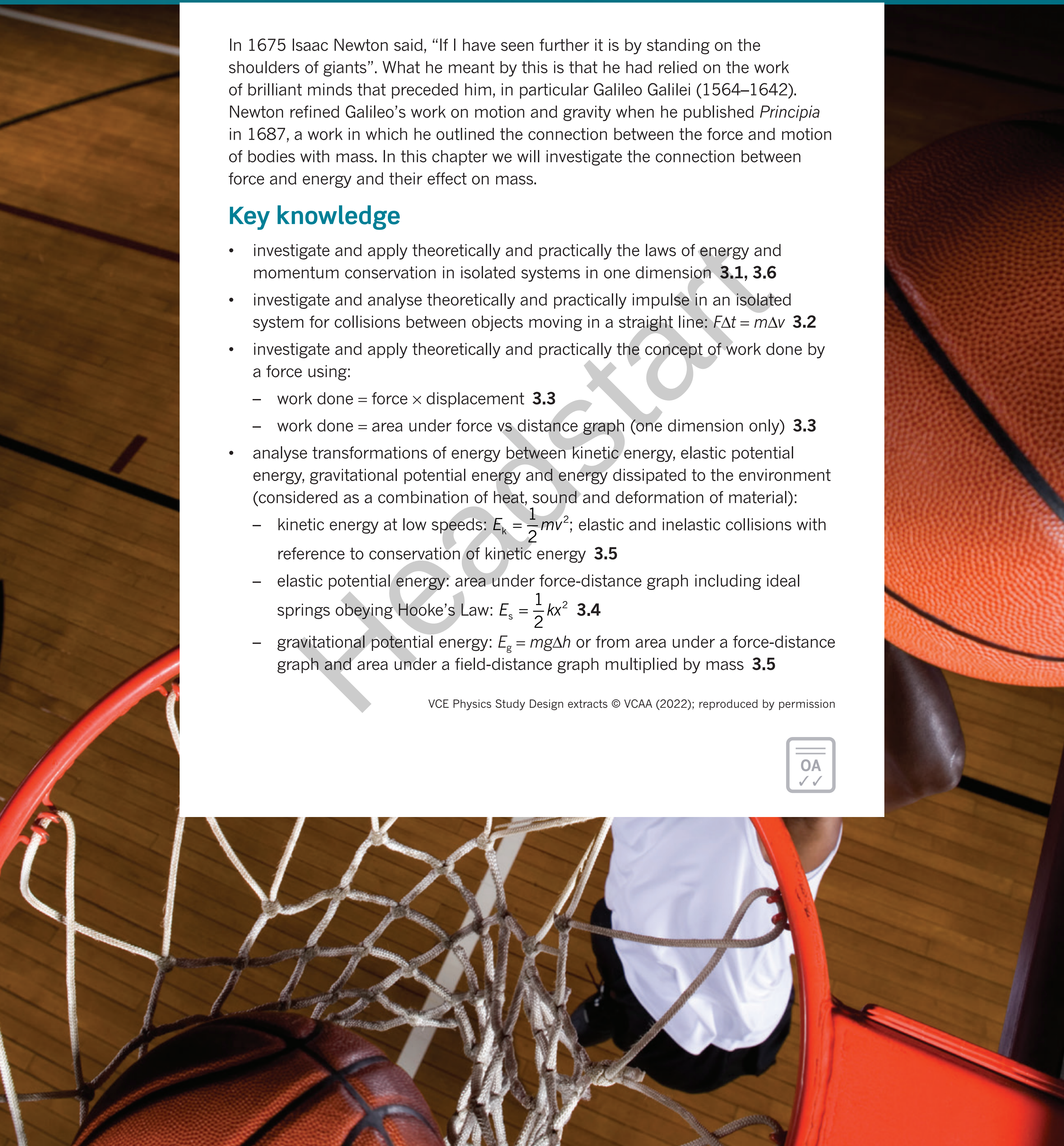
CHAPTER

03 The relationship between force, energy and mass



In 1675 Isaac Newton said, “If I have seen further it is by standing on the shoulders of giants”. What he meant by this is that he had relied on the work of brilliant minds that preceded him, in particular Galileo Galilei (1564–1642). Newton refined Galileo’s work on motion and gravity when he published *Principia* in 1687, a work in which he outlined the connection between the force and motion of bodies with mass. In this chapter we will investigate the connection between force and energy and their effect on mass.

Key knowledge

- investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension **3.1, 3.6**
- investigate and analyse theoretically and practically impulse in an isolated system for collisions between objects moving in a straight line: $F\Delta t = m\Delta v$ **3.2**
- investigate and apply theoretically and practically the concept of work done by a force using:
  - work done = force × displacement **3.3**
  - work done = area under force vs distance graph (one dimension only) **3.3**
- analyse transformations of energy between kinetic energy, elastic potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
  - kinetic energy at low speeds: $E_k = \frac{1}{2}mv^2$; elastic and inelastic collisions with reference to conservation of kinetic energy **3.5**
  - elastic potential energy: area under force-distance graph including ideal springs obeying Hooke’s Law: $E_s = \frac{1}{2}kx^2$ **3.4**
  - gravitational potential energy: $E_g = mg\Delta h$ or from area under a force-distance graph and area under a field-distance graph multiplied by mass **3.5**

3.1 Conservation of momentum

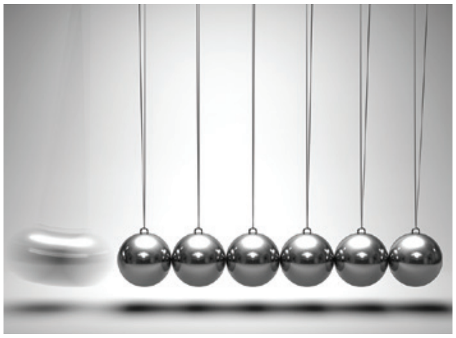


FIGURE 3.1.1 Newton's cradle is a popular illustration of almost perfect conservation of momentum. As the raised sphere collides with the other spheres, the sphere's momentum is passed on until the final sphere continues with the same momentum as the original.

Where there are moving objects, there are bound to be collisions. These can range from the interaction of sub-atomic particles to events on a galactic scale. Newton's cradle (Figure 3.1.1) provides another example of a collision. It also provides a demonstration of the law of conservation of momentum, a powerful tool with which to analyse collisions.

THE LAW OF CONSERVATION OF MOMENTUM

The product of the mass of an object and its velocity is called its **momentum**, and is given by the following equation.

$p = mv$

where p is momentum (kg m s^{-1})

m is the mass of the object (kg)

v is the velocity of the object (m s^{-1})

Given that velocity is a vector quantity and momentum is calculated from velocity, it follows that momentum is also a vector quantity.

The **law of conservation of momentum** states that in any collision or interaction between two or more objects in an isolated system, the total momentum of the system will be **conserved** (that is, it will remain constant). The total initial momentum is equal to the total final momentum.

sum of the initial momentum ($\Sigma p_{\text{initial}}$) = sum of the final momentum (Σp_{final})

where Σ is the mathematical symbol representing the addition of each factor

Hence $\Sigma p_{\text{initial}}$ is the sum of the initial momentum of each object in the system and Σp_{final} is the sum of the final momentum of each object in the system.

Another way of putting this is that the total change in momentum of the system is zero. This is often found by adding up the change in momentum of all the parts of the system:

$$\Sigma \Delta p = 0$$

In physics, a collision refers to a situation where two objects interact and exert action–reaction forces on each other. They do not have to make physical contact. For instance, two identically charged particles could approach and repel one another, moving off in opposite directions without ever making physical contact. This is still considered a collision and the law of conservation of momentum still applies.

Note that the law refers to objects in an **isolated system**. For a system to be isolated, there are only internal forces acting between the objects, with no interaction with any objects outside the system. In reality, perfectly isolated systems cannot exist on Earth because of friction and gravity. There are, however, many situations where treating a system as isolated is a useful approximation.

In the rear-end collision between a car and a bus examined in Worked example 3.1.1 on the next page friction is relatively small compared to the forces exerted by the vehicles on one another. Therefore the vehicles can be treated as an isolated system.

PHYSICSFILE

Discovering the neutrino

Conservation of momentum helped scientists discover the neutrino. In the 1920s, it was observed that in beta decay a nucleus emitted a beta particle. However, when the nucleus recoiled, it was not in the exact opposite direction to the emitted electron. Thus the momentum of these particles did not appear to comply with the law of conservation of momentum. In 1930, Wolfgang Pauli proposed that another particle must have also been emitted in order to conserve the total momentum of the system. This particle, the neutrino, was not detected experimentally until 1956. As you read this, billions of neutrinos originating from the Sun are passing through your body and the Earth.

Worked example 3.1.1

CONSERVATION OF MOMENTUM

In a head-on collision on a demolition derby track, a car of mass 1000 kg travelling east at 20.0 m s^{-1} crashes into a mini-bus of mass 5000 kg travelling west at 8.00 m s^{-1}. Assume that the car and mini-bus lock together on impact and ignore the effect of friction.

a Calculate the final common velocity of the vehicles.

Thinking	Working
First assign a direction that will be considered positive. Note: as long as directions are assigned positive or negative consistently in the same problem, it does not matter which direction is assigned positive.	In this case we will consider vectors directed eastwards to be positive. $m_c = 1000$ kg $u_c = 20.0$ m s^{-1} $m_b = 5000$ kg $u_b = -8.00$ m s^{-1}
Apply the law of conservation of momentum.	$\Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$ $m_c u_c + m_b u_b = (m_c + m_b)v$ $(1000)(20.0) + (5000)(-8.00) = (1000 + 5000)v$ $(-20\,000) = (6000)v$ $v = \frac{(-20\,000)}{(6000)}$ $= -3.33333$ $= 3.33 \text{ m s}^{-1}$ west

b Calculate the change in momentum of the car.

Thinking	Working
The change in momentum of the car is its final momentum minus its initial momentum.	$\Delta p_c = p_{\text{final}} - p_{\text{initial}}$ $= m_c(v - u)$ $= 1000(-3.33333 - 20.0)$ $= -23\,333.3$ $= 2.33 \times 10^4 \text{ kg m s}^{-1}$ west

c Calculate the change in momentum of the bus.

Thinking	Working
The change in momentum of the bus is its final momentum minus its initial momentum.	$\Delta p_b = p_{\text{final}} - p_{\text{initial}}$ $= m_b(v - u)$ $= 5000(-3.33333 - (-8.00))$ $= 23\,333.3$ $= 2.33 \times 10^4 \text{ kg m s}^{-1}$ east

d Verify that the momentum of the system is constant.

Thinking	Working
The total change in the momentum of a system is the vector sum of the change of momentum of its parts. This should be zero from the conservation of momentum.	$\Delta p_c + \Delta p_b = (-2.33 \times 10^4) + (2.33 \times 10^4) = 0$ Therefore the momentum of the system is constant (i.e. conserved) as expected.

Worked example: Try yourself 3.1.1

CONSERVATION OF MOMENTUM

In a safety-rating test of head-on collisions, a car of mass 1200 kg travelling east at 22.0 m s^{-1} crashes into a bus of mass 7000 kg travelling west at 15.0 m s^{-1}. In answering the following questions, assume that the car and bus lock together on impact. You can ignore the effect of friction.

a Calculate the final common velocity of the vehicles.

b Calculate the change in momentum of the car.

c Calculate the change in momentum of the bus.

d Verify that the momentum of the system is constant.

CONSERVATION OF MOMENTUM FROM NEWTON'S LAWS

The principle of conservation of momentum follows directly from Newton's second and third laws. This can be shown in the following way.

