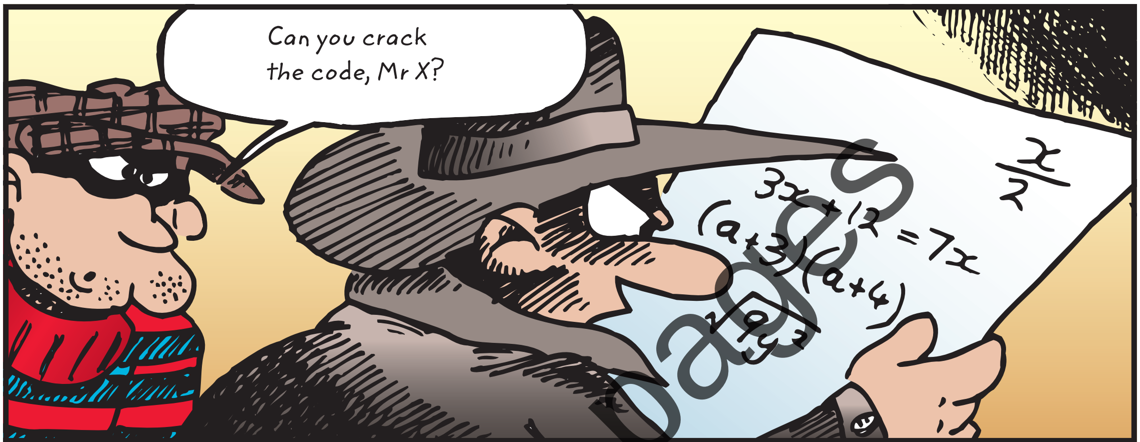
ALGEBRA AND EQUATIONS

2



Contents

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Algebraic Techniques*** and ***Equations*** [Stage 5.2]

- Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)
- Apply the four operations to algebraic fractions with pronumerals in the denominator (NSW)
- Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)
- Solve simple quadratic equations using a range of strategies (ACMNA241)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

Before starting this chapter it would be beneficial to review the algebra and equations met in Year 9 by completing sections 1:02, 1:05, 1:06 and 1:10.

2:01 Further algebraic fractions

PREP QUIZ 2:01

Rewrite these fractions in their simplest form.

1 $\frac{6}{8}$ 2 $\frac{4a}{5a}$ 3 $\frac{10x}{5xy}$ 4 $\frac{np}{mn}$ 5 $\frac{9p^2}{12pq}$

Simplify these expressions.

6 $\frac{5x}{6} \times \frac{3}{x}$ 7 $\frac{ay}{m} \div \frac{y}{am}$ 8 $\frac{x}{5} + \frac{3x}{5}$ 9 $\frac{n}{6} + \frac{n}{4}$ 10 $\frac{2p}{3} - \frac{p}{2}$

In Year 9 you were shown how to simplify algebraic fractions as well as how to perform the four basic operations.

The Prep quiz above should have reminded you of these skills. In this section we will extend the addition and subtraction of fractions to those with pronumerals in the denominator.

Addition and subtraction

When adding or subtracting two fractions you should remember this rule.

Rewrite each fraction as two equivalent fractions with a common denominator, then add or subtract the numerators.

WORKED EXAMPLE 1

If the denominators are the same, simply add or subtract the numerators.

a $\frac{2x}{5} + \frac{3x}{5} = \frac{2x + 3x}{5}$

$= \frac{{}^{1}\cancel{5}x}{{}_{1}\cancel{5}}$

$= x$

b $\frac{5}{a} - \frac{3}{a} = \frac{5 - 3}{a}$

$= \frac{2}{a}$

WORKED EXAMPLE 2

If the denominators are different, first find the lowest common multiple (LCM).

a $\frac{x}{4}+\frac{2x}{3}=\frac{x\times 3}{4\times 3}+\frac{2x\times 4}{3\times 4}$

$=\frac{3x}{12}+\frac{8x}{12}$

$=\frac{11x}{12}$

b $\frac{5}{2x}+\frac{2}{3x}=\frac{5\times 3}{2x\times 3}-\frac{2\times 2}{3x\times 2}$

$=\frac{15}{6x}-\frac{4}{6x}$

$=\frac{11}{6x}$

c $\frac{5n}{6}-\frac{3n}{4}=\frac{10n}{12}-\frac{9n}{12}$

$=\frac{n}{12}$

d $\frac{2a}{n}-\frac{3a}{4n}=\frac{8a}{4n}-\frac{3a}{4n}$

$=\frac{5a}{4n}$

Multiplication and division

When multiplying:
- cancel any common factors
- multiply the numerators together and multiply the denominators together.

When dividing:
- turn the second fraction upside down
- multiply as above (i.e. invert and multiply).

Remembering the index laws can also be useful.

