – 11.



So, to calculate -97 + 64, you could simply do 97 – 64, then change the sign of the answer.

97 – 64 = 33 -97 + 64 = -33

Similarly, to calculate -5 – 9, you could consider the mirror image situation: 5 + 9.



So, to calculate -53 – 49, you could simply do 53 + 49, and change the sign of the answer.

53 + 49 = 102 -53 – 49 = -102

1.1 Integers review

Navigator

Answers p. 616

1 (columns 1–3), 2, 3 (columns 1–3), 4, 5, 6, 8, 9, 10, 13, 15, 17	1 (columns 3–4), 3 (columns 3–4), 4, 5 (row 2), 6, 7, 8, 10, 11, 14, 15, 16, 17	1 (columns 3–4), 3 (columns 3–4), 4, 5 (row 2), 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19

Fluency

W.E. 1

1 Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) +4 + (+9)	**(b)** -7 + (+5)	**(c)** +5 + (+3)	**(d)** +2 – (+6)
(e) +10 – (+13)	**(f)** -3 + (+8)	**(g)** +12 + (+6)	**(h)** -1 – (+9)
(i) -15 + (+8)	**(j)** +8 – (+9)	**(k)** -8 – (+13)	**(l)** -19 + (+11)
(m) +16 – (+9)	**(n)** -14 + (+7)	**(o)** -4 – (+22)	**(p)** -17 + (+23)

2 Use your calculator to evaluate the following.

(a) -3 + -5 **(b)** 22 – 12 **(c)** 13 + -29

(d) 84 + 12 **(e)** 39 + +6 **(f)** -46 – -12

(g) -52 + -16 + 12 **(h)** 22 – 13 + 6 **(i)** 46 – +24 + -13 – -20

3 Simplify the following by writing a single symbol between the two numbers, then calculate the answer. W.E. 2

(a) +5 + (-4) **(b)** +7 + (-9) **(c)** +5 – (-11) **(d)** -7 – (-6)

(e) -4 – (-9) **(f)** -3 – (+8) **(g)** -14 – (+3) **(h)** +12 – (-8)

(i) +19 – (+13) **(j)** 0 – (-3) **(k)** -8 – (-16) **(l)** +15 + (-22)

(m) -11 – (-7) **(n)** -13 + (-9) **(o)** +25 + (-31) **(p)** -27 – (-16)

4 Write a negative or a positive integer to describe the following situations.

(a) 350 m above sea level

(b) a loss of $4800

(c) rewinding 6 seconds of an audio recording

(d) depositing $73 into your bank account

(e) 2 levels below the ground floor of a building

(f) 19 metres under water

(g) a company profit of $10 750

(h) skipping 34 minutes of a TV episode

(i) withdrawing $200 from your bank account

(j) a plane flying at an altitude of 8100 metres

5 Use the symbols greater than (>) or less than (<) between the following pairs of integers to show their relationship. A number line may be useful.

(a) +2 ___ +6 **(b)** -3 ___ +1 **(c)** +5 ___ -4 **(d)** -1 ___ -3

(e) +5 ___ -10 **(f)** -6 ___ -8 **(g)** +16 ___ -16 **(h)** -33 ___ -12

6 Write the following in descending order (largest to smallest).

(a) +4, 0, -7, +11, -2 **(b)** -23, 1, 0, -9, +7

(c) -3, 4, 0, 11, -15, 1 **(d)** -5, 8, 19, -43, -2, 6

(e) 14, -72, 5, 26, -1, -38 **(f)** 32, -19, 0, 17, -56, 4

7 Calculate:

(a) 2 + 7 – 5 **(b)** -3 + 10 – 5 **(c)** -6 + 4 – 8 **(d)** -15 + 9 + 8

(e) 11 + 14 – 23 **(f)** -7 – 8 – (-9) **(g)** 4 + 5 – (-5) **(h)** -6 + (-9) – (+9)

Understanding

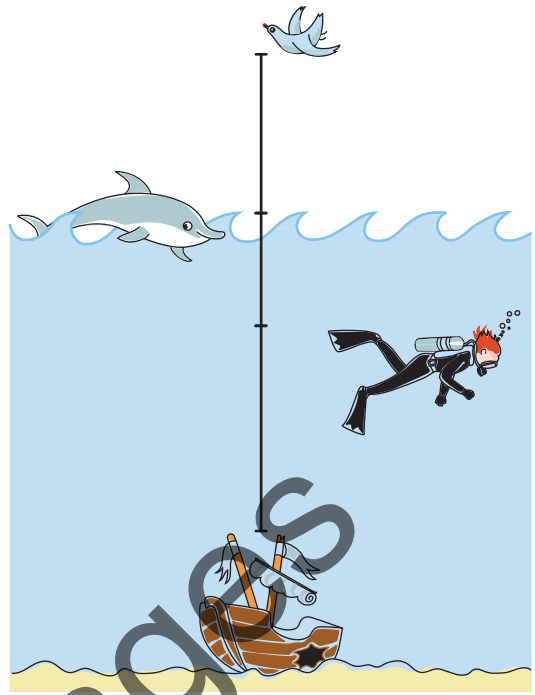
8 **(a)** Copy the diagram's vertical line and label it according to the following information.