Consider a bowling ball of mass m_b moving with an initial velocity of u_b. It collides with a stationary pin of mass m_p. The velocity of both the ball and pin changes. The pin's final velocity is represented by v_p in Figure 3.1.2.

When the ball and pin collide, they exert action–reaction forces on each other and, according to Newton's third law:

$$F_{bp} = -F_{pb}$$

The forces cause the ball to decelerate and the pin to accelerate. Thus from Newton's second law ($F = ma$):

$$m_b a_b = -m_p a_p$$

The ball and pin are in contact for time Δt. Thus we can rewrite acceleration in terms of velocity:

$$m_b \frac{(v_b - u_b)}{\Delta t} = -m_p \frac{(v_p - u_p)}{\Delta t}$$

The times are the same and so they cancel out, leaving:

$$m_b(v_b - u_b) = -m_p(v_p - u_p)$$

Expanding and rearranging gives:

$$m_b u_b + m_p u_p = m_b v_b + m_p v_p$$

The left-hand side of this equation describes the initial momentum of the system and the right-hand side represents the final momentum of the system. Thus an application of Newton's second and third laws has produced the equation for the conservation of momentum:

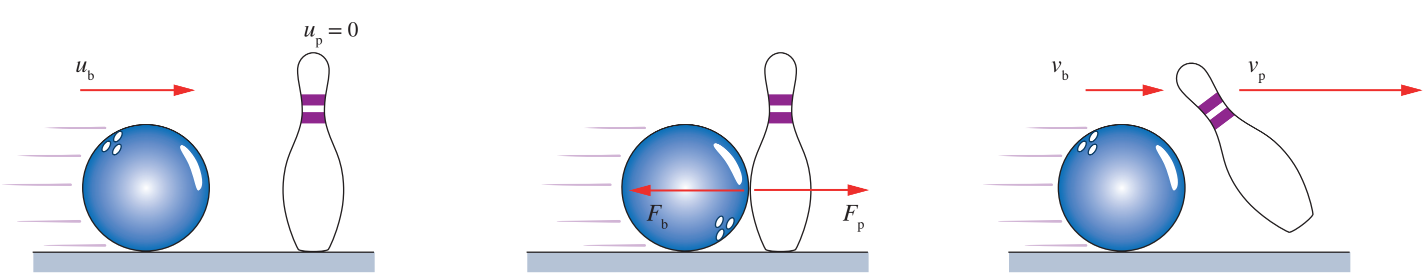


FIGURE 3.1.2 When a bowling ball collides with a pin, they exert equal but opposite forces on each other. These forces cause the ball to lose some momentum and the pin to gain an equal amount of momentum.

Worked example 3.1.2

REBOUNDING

In a football game, a player A of mass 80.0 kg travelling towards the goal at 9.00 m s^{-1} collides with an opposition player (player B) of mass 72.0 kg travelling away from the goal at 6.50 m s^{-1}. After the collision, player A is travelling towards the goal at 0.50 m s^{-1}. Assume that the two players rebound off each other on impact and ignore the effects of friction.

a Calculate the sum of the momentum of the two players before the collision.

Thinking	Working
First assign a direction that will be considered positive.	In this case we will consider vectors directed towards the goals to be positive. $m_A = 80.0$ kg $u_A = 9.00$ m s^{-1} $v_A = 0.50$ m s^{-1} $m_B = 72.0$ kg $u_B = -6.50$ m s^{-1}
Use the equation of momentum for each player and substitute the values.	$\Sigma p_{initial} = p_A + p_B$ $= m_A u_A + m_B u_B$ $= (80.0)(9.00) + (72.0)(-6.50)$ $= (720) + (-468)$ $= 252$ kg m s^{-1} towards the goal

b Calculate the final velocity of the opposition player (player B).

Thinking	Working
The sum of the momentum after the collision is equal to the sum of the momentum before the collision.	$\Sigma p_{initial} = \Sigma p_{final}$ $\Sigma p_{initial} = m_A v_A + m_B v_B$ $252 = (80.0)(0.50) + (72.0)v_B$ $v_B = \frac{252 - 40.0}{72.0}$ $= 2.94444$ m s^{-1} towards the goal

c Calculate the change in momentum of player A.

Thinking	Working
The change in momentum of player A is their final momentum minus their initial momentum.	$\Delta p_A = p_{final} - p_{initial}$ $= m_A(v_A - u_A)$ $= (80.0)(0.50 - 9.00)$ $= -680$ kg m s^{-1} $= 680$ kg m s^{-1} away from the goal

d Calculate the change in momentum of player B.

Thinking	Working
The change in momentum of player B is their final momentum minus their initial momentum.	$\Delta p_B = p_{final} = p_{initial}$ $= m_B(v_B - u_B)$ $= (72.0)(2.94444 - (-6.50))$ $= 680$ kg m s^{-1} towards the goal

Worked example: Try yourself 3.1.2

REBOUNDING

In a child's toy, a blue marble rolls along a track and collides with a red marble rolling the other way. The blue marble has a mass of 0.00320 kg and is travelling south at 0.800 m s^{-1} as it hits the red marble. The red marble has a mass of 0.00150 kg and is travelling north at 1.00 m s^{-1} when it hits the blue marble. After the collision the blue marble is now travelling towards the south at 0.450 m s^{-1}. In answering the following questions, assume that the two marbles bounce off each other on impact. You can ignore the effect of friction.

a Calculate the sum of the momentum of the two marbles before they hit.

b Calculate the final velocity of the red marble.

c Calculate the change in momentum of the blue marble.

d Calculate the change in momentum of the red marble.

Worked example 3.1.3

EXPLOSIVE MOMENTUM

Two friends are standing on their stationary skateboards facing each other with their hands gripped together. They place their feet together and push with their legs as they release their hands. After releasing their grip, skater A, of mass 50.0 kg, travels towards the east at 3.50 m s^{-1}. Skater B, of mass 34.0 kg, travels in the opposite direction. You can ignore the effect of friction.

a Calculate the sum of the momentum of the two skaters before they release their grip.

Thinking	Working
Assign a direction that will be considered positive.	In this case we will consider vectors directed towards the east to be positive. $m_A = 50.0$ kg $u_A = 0$ $v_A = 3.50$ m s^{-1} $m_B = 34.0$ kg $u_B = 0$ m s^{-1}
Use the equation of momentum for the combined mass of the skaters.	$\Sigma p_{initial} = p_A + p_B$ $= (m_A + m_B)u$ $= (50.0 + 34.0)(0)$ $= (84.0)(0)$ $= 0$ kg m s^{-1}

b Calculate the final velocity of skater B.

Thinking	Working
The sum of the momentum after the skaters release their grip is equal to the sum of the momentum before the release.	$\Sigma p_{initial} = \Sigma p_{final}$ $\Sigma p_{initial} = m_A v_A + m_B v_B$ $(0) = (50.0)(3.50) + (34.0)v_B$ $(34.0)v_B = (-175)$ $v_B = -5.14706$ $= 5.15$ m s^{-1} west

c Calculate the change in momentum of skater A.	
Thinking	**Working**
The change in momentum of skater A is their final momentum minus their initial momentum.	$\Delta p_A = p_{final} - p_{initial}$ $= m_A(v_A - u_A)$ $= (50.0)(3.50 - 0)$ $= 175$ $= 175\ \text{kg m s}^{-1}$ east

d Calculate the change in momentum of skater B.	
Thinking	**Working**
The change in momentum of skater B is their final momentum minus their initial momentum.	$\Delta p_B = p_{final} - p_{initial}$ $= m_B(v_B - u_B)$ $= (34.0)(-5.14706 - 0)$ $= -175$ $= 175\ \text{kg m s}^{-1}$ west

Worked example: Try yourself 3.1.3

EXPLOSIVE MOMENTUM

Two ice dancers are standing still in the centre of an ice rink facing each other with their palms together. They then begin their routine by pushing with their hands. After pushing away, ice dancer A, of mass 62.0 kg, travels towards the north at 2.20 m s^{-1}. Ice dancer B, of mass 98.0 kg, travels towards the south. You can ignore the effect of friction.

a Calculate the sum of the momentum of the two ice dancers before they push away.

b Calculate the final velocity of the dancer B.

c Calculate the change in momentum of ice dancer A.

d Calculate the change in momentum of ice dancer B.

PHYSICSFILE

Not so strongman

Traditionally, circus strongmen would often perform a feat where they place a large rock on their chest and then invite another person to smash the rock with a sledgehammer. This might seem at first to be an act of extreme strength and daring. However, a quick analysis using the principle of conservation of momentum will show otherwise. Assume that the rock has a mass of 27 kg and that a sledgehammer of mass 3.0 kg strikes it at 5.0 m s^{-1}. From the law of conservation of momentum, we can show that the rock and sledgehammer will move together at just 0.50 m s^{-1} after impact:

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$(3.0 \times 5.0) + (27 \times 0) =$

$(3.0 + 27) \times v$

$15 = 30v$

$v = 0.50\ \text{m s}^{-1}$

This is a very low speed. The large mass of the rock means that the final common speed is too low to hurt the strongman. A more daring feat would be to use the sledgehammer to smash a pebble.

3.1 Review

SUMMARY

- The momentum of an object is the product of its mass and its velocity: $p = mv$. Momentum is measured in kg m s^{-1}.
- The total momentum of an isolated system is conserved, that is, the sum of the momentum of the parts of a system before a collision is equal to the sum of the momentum after the collision: $\Sigma p_{initial} = \Sigma p_{final}$.
- In a simple collision between two objects of mass m_1 and m_2, this equation becomes:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

- A collision between two objects of mass m_1 and m_2 can be described as either:
  - combined, where two separate masses lock together on impact
  - rebound, where two separate masses bounce off each other and remain separate
  - explosive, where two combined masses separate, moving away from each other.

KEY QUESTIONS

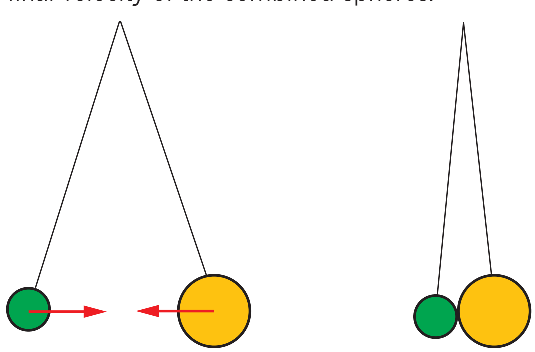
Knowledge and understanding

1 Using the concepts investigated in this section, explain why you are more likely to end up in the water when attempting to step onto a dock from a stand-up paddle board than when attempting to step onto a dock from a river ferry.

2 Two toy cars have Velcro attached such that they will stick together on contact. A child makes the cars collide and both cars come to rest as they stick together. Explain how momentum is conserved in this situation.

Analysis

3 In an experiment, a student hangs a green sphere (25.0 kg) from a long rope and an orange sphere (50.0 kg) from another long rope. Each sphere has a magnet attached. The spheres are pushed towards each other. Just before they collide, the green sphere is moving east at 3.50 m s^{-1} and the orange sphere is moving west at 6.00 m s^{-1}. They collide in mid-air and remain locked together due to the magnets. Calculate the final velocity of the combined spheres.



4 While shunting empty railway carriages, an 11.0 t passenger carriage travelling at 7.50 m s^{-1} north collides with a diner carriage of 16.0 t that is also travelling north but at 3.50 m s^{-1}. The two carriages lock together after the collision. Ignoring friction, find their combined velocity.