When multiplying, add the indices.
e.g. $a^5\times a^3=a^{5+3}=a^8$

When dividing, subtract the indices.
e.g. $a^5\div a^3=a^{5-3}=a^2$

WORKED EXAMPLES

1 $\frac{2ab}{3}\times\frac{9}{4b}=\frac{{}^1\cancel{2}a\,{}^1\cancel{b}}{{}_1\cancel{3}}\times\frac{{}^3\cancel{9}}{{}_2\cancel{4}\,\cancel{b}_1}$

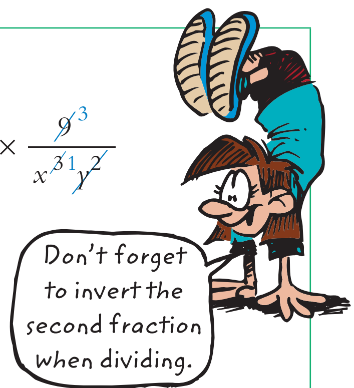
$=\frac{a\times 3}{1\times 2}$

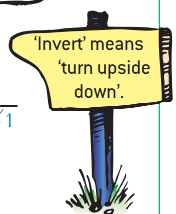
$=\frac{3a}{2}$

2 $\frac{x^2y^4}{6}\times\frac{9}{x^3y^2}=\frac{\cancel{x^2}\,y^{\cancel{4}\,2}}{{}_2\cancel{6}}\times\frac{\cancel{9}^3}{x^{\cancel{3}\,1}\,\cancel{y^2}}$

$=\frac{y^2\times 3}{2\times x}$

$=\frac{3y^2}{2x}$





3 $\frac{2mp}{5n}\div\frac{8p}{15mn}=\frac{{}^1\cancel{2}m\,\cancel{p}^1}{{}_1\cancel{5}\,\cancel{n}_1}\times\frac{{}^3\cancel{15}m\,\cancel{n}^1}{{}_4\cancel{8}\,\cancel{p}_1}$

$=\frac{m\times 3m}{1\times 4}$

$=\frac{3m^2}{4}$

4 $\frac{3x^2}{8y^5}\div\frac{15x^3}{4y}=\frac{{}^1\cancel{3}\,\cancel{x^2}}{{}_2\cancel{8}\,y^{\cancel{5}\,4}}\times\frac{{}^1\cancel{4}\,\cancel{y}}{{}_5\cancel{15}\,x^{\cancel{3}\,1}}$

$=\frac{1\times 1}{2y^4\times 5x}$

$=\frac{1}{10xy^4}$

Exercise 2:01

Foundation worksheet 2:01A & B
Simplifying algebraic fractions

1 Simplify the following.

a $\frac{3a}{2}+\frac{a}{2}$ **b** $\frac{3x}{5}-\frac{2x}{5}$ **c** $\frac{a}{3}+\frac{4a}{3}$ **d** $\frac{9m}{10}-\frac{3m}{10}$

e $\frac{x}{4}+\frac{y}{4}$ **f** $\frac{5a}{3}-\frac{2b}{3}$ **g** $\frac{2}{a}+\frac{3}{a}$ **h** $\frac{7}{x}+\frac{1}{x}$

i $\frac{3}{y}-\frac{2}{y}$ **j** $\frac{9}{m}-\frac{1}{m}$ **k** $\frac{5a}{x}+\frac{2a}{x}$ **l** $\frac{2x}{y}-\frac{3x}{y}$

m $\frac{5}{3n}+\frac{7}{3n}$ **n** $\frac{3}{2x}-\frac{1}{2x}$ **o** $\frac{8a}{5b}+\frac{2a}{5b}$ **p** $\frac{7m}{4x}-\frac{3m}{4x}$

2 Reduce each of these expressions to its simplest form.

a $\frac{x}{3}+\frac{x}{5}$ **b** $\frac{a}{2}+\frac{a}{5}$ **c** $\frac{y}{3}-\frac{y}{4}$ **d** $\frac{m}{2}-\frac{m}{4}$

e $\frac{2a}{3}+\frac{a}{2}$ **f** $\frac{5x}{3}+\frac{2x}{4}$ **g** $\frac{3n}{8}-\frac{n}{4}$ **h** $\frac{4p}{5}-\frac{3p}{10}$

i $\frac{x}{4}+\frac{y}{3}$ **j** $\frac{2a}{3}-\frac{3b}{2}$ **k** $\frac{3m}{5}-\frac{n}{2}$ **l** $\frac{k}{6}-\frac{2l}{4}$

m $\frac{2}{x}+\frac{4}{3x}$ **n** $\frac{1}{3a}+\frac{2}{4a}$ **o** $\frac{7}{2m}-\frac{2}{5m}$ **p** $\frac{5}{8x}-\frac{1}{2x}$

q $\frac{2a}{3x}+\frac{3a}{2x}$ **r** $\frac{x}{3m}-\frac{2x}{m}$ **s** $\frac{5m}{2n}+\frac{3m}{4n}$ **t** $\frac{2x}{3a}+\frac{y}{4a}$