A bird is 24 metres above the water.

A scuba diver is 17 metres below the surface.

The wreck of a ship is 31 metres below the diver.

A dolphin is just on the surface of the water.



(b) Use your diagram to find out:

(i) the distance between the bird and the diver

(ii) the distance between the wreck and the dolphin.

9 Ahmed has saved $110 to buy a new bicycle, but the price is $200. Ahmed's parents lend him the rest of the money, to be repaid later at $15 per month.

(a) Write an integer expression to show how much Ahmed needs to borrow from his parents.

(b) Complete and extend the following table to find how long it would take Ahmed to repay his loan.

Month	Initially	After 1 month	After 2 months	After 3 months
Money owing				

10 The maximum temperatures and minimum temperatures recorded during a week in June on Mount Kosciuszko are shown in this table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (°C)	11	13	8	6	9	12	13
Minimum (°C)	-3	-1	-7	-5	-4	-1	0

(a) On which day was the lowest minimum recorded?

(b) On which day was the highest minimum recorded?

(c) What was the difference between the maximum and the minimum temperatures on:

(i) Thursday **(ii)** Friday?

(d) On which day was the difference between the maximum and the minimum temperatures the greatest?

11 On Monday, Rachel withdrew $120 from her bank account at an ATM. On Tuesday, she used her bankcard to pay $87 for her shopping from the same account. On Thursday, her employer deposited her salary of $243 into the account, and her mum also deposited $50 for her birthday. On Friday, Rachel used the account to pay a $109 bill online.

(a) Write one long integer calculation that shows each of the events mentioned above as an addition or a subtraction. (Hint: Begin by writing the $120 withdrawal as -120.)

(b) Complete your calculation to find the following.

(i) Did Rachel have more or less money in her account by the end of the week?

(ii) How much more or less?

12 Puerto Rico Trench is the deepest trench on the floor of the Atlantic Ocean. The bottom of the trench is 8605 metres below sea level. Mount McKinley is the highest mountain in North America. It is 6194 metres above sea level. If Mount McKinley could be picked up and dropped into the Puerto Rico Trench, what depth of water would be between the top of the mountain and sea level?

Reasoning

13 Using the number lines on page 5, find the number you would have to add to each of the following to get an answer of +5.

(a) 0 **(b)** -3 **(c)** -12 **(d)** -10 **(e)** +6 **(f)** +16 **(g)** -9

14 A magic square is one for which the numbers in each row, column and diagonal add up to the same 'magic' total.

Complete the following magic squares, by first working out the magic total.

(a)

-6		-2
	-3	
		0

(b)

6			-18
	2		8
	0	-8	
12		-2	-12

15 Calculate the following by considering the 'mirror image' of each on the opposite side of the number line.

(a) -27 + 14 **(b)** -59 + 36 **(c)** -87 + 62

(d) -31 – 29 **(e)** -68 – 43 **(f)** -75 – 58

(g) -47 + (-62) **(h)** -71 – (-26) **(i)** -96 – (+31)

16 **(a)** Complete the following calculations.

(i) -2 + 3 **(ii)** -5 + 11 **(iii)** -8 + 17 **(iv)** -21 + 34

(b) Now, reverse the order of the two numbers in the above additions, and calculate the resulting subtractions (e.g. -4 + 6 becomes 6 + -4, or 6 – 4).

(c) What do you notice? Comment on your observation.

(d) What is the name for the property of numbers that makes it possible to add them or multiply them in any order?

(e) Use your observation from part **(c)** to calculate the following.

(i) -34 + 45 **(ii)** -53 + 69 **(iii)** -72 + 99 **(iv)** -98 + 113

Open-ended

17 The thermometer at the weather station on Mount Wellington reads -4 °C at 5 am one day in July. By 2 pm, the temperature has reached the day's maximum of 9 °C. Suggest what the thermometer reading might have been at:

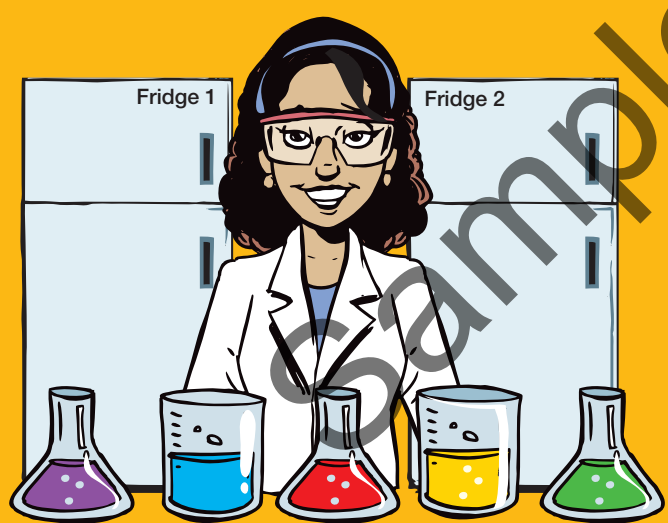
(a) 8:30 am **(b)** midday.

