# Methemetics

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#### STUDENT COMPANION





# Pearson Secondar Teach Maths

**Student Companion** 

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We pay our respects to Elders, past and present.

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#### How to use this Student Companion

The *Student Companion* is a complementary resource that offers a print medium for corresponding lessons in *Pearson Secondary Teaching Hub*. It is designed to support teaching and learning by providing learners with a place to create a portfolio of learning to suit their individual needs, whether you are:

- supporting a blended classroom using the strengths of print and digital
- preparing for exams by creating a study guide or bound reference
- needing a tool to differentiate learning or
- looking for meaningful homework tasks.

Learners can develop their portfolio of learning as part of classroom learning or at home as an additional opportunity to engage and re-engage with the knowledge and skills from the lesson.

This could be done as prior learning in a flipped classroom environment or as an additional revision or homework task.

#### Learning intention and success criteria

#### Apply irrational numbers

Learning intention: To be able to apply irrational number

Success criteria:

SC 1: I can write approximate values for irrational numbers. SC 2: I can apply irrational numbers in the solution of problems.

SC 1: I can write approximate values for irrational numbers

Worked example: Writing a decimal approximation for an

**Learning intentions** are provided for every lesson. The learning intentions are goals or objectives that align to the corresponding digital lesson. They describe what learners should know, understand or be able to do by the end of the lesson.

**Success criteria** clarify expectations and describe what success looks like. The success criteria are specific, concrete and measurable so learners can actively engage with and reflect on their evidence of learning within each lesson.

#### **Worked examples**

Worked examples provide learners with a step-by-step solution to a problem. The worked examples in the *Student Companion* correspond to those in the digital lesson and are provided for each skill to:

- scaffold learning
- support skill acquisition
- reduce the cognitive load.

The **worked examples** are an effective tool to demonstrate what success looks like. The 'try yourself' format of the worked examples in the *Student Companion* support the gradual release of responsibility. Learners can view a completed worked example and a video walkthrough of the worked example in the corresponding digital lesson and then apply the scaffolded steps themselves to solve a unique problem.



SC 2: I can apply irrational numbers in the solution of problems

#### Worked example: Calculating exact lengths that have irrational values and then give a decimal approximation

Working

Calculate the following lengths in exact form, then approximate the lengths correct to 1 decimal place.

(a) The exact circumference of a circle of diameter 8

Thinking	
Recall the formula for th a circle.	ne circumference of
Substitute the known va	lues and simplify.
Write the answer.	

(b) The exact length of the diagonal of a square of side length 6  ${
m cm}$ 

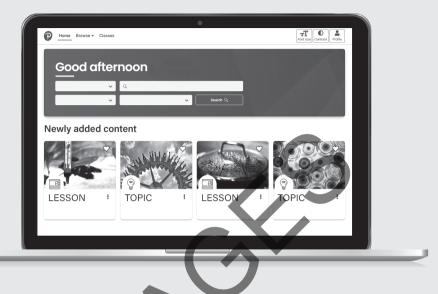
	Thinking	Working
ow	Draw a diagram showing a right-angled triangle, marking the length to be calculated as $x$ .	
e	Recall Pythagoras' theorem for right-angled triangles.	
ked	Substitute the known values and simplify.	
al	Take the square root of the number, ignoring the possibility of a negative value, since lengths are positive.	
	Write the answer.	
	• 1 For the circle shown, use the following rules.	
	(a) $C = 2\pi r$ to calculate the circumference	
	<b>(b)</b> $A = \pi r^2$ to calculate the area	
	Give your answers	
	(i) in exact form, and	
r	(ii) accurate to 2 decimal places.	
on iitive	(a)	(b)
n	( <u>6 cm</u>	
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Practice questions are provided in the Student Companion so that learners can apply the knowledge and skills obtained in the worked example given. These questions are designed to ensure learners build confidence and demonstrate efficiency. They foll on from the Check your understanding questions beside th corresponding work example in the digit lesson.