3 Simplify these products.

a $\frac{x}{2}\times\frac{y}{3}$ **b** $\frac{a}{4}\times\frac{b}{3}$ **c** $\frac{m}{2}\times\frac{m}{5}$ **d** $\frac{a}{4}\times\frac{a}{10}$

e $\frac{3}{a}\times\frac{4}{m}$ **f** $\frac{2}{x}\times\frac{1}{y}$ **g** $\frac{1}{p}\times\frac{4}{p}$ **h** $\frac{1}{n}\times\frac{1}{3n}$

i $\frac{p}{q}\times\frac{x}{y}$ **j** $\frac{2}{a}\times\frac{a}{4}$ **k** $\frac{m}{5}\times\frac{10}{n}$ **l** $\frac{3x}{5}\times\frac{2}{9x}$

m $\frac{ab}{3}\times\frac{2}{b}$ **n** $\frac{x}{y}\times\frac{y}{x}$ **o** $\frac{6m}{5a}\times\frac{15a}{2m}$ **p** $\frac{8x}{5p}\times\frac{2a}{3x}$

4 Simplify these divisions.

a $\frac{m}{2}\div\frac{m}{4}$ **b** $\frac{n}{3}\div\frac{n}{5}$ **c** $\frac{5a}{3}\div\frac{2a}{9}$ **d** $\frac{x}{5}\div\frac{3x}{10}$

e $\frac{5}{a}\div\frac{2}{a}$ **f** $\frac{3}{2m}\div\frac{1}{3m}$ **g** $\frac{a}{b}\div\frac{2a}{b}$ **h** $\frac{3x}{5y}\div\frac{x}{10y}$

i $\frac{a}{b}\div\frac{x}{y}$ **j** $\frac{2p}{3q}\div\frac{8p}{9q}$ **k** $\frac{10k}{3n}\div\frac{2k}{9n}$ **l** $\frac{a}{2}\div\frac{a}{3}$

m $\frac{xy}{2}\div\frac{y}{4}$ **n** $\frac{b}{2}\div\frac{ab}{6}$ **o** $\frac{xy}{c}\div\frac{y}{cx}$ **p** $\frac{9a}{b}\div\frac{4a}{3b}$

Simplify these expressions.

5

a $\frac{a}{3} \times \frac{12}{5a}$　b $\frac{2}{p} \times \frac{p}{3}$　c $\frac{15}{x} \div 5$　d $3b \div \frac{6}{b}$

e $\frac{xy}{z} \times \frac{2z}{x}$　f $\frac{ab}{c} \div \frac{a}{c}$　g $\frac{9m}{2} \times \frac{4m}{3}$　h $\frac{2x}{y} \div \frac{x}{2y}$

i $\frac{4}{pq} \times \frac{p}{q}$　j $\frac{3}{a} \times \frac{2}{b}$　k $\frac{4ab}{x} \times \frac{xy}{2ac}$　l $\frac{9bc}{2a} \div \frac{6b}{4a}$

m $\frac{2}{x} \times \frac{x}{3} \times \frac{9}{4}$　n $\frac{b}{c} \times \frac{c}{a} \times \frac{a}{b}$　o $\frac{8bc}{3a} \times \frac{9a}{b} \times \frac{1}{4c}$　p $\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3}$

6

a $\frac{y^5}{3} \times \frac{6}{y^2}$　b $\frac{a^4}{8} \times \frac{6}{a^6}$　c $\frac{m^2}{4n^5} \times \frac{10n^6}{5m^3}$

d $\frac{x^2y^3}{12} \times \frac{6}{x^3y^2}$　e $\frac{a^3b^2}{3a} \times \frac{6b}{b^5}$　f $\frac{5m^4}{3n^3} \times \frac{12n^6}{15m^2}$

g $\frac{4a^3}{x^2z^3} \times \frac{6x^5z^3}{3a^2}$　h $\frac{k^3n^2}{5k^5} \times \frac{10kn}{n^5}$　i $\frac{pq^2}{qr^3} \times \frac{pr^6}{p^2q^2r^2}$

7

a $\frac{3x^5}{2} \div \frac{9x^3}{8}$　b $\frac{5m^2}{6} \div \frac{10m^4}{9}$　c $\frac{3}{2y^3} \div \frac{5}{4y^6}$

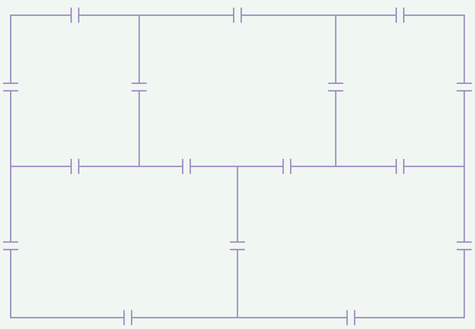
d $\frac{4x^6}{3y^5} \div \frac{5x^2}{6y}$　e $\frac{5a^3}{3b^4} \div \frac{10a^2}{6b}$　f $\frac{p^6}{4q^5} \div \frac{5p^2}{6q^4}$

g $\frac{x^6y^2}{z^3} \div \frac{x^4}{yz^5}$　h $\frac{fg^3}{3h^5} \div \frac{f^3g}{6h^2}$　i $\frac{5ax^3}{4by^5} \div \frac{15b^2x^2}{8ay^7}$

FUN SPOT 2:01 AN A-MAZE-ING HOUSE!

The diagram shows the plan of a house with five rooms. There is a doorway between adjacent rooms as well as nine doors opening to the outside.

Is it possible to walk through each and every door of the house without going through any door twice? You can start anywhere, inside or outside the house.