5 A sports car of mass 1000 kg travelling east at 36.0 km h^{-1} approaches a station wagon of mass 2000 kg moving west at 18.0 km h^{-1}.

a **i** Calculate the momentum of the sports car.

ii Calculate the momentum of the station wagon.

iii Determine the sum of the momentum of these vehicles.

b The two vehicles now collide head-on on an icy stretch of road where there is negligible friction. The vehicles remain locked together after the collision.

i Calculate the common velocity of the two vehicles after the collision.

ii Where has the initial momentum of the vehicles gone?

iii Determine the change in momentum of the sports car.

iv Determine the change in momentum of the station wagon.

6 A 155 g pink snooker ball travelling with initial velocity 5.00 m s^{-1} to the right collides with a stationary green ball of mass 132 g. The two balls rebound off each other. If the final velocity of the pink ball is 3.00 m s^{-1} to the left, calculate the velocity of the green ball after the collision.

7 Two students at a fun fair run at each other with large fitness balls held in front of them. One student has a yellow ball and is running at 4.20 m s^{-1} east with a total mass of 71.0 kg. The other student has an orange ball and is running at 5.30 m s^{-1} west with a total mass of 65.0 kg. After they collide and bounce off each other, the final velocity of the student with the orange ball is 1.40 m s^{-1} east. Calculate the final velocity of the student with the yellow ball.

8 In a training exercise a group of astronauts investigate what would happen in the event of a failed coupling of a cargo ship with the International Space Station (ISS). In a large hangar a model of the ISS, with a mass of 4.20×10^5 kg, is suspended from the ceiling and is stationary. A 3.20×10^4 kg model of a supply ship is nearby and also suspended. The model of the supply ship is pushed towards the model of the ISS. If it is moving at 5.00 m s^{-1} south as it strikes the model of the ISS and rebounds at 5.00 m s^{-1} north, with what velocity does the model of the ISS move after the collision?

9 A stationary 1000 kg cannon mounted on wheels fires a 10.0 kg shell east with a horizontal speed of 505 m s^{-1}. Assuming that friction is negligible, calculate the recoil velocity of the cannon.

10 An astronaut is floating in deep space while holding a toolbox. The total mass of the astronaut, including their suit, is 235 kg and the mass of the toolbox is 46.0 kg. The combined astronaut and toolbox are drifting away from the spaceship at 0.750 m s^{-1}. With no other way to get back to the spaceship the astronaut decides to sacrifice the toolbox and throw it as fast as possible in a direction away from the spaceship. With what speed should the astronaut throw the toolbox if they hope to move towards the spaceship at 0.300 m s^{-1} after the throw?

3.2 Impulse

In a car crash, it is not only how fast the car travels that determines the damage, but how quickly it stops. This is a direct consequence of Newton's second law of motion. From $F = ma$, if a is small you can conclude that the force required to bring the car to a stop will also be relatively small. On the other hand, if the car is brought to a halt very rapidly (Figure 3.2.1), there will be a large deceleration requiring a large force. The force determines the damage. Ignoring the likelihood of injury caused by a large force that acts for a short time, such as in a car crash, a small force acting for a longer time has the same effect: suddenly applying the brakes and gradually applying the brakes both bring the car to rest. One way to quantify the similarity between these situations is to describe the impulse in a collision, which considers both the force and the time over which the force acts.

FIGURE 3.2.1 Rapid deceleration requires a large force and often results in damage and injury.

CHANGE IN MOMENTUM

Newton's original formulation of his second law was not expressed in terms of acceleration. Rather, he spoke of the 'motion' of a body that would be altered when a force acted on that body over a time interval. This is very close to saying that the momentum of the body changes when a resultant force acts on it. This is equivalent to the more familiar $F = ma$ formulation of Newton's second law, as will now be shown.

Consider a body of mass m with a resultant force F acting on it for time Δt. The mass will accelerate as described by Newton's second law:

$$F = ma$$

$\therefore F = \dfrac{m\Delta v}{\Delta t}$ after substituting the definition of acceleration.

By rearranging this equation, we can write $F\Delta t = m\Delta v = \Delta p$.

The term $m\Delta v$ is the change in the momentum of the body. It is also called the **impulse**. The force involved in a collision can change in value during the collision, so the average force is used. The average force acting on the body for a time Δt causes a change in the momentum.

The term $F\Delta t$ is called the impulse of the resultant force and is equal to the change in momentum of the object. That is:

impulse = $F\Delta t = \Delta p$

where F is the average force acting on the object (N)

Δt is the time over which the force acts (s)

Δp is the change in momentum of the object ($kg\,m\,s^{-1}$).

Impulse is measured in newton seconds (N s)

It is important to note that impulse is a vector quantity. Its direction is the same as that of the average force or of the change in momentum (or velocity).

Momentum units

Since impulse can be expressed in terms of a momentum change, the units for momentum ($kg\,m\,s^{-1}$) and impulse (N s) must be equivalent. This can be shown using Newton's second law.

Given that $1\,N = 1\,kg\,m\,s^{-2}$ (from $F = ma$), it follows that $1\,N\,s = 1\,kg\,m\,s^{-2} \times s$

That is, $1\,N\,s = 1\,kg\,m\,s^{-1}$.

Even though the units are equivalent, they should be used with the appropriate quantities as a reminder of the quantity that is being considered: momentum or impulse. The newton second (N s) is the product of a force and a time interval and so should be used with impulse. The kilogram metre per second ($kg\,m\,s^{-1}$) is the product of a mass and a velocity and so should be used with momentum. Even so, it is not uncommon to see newton seconds used as the unit of momentum.

Worked example 3.2.1

CALCULATING THE IMPULSE

Calculate the impulse of a tree on a 1480 kg sports car if the vehicle is travelling at $93.0\,km\,h^{-1}$ in a northerly direction when the driver loses control of the vehicle on an icy road. The car comes to rest against the tree.

Thinking	Working
Convert the speed to $m\,s^{-1}$.	$93.0\ km\,h^{-1} = \frac{93.0}{3.6}\,m\,s^{-1}$ $= 25.8333\,m\,s^{-1}$
Calculate the change in momentum. The negative sign indicates that the change in momentum, and therefore the impulse, is in a direction opposite to the initial momentum (with north as positive).	$\Delta p = m(v-u)$ $= (1480)(0 - 25.8333)$ $= -3.82333 \times 10^4$ $= 3.82 \times 10^4$ N s south
The impulse is equal to the change in momentum.	Impulse = 3.82×10^4 N s south

Worked example: Try yourself 3.2.1

CALCULATING THE IMPULSE

Calculate the impulse of the braking system on the 1480 kg sports car if the vehicle was travelling at $95.5\,km\,h^{-1}$ in a north-easterly direction before coming to an abrupt halt.

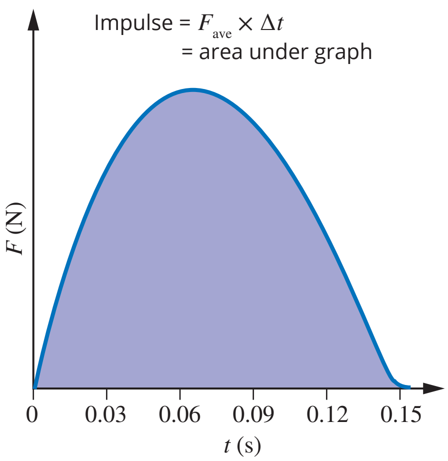


FIGURE 3.2.2 The force changes with time as the racquet strikes the ball.

FORCE VERSUS TIME GRAPHS

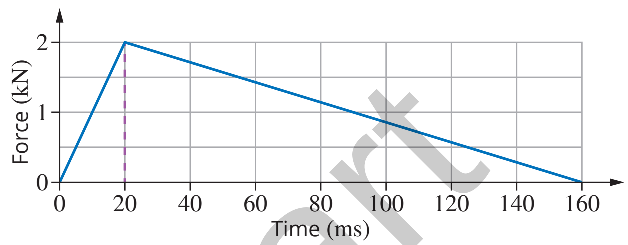
In many situations the force applied to an object is not constant. For example, when a tennis player hits a ball, the initial force exerted by the racquet is relatively small. As the strings stretch and the ball deforms, this force builds up to a maximum before decreasing again as the ball rebounds from the racquet. Where the force changes over time, the relationship can be represented graphically (Figure 3.2.2).

The impulse of the ball, or the change in momentum, can be found from the product $F\Delta t$. This is simply the area under the force vs time graph.

Worked example 3.2.2

IMPULSE OF RUNNING SHOES

A running-shoe company plots the following force vs time graph for a running shoe. Use the data to calculate the magnitude of the impulse.

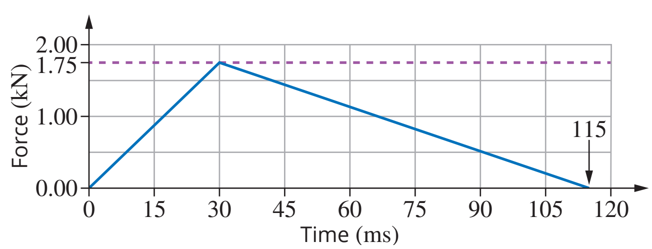


Thinking	Working
Recall that impulse = $F\Delta t$. This is the area under the force vs time graph.	impulse $= \frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2} \times 160 \times 10^{-3} \times 2 \times 10^{3}$ $= 160$ N s

Worked example: Try yourself 3.2.2

IMPULSE OF RUNNING SHOES

A running-shoe company plots the following force vs time graph for an alternative design intended to reduce the peak force on the heel. Calculate the magnitude of the impulse.



APPLICATIONS OF IMPULSE

The connection between impulse, force and collision duration is useful in analysing collisions. When a vehicle collides with another object and comes to rest, the vehicle and occupants undergo a rapid deceleration. The impulse depends on the initial speed of the vehicle and on its mass.

Since impulse $= \Delta p = F\Delta t$, a large force is exerted to bring the vehicle to rest in a very short time. Extending the time taken for a vehicle to stop reduces the force exerted. Examples of increased stopping times in different activities are shown in Figure 3.2.3 on page 121.

FIGURE 3.2.3 (a) The landing mat extends the time over which the athlete comes to rest, reducing the size of the stopping force. If the high jumper missed the mat and landed on the ground, the force would be larger, but their momentum change would be the same. (b) Thick padding around the goal post extends the time over which a player who collides with it comes to rest, thereby reducing the size of the stopping force. (c) Wicketkeepers follow the ball's final trajectory with their gloves when keeping. This extends the ball's stopping time, reduces the stopping force and softens the blow on the gloves.

Worked example 3.2.3

BRAKING FORCE

A 2520 kg truck is travelling at 30.0 m s^{-1} before the brakes are applied. Calculate the magnitude of the average force exerted by the brakes to bring the vehicle to rest in 12.0 s.

Thinking	Working
Calculate the change in momentum. The negative sign indicates that the change in momentum, and therefore the braking force, is in the direction opposite to the initial momentum.	$\Delta p = m(v - u)$ $= 2520(0 - 30.0)$ $= -75600$ $= -7.56 \times 10^4$ kg m s^{-1}
Transpose $\Delta p = F\Delta t$ to find the force. The sign of the momentum can be ignored, since you are only finding the magnitude of the average force.	$F = \frac{\Delta p}{\Delta t}$ $= \frac{75600}{12.0}$ $= 6.30 \times 10^3$ N

Worked example: Try yourself 3.2.3

BRAKING FORCE

The same 2520 kg truck travelling at 30.0 m s^{-1} needs to stop in 1.50 s because a vehicle in front has suddenly stopped. Calculate the magnitude of the average braking force required to stop the truck in that time.

Safety features in cars—such as crumple zones and airbags (Figure 3.2.4)—are designed to extend Δt. This reduces the force on the occupants of the vehicle, potentially saving lives and preventing injuries.