18 List three integers that give a negative answer when added to -17.

19 List three integers that give a positive answer when subtracted from -17.

Problem solving

Lab maths



A	B	C	D	E
Max: 2 °C Min: -3 °C	Max: 0 °C Min: -4 °C	Max: -4 °C Min: -9 °C	Max: -3 °C Min: -7 °C	Max: 13 °C Min: 2 °C

Professor Berner has two fridges in her laboratory in which she needs to safely store five different chemicals, A, B, C, D and E.

Each chemical has a maximum and a minimum safe temperature at which it can be stored—anything outside this range and it will explode!

Each fridge can be set to any temperature Professor Berner chooses.

Use the information on the chemical labels to decide Professor Berner's temperature settings and which chemicals are stored in each fridge.

Strategy options

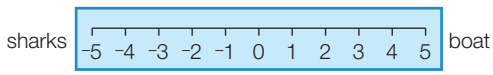
- Guess and check.
- Work backwards.

Investigation

Walking the plank

Equipment required: 1 large number line from -5 to +5 (optional), 1 die marked with +3, +2, +1, -1, -2, -3 and 1 die marked with +, +, +, –, –, –

You have been captured by a gang of pirates who plan to make you walk the plank. However, the pirate captain decides to give you a chance to save yourself. The plank is marked as shown below.



The pirate captain has two dice: one with the sides marked +, +, +, –, –, – (called the 'operations die'), and one marked -3, -2, -1, +1, +2, +3 (called the 'number die'). The captain places you at point 0 and rolls the dice.

Depending on the roll of the dice, you'll have to either move forwards or backwards, according to the following instructions.

- If the operations die shows a '+', you turn to face the boat (the positive end of the number line).
- If it shows a '–', you face the sharks (the negative end).
- If the number die shows a positive number, you walk forwards that number of steps in the direction you are facing.
- If it shows a negative number, you walk backwards that number of steps.

For example, if the captain rolls (–) (+1), you face the sharks and walk forward one step to -1. If the captain rolls (–) (-1), you face the sharks and walk backwards one step to +1.

The captain keeps rolling and you keep moving until you move past +5 or -5. If you go past +5, you are safely back on the boat, but if you go past -5, you join the sharks.

The Big Question

Is this game fair? How many moves will it take to save you?

Engage

1 Copy the diagram of the plank. Starting at 0, use it to find where you would be if the pirate captain rolled (+) (-3), followed by (–) (-2).

2 Return to 0. Roll the two dice again, and move as they tell you. Keep rolling and moving until you are either with the sharks or back on the boat. Keep track of how many moves it takes.

Explore

3 (a) A 'fair game' means that both you and the captain have an equal chance of winning. To decide if the game is fair, make a list of all the possible outcomes of rolling the dice.

(b) To find out a reasonable number of moves in which you could expect to get a result (sharks or boat), you will need to play the game several times. (Take turns being the pirate captain and the 'victim'!)

Strategy options

- Make a table.
- Act it out.

Explain

4 (a) What is the minimum number of moves possible that would bring you to safety?

(b) What was the largest number of moves you needed to complete a game?

Elaborate

5 Complete the following to answer the Big Question.

(a) State whether you and the captain have an equal chance of winning, and explain how you decided.

(b) What is a 'reasonable' number of moves needed to complete the game? How did you decide this?

(c) How often would you expect to get a result in the 'reasonable' number of moves you have chosen?

6 Imagine you are teetering on the edge of the plank at -5. Show how it is possible to still be on -5 after four different rolls of the dice.

Evaluate

7 How confident are you in the answer you gave for a 'reasonable number of moves'? What could you do to become more confident?

8 Did playing this game help you get better at adding and subtracting integers? Could it be useful for someone new to the topic? Explain your answer.

Extend

9 How would you expect your 'reasonable number of moves' to change if the plank extended from -8 to +8? Make a prediction and then test it by playing the game.

Powers of powers, products and quotients

1.6

Raising a number in index form to a power

Consider $(7^2)^3$. This is 7^2 multiplied by itself three times (that is, multiplied so that it appears in the multiplication three times). You can write this as follows.

$(7^2)^3 = (7 \times 7) \times (7 \times 7) \times (7 \times 7)$
$= 7^6$

or:

$(7^2)^3 = 7^2 \times 7^2 \times 7^2$
$= 7^{(2+2+2)}$
$= 7^6$

Notice that 2×3 equals 6. So, you can simplify by multiplying the indices together:

$(7^2)^3 = 7^{(2 \times 3)}$
$= 7^6$

When raising a number in index form to a power, keep the base and multiply the indices; e.g. $(3^4)^3 = 3^{12}$.

Worked example 13 W.E. 13

Simplify $(2^3)^5$

Thinking

Keep the base and multiply the indices.

Working

$(2^3)^5$
$= 2^{3 \times 5}$
$= 2^{15}$

Raising a product to a power

How could you calculate $(4 \times 7)^3$?