2:02 Expanding and factorising

PREP QUIZ 2:02

Expand: **1** $3(a + 5)$ **2** $m(m - 6)$ **3** $2y(3y + z)$ **4** $-5(4x - 1)$

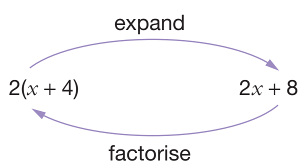
Simplify: **5** $p^3 \times p^4$ **6** $5x^2 \times 3x^5$ **7** $3m^2n^4 \times 4mn^3$

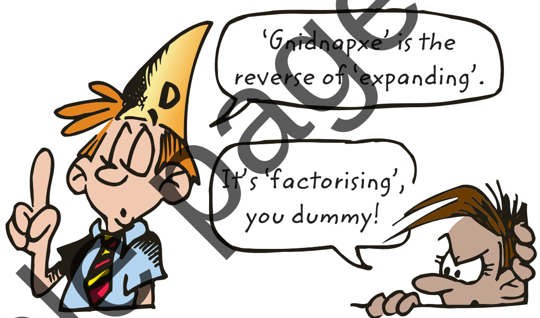
What is the highest common factor (HCF) of:

8 12 and 20 **9** $5ab$ and $10bc$ **10** x^2y^5 and x^3y^4

The Prep quiz above should have reminded you about expanding algebraic expressions. Each number inside the grouping symbols is multiplied by the term outside.

If we expand the expression $2(x + 4)$, we obtain $2x + 8$. To factorise $2x + 8$, we simply reverse this procedure. We notice that 2 is the highest common factor of $2x$ and 8, so 2 is written outside the parentheses and the remainder is written inside the parentheses: $2x + 8 = 2(x + 4)$.





This section is a review of these two skills that are needed throughout this chapter. The worked examples below also include expressions that involve skills used when multiplying with indices.

WORKED EXAMPLE 1

Expand, and simplify, each expression.

a $x(x + 5) = x \times x + x \times 5$
$= x^2 + 5x$

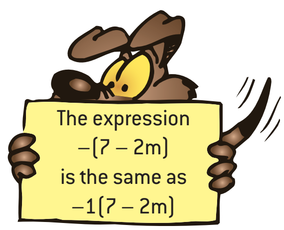
b $5a(3a - 2b) = 5a \times 3a - 5a \times 2b$
$= 15a^2 - 10ab$

c $3p(p + 4) - 5(2p - 3) = 3p \times p + 3p \times 4 - 5 \times 2p - 5 \times (-3)$
$= 3p^2 + 12p - 10p + 15$
$= 3p^2 + 2p + 15$

d $y^2(y^3 + 5) = y^5 + 5y^2$

e $4n^2m^3(2n^3 - 5m^4) = 8n^5m^3 - 20n^2m^7$

f $w^3(w^2 - 4) - (w^3 + 2) = w^5 - 4w^3 - w^3 - 2$
$= w^5 - 5w^3 - 2$



WORKED EXAMPLE 2

Factorise these expressions by taking out the highest common factor.

a $9a + 12b = 3 \times 3a + 3 \times 4b$ HCF = 3
$= 3(3a + 4b)$

b $10x^2 - 5xy = 5x \times 2x - 5x \times y$ HCF = $5x$
$= 5x(2x - y)$

c $6m^2 - 9m + 3mn = 3m(2m - 3 + n)$

d $-p^2 + 5pq = -p \times p - (-p) \times 5q$ HCF = $-p$
$= -p(p - 5q)$

e $x^5 + 6x^2 = x^2(x^3 + 6)$ HCF = x^2

f $4m^5n^3 - 6m^2n^6 = 2m^2n^3(2m^3 - 3n^3)$ HCF = $2m^2n^3$

g $3x^2y^3 - 12x^3y^4 + 9x^4y^2 = 3x^2y^2(y - 4xy^2 + 3x^2)$ HCF = $3x^2y^2$

Exercise 2:02

P Foundation worksheet 2:02A
Grouping symbols
Foundation worksheet 2:02B
Common factors

1 Expand the following.

a $a(a + 7)$
b $x(x - 1)$
c $y(2y + 7)$
d $5p(p - 4)$
e $6z(3 + 7z)$
f $2a(3a - 1)$
g $4k(2k + 3m)$
h $8m(n - 3m)$
i $-2(x + 5)$
j $-7(y - 5)$
k $-(m + 4)$
l $-4(3k - 1)$
m $-p(p + 1)$
n $-x(x - y)$
o $-2z(3z + 5)$
p $-ab(a - b)$

2 Simplify:

a $2(a + 3) + 5a + 2$
b $3(x + 5) + 7x - 8$
c $5(y - 2) + 3y + 7$
d $4(a - 1) + 6a - 5$
e $3(p + 2) - 2p + 4$
f $10(m + 3) - 11m - 15$
g $5a + 6 + 2(a + 7)$
h $2x + 7 + 5(x - 1)$
i $7n - 4 + 3(n - 1)$
j $4h - 1 + 7(h + 2)$
k $6x + 2(x + 1) + 5$
l $4y + 6(y + 2) - 10$
m $3a + 10 - 2(a + 1)$
n $10m + 4 - 5(m + 4)$