FIGURE 3.2.4 Airbags reduce the force on passengers by extending the time it takes for them to stop in the event of a collision.

Figure 3.2.5 shows a force vs time graph for a collision where an airbag is inflated compared with one where there is no airbag. The change in momentum, or impulse, of the passenger is the same in each case. Thus the area under each curve should be equal. Note, however, that both the peak force and the average force are significantly higher where there is no airbag. The broader peak for the airbag indicates that the passenger is losing their momentum over a longer time and thus requiring a lower force.

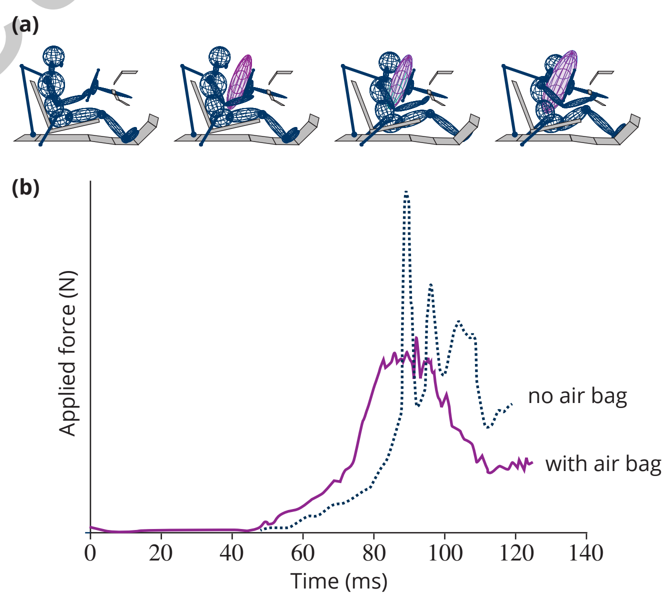


FIGURE 3.2.5 (a) Diagram of an airbag being inflated in a collision. (b) Graph illustrating the difference in the force on a passenger over time when an airbag is inflated in a collision (solid line), and when no airbag is present (dotted line).

CASE STUDY ANALYSIS

Car safety and crumple zones

Worldwide, car accidents are responsible for millions of deaths each year. Many times this number of people are injured. One way of reducing the road toll is to design safer vehicles. Modern cars employ a variety of safety features that help to improve the occupants' chances of surviving an accident. Some of these safety features are the antilock braking system (ABS), electronic stability control (ESC), inertia reel seatbelts, variable-ratio-response steering systems, collapsible steering columns, head rests, shatterproof windscreen glass, padded dashboards, front and side air bags, front and rear crumple zones and a rigid passenger compartment.

Some cars today are equipped with collision avoidance systems. These have radar, laser or infrared sensors that advise the driver of a hazardous situation. They may even take control of the car when an accident appears likely.

The purpose of such safety features as inertia reel seatbelts, collapsible steering columns, padded dashboards, air bags and crumple zones is not to reduce the size of the impulse, but to reduce the size of the forces that act to bring the car to a stop. Automotive engineers strive to achieve this by extending the time over which the driver loses momentum.

Crumple zones

A popular misconception is that cars would be much safer if they were sturdier and more rigid. Drivers often complain that cars seem to collapse too easily during collisions, and that it would be better if cars were structurally stronger—more like an army tank. In fact, cars are specifically designed to crumple to some extent (Figure 3.2.6). This makes them safer and actually reduces the seriousness of injuries suffered in car accidents.

FIGURE 3.2.6 Cars are designed with weak points in their chassis that enable the car to crumple in the event of a collision. This extends the time over which the cars come to rest and so reduces the size of the forces acting on the occupants.

Army tanks are designed to be extremely sturdy and rigid vehicles. They are able to withstand the effect of collisions without suffering serious structural damage. If a tank travelling at 60.0 km h^{-1} crashed into a solid obstacle, the tank would be relatively undamaged. However, its occupants would very likely be killed. This is because the tank has no give in its structure and so the tank and its occupants would stop in an extremely short time interval. The occupants would lose all of their momentum in an instant, which means that the forces acting on them would necessarily be very large. These large forces would cause the occupants of the tank to sustain very serious injuries, even if they were wearing seatbelts.

Cars today have strong and rigid passenger compartments; however, they are also designed with non-rigid sections— such as bonnets and boots—that crumple if the cars are struck from the front or rear (Figure 3.2.7). The chassis contains parts that have grooves or beads cast into them. In a collision, these grooves or beads act as weak points, causing the chassis to crumple in a concertina shape.

FIGURE 3.2.7 The Australian New Car Assessment Program (ANCAP) assesses the crashworthiness of new cars. This car has just crashed at 50 km h^{-1} into a 5 t concrete block. The crumpling effect can clearly be seen.

By crumpling the front or rear of the car, the time interval over which the car and its occupants come to a stop is extended. This stopping time is typically longer than 0.1 s in a 50 km h^{-1} crash. Because the occupants' momentum is lost more gradually, the peak forces that act on them are smaller and so the chances of injury are reduced.

continued over page

CASE STUDY ANALYSIS *Continued*

Analysis

1 Consider the driver of a car that crashes into a tree while driving north at 60.0 km h^{-1}. If the driver has a mass of 90.0 kg, calculate the momentum of the driver just before the collision.

2 If the driver comes to a complete stop as a result of the collision, calculate their change in momentum (i.e. the impulse).

3 How would the impulse change if the collision occurred over a longer period of time?

4 Compare the impulse experienced by a 90.0 kg driver of a car to the impulse experienced by a 90.0 kg driver of a tank if both were to crash and come to a stop from 60.0 km h^{-1}.

5 Calculate the force on the 90.0 kg car driver if the impulse experienced by the driver occurred over a period of 985 ms.

6 Calculate the force on the 90.0 kg tank driver if the impulse experienced by the driver occurred over a period of 81.5 ms. Assume that the tank, like the car, is travelling north before the impact.

7 Given that the conditions of a collision are identical except for the period of time over which the collision occurs, how would you describe the relationship between the force experienced and the period of time?

3.2 Review

OA ✓✓

SUMMARY

- When a force is exerted on an object over a time interval, Δt, it brings about a change in momentum, Δp, by changing the velocity of the object:

$$F\Delta t = m\Delta v = \Delta p$$

- Impulse is the change in the momentum of an object.
- The unit of Impulse is the newton second (N s) and the unit of change in momentum is kg m s^{-1}. These units are equivalent.
- Impulse can be calculated from the area under a force vs time graph.
- When designing for safety, measures are taken to increase the time of an interaction in order to reduce the maximum force experienced during that interaction.

KEY QUESTIONS

Knowledge and understanding

1 A 165 g cricket ball flies past the wicket at 155 km h^{-1} and is stopped by the wicket keeper. Calculate the magnitude of the impulse delivered by the ball to the wicket keeper.

2 When a tennis player serves, she hits a 57.0 g tennis ball at the top of its flight when the ball is momentarily stationary. It then leaves the racquet at 144 km h^{-1}. If the ball and racquet are in contact for 0.0600 s, calculate the magnitude of the average force exerted by the racquet on the ball.

3 A basketball of mass 0.625 kg is bounced against the court at a speed of 32.0 m s^{-1}. It rebounds at 24.5 m s^{-1}. Calculate the average force exerted by the court on the ball if the interaction lasts 16.5 ms.

4 Consider a 100 t train travelling at 50.0 km h^{-1}.

a Calculate the momentum of the train.

b Calculate the magnitude of the impulse if the train were to collide with a 5.00 t truck at a level crossing and push the truck for 15.0 m before coming to rest.

Analysis

5 Three balls of identical mass are thrown against a surface at the same speed.
Ball A stops on impact.
Ball B rebounds with 75% of its initial speed.
Ball C rebounds with 50% of its initial speed.
Order the balls in terms of their change in momentum, from least to most.

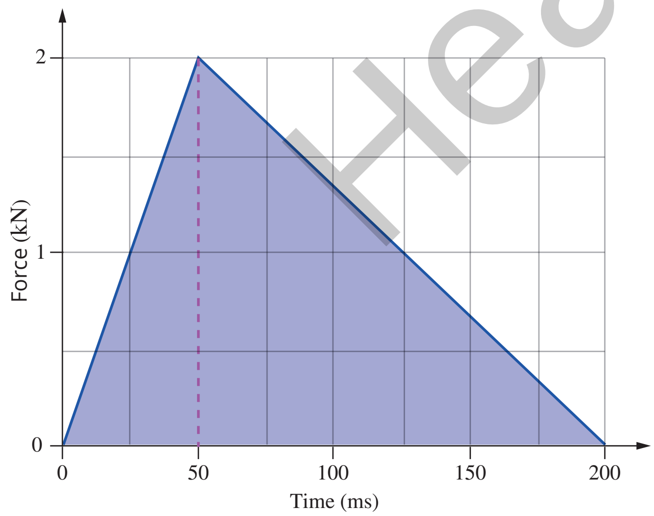
6 A child wearing a backpack jumps from a tree and lands on her feet. Describe at least three factors that will influence the force on her knees and ankles when she lands. In your response, you may wish to refer to the child's footwear, her landing technique and the surface on which she lands.

7 Two crash-test cars of identical mass are travelling at $22.0\,\text{m s}^{-1}$ towards a solid concrete block. Car A is designed with crumple zones built into the front of the chassis and car B is built with a rigid chassis. The passenger compartment of car A comes to rest in 0.0896 s after hitting the concrete block, while the passenger compartment in car B comes to rest in 0.00400 s. By what factor does the average force on car B compare to car A?

8 The graph below represents the force exerted by an athlete's foot over the 200 ms that his foot is in contact with the ground.

a Calculate the magnitude of the impulse of the athlete on the ground.

b Calculate the magnitude of the average force exerted by his foot over the duration of the contact.



9 A tennis ball of mass 57.5 g is tested for compliance with tennis regulations by being dropped from a height of 251 cm onto concrete. A bounce height of 146 cm is deemed acceptable. Find the magnitude of the average force on a ball that just reaches the acceptable height if it is in contact with the concrete for 0.0550 s.

3.3 Work done

In everyday language, the concept of work is associated with effort and putting energy into something, whether it be your studies, sports or a part time job. Although the word **work** has a much more specific meaning in physics, it is still connected with energy.

When an unbalanced force acts on an object over a time interval, the object accelerates and its momentum changes. When the force causes a displacement in the direction of the object, the energy of the object changes, and we say that work has been done. The weightlifter in Figure 3.3.1 does work by exerting a force and causing the barbell to undergo a displacement. The **gravitational potential energy** of the barbell is increased and the store of chemical energy in the muscles of the weightlifter is decreased.

FIGURE 3.3.1 Suamili Nanai broke the Australian clean-and-jerk record in July 2021 by lifting 201 kg above his head in two movements.

CALCULATING WORK

Work is the transfer of energy from one object to another and/or the transformation of energy from one form to another. A force does work on an object when it acts on that object and causes a displacement in the direction of the force. Where the force is constant, the work done by the force is given by the following equation.

$W = Fs$

where W is the work done by the force (J)

F is the magnitude of the constant force (N)

s is the displacement (m)

If the force is applied at an angle to the displacement, only the component of the force in the direction of the displacement contributes to the work done. If the force and displacement vectors are at an angle θ to each other, then $F \cos \theta$ is the component of force that does work.

$$W = Fs \cos \theta$$

where W is the work done by the force (J)
F is the magnitude of the constant force (N)
s is the displacement (m)
θ is the angle between the force and displacement vectors

While both force and displacement are vectors, work and energy are scalar quantities and are measured in joules (J).