You could find the product in brackets first, then raise it to the power:

$(4 \times 7)^3 = (28)^3$
$= 21\,952$

Or, you could raise each factor in the product to the power. You can show this by writing the expression in expanded form first: $(4 \times 7)^3 = (4 \times 7) \times (4 \times 7) \times (4 \times 7)$

$= 4 \times 4 \times 4 \times 7 \times 7 \times 7$
$= 4^3 \times 7^3$
$= 64 \times 343$
$= 21\,952$

If a product of factors in brackets has been raised to a power, then each factor in the brackets is raised to that power.

e.g. $(3 \times 5)^4 = 3^4 \times 5^4$

Raising a quotient to a power

What does $\left(\frac{2}{5}\right)^3$ simplify to? It has a base of $\frac{2}{5}$ and an index of 3.

$$\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$

$$\left(\frac{2}{5}\right)^3 = \frac{2 \times 2 \times 2}{5 \times 5 \times 5}$$

$$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$$

Each number in the brackets has been raised to the power of 3.

If a quotient in brackets has been raised to a power, then each number in the brackets is raised to that power.

e.g. $\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$

Worked example 14

W.E. 14

Expand the brackets in the following.

(a) $(2 \times 3)^4$ **(b)** $\left(\frac{5}{7}\right)^2$

Thinking	Working
(a) Raise every factor in the brackets to the power outside the brackets.	(a) $(2 \times 3)^4$ $= 2^4 \times 3^4$
(b) Raise every number in the brackets to the power outside the brackets.	(b) $\left(\frac{5}{7}\right)^2$ $= \frac{5^2}{7^2}$

The zero power

To simplify $5^3 \div 5^3$, you can use two methods.

Method 1

Use the rule for dividing index numbers, which is to keep the base and subtract the indices.

$5^3 \div 5^3 = 5^{(3-3)}$
$= 5^0$

Method 2

Write in expanded form and cancel common factors.

$5^3 \div 5^3 = \frac{5^3}{5^3} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5}} = 1$

(Don't forget that any number divided by itself is 1.)

From Method 1, you can see that $5^3 \div 5^3 = 5^0$

From Method 2, you can see that $5^3 \div 5^3 = 1$

Therefore, $5^0 = 1$.

Try this with $\frac{8^4}{8^4}$. Do you get $8^0 = 1$?

Any number raised to the power of zero equals one.
(The exception is zero itself: 0^0 is undefined.)

1.6 Powers of powers, products and quotients

Navigator

1, 2 (columns 1–2), 3 (row 1), 4, 5, 7 (column 1), 8, 11, 13 (a–b), 14, 19	1, 2 (columns 2–3), 3 (row 2), 4, 5, 6, 7 (columns 2–3), 8, 9, 11, 12 (row 1), 13 (a–d), 14, 16, 19	1, 2 (columns 3–4), 3 (row 2), 4, 5, 6, 7 (columns 3–4), 8, 9, 10, 12, 13 (c–f), 15, 16, 17, 18

Answers p. 620

Equipment required: calculator for Questions **11**, **17**

Fluency

1 Simplify the following.

W.E. 13

(a) $(9^6)^2$ **(b)** $(14^7)^3$ **(c)** $(6^8)^8$ **(d)** $(10^5)^5$

(e) $(4^2)^3$ **(f)** $(5^2)^0$ **(g)** $(2^4)^5$ **(h)** $(7^0)^2$

2 Expand the brackets in the following.

W.E. 14

(a) $(4 \times 3)^2$ **(b)** $(2 \times 5)^3$ **(c)** $(3 \times 2)^5$ **(d)** $(4 \times 7)^4$

(e) $(7 \times 10)^3$ **(f)** $(8 \times 9)^1$ **(g)** $(2 \times 3 \times 1)^6$ **(h)** $(4 \times 5 \times 6)^2$

(i) $\left(\frac{3}{5}\right)^3$ **(j)** $\left(\frac{2}{9}\right)^2$ **(k)** $\left(\frac{10}{13}\right)^3$ **(l)** $\left(\frac{12}{13}\right)^2$

(m) $\left(\frac{1}{2}\right)^5$ **(n)** $\left(\frac{1}{10}\right)^3$ **(o)** $\left(\frac{1}{11}\right)^6$ **(p)** $\left(\frac{1}{12}\right)^2$

3 Evaluate the following.

(a) 7^0 **(b)** 20^0 **(c)** $\left(\frac{5}{6}\right)^0$ **(d)** $\left(-3\frac{1}{2}\right)^0$ **(e)** $(-0.5234)^0$

(f) 3×5^0 **(g)** $2^2 \times 6^0$ **(h)** -3^0 **(i)** -6×5^0 **(j)** $3 \times (-6)^0$

4 **(a)** $(5^6)^2$ simplifies to:

A 5^8 **B** 5^4 **C** 5^3 **D** 5^{12}

(b) $(4 \times 3)^0$ simplifies to:

A 1 **B** 2 **C** 7 **D** 12

5 What is $(2^3)^2 \times 2^3$ equal to?

A 4^9 **B** 2^8 **C** 2^9 **D** 2^{35}

6 State whether the following equations are true (T) or false (F). If false, where possible state the correct answer in index form.