3 Simplify each expression by expanding the grouping symbols and then collecting like terms.

a $3(x - 2) - 2(x + 1)$
b $5(y + 2) + 3(y + 4)$
c $2(a - 1) + 5(a - 1)$
d $8(m - 3) + 5(m + 2)$
e $4(3x + 2) + 5(x - 4)$
f $6(x + 7) + 2(2x - 1)$
g $5(x - 7) - 3(x - 4)$
h $6(m + 1) - 3(m + 2)$
i $9(a + 5) - 7(a - 3)$
j $5(n - 5) - 3(n + 7)$
k $x(x + 3) + 3(x + 1)$
l $a(a + 3) + 7(a - 3)$
m $m(m + 3) - 4(m + 3)$
n $t(t - 5) - 4(t - 5)$
o $a(a + 2b) + a(2a + b)$
p $x(x - y) + y(x + y)$

4 Expand and simplify:

a $x^2(x^2 - 1)$
b $a^3(5 - a^2)$
c $a^2(5a - a^3)$
d $x(x^2 + y)$
e $m(7 - m^2)$
f $y(y^2 - xy)$
g $3a^2(2a^3 + 3a)$
h $5x(3x^2 - x)$
i $2m^3(n^2 - m^2)$
j $x(5x^2 - 3x + 7)$
k $x^2(2x^2 + 7x - 14)$
l $y(y^2 - 7y - 1)$
m $a^2b^3(a^3 - b^4)$
n $2x^2y(2x^3 - 3y^4)$
o $6nm^3(2n^3m - 5nm^4)$
p $x^3(x^2 - 3) + 4(x^3 + 2)$
q $m^3(m^3 - 1) - m^4(m^2 + 2)$
r $z^5(z^2 - 4) - (z^5 - 1)$

5 Factorise fully the following expressions.

a $9x + 6$
b $10 + 15a$
c $4m - 6n$
d $x^2 + 7x$
e $2a^2 - 3a$
f $12y - 6y^2$
g $ab - bx$
h $st - s$
i $4ab + 10bc$
j $-4m + 6n$
k $-x^2 - 3x$
l $-15a + 5ab$
m $3x + x^2 - ax$
n $ax + ay + az$
o $4m - 8n + 6p$
p $5ab - 15ac + 10ad$
q $x^2 - 7x + xy$
r $a(a + 3) - (a + 3)$

6 Using the rule for multiplying with indices, factorise the following expressions.

a $x^3 + 4x^2$
b $a^5 - 6a^3$
c $p^6 + p^4$
d $y^5 - 3y^2$
e $2x^3 + 6x^4$
f $5a^5 - 10a^4$
g $6p^6 + 3p$
h $5y - 3y^2$
i $x^3y^2 + 4x^2y$
j $a^5b^3 - a^3b^5$
k $p^6q^2 + p^4q^3$
l $y^5z - y^2z^4$
m $3m^3n + 9m^2n^2$
n $8a^5b^3 - 6a^3b$
o $9p^6q^5 + 12p^3q^5$
p $10xy^5 - 4x^4y^2$

7 Factorise these expressions.

a $x^2y^3 + x^3y^2 + x^4y^4$
b $3a^2b^3 - 6a^3b^5 + 9a^4b^4$
c $8m^3n^2 + 12m^5n + 4m^4n^2$
d $5x^2z^2 - 10x^3z^3 + 15x^4z^4$
e $x^2y^3z^4 - x^3y^4z^2 + x^4y^2z^3$
f $9a^2b^3c^5 + 3a^5b^4c^3 + 6a^4b^2c^4$

CHALLENGE 2:02 GROUPING IN PAIRS

For some algebraic expressions there may not be a factor common to every term. For example, there is no factor common to every term in the expression:

$$3x + 3 + mx + m$$

The first pair of terms have a common factor of 3 and the second pair of terms have a common factor of m. So:

$$3x + 3 + mx + m = 3(x + 1) + m(x + 1)$$

Now, it can be seen that $(x + 1)$ is a common factor for each term.

$$3(x + 1) + m(x + 1) = (x + 1)(3 + m)$$

Therefore:

$$3x + 3 + mx + m = (x + 1)(3 + m)$$

The original expression has been factorised by grouping the terms in pairs.

Exercises

1 Complete the factorisation of each expression.

a $5(a + 1) + b(a + 1)$
b $4(y - 3) + x(y - 3)$
c $p(q + 7) - 5(q + 7)$
d $m(p + q) + n(p + q)$
e $a(a - b) + b(a - b)$
f $2x(y + 3) - (y + 3)$

$$ab + ac + bd + cd = a(b + c) + d(b + c)$$
$$= (b + c)(a + d)$$

2 Factorise these expressions.

a $6x + 6 + ax + a$
b $8p - 8q + mp - mq$
c $ab + 3bc + 5a + 15c$
d $x^2 + xy + xz + yz$
e $ab + b + 4a + 4$
f $12m^2 + 16m + 3mn + 4n$
g $mn - m + n - 1$
h $x^3 + x^2 + x + 1$
i $a^2 + bc + ac + ab$

These will be treated further in Chapter 10.