To find the work done on an object, it is the net force that needs to be used. For instance, if a person pushes a heavy couch across a carpeted floor, the work done on the couch depends on the force applied by the person less the frictional force that opposes the motion:

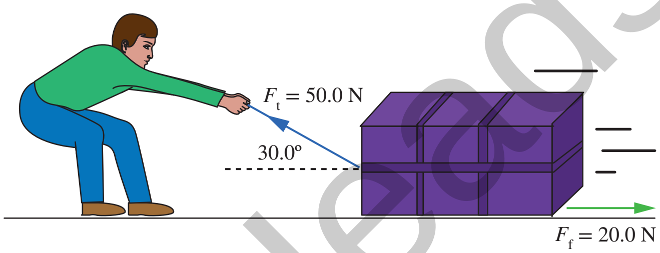
$$W = \Delta E = F_{net} s$$

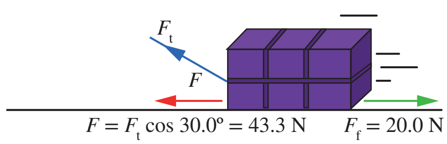
In this section we will assume that any force that acts to do work on an object is the net force.

Worked example 3.3.1

FORCE APPLIED AT AN ANGLE TO THE DISPLACEMENT

A rope that is 30.0° to the horizontal is used to pull a 10.0 kg crate across a rough floor. The crate is initially at rest and is dragged 4.00 m along the floor. The tension, F_t, in the rope is 50.0 N and the frictional force, F_f, opposing the motion is 20.0 N.



a Determine the work done by the person.	
Thinking	**Working**
Draw a diagram of the forces in action.	
Find the component of the tension in the rope that is in the direction of the displacement (shown by the red arrow).	$F = 50.0 \times \cos 30.0° = 43.3013$ N
Find the work done by the person.	$W = Fs$ $= 43.3013 \times 4.00$ $= 1.73205 \times 10^2$ $= 1.73 \times 10^2$ J

PHYSICSFILE

How much work does it take to break a record?

In July 2021, weightlifter Suamili Nanai (Figure 3.3.1) became the male Australian record holder for the clean-and-jerk when he lifted 201 kg from the ground to above his head. While it is difficult to determine how much time and effort had gone into Nanai's preparations for breaking the record, we can calculate the work he did against gravity to lift the record mass. Assuming that he lifted the mass from the ground to 205 cm above the ground, we can calculate the work done as:

$$\begin{aligned} W &= Fs \\ &= mgs \\ &= 201 \times 9.8 \times 2.05 \\ &= 4038.1 \\ &= 4040 \text{ J} \end{aligned}$$

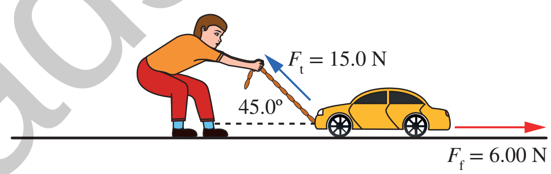
b Calculate the work done on the crate.	
Thinking	**Working**
The work done on the crate is the net force acting on it multiplied by the displacement. (This is also the increase in the kinetic energy of the crate.)	$W = Fs$ $= (F - F_f)s$ $= (43.3013 - 20.0) \times 4.00$ $= 93.20$ $= 93.2$ J

c Calculate the energy transformed to heat and sound due to the frictional force.	
Thinking	**Working**
Energy transformed to heat and sound due to the frictional force is the difference between the work done by the person and the energy gained by the crate.	$E = 173.2 - 93.20$ $= 79.99$ $= 80.0$ J
This is equal to the work done against friction, which can also be calculated from the frictional force.	$W_f = F_f s$ $= 20.0 \times 4.00$ $= 80.0$ J

Worked example: Try yourself 3.3.1

FORCE APPLIED AT AN ANGLE TO THE DISPLACEMENT

A boy moves a toy car by pulling on a cord that is attached to the car at 45.0° to the horizontal. The boy applies a force of 15.0 N and pulls the car for 10.0 m along a path against a frictional force of 6.00 N.



a Determine the work done by the boy pulling on the cord.

b Calculate the work done on the toy car.

c Calculate the energy transformed into heat and sound due to the frictional force.

When a force performs no work

It is important to remember that work is only done when a force, or a component of a force, is applied in the direction of displacement. Hence it is possible to exert a force and feel very tired without doing work. This would mean no energy has been transferred. For example, if you hold a heavy object in outstretched arms you will get tired very quickly, but you are not doing any work on the object.

Similarly, an object moving in a circular path in a horizontal plane is constantly accelerated by a centripetal force. Because this force is perpendicular to the displacement at each instant, the force does no work, and no energy is transferred to the object. It does not get faster or slower; it only changes direction (Figure 3.3.2).

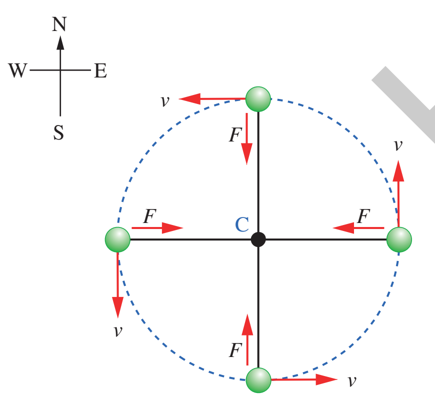


FIGURE 3.3.2 A body moving in a circular path has a force directed towards the centre of the path. The displacement is in the direction of the velocity. There is therefore no force in the direction of the displacement and thus no work is done.

FORCE VS DISTANCE GRAPHS

When the force is constant, the work done is easily calculated. However, in many situations the net force is changing. In these situations, a graph can be used to calculate the work done. Where the force vs distance relationship is represented graphically, the work done is the area under the graph. This principle is very similar to the way in which impulse can be calculated from the area under a force vs time graph. However, it is important not to confuse these two quantities.

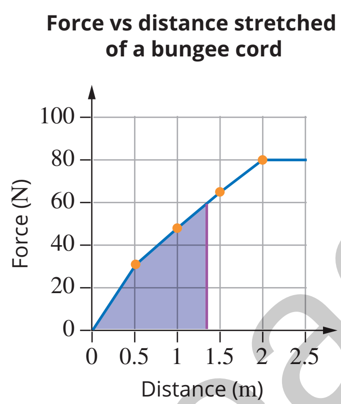
When graphed, the relationship between the force and the distance stretched of an **elastic** object, such as a spring, offers a way to calculate the work done in stretching the material: the work done is the area under the force vs distance stretched graph.

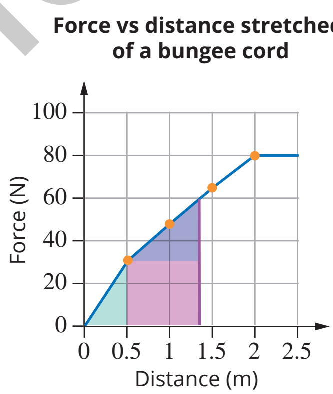
If the force vs distance graph, or force vs distance stretched graph, is not linear, the area can be calculated by counting squares. It is important to take careful note of the units in order to calculate the work represented by each square.

Worked example 3.3.2

CALCULATING WORK DONE FROM A GRAPH

The force required to stretch a piece of bungee cord is represented in the graph below. Calculate the work done when a 60 N force is applied to the cord.



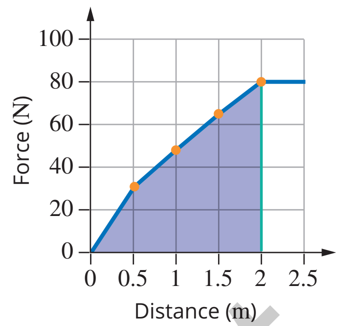
Thinking	Working
The work done is the area under the force vs distance graph. This may be found by calculation, or by counting squares. In this case it is best to divide the area into triangles and rectangles and sum the individual areas.	
Add the areas together to calculate the work done.	Area $= (\frac{1}{2} \times 0.50 \times 30) + (\frac{1}{2} \times 0.90 \times 30) + (30 \times 0.9$ Work done = 48 J

Worked example: Try yourself 3.3.2

CALCULATING WORK DONE FROM A GRAPH

The force required to elongate a piece of rubber tubing is represented in the graph below. Calculate the work done when the tubing is stretched by 2.0 m.

Force vs distance stretched of rubber tubing



3.3 Review

SUMMARY

- When a force does work on an object, there is a change in the displacement and energy of the object.
- Work, W, is a scalar and is measured in joules (J).
- The work done on an object is the net force on the object multiplied by the displacement moved in the direction of the force: $W = Fs$.
- When the force is applied to an object at an angle to the displacement, work is only done by the component of the force in the direction of the displacement: $W = Fs \cos \theta$.
- A centripetal force does no work on an orbiting object, as the force and displacement are perpendicular.
- The work done by a varying force is the area under the force vs distance graph.

KEY QUESTIONS

Knowledge and understanding

1 Describe a scenario in which a force is applied but no work is done.

2 If we consider the Earth to orbit the Sun in a circular path with a constant gravitational force of attraction, justify the statement that the Sun does no work on the Earth.

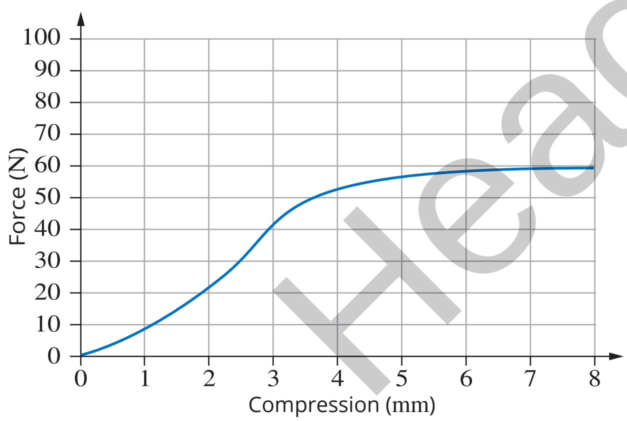
3 A child uses a string to drag a 2.00 kg toy across a floor. The string is held at an angle of 60.0° to the horizontal and the child applies a force of 30.0 N on the toy, which is initially at rest. A constant frictional force of 10.0 N acts on the toy as it is dragged 2.40 m along the floor.

a Calculate the work done by the horizontal component of the 30.0 N force.

b Calculate the work that the child does in overcoming friction.

c Calculate the kinetic energy gained by the toy.

4 The graph below shows the force vs distance graph as a sports shoe is compressed during the stride of an athlete. Calculate the work done in compressing the shoe by 7 mm.



5 A weightlifter raises a 155 kg barbell to a height of 1.20 m at constant speed. Calculate the work done by the weightlifter.

6 Krisha pushes a lawnmower at constant speed across 15.0 m of lawn. She applies a force of 68.0 N at an angle of 60.0° to the horizontal. Calculate the work she does against friction.

Analysis

7 An engineer is testing a new material for its elastic properties. By applying various forces on a sample and measuring its corresponding distance stretched, the following data were obtained.

Force (kN)	Distance stretched (mm)
0.0	0.00
5.0	2.00
10.0	6.00
15.0	8.00
20.0	7.50
25.0	5.50
30.0	3.00
35.0	1.00
40.0	0.00

a Construct a graph of the data with force on the *y*-axis and distance stretched on the *x*-axis. Ensure that you draw a smooth curve of best fit.

b Use the graph to calculate the work done up to the point of maximum distance stretched.

8 An 806 g javelin is released at an angle of 45.0° from a height of 1.90 m and at a speed of 108 km h^{-1}. Calculate the work done by the gravitational force on the javelin from its release to the point where it lands on the ground.