Each lesson in the Student Companion contains a space for students to reflect on their understanding. The simple and intuitive design of the **lesson reflection tool** allows students to scale their confidence, reflect on their learning and identify areas in which they need support.



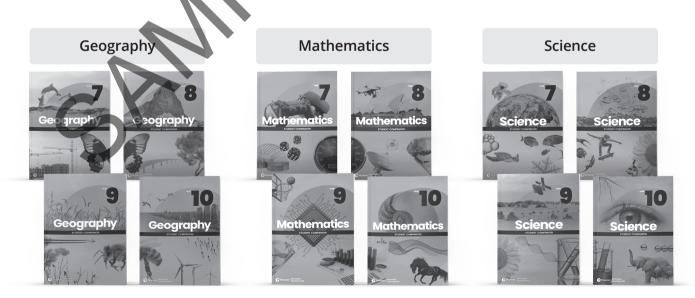
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#### **Real numbers**

#### Identify and define irrational numbers

Learning intention: To be able to identify and define irrational numbers

#### Success criteria:

- SC 1: I can define a rational number and determine whether a number is rational or irrational.
- SC 2: I can use a number line to indicate the solution interval for inequalities.

### SC 1: I can define a rational number and determine whether a number is rational or irrational

#### Worked example: Identifying rational numbers in a list

Label each of the following numbers as either rational or irrational.

 $\sqrt{64}$ , 15 $\pi$ , -5.27, 2.7,  $\sqrt{125}$  and  $\sqrt[3]{-27}$ 

Thinking	Working
Recall the definition of a rational number.	A number that is rational can be written in the form $\frac{a}{b}$ .
Examine the square roots.	
Examine the cube root.	
Identify any recurring and terminating decimals. Recurring and terminating decimals are rational.	
Identify any terms that contain transcendental numbers ( $\pi$ or $e$ ), which are irrational.	

1 Label each of the following numbers as rational or irrational. For the rational numbers, show the value expressed in the form  $\frac{a}{b}$ , where a and b are integers.

(a)	$5\frac{1}{3}$	(b)	√1000	(c)	∛800
(d)	58π	(e)	6.Ż	(f)	-\sqrt{400}
_		following numb	ers in the form $\frac{a}{2}$ thus c	onfirmi	ng they are rational. In eacl
Exp cas	press each of the se, express the fr	raction in simples	st form. $b^{\dagger}$	, on the second s	ng they are rational. In cuo

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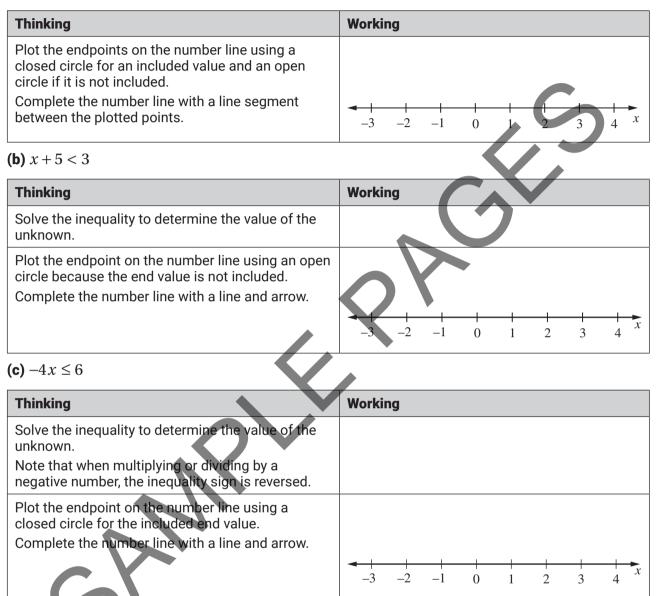
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#### SC 2: I can use a number line to indicate the solution interval for inequalities

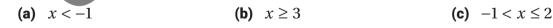
#### Worked example: Showing a solution interval on a number line

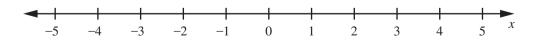
Show the following intervals on a number line.