(a) $(3 + 4)^2 = 3^2 + 4^2$ **(b)** $(12 - 8)^2 = 4^2$ **(c)** $(-3)^2 + (6)^3 = (-18)^2$

(d) $(-4)^3 \times (-2)^3 = 8^3$ **(e)** $(5)^2 \times (2)^3 = 10^3$ **(f)** $8^7 \div 8^0 = 8^7$

Understanding

7 Use a combination of the rules for working with index numbers to simplify the following. Leave your answers in index form (do not evaluate).

(a) $(8^2 \times 7^3)^2$ **(b)** $\left(\frac{6^2}{7^2}\right)^3$ **(c)** $\frac{(2 \times 4)^5}{2}$ **(d)** $\frac{(5^3)^2}{5^2}$

(e) $\frac{3^4 \times 4^7}{9^2 \times 3^4}$ **(f)** $\frac{3^7 \times 4^0}{3^5 \times 4^5}$ **(g)** $\frac{(7 \times 8)^4}{7^2}$ **(h)** $\frac{(8 \times 9 \times 10)^3}{(9 \times 8)^3}$

(i) $\frac{(4^3 \times 5)^4}{(5^2 \times 4)^2}$ **(j)** $(2 \times 3)^4 \times \left(\frac{1}{3}\right)^2$ **(k)** $\left(\frac{3}{2}\right)^4 \times \left(\frac{2}{3}\right)^3$ **(l)** $\left(\frac{1}{3^5}\right)^2 \times \left(\frac{2}{3}\right)^2$

8 Write the following as the product of two or three prime factors, raised to a single power; e.g. $15^2 = (5 \times 3)^2$.

(a) 10^6 **(b)** 21^3 **(c)** 35^2 **(d)** 45^4 **(e)** 70^5 **(f)** 77^7

9 Write the following as the product of two or more prime factors in index form. The index of each prime factor may be different; e.g. $24 = 2^3 \times 3$.

(a) 36 **(b)** 48 **(c)** 63 **(d)** 72 **(e)** 75 **(f)** 84

10 Evaluate the following without a calculator by writing the numerators and denominators as products of prime factors, then simplifying.

(a) $\frac{9 \times 14}{15 \times 21}$ **(b)** $\frac{12 \times 18}{27 \times 24}$ **(c)** $\frac{45 \times 21}{35 \times 36}$ **(d)** $\frac{54 \times 60}{48 \times 33}$

11 **(a)** By trial and error, find the largest number that is a power of 2 and that will fit on a 10-digit calculator display.

(b) Find the largest power of 9 that will fit on a 10-digit calculator display.

12 Pronumerals are the symbols used to represent unknown numbers. You can apply the rules for working with index numbers to pronumerals. Use one of the index number rules you have learnt in this section to simplify the following.

(a) $(a^3)^2$ **(b)** $\left(\frac{c}{d}\right)^4$ **(c)** $(xy)^5$ **(d)** m^0

(e) $(2x)^2$ **(f)** $5p^0$ **(g)** $\left(\frac{b^2}{c}\right)^5$ **(h)** $\left(\frac{3a}{b}\right)^3$

Reasoning

13 Use the appropriate index law to find the value of x in each of the following.

(a) $10^{12} \div 10^5 = 10^x$ **(b)** $(5^6)^3 = 5^x$ **(c)** $(6^x)^2 = 6^{20}$

(d) $\dfrac{7^4 \times 7^x}{7^{11}} = 1$ **(e)** $\dfrac{(3^4)^x}{3^{12}} = 1$ **(f)** $\dfrac{(5^6)^x \times (5^x)^2}{5^4} = 5^4$

14 Kelvin and Terry were given the following question as part of their indices homework.

Evaluate: $\left(\dfrac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

Here is Kelvin's working:

$\left(\dfrac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

$= \left(\dfrac{27 \times 32}{9 \times 8}\right)^2$

$= \dfrac{729 \times 1024}{81 \times 64}$

$= \dfrac{746\,496}{5184}$

$= 144$

Here is Terry's working:

$\left(\dfrac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

$= (3 \times 2^2)^2$

$= (3 \times 4)^2$

$= 12^2$

$= 144$

'Wow', said Kelvin. 'We both got the same answer but your working doesn't look as complicated as mine'. 'No', said Terry. 'And I didn't have to use my calculator either!'

(a) How is Terry's method different from Kelvin's?

(b) Both answered the question correctly, but who did it more efficiently? Give a reason for your choice.

15 Write the following numbers as index numbers raised to a second index; for example, $64 = (2^2)^3$.

(a) 16 **(b)** 81 **(c)** 625

(d) 729 **(e)** 256 **(f)** 1 000 000

Open-ended

16 Find two or three ways of writing the following as a product of two or three factors, raised to a single power.

(a) 24^3 **(b)** 36^5 **(c)** 48^2 **(d)** 80^4

17 1000 can be written as 10^3, or as $5^3 \times 2^3$. Find at least two different ways of writing 625 000 using indices.