3.4 Elastic potential energy

In everyday life, you frequently encounter situations in which work is done to stretch or compress materials. Think of bungee jumping, pole vaulting (Figure 3.4.1), trampolining and tennis, where the elastic properties of materials are harnessed to generate thrills for spectators and participants. Computer keyboards have tiny springs in the keys, and wind-up toys, old-fashioned watches, door-closing mechanisms and car suspensions are some of the other devices that use elastic springs.

Elastic potential energy is the energy stored in a material when it is stretched or compressed. If the material is elastic, this energy can be returned to the system, but in inelastic materials permanent change occurs.

FIGURE 3.4.1 The elastic potential energy stored in the pole is what allows the pole vault competitor to propel herself over the bar.

PHYSICSFILE

Recovery straps and tow ropes

Recovery straps are used to pull bogged cars out of their predicament using the energy stored in the elasticised straps. When the recovery vehicle moves forward, the kinetic energy of the recovery vehicle causes the strap to stretch. The energy is stored in the recovery strap and is then transferred to the bogged vehicle over an extended period of time, which pulls it out of the sand or mud. Using a tow rope, which has no elasticity, would cause a greater force over a shorter period, which could cause damage to the recovery vehicle, the bogged vehicle, or the tow rope itself.

Conversely, you would not use a recovery strap to tow a broken down vehicle, as the energy stored in the strap would cause the vehicle in tow to 'bounce' forwards and backwards, which would make it difficult to control. Using a tow rope with less stretch means that the kinetic energy of the towed vehicle would not fluctuate and so its velocity could be maintained safely.

HOOKE'S LAW

It is relatively easy to start stretching a spring, but more and more force is required for each incremental amount of extension (distance stretched). This is expressed in Hooke's law.

$F = -kx$

where F is the force exerted by the spring (N)

k is the spring constant ($\mathrm{N\,m^{-1}}$)

x is the displacement (the extension or compression) of the spring (m)

Hooke's law describes how the force exerted by a spring is directly proportional to, but opposite in direction to, the distance that the spring is extended or compressed. The **spring constant** k is a measure of the stiffness of the spring. The behaviour of a spring under force is often illustrated graphically by plotting the force applied versus the extension achieved (Figure 3.4.2). The spring constant is represented by the gradient of the graph. Notice that a stiffer spring has a greater gradient and thus a larger spring constant.

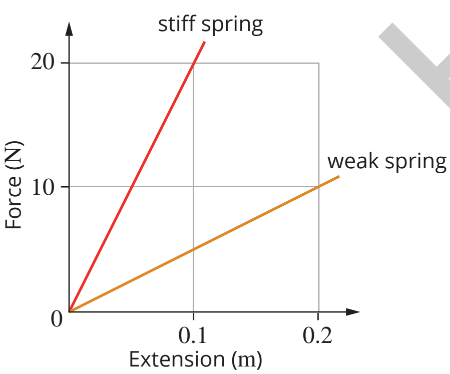


FIGURE 3.4.2 Both springs represented in this graph are ideal (i.e. they obey Hooke's law). The springs obey Hooke's law because they both have linear graphs, but they have different degrees of stiffness. The stiff spring has a spring constant of $200\,\mathrm{N\,m^{-1}}$. The spring constant of the other more elastic spring is just $50\,\mathrm{N\,m^{-1}}$. The stiffer spring has the higher gradient (i.e. a steeper line) on the graph.

When considering the work done in deforming a spring, the force applied is in the direction of the displacement and hence the negative sign in $F = -kx$ can be ignored. The applied force is directly proportional to displacement and, as discussed in the previous section, when force is not constant, the work done by the force can be calculated (or estimated) by determining the area under the force vs distance graph.

An expression for the work done in extending or compressing a spring, and the elastic potential energy which is then stored in the spring, can now be derived. Consider the graph in Figure 3.4.3. The elastic potential energy when the spring is extended by 5 metres is represented by the area of the shaded triangle.

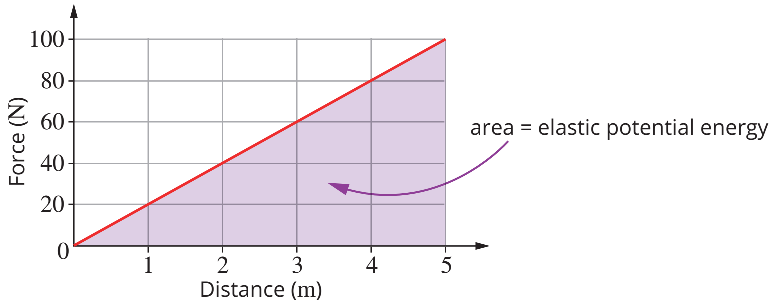


FIGURE 3.4.3 The elastic potential energy is calculated by the area under the force vs distance graph.

The elastic potential energy, E_s, is calculated as follows:

$$\begin{aligned} E_s &= \frac{1}{2} \times \text{ base } \times \text{ height} \\ &= \frac{1}{2} \times x \times F \\ &= \frac{1}{2} \times x \times kx \\ &= \frac{1}{2} kx^2 \end{aligned}$$

$$E_s = \frac{1}{2} kx^2$$

where k is the spring constant (N m^{-1})

x is the distance the spring is extended, also called extension (m)

Elastic potential energy is the energy stored in any elastic medium—such as a rope, spring or rubber band—due to forces stretching or compressing the bonds between atoms.

We call the directly proportional relationship between force and extension ‘elastic behaviour’. Elastic behaviour obeys Hooke’s law. Springs that exhibit elastic behaviour will be able to do work with the elastic potential energy when the applied force is removed.

It is possible to exceed the **elastic limit** of a spring or other elastic material. At this point permanent **deformation** occurs, that is, the spring no longer returns to its initial shape. If the force is increased further, the **breaking point** is reached, at which point the material fails or breaks down (Figure 3.4.4).

While the work done in permanently deforming a spring can still be calculated from the area under the force vs distance curve, the energy stored may not all be recoverable, as work has been done to permanently change the material.

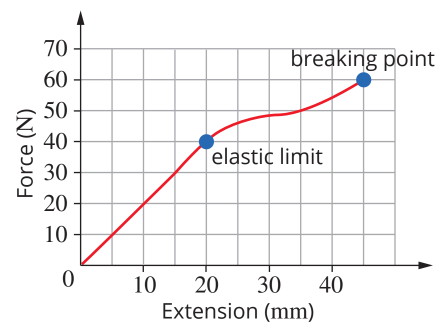
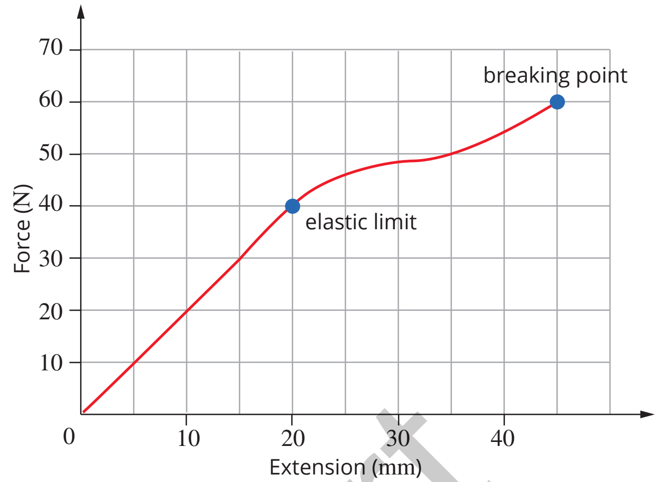


FIGURE 3.4.4 The point at which the force vs distance curve first deviates from linear behaviour is the elastic limit, i.e. the point where permanent damage is done to the spring.

Worked example 3.4.1

CALCULATING THE SPRING CONSTANT, ELASTIC POTENTIAL ENERGY AND WORK

A fine steel wire has the force and extension properties shown in the graph below.



a Calculate the spring constant, k, for the wire.	
Thinking	**Working**
The spring constant is the gradient of the first linear section of the graph (in units $N\,m^{-1}$).	$k = \frac{\Delta F}{\Delta x}$ $= \frac{40}{0.020}$ $= 2000\ N\ m^{-1}$

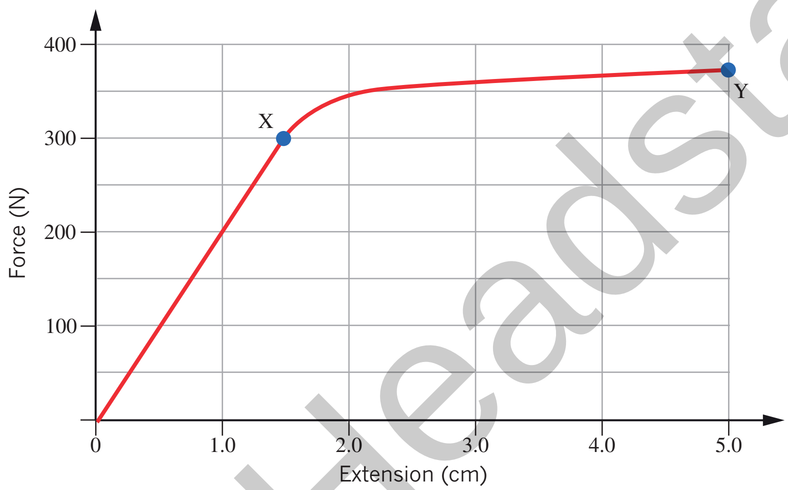
b Calculate the elastic potential energy that the wire can store before permanent deformation begins.	
Thinking	**Working**
The elastic potential energy is the area under the curve up to the elastic limit.	$E_s = \frac{1}{2}\text{height} \times \text{base}$ $= \frac{1}{2} \times 40 \times 0.020$ $= 0.40\ J$
This value can also be obtained using the formula for elastic potential energy.	$E_s = \frac{1}{2}kx^2$ $= \frac{1}{2} \times 2000 \times (0.020)^2$ $= 0.40\ J$

c Calculate the work done to break the wire.	
Thinking	**Working**
Add up the number of squares under the curve up to the breaking point.	Number of squares = 33 (approx.)
Calculate the energy per square. This is given by the area of a square. Remember to convert mm to m.	Energy for one square = 10×0.005 $= 0.050\,J$
Multiply the energy per square by the number of squares.	Work = energy per square × number of squares $= 0.050 \times 33$ $= 1.650$ $= 1.7\,J$ (approx.)

Worked example: Try yourself 3.4.1

CALCULATING THE SPRING CONSTANT, ELASTIC POTENTIAL ENERGY AND WORK

An alloy sample is tested under tension, giving the force vs extension graph shown below. X indicates the elastic limit and Y indicates the breaking point.



a Calculate the spring constant, *k*, for the sample.

b Calculate the elastic potential energy that the alloy can store before permanent deformation begins.

c Calculate the work done to break the sample.

3.4 Review

SUMMARY

- Hooke's law states that the force exerted by a spring is $F = -kx$. The negative sign indicates that the force opposes the displacement.
- k is the spring constant and is measured in $N\,m^{-1}$. This can be calculated (or estimated) from the gradient of the linear section (or the first linear section if there is more than one) of a force–displacement graph.
- The work done to a spring is equal to the elastic potential energy stored in the spring:

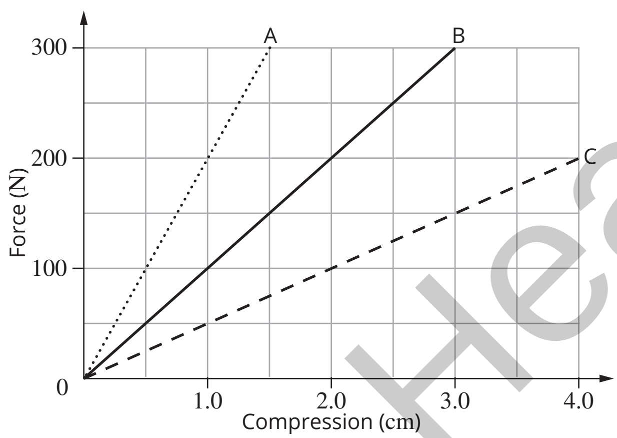
$$E_s = \frac{1}{2}kx^2$$

- The elastic potential energy (E_s) is measured in J or N m. This can be calculated (or estimated) from the area under a force–displacement graph.
- When a material displays elastic behaviour, it obeys Hooke's law, and the elastic potential energy stored is returned when the force is removed.
- When a material exceeds its elastic limit, permanent deformation occurs and not all the elastic potential energy is returned when the force is removed.