2:03 Binomial products

PREP QUIZ 2:03

Simplify:
1. $5x + 7x$
2. $2a - a$
3. $x^2 + 3x - 5x + 3$

Expand:
4. $2(x + 5)$
5. $x(x - 2)$
6. $-3(a + 1)$
7. $-y(5 - y)$

Expand and simplify:
8. $x(x + 1) + 3(x + 1)$
9. $5(a + 5) - a(a + 5)$
10. $2x(3x - 2) - 5(3x + 2)$

A binomial expression contains two terms, e.g. $2x - 7$ or $a + b$.

A binomial product is the product of two such expressions, e.g. $(2x - 7)(a + 5)$.

Multiplying binomial expressions

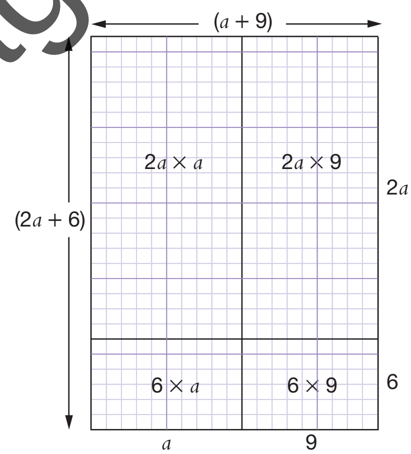
The expansion of binomial products may be demonstrated by considering the area of a rectangle with dimensions $(2a + 6)$ and $(a + 9)$.

- The area of the whole rectangle must be equal to the sum of the four smaller areas.
- Area $= (2a + 6)(a + 9)$

$$= 2a(a + 9) + 6(a + 9)$$
$$= 2a^2 + 18a + 6a + 54$$
$$= 2a^2 + 24a + 54$$

- We can see that the product of two binomials has four terms. Often two of these may be added together to simplify the answer.

(2a + 6)(a + 9) by areas



WORKED EXAMPLES

1 $(a + 2)(b + 4) = a(b + 4) + 2(b + 4)$
$= ab + 4a + 2b + 8$

2 $(a - 2)(a + 7) = a(a + 7) - 2(a + 7)$
$= a^2 + 7a - 2a - 14$
$= a^2 + 5a - 14$

3 $(x + 2y)(2x + y) = x(2x + y) + 2y(2x + y)$
$= 2x^2 + xy + 4xy + 2y^2$
$= 2x^2 + 5xy + 2y^2$

4 $(1 - x)(x - 3) = 1(x - 3) - x(x - 3)$
$= x - 3 - x^2 + 3x$
$= 4x - x^2 - 3$

You should notice that each term in the first binomial is multiplied by each term in the second.

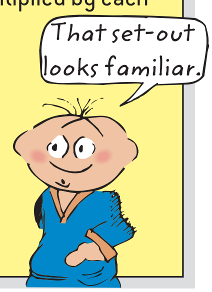
$2x^2$ -15

$(x + 5)(2x - 3)$

$10x$

$-3x$

$= 2x^2 + 10x - 3x - 15$
$= 2x^2 + 7x - 15$



$$(a + b)(c + d) = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$

Exercise 2:03

1 Expand the following binomial products.

a $(a+2)(b+3)$ b $(x+1)(y+4)$ c $(m+7)(n+5)$ d $(a+3)(x+2)$
e $(p+5)(q+4)$ f $(2x+1)(y+3)$ g $(a+6)(3p+2)$ h $(4x+1)(2y+3)$
i $(3a+1)(2b-7)$ j $(7x+5)(2p+1)$ k $(5p+3)(x-4)$ l $(2x+y)(a+2b)$

2 Expand the following and collect the like terms.

a $(a+2)(a+3)$ b $(x+1)(x+5)$ c $(n+3)(n+4)$ d $(p+2)(p+5)$
e $(m+1)(m-3)$ f $(y+7)(y-2)$ g $(x+1)(x-6)$ h $(t+2)(t-4)$
i $(x-2)(x-4)$ j $(n-7)(n-1)$ k $(a-6)(a-3)$ l $(x-10)(x-9)$
m $(y-11)(y+7)$ n $(a-2)(a+1)$ o $(x-8)(x-8)$ p $(m-9)(m-2)$
q $(a-3)(a+3)$ r $(x-7)(x+3)$ s $(y+12)(y+5)$ t $(a-8)(a+8)$
u $(q+5)(q+5)$ v $(x-1)(x-9)$ w $(t+3)(t+10)$ x $(k-8)(k+11)$

3 Find these products and simplify.

a $(a+3)(2a+1)$ b $(2x+1)(x+2)$ c $(3m+2)(m+5)$ d $(y+3)(4y+1)$
e $(2x+1)(2x+3)$ f $(3n+2)(2n+1)$ g $(2x+3)(4x+3)$ h $(5t+2)(2t+3)$
i $(2x-2)(5x-1)$ j $(8p+1)(3p-2)$ k $(5m-2)(2m-5)$ l $(3q+1)(7q-2)$
m $(3x+2)(6x-2)$ n $(2n+3)(2n-3)$ o $(8y-1)(8y+1)$ p $(3k-2)(5k-3)$
q $(7p-1)(7p-1)$ r $(3x-1)(5x-3)$ s $(5x+4)(5x+4)$ t $(9y-4)(3y+2)$
u $(5p+2)(p-7)$ v $(10q-1)(q-10)$ w $(4a+3)(3a+4)$ x $(7p+5)(7p-5)$