18 Write three different sets of values for m and n so that $(a^m)^n$ simplifies to a^{24}.

19 The index laws are sometimes used incorrectly. Below is a set of calculations that look reasonable, but are actually incorrect.

In each case, explain the mistake and give the correct answer. Explain why your answer is correct.

(a) $3^2 \times 3^4 = 3^8$ (b) $\dfrac{7^9}{7^3} = 7^3$ (c) $(5^2)^4 = 5^{2^4} = 5^{16}$ (d) $13 \times 4^0 = 1$

Challenge 1

1 The mean of -5, -3, 0, 4 and 9 is:

A -1 **B** 1 **C** $3\frac{4}{5}$ **D** $4\frac{3}{4}$

2 Reece bought a new table for $100. He then changed his mind and sold it for $110. He changed his mind again and bought it back for $130, then sold it again for $150. What overall profit or loss did Reece make?

A $30 loss **B** $10 loss **C** $20 profit **D** $30 profit

3 Three consecutive numbers are such that twice the greatest added to three times the least is -31. Find the numbers.

4 My father is 30 years older than I am, and my mother is 24 years older than I am. How old was I when my father's age was double my mother's? Explain your answer.

5 If $x^y = 64$, find all the possible pairs of values for x and y.

6 **(a)** By writing 2^{15} as $(2^3)^5$ and 3^{10} as $(3^2)^5$, find out whether $2^{15} > 3^{10}$ is a true or a false statement. Do not use a calculator.

(b) Use the same method to decide which is larger, 2^{27} or 3^{18}.

7 If the numbers $a = 2^{80}$, $b = 3^{60}$ and $c = 5^{40}$ are written in ascending order, then what order do they appear in?

A a, b, c **B** a, c, b **C** b, c, a **D** c, a, b

8 What is the value of $(-1)^5 - (-1)^4$?

A 1 **B** 0 **C** -1 **D** -2

9 $2^3 \times 2^2 \times 3^3 \times 3^2$ is equal to:

A 6^5 **B** 6^6 **C** 36^5 **D** 36^{10}

10 If $10^x - 10 = 99990$, then x is equal to:

A 4 **B** 5 **C** 6 **D** 9

11 The value of $\frac{8^2}{2^8}$ is:

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 2 **D** 4

12 If $y^x = 256$, find all the possible whole number pairs of values for x and y.

13 If $x = -3$, which of the following expressions has the largest value?

A $x^2 - 3$ **B** $(x - 3)^2$ **C** $(x + 3)^3$ **D** $x^2 + 3$

14 What is the last digit in the number represented by 4^{3827}?

15 What is the last digit in the number represented by 3^{2004}?

Chapter review 1

Maths literacy

base	exponent	integers	simplify
cube	index	negative	square
evaluate	index form	positive	
expanded form	indices	power	

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

2 Two negative numbers multiply to give a _____ result.

3 A _____ number divided by a positive number gives a negative result.

4 5^3 is written in _____, while $5 \times 5 \times 5$ is written in _____.

5 6^7 is read as 'six to the _____ of 7'.

6 The number that is raised to a power is called the _____.

7 Other names for 'power' are _____ and _____.

8 The addition of two negative numbers will always give a _____ answer.

9 To _____ means to find a value by doing a calculation.

10 Any number (except zero) raised to the _____ of zero is equal to 1.

11 When you _____ an expression, you do not need to calculate an actual value.

Fluency

1 Write a positive or a negative integer to represent each of the following. 1.1

(a) The bottom of a lake is 23 metres below sea level.

(b) A business made a profit of $840 000.

(c) You deposit $350 into your bank account.

(d) The value of a share in a mining company rose by $4.

2 Place the following in ascending order. 1.1

(a) -6, 9, 14, -23, 0 (b) 8, -15, 5, -7, -2 (c) 34, -11, 0, 6, 12

3 Calculate: 1.1

(a) -3 + (+10) (b) 7 – (+9) (c) -5 – (+6) (d) -11 + (+4)

(e) 8 + (-5) (f) -13 + (-11) (g) 2 – (-18) (h) -12 – (-4)

4 Calculate: 1.1

(a) 4 + 7 – 9 (b) -2 + 5 – 1 (c) -15 + 23 – 8

(d) 6 + (-3) – (+10) (e) -7 – (-8) + 3 (f) 5 – (-14) – 20

5 Find the following products. **1.2**

(a) -14×2 (b) 4×-5 (c) -6×-6 (d) -7×-12

(e) 15×-9 (f) -22×8 (g) -60×30 (h) -28×-200

(i) $(-6)^2$ (j) -7^2 (k) $-2^2 \times 3^2$ (l) $(-4)^2 \times (-1)^2$

6 Find the following quotients. **1.3**

(a) $-36 \div 3$ (b) $55 \div -11$ (c) $\frac{-28}{-4}$ (d) $-27 \div 3$

(e) $66 \div -11$ (f) $\frac{-91}{-7}$ (g) $72 \div -9$ (h) $-45 \div -5$

(i) $\frac{-80}{4}$ (j) $440 \div -10$ (k) $-320 \div 8$ (l) $\frac{-78}{6}$

7 Evaluate the following expressions. **1.4**

(a) $45 \div 9 \times -2 - 4$ (b) $46 + (-6 \times 7) + 20 \div -5$

(c) $-9 \times -5 - 3 \times 4 + 2$ (d) $-8 + (-18) \div -3 - 4 \times -4$

(e) $-4 + (-6)^2 \div 9$ (f) $-7 \times -8 - 5^2 + (-10)$

8 Write the following in index form and find the value of each. **1.5**

(a) $9 \times 9 \times 9 \times 9 \times 9$ (b) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