KEY QUESTIONS

Knowledge and understanding

1 Rank the springs below in order of increasing stiffness.



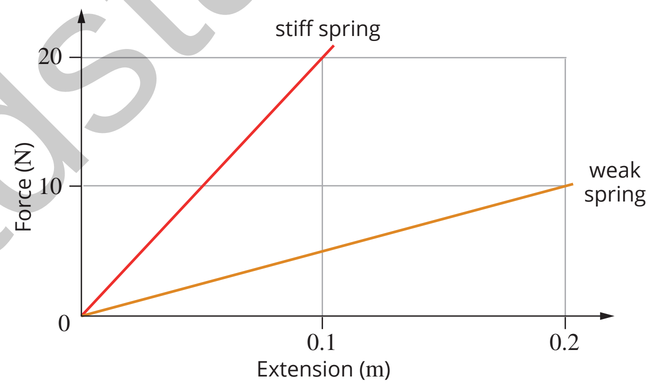
2 Consider the following tasks and decide whether you would prefer a rope with a high, medium or low spring constant.

a lowering a prefabricated concrete panel into place on a high-rise building site

b towing a bogged car out of a muddy track

c making a cargo net to secure various loads in a trailer

3 The graph of the stretching force versus extension for two springs is shown below.



a Calculate the spring constant of each spring.

b Find the difference between the elastic potential energy stored when each of the springs is extended by 20 cm. Assume the elastic limit has not been reached.

4 A 1.00 m piece of rubber has a spring constant of 50.0 N m^{-1}. Calculate how much the rubber will stretch if a force of 4.00 N is exerted on it.

5 A stretched rubber band is used to launch a toy plane into the air. The rubber band is stretched by 25.0 cm and has a spring constant of 128 N m^{-1}. Assume that the rubber band follows Hooke's law and ignore its mass.

a Calculate the magnitude of the force applied to the rubber band to stretch it by 25.0 cm.

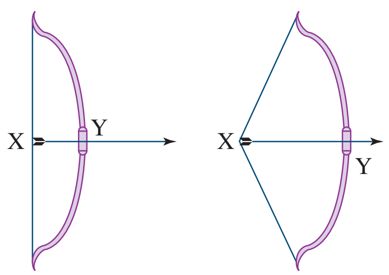
b Calculate the elastic potential energy stored in the stretched rubber band.

Analysis

6 An archer purchased a new bow for the Olympics. The table below shows the force required to pull the string back by various distances (the distance between X and Y in the diagrams).

Force (N)	Distance between bow and string (m)
0.0	0.100
30.0	0.150
40.0	0.200
45.0	0.250
50.0	0.300

The illustrations below show the bow and its string when (a) no force is applied and (b) when some force is applied.



(a) **(b)**

Answer the following questions in the case where the archer has drawn the string back so that the distance between the bow and the string (XY) is 30.0 cm.

a Construct a graph of the force (N) vs XY distance (m).

b Use the graph to calculate the elastic potential energy stored in the stretched string.

c Calculate the work done by the archer.

d Does the string obey Hooke's law as it is drawn back until the distance between X and Y is 30.0 cm? Justify your answer.

e Where on the graph is the elastic limit of the string?

3.5 Kinetic and potential energy

FIGURE 3.5.1 The bungee jumper is in free fall until the cord starts to take up some of the kinetic energy and convert it to potential energy.

A bungee jumper stakes their life on the principle of the conservation of energy (Figure 3.5.1). The gravitational potential energy they lose as they begin their jump is rapidly converted to kinetic energy. As the bungee jumper approaches the ground, the kinetic energy is converted to elastic potential energy in the bungee cord. The jumper is then jerked back upwards (no doubt relishing the adrenalin rush) as the elastic potential energy is converted back to kinetic and potential energy. The calculations that ensure their safety are the subject of this section.

KINETIC ENERGY

Kinetic energy (E_k) is the energy of motion of a body. For low speeds, it is calculated using the following equation.

$$E_k = \frac{1}{2}mv^2$$

where E_k is the kinetic energy of the object (J)
m is the mass of the object (kg)
v is the velocity of the object (m s^{-1})

This equation can be derived from the definition of work. Recall that if a force, F, acts on a body of mass m and causes a horizontal displacement of s, the work done is given by the formula $W = Fs$, which is equivalent to $W = mas$.

Start by rearranging the equation $v^2 = u^2 + 2as$ to make s the subject:

$$s = \frac{v^2 - u^2}{2a}$$

Substitute this into the second equation for work given above: $W = mas$. This yields:

$$\begin{aligned} W &= ma\left(\frac{v^2 - u^2}{2a}\right) \\ &= m\left(\frac{v^2 - u^2}{2}\right) \\ &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \end{aligned}$$

As the work is done to change the kinetic energy, then:

$$\Delta E_k = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

For a particular speed, the equation can be simplified to:

$$E_k = \frac{1}{2}mv^2$$

Kinetic energy in collisions

In perfectly **elastic collisions**, kinetic energy is transferred between objects and no energy is **transformed** into heat, sound or deformation. In these cases the following relationship holds:

$$E_k \text{ (before)} = E_k \text{ (after)}$$

In Section 3.1 you saw that, in a closed system, momentum is always conserved in a collision. The total energy is also conserved in a closed system. However, in general, kinetic energy is not conserved in collisions. These collisions are called **inelastic collisions**.

Perfectly elastic collisions do not exist in everyday situations, but they do exist in the interactions between atoms and subatomic particles. A collision between two billiard balls or the spheres in Newton's cradle is almost perfectly elastic, because very little of their kinetic energy is transformed into heat and sound energy.

Collisions such as a bouncing basketball, a gymnast bouncing on a trampoline or a tennis ball being hit are moderately elastic, with about half the kinetic energy of the system being retained. Perfectly inelastic collisions are those in which the colliding bodies stick together after impact with no kinetic energy. Some car crashes, a collision between a meteorite and the Moon, and a collision involving two balls of plasticine could all be perfectly inelastic. In these collisions, most—and sometimes all—of the initial kinetic energy of the system is transformed into other forms of energy.

Worked example 3.5.1

ELASTIC OR INELASTIC COLLISION?

A car of mass 1000 kg travelling west at 20.0 m s^{-1} crashes into the rear of a stationary bus of mass 5000 kg. The vehicles lock together on impact. Using appropriate calculations, show whether the collision is elastic or inelastic.

Thinking	Working
Use conservation of momentum to find the final velocity of the wreck.	$\Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$ $p_{\text{initialc}} + p_{\text{initialb}} = p_{\text{final(c+b)}}$ $m_c u_c + m_b u_b = m_{c+b} u_{c+b}$ $1000 \times 20.0 + 5000 \times 0 = (1000 + 5000)v$ $20000 = 6000v$ $v = 3.33\,\text{m s}^{-1}$
Calculate the total initial kinetic energy before the collision.	Initially: $E_k = \frac{1}{2}mu^2$ $= \frac{1}{2} \times 1000 \times 20.0^2$ $= 2.00 \times 10^5$ J
Calculate the total final kinetic energy of the joined vehicles.	Finally: $E_k = \frac{1}{2}mv^2$ $= \frac{1}{2} \times (1000 + 5000) \times 3.33^2$ $= 33266.7$ $= 3.33 \times 10^4$ J
Compare the kinetic energy before and after the collision to determine whether the collision is elastic or inelastic.	The kinetic energy after the collision is less than the kinetic energy before it. Therefore the collision is inelastic. The missing energy has been transformed into heat, sound and deformation of the vehicles.

Worked example: Try yourself 3.5.1

ELASTIC OR INELASTIC COLLISION?

A 209 g softball with initial velocity 9.00 m s^{-1} to the right collides with a stationary baseball of mass 112 g. After the collision, both balls move to the right and the 209 g softball has a speed of 3.00 m s^{-1}. Using appropriate calculations, show whether the collision is elastic or inelastic.

POTENTIAL ENERGY

The gravitational potential energy of an object, E_g, is the energy stored in it due to its position in a gravitational field above a reference point. It is directly proportional to the mass of the object, m, its height above the reference point, Δh, and the strength of the gravitational field, g. This is combined in the following equation.

$$E_g = mg\Delta h$$

where E_g is the gravitational potential energy (J)
m is the mass of the object (kg)
g is gravitational field strength ($N\,kg^{-1}$)
Δh is the height above the reference point (m)

This equation is derived from the fact that, in order to lift an object of mass m through a distance Δh, work needs to be done against the force of gravity. Close to the surface of the Earth, this force is simply $F = mg$ (where $g = 9.8\,N\,kg^{-1}$) and the distance travelled, s, is Δh. Thus the work done is $W = Fs$, which is equal to the potential energy gained.

Calculating changes in gravitational potential energy from a force graph

When the gravitational force acting on an object varies, the gravitational potential energy can be calculated using a graph (in the same way that you calculated the work done by a varying force in sections 3.3 and 3.4). If the force is plotted as a function of distance, a graph like the one in Figure 3.5.2 is obtained.

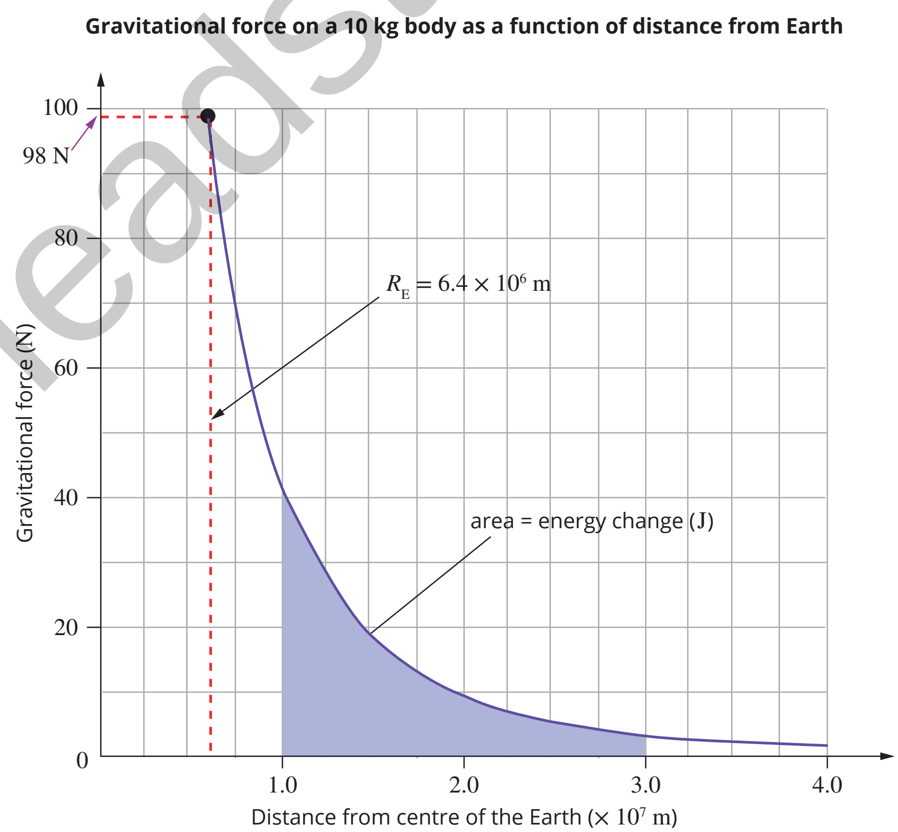


FIGURE 3.5.2 Plot of the gravitational force acting on a 10 kg body as a function of the distance from the centre of the Earth. The shaded area represents the work done in moving the body from 1.0×10^7 m to 3.0×10^7 m above the centre of the Earth.