4 Expand and simplify:

a $(3+x)(4+x)$ b $(5-a)(2-a)$ c $(7+m)(1-m)$
d $(3-n)(3+n)$ e $(4+y)(y+5)$ f $(x-7)(5-x)$
g $(9+k)(k+10)$ h $(2a+1)(3+a)$ i $(3n+1)(7-2n)$
j $(x+y)(x+2y)$ k $(2n+m)(n+2m)$ l $(a-b)(2a+3b)$
m $(2p-q)(2p+q)$ n $(3x+y)(2x-5y)$ o $(3a+2b)(2a+3b)$
p $(9w-5x)(9w-5x)$

2:04 Factorising quadratic trinomials

PREP QUIZ 2:04

Expand: **1** $(x+2)(x+3)$ **2** $(a-1)(a+3)$ **3** $(m-7)(m-2)$
4 $(x+5)^2$ **5** $(a-2)^2$

Find two numbers a and b where:

6 $a+b=5$ and $ab=6$ **7** $a+b=9$ and $ab=20$
8 $a+b=-2$ and $ab=-15$ **9** $a+b=3$ and $ab=-4$
10 $a+b=7$ and $ab=-18$

ASSIGNMENT 2A Chapter review

1 Simplify each expression.

a $\frac{2x}{5} + \frac{x}{5}$

b $\frac{2a}{3} - \frac{5a}{9}$

c $\frac{7x}{8} + \frac{x}{6}$

d $\frac{6}{n} - \frac{5}{n}$

e $\frac{2}{3y} + \frac{3}{2y}$

f $\frac{4a}{5w} - \frac{2a}{3w}$

g $\frac{2p}{3} \times \frac{q}{4}$

h $\frac{2a}{3b} \times \frac{6b}{5a}$

i $\frac{2x}{5} \div \frac{4x}{5}$

j $\frac{7n}{5m} \div \frac{3n}{10m}$

k $\frac{k^4t}{12} \times \frac{9}{k^3t^2}$

l $\frac{4a^3b^4}{c^3} \times \frac{2c^2}{3a^2b^5}$

m $\frac{4x^4}{5y^3} \div \frac{6x^3}{15y}$

n $\frac{8p^4q^7}{5q^4} \div \frac{6p^3q^2}{10p}$

2 Expand, and simplify where possible.

a $(x - 1)(x + 2)$

b $5x + 3(x - 1)$

c $2(x + 3) - 2x - 3$

d $(2x + 1)(x - 7)$

e $(x + 5)(x - 5)$

f $(3x + 2)^2$

g $x(x - 3) + 2(x + 1)$

h $(2 - x)(3 - x)$

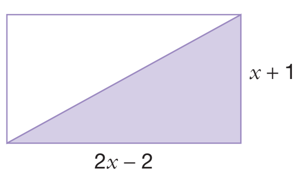
i $(x + y)(y - x)$

j $(2x - y)^2$

k $x^3(x^3 - 1)$

l $3x^2y^4(2xy^3 + 5x^3y)$

3 Find an expression for the shaded area of this rectangle. Expand and simplify your answer.



4 Take out the highest common factor to factorise these expressions.

a $m^3 + 4m^2$

b $4n^5 - 6n$

c $a^3b^2 - a^2b^3$

d $9p^6q^5 + 6p^5q^6 - 12p^4q^4$

5 Factorise these expressions.

a $10x^2 - 5x$

b $a^2 + ab - ac$

c $y^2 + 9y + 20$

d $n^2 - n - 12$

e $8 - 6k + k^2$

f $a(a + 4) - 5(a + 4)$

g $x^3 + 5x^2 + 6x$

6 Solve these equations.

a $a^2 - 36 = 0$

b $4t^2 - 9 = 0$

c $m^2 + 25 = 0$

d $(x + 2)(x - 3) = 0$

e $(n - 5)(n + 5) = 0$

f $(x - 3)^2 = 0$

g $5w^2 - 10w = 0$

h $x^2 - 7x + 12 = 0$

i $y^2 - 4y - 45 = 0$

j $15 + 8p + p^2 = 0$

7 a Rearrange $10 + 3y - y^2 = 0$ in the form $0 = y^2 - 3y - 10$, and then solve the equation.

b Solve the equation:

$42 - m - m^2 = 0$

8 Rearrange each equation and solve.

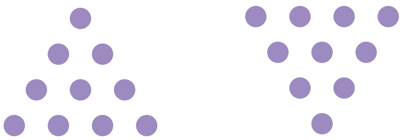
a $y^2 = 4y + 21$

b $n^2 + 2n = 80$

c $z + 110 = z^2$

d $x^2 + 5x = 2x + 70$

ASSIGNMENT 2B Working mathematically

1 

Move three dots in the diagram on the left to obtain the diagram on the right.