(c) $3 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$ (d) $8 \times 8 \times 10 \times 10 \times 10$

9 Write the following in expanded form. **1.5**

(a) $2^4 \times 5^3$ (b) $13^3 \times 8^6$ (c) $4^2 \times 7^2 \times 9^5$ (d) $6^6 \times 10^2 \times 17$

10 Simplify the following. **1.5**

(a) $7^3 \times 7^2$ (b) $3^6 \times 3^2$ (c) $5^2 \times 5^2$ (d) $2^2 \times 2^6$

(e) $3^6 \times 2^6 \times 3^3 \times 2^4$ (f) $4^2 \times 7^2 \times 4^3 \times 7$ (g) $5^2 \times 2^3 \times 2^3 \times 5^4$ (h) $7^5 \times 11^5 \times 7^2 \times 11$

11 Simplify the following. **1.5**

(a) $5^6 \div 5^2$ (b) $\frac{7^4}{7}$ (c) $11^5 \div 11^3$ (d) $\frac{2^6}{2^3}$

(e) $(3^6 \times 4^3) \div (3^5 \times 4^2)$ (f) $\frac{7^3 \times 10^5}{7 \times 10^3}$ (g) $(8^2 \times 5^7) \div (8 \times 5^4)$ (h) $\frac{2 \times 9^3 \times 13^3}{9^2 \times 13}$

12 Evaluate the following. **1.5**

(a) $(-3)^4$ (b) -3^4 (c) $(-5)^3$

(d) -1^7 (e) $-4^2 \times (-3)^2$ (f) $(-2)^4 \times (-3)^4$

13 Simplify, then evaluate the following. **1.6**

(a) $(3^5)^2$ (b) $(7^2)^3$ (c) $(3^5)^3$ (d) $(2^4)^4$

(e) 2^0 (f) 6×3^0 (g) $\frac{5^6}{5^6}$ (h) $\frac{2^{10}}{2^{10}}$

14 Expand the brackets. **1.6**

(a) $(4 \times 11)^3$ (b) $(8 \times 9)^5$ (c) $(3 \times 5)^7$ (d) $(7 \times 10)^4$

(e) $\left(\frac{1}{2}\right)^3$ (f) $\left(\frac{3}{4}\right)^2$ (g) $\left(\frac{5}{6}\right)^4$ (h) $\left(\frac{8}{9}\right)^7$

Understanding

15 The minimum overnight temperatures for 1 week at Mt Hotham were -2 °C, -3 °C, 1 °C, 2 °C, -3 °C, -2 °C, 0 °C. Find the mean minimum overnight temperature for the week. (The mean is found by adding all values, then dividing the result by the number of values.) **1.1**

16 In the game called Count'Em Up, red tokens are worth 5 points, black tokens are worth -3 points and white tokens are worth -1 point. Calculate the total point score at the end of a round for each of the following players. **1.2**

Ava: 2 red, 3 black and 1 white
Georgia: 3 red, 4 black and 2 white
Rose: 3 red, 5 black and 4 white
Wei: 2 red, 5 black and 2 white

17 A scientist is observing the behaviour of bacterial cells. She finds that each cell divides in two every 24 hours. **1.5**

(a) If the scientist isolates a single cell in a dish, complete the following table to show how many cells there will be after a certain number of days.

Time (number of days)	0	1	2	3	4
Number of cells	1				

(b) Extend the pattern in your table to find how many cells there will be after 1 week.

(c) Write the 'Number of cells' row of the table as a series of index numbers.

(d) Use the pattern to find how many cells there will be after 2 weeks.

18 Three friends invest a total of $270 in a lottery (they put in $90 each). If they collect prizes worth a total of: **1.3**

(a) $60, find the loss for each friend

(b) $300, find the profit for each friend

(c) $28 500, find the profit for each friend.