(a) −2 ≤ *x* < 3

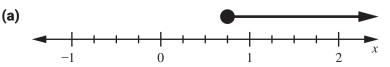


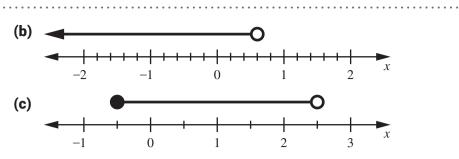
1 Draw and label the following solution intervals on the number line below.



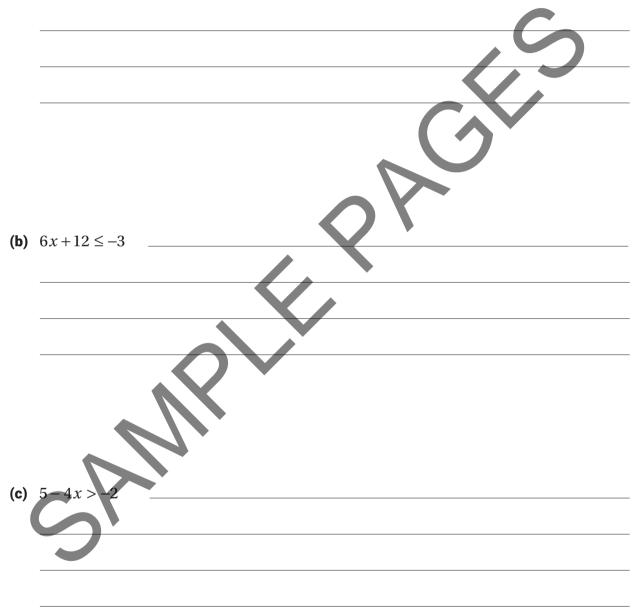


2 State the solution interval shown in each of the following diagrams.





- **3** Solve each of the following inequalities and show the solution interval on a number line.
  - (a)  $2x+1 \ge 4$



#### **Apply irrational numbers**

Learning intention: To be able to apply irrational numbers

#### Success criteria:

- SC 1: I can write approximate values for irrational numbers.
- SC 2: I can apply irrational numbers in the solution of problems.

#### SC 1: I can write approximate values for irrational numbers

# SC 1: I can write approximate values. Worked example: Writing a decimal approximation for an irrational number

Т	hinking	Working
ir E tl 5 E	lse your calculator to obtain a value for the rational number. xamine the third digit after the decimal point. If ne digit is 0–4, round down. If the digit is –9, round up. xamine the fifth digit after the decimal point nd round as before.	
1	Write each of the following irrational numbers	as a decimal correct to
	(i) 1 decimal place	
	(ii) 4 decimal places.	
	(a) $\sqrt{13}$ (b) $\sqrt{40}$	(c) $\sqrt[3]{16}$ (d) $\sqrt[3]{75}$
2	Write each of the following irrational numbers (i) 1 decimal place (ii) 4 decimal places.	as a decimal correct to
	(a) $5\pi + 9$	(b) $\frac{7\pi}{8}$
	(c) e+6	(d) $\frac{5e+2}{4}$
	RATE MY EARNING I need some help I am	getting there I get it I am confident

#### SC 2: I can apply irrational numbers in the solution of problems

#### Worked example: Calculating exact lengths that have irrational values and then give a decimal approximation

Calculate the following lengths in exact form, then approximate the lengths correct to 1 decimal place.

(a) The exact circumference of a circle of diameter 8 cm

Thinking	Working
Recall the formula for the circumference of a circle.	Co
Substitute the known values and simplify.	
Write the answer.	
(b) The exact length of the diagonal of a square of s	side length 6 cm
Thinking	Working
Draw a diagram showing a right-angled triangle, marking the length to be calculated as <i>x</i> .	
Recall Pythagoras' theorem for right-angled triangles.	
Substitute the known values and simplify.	
Take the square root of the number, ignoring the possibility of a negative value, since lengths are positive.	
Write the answer.	
1 For the circle shown, use the following rules.	
(a) $C = 2\pi r$ to calculate the circumference	
(b) $A = \pi r^2$ to calculate the area	
Give your answers	
(i) in exact form, and	
(ii) accurate to 2 decimal places.	