2 A ladder hangs over the side of a ship. The rungs in the ladder are each 2·5 cm thick and are 18 cm apart. The fifth rung from the bottom of the ladder is just above the water level. If the tide is rising at a rate of 15·5 cm per hour, how many rungs will be under water in 3 hours?

3 A set of Australian coins consists of a 10c, 20c, 50c, \$1 and \$2 coin. How many different sums of money can be obtained by taking any three of the coins?

4 Roger started a trip into the country between 8 am and 9 am, when the hands of the clock were together. He arrived at his destination between 2 pm and 3 pm, when the hands of the clock were exactly 180° apart. For how long did he travel?

5 What is the smallest whole number that, when multiplied by 7, will give you an answer consisting entirely of 8s?

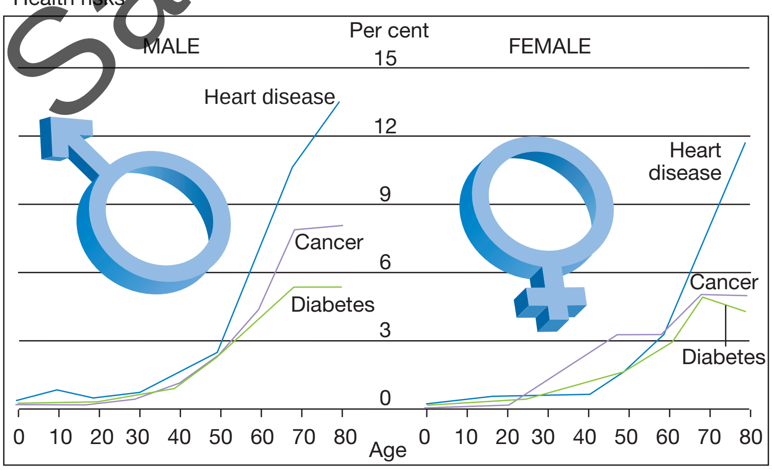
6 a From the data in the graph below, who has the greater chance of having heart disease: a 60-year-old woman or a 60-year-old man?

b Who has the greater chance of having cancer: a 50-year-old woman or a 50-year-old man?

c Which of the three diseases reveals the greatest gender difference for the age range of 20 to 50 years?

d Would the number of 80-year-old men suffering from heart disease be greater or less than the number of 80-year-old women suffering from heart disease? Give a reason for your answer.

Health risks



Source: Australian Institute of Health and Welfare

ASSIGNMENT 2C Cumulative revision

1 **a** Find 15% of \$125. 1:01D
b What percentage is \$25 of \$125?
c Decrease a price of \$125 by 30%.
d 15% discount of a price is equal to \$24. What is the full price?

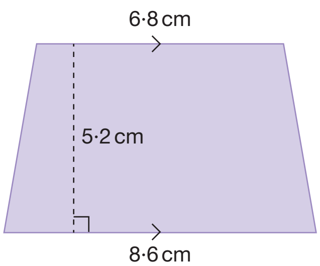
2 Simplify these ratios. 1:01E
a $20:45$ **b** $3{\cdot}5:4{\cdot}2$ **c** $2\frac{1}{3}:1\frac{1}{2}$

3 Simplify the following expressions. 1:02
a $5x - 2y - x + y$ **b** $6ab \times 3ac$ **c** $10ax \div 5a$ **d** $\frac{2a}{3} + \frac{3a}{5}$

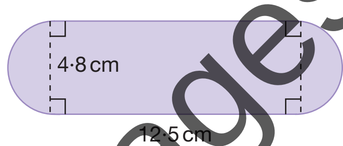
4 A card is drawn from a standard pack of 52 playing cards. 1:03
What is the probability the card is:
a red **b** a club **c** a Jack **d** the 7 of spades?

5 Find the area of each shape. 1:04

a



b

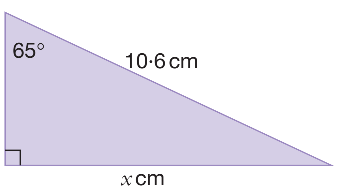


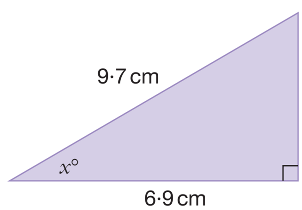
6 Evaluate: 1:05
a $5^2 \times 2^5$ **b** $4^5 \div 4^4$ **c** 4^{-2} **d** $(2^3)^{-1}$

7 Solve these equations: 1:06
a $9m - 4 = 14$ **b** $5x - 7 = 3x - 1$ **c** $6(2a + 3) = 2(5a + 11)$

8 Yvonne is paid an hourly rate of \$28.40 for a 36 hour week. The first 6 hours overtime are paid at time-and-a-half; after that extra hours worked are paid double-time. Find Yvonne's wage for a week in which she works 45 hours. 1:07

9 For these triangles, find: 1:11
a the value of x to one decimal place **b** the value of x to the nearest degree.





10 For the set of scores 3 5 4 7 5 4 8 3 4, find the: 1:12
a range **b** mode **c** median **d** mean **e** Q_1 **f** Q_3
g interquartile range.