19 An ice-cube tray filled with water at a temperature of 21 °C is put in the freezer, where it takes 3 hours to freeze solid at 0 °C. What is the average hourly drop in temperature? **1.4**

20 Choose the best answer. **1.5**

(a) $5^3 \times 2^3$ is the same as:

A 10^3 **B** $(5 \times 2)^6$ **C** 10^6 **D** both B and and C

(b) $7^6 \times 7^6$ is the same as:

A 49^6 **B** 7^{12} **C** both A and and B **D** 49^{12}

21 Simplify if possible, then evaluate: **1.5, 1.6**

(a) $\left(\frac{3}{7}\right)^0$ **(b)** $(-8)^0$ **(c)** $\frac{(-6)^5}{(-6)^5}$ **(d)** $\left(-\frac{2}{5}\right)^0$

(e) $\frac{(-3)^6}{(-3)^5}$ **(f)** $\frac{(-5)^8}{(-5)^6}$ **(g)** $\frac{(-2)^{10}}{(-2)^5}$ **(h)** $\frac{(-3)^{20}}{(-3)^{17}}$

(i) $\frac{2^3 \times 2^4}{2^5}$ **(j)** $\frac{10^5 \times 10}{-10^2}$ **(k)** $\frac{3^4 \times 4^2 \times 3^2 \times 4}{(3 \times 4)^3}$ **(l)** $\left(\frac{1}{2}\right)^3 \times \left(\frac{3}{4}\right)^2$

22 Evaluate the following without using a calculator by writing the numbers as products of prime factors and simplifying the calculation. **1.6**

(a) $\frac{25 \times 12}{15 \times 18}$ **(b)** $\frac{10^2 \times 45}{21^2 \times 25}$ **(c)** $\frac{9 \times 7}{10^4 \times 21}$

Reasoning

23 Calculate the value of x in the following equations. **1.5**

(a) $15^7 \div 15^4 = 15^x$ **(b)** $12^{12} \div 12^x = 12^4$ **(c)** $34^x \div 34^{10} = 1$

24 16 can be written as $(2^2)^2$. Rewrite the following as an index number raised to a second index, using the base given in brackets. **1.6**

(a) 36^2 (6) **(b)** 128^4 (2) **(c)** $(1000^3)^2$ (10) **(d)** 4900^7 (70)

25 **(a)** Find the magic sum for the following 4×4 magic square. **1.1, 1.2, 1.3**

		2	-20
-18		-8	
	-2		8
10	-12	-4	

(b) Complete the magic square.

(c) Make a new 4×4 magic square by dividing each number in the completed square by -2. What is the magic sum of this magic square?

(d) Make another new 4×4 magic square by multiplying each number in the original magic square by -2. What is the magic sum of this magic square?

26 Van is organising a table tennis tournament for the local clubs in his area. The tournament will have 3 rounds, then the semifinals, then the final. The losers in each round leave the competition. How many players will Van need to invite to compete in round 1 in order to end up with 2 players in the final? **1.5**

27 Insert brackets where necessary to make the following statements true. **1.4**

(a) $-3 - 4 \times -2 = 14$ **(b)** $2 + -3 \times 4 + -2 = -6$

(c) $16 \div -4 + 3 \times -2 + 1 = -7$ **(d)** $4 + -3 \times -2 \div -6 \times -1 + 5 = 0$

(e) $2 - 3 + 6 \times 3 + 2 \times -1 - 6 = -37$ **(f)** $24 \div -2 \div -3 + 4 \times -2 - 5 = -24$

28 State whether the following are true (T) or false (F). **1.5, 1.6**

(a) $5^3 \times 5^2 = 5^6$ **(b)** $8^3 \times 8^4 \times 8^6 > 8^9 \times 8^4$ **(c)** $17 + 17 + 17 + 17 = 17^4$

(d) $(3^6)^3 = (3^3)^6$ **(e)** $(4 + 6)^4 = (10^2)^2$ **(f)** $(4 + 3)^2 = 4^2 + 3^2$

Numeracy practice 1

Non-calculator

1 Two numbers added together equal -5. The same two numbers multiply together to give -24. What are the two numbers?

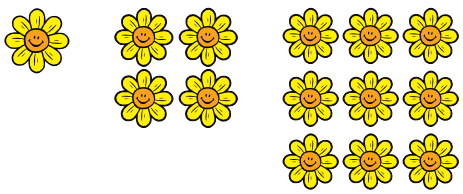
2 You have $5 in your pocket. You owe a friend $8, and another friend owes you $12. If everyone pays what they owe, how much money do you have now?

3 Which of the following is equal to $9^2 \times 3^3$?

A $9 \times 2 \times 3 \times 3$ **B** $3 \times 3 \times 3 \times 3 \times 3$

C $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ **D** 27^5

4 Dora is making this flower pattern, made of an increasing number of shapes.



(a) Complete the table to show the number of flowers she needs for each shape in her pattern.

Shape	1	2	3	4	5
Number of flowers					

(b) What sequence of numbers does the second row of the table form?

(c) How many flowers would she need for Shape 10?

Calculator allowed

5 $(-2)^2 - 9$ is equal to:

A -13 **B** -5 **C** 5 **D** 13

6 A microwave oven can heat a frozen meal by 10 °C every minute. A meal taken out of a freezer is at -20 °C. How many minutes should the meal be heated to be at 50 °C when served?

7 If $A^3 = B^2 = C^6 = 64$, then what are the values of A, B and C?

8 Which of the following is not equal to $\sqrt{4096}$?

A $\frac{2^8}{2^2}$ **B** $(2^3)^2$ **C** 2048 **D** 4^3