(a)		(b) 	
RATE MY LEARNING I need some help	I am getting there	l get it	I am confident

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# Plot and locate irrational numbers on a number line by estimation and by construction

**Learning intention:** To be able to plot and locate irrational numbers on a number line by estimation and by construction

#### Success criteria:

- SC 1: I can estimate the location of irrational numbers on a number line.
- SC 2: I can use construction techniques to accurately locate irrational numbers on a number line.

SC 1: I can estimate the location of irrational numbers on a number line

## Worked example: Plotting an approximate value for an irrational number on a number line

Write the value of  $2\pi$  correct to 2 decimal places and plot its position on a number line.

Thinking	Working
Write the approximate value of the irrational number.	
Identify more rounded values above and below the approximation.	
Draw a number line showing the approximate value.	

1 Plot each of the following irrational numbers on the number line below.

$$\sqrt{10}, \sqrt{5}, \frac{3\pi}{4}, \frac{e^2}{2}$$

- **2** List the following irrational numbers  $\sqrt{12}$ ,  $\sqrt[3]{-15}$ ,  $-\frac{5\pi}{2}$ ,  $\sqrt{14}$ ,  $-\sqrt[3]{20}$ ,  $\sqrt[3]{6}$ ,  $-\frac{3e}{2}$  in
  - (a) ascending order
  - (b) descending order.

## SC 2: I can use construction techniques to accurately locate irrational numbers on a number line

## Worked example: Using construction to show the location of an irrational number on a number line

(a) Construct a right-angled triangle on a number line and use the construction to plot the position of  $\sqrt{5}$ .

Thinking	Working
Write 5 as the sum of two square numbers.	
Use Pythagoras' theorem to determine the side lengths.	
Draw a right-angled triangle on grid paper, with base 2, height $1$ and hypotenuse $\sqrt{5}.$	
Set the compass radius to the length of the hypotenuse ( $\sqrt{5}$ units).	
With the compass point at 0, draw an arc from the vertex to the number line.	
Mark the point at which the arc intersects the number line as the location of $\sqrt{5}$ .	

(b) Use the construction in (a) to plot the position of  $\sqrt{10}$  on the number line.

Thinking	Working
Write 10 as the sum of two squares of either whole numbers, or lengths on the original construction.	
Draw a right-angled triangle on grid paper, with	
base $\sqrt{5}$ , height $\sqrt{5}$ and hypotenuse $\sqrt{10}.$	
Set the compass radius to the length of the	
hypotenuse ( $\sqrt{10}$ units).	
With the compass point at 0, draw an arc from the vertex to the number line.	
Mark the point at which the arc intersects the	▲ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
number line as the location of $\sqrt{10}$ .	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

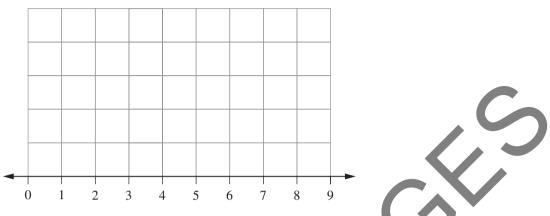
1 Write each of the following numbers as the sum of two positive square numbers, and hence write the side lengths of a right-angled triangle for which the lengths of the shorter sides are whole numbers of units and the length of the hypotenuse involves a surd.

(a)	26	(b)	32	(c)	34

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#### **Real numbers**

- **2** On the number line and grid below:
  - (a) construct a right-angled triangle with a hypotenuse of  $\sqrt{45}$  units and draw an arc to the number line
  - (b) use the construction in part (a) to form a right-angled triangle to enable  $\sqrt{61}$  to be plotted on the number line.



3 (a) Write your own instructions for how you can construct a length of  $\sqrt{130}$  cm using a ruler and a compass.

(b) Draw the construction from part (a).