Worked example 3.5.2

DETERMINING CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE VS DISTANCE GRAPH

Using the graph in Figure 3.5.2, estimate the work done against the gravitational force in moving the 10 kg object from an orbital radius of 1.0×10^7 m to 3.0×10^7 m, and hence find the gravitational potential energy gained.

Thinking	Working
Find the energy represented per square in the graph.	One square represents: $10 \times 0.25 \times 10^7 = 2.5 \times 10^7$ J
Identify the two values of distance that are relevant to the question.	The object starts at 1.0×10^7 m and finishes at 3.0×10^7 m.
Count the squares under the curve between the two distances and multiply the number by the energy per square.	Work done: 10.5 squares (approx.) $\times 2.5 \times 10^7$ $= 2.6 \times 10^8$ J (approx.)
Potential energy gained = work done	2.6×10^8 J (approx.)

Worked example: Try yourself 3.5.2

CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE VS DISTANCE GRAPH

Using the graph in Figure 3.5.2, calculate the gravitational potential energy gained if the 10 kg object is moved from the surface of the Earth to 2.0×10^7 m above the centre of the Earth.

The disadvantage of the graph in Figure 3.5.2 is that it is specific to the mass of the object under consideration. Further, to construct the graph, the force on the 10 kg object has to be calculated at each distance.

Sometimes it is more useful to create a graph of the force exerted per unit mass. Recall Newton's law of universal gravitation:

$$F_g = \frac{GMm}{r^2}$$

This can be rearranged as:

$$\frac{F_g}{m} = g = \frac{GM}{r^2}$$

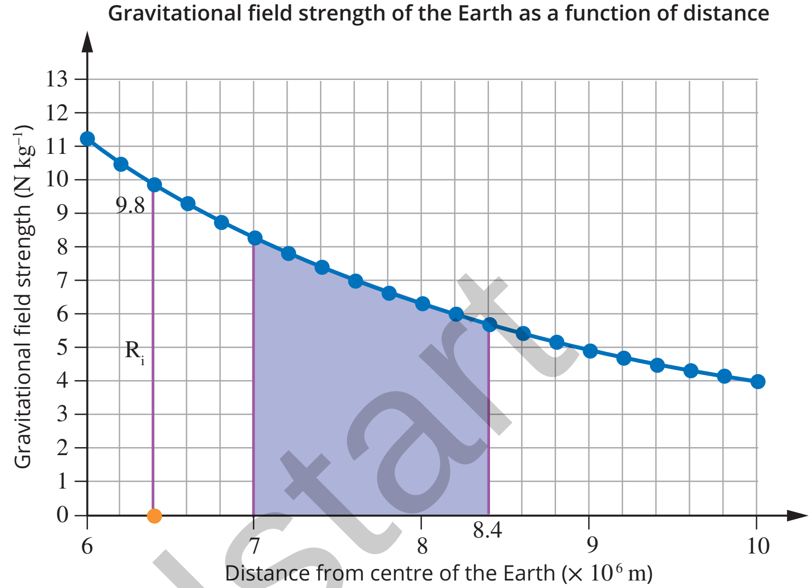
This is often called the gravitational field strength equation and it is dependent only on the mass that is generating the gravitational field. Such a graph can be used to calculate the work done on any mass in the field.

Worked example 3.5.3

CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH GRAPH

A decommissioned satellite of mass 1000 kg has an elliptical orbit around the Earth. At its closest approach (its perigee), it is 600 km above the Earth's surface. At its furthest point (its apogee) it is 2000 km from the Earth's surface. The Earth has a radius of 6.4×10^6 m. The gravitational field strength of the Earth is shown in the graph.

Gravitational field strength of the Earth as a function of distance



a Calculate the change in potential energy of the satellite as it moves from its perigee to its apogee.

Thinking	Working
Find the energy represented by each square in the graph.	One square represents $1.0 \times 0.20 \times 10^6 = 2.0 \times 10^5\ \text{J kg}^{-1}$
Count the squares under the curve for the relevant area, and multiply the total by the energy per kg represented by each square.	49 squares (approx.) $\times 2.0 \times 10^5$ $= 9.8 \times 10^6\ \text{J kg}^{-1}$
Calculate the potential energy gained by the satellite by multiplying the work done by the mass of the satellite.	Energy gained: $E_g = 9.8 \times 10^6 \times 1000$ $= 9.8 \times 10^9$ J (approx.)

b The satellite is moving with a speed of 15 km s^{-1} at its perigee. How fast is it travelling at its apogee?

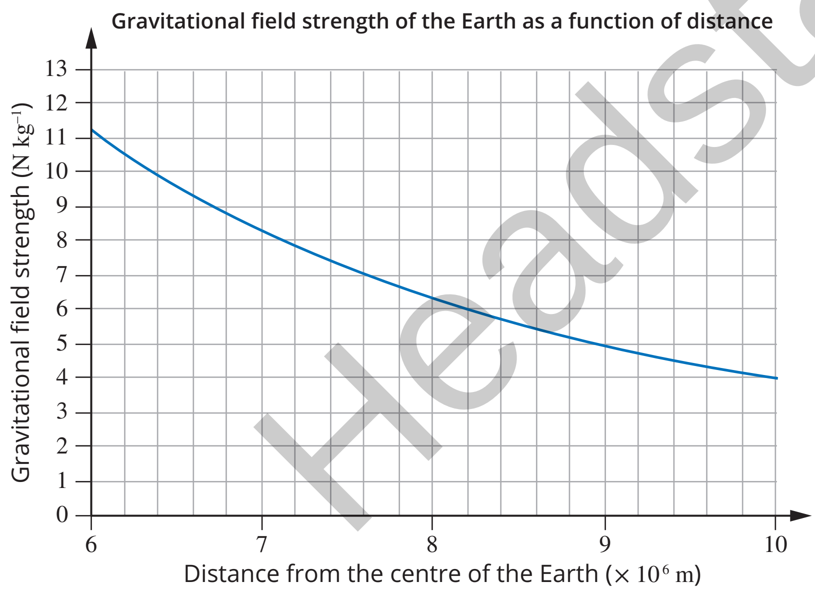
Thinking	Working
First calculate the satellite's kinetic energy at its apogee.	$E_{kp} = \frac{1}{2} m v_p^2$ $= \frac{1}{2} \times 1000 \times (15 \times 10^3)^2$ $= 1.125 \times 10^{11}$ J

The gain in gravitational potential energy at its apogee is at the expense of kinetic energy. Calculate the kinetic energy of the satellite at its apogee.	$E_{ka} = E_{kp} - E_g$ $= 1.125 \times 10^{11} - 9.8 \times 10^{9}$ $= 1.0 \times 10^{11}$ J
Calculate the speed of the satellite at its apogee.	$E_{ka} = \frac{1}{2} m v_a^2$ $1.0 \times 10^{11} = \frac{1}{2} \times 1000 \times v_a^2$ $v_a = \sqrt{\frac{2 \times 1.0 \times 10^{11}}{1000}}$ $= 14142.1 \text{ m s}^{-1}$ $= 14 \text{ km s}^{-1}$

Worked example: Try yourself 3.5.3

CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH GRAPH

A satellite of mass 1100 kg is in an elliptical orbit around the Earth. At its closest approach (perigee), it is just 600 km above Earth's surface. Its furthest point (apogee) is 2600 km from the Earth's surface. The Earth has a radius of 6.4×10^6 m. The gravitational field strength of the Earth is shown in the graph.



a	Calculate the change in potential energy of the satellite as it moves from its perigee to its apogee.

b	The satellite is moving with a speed of 8.0 km s^{-1} at its perigee. How fast is it travelling at its apogee?

WORK AND ENERGY

Both work and energy are scalar quantities and thus have only magnitude. It is important, however, that you keep account of whether kinetic energy is being gained or lost by an object or whether gravitational potential energy is being gained or lost by the gravitational field. If work is being done by a body, it could lose kinetic energy as it slows down, or, if work is being done by the gravitational field, the field loses gravitational potential energy as the object falls. Conversely, if work is done on the body by an external force, the body would gain kinetic energy as it speeds up, or the gravitational field would gain gravitational potential energy as the object rises.

A weightlifter loses chemical potential energy as they exert a force on a barbell to lift the bar. If they lift the bar at constant speed, the bar does not gain kinetic energy, but the gravitational field gains gravitational potential energy. In drawing back an arrow, an archer does work on the bow, and this elastic potential energy is transformed to the kinetic energy of the arrow when the string does work on the arrow as it is released (Figure 3.5.3).

FIGURE 3.5.3 The archer does work on the bow and elastic potential energy is stored. This is later transformed into the kinetic energy of the arrow.

3.5 Review

SUMMARY

- Kinetic energy is the energy of motion of a body:

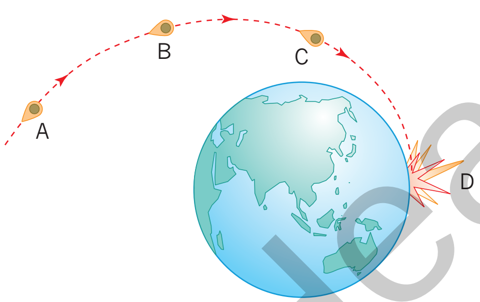
$$E_k = \frac{1}{2}mv^2$$

- For perfectly elastic collisions, the kinetic energy before the collision is equal to the kinetic energy after the collision.
- Close to the surface of the Earth, where the force of gravity can be taken as constant, the change in gravitational potential energy of an object of mass m is $E_g = mg\Delta h$, where the height changes by Δh.
- For a non-constant gravitational force, the gravitational potential energy can be calculated from the area under a graph of force versus distance.
- For convenience, force–distance graphs are often plotted as force per unit mass (for example, gravitational field strength) versus distance. This enables the same graph to be used for any mass. In this case the area under the graph is the potential energy per unit mass.

KEY QUESTIONS

Knowledge and understanding

1 The figure below shows a meteor plunging towards the Earth, partially burning up in the atmosphere on its way.
Choose which statements are correct. More than one correct answer is possible.



A The kinetic energy of the meteor increases as it travels from A to D.
B The gravitational potential energy of the meteor relative to the surface of the Earth increases as it travels from A to D.
C The total energy of the meteor increases as it travels from A to D.
D The total mechanical energy of the meteor remains constant.
E The gravitational potential energy of the meteor relative to the surface of the Earth decreases as it travels from A to D.

2 In a cable car system, two cars of the same mass are attached to a moving cable that is powered by a motor at one end. As car A is pulled upwards, car B descends, both at the same speed. Select the statements that are correct. More than one correct answer is possible.
A Car A and car B each have constant kinetic energy.
B Car A and car B each have constant gravitational potential energy.
C As the gravitational potential energy of car A increases, that of car B decreases.
D The motor does work on the cable.

3 Calculate the gravitational potential energy of a 115 kg climber standing at the top of Mount Kosciuszko 2228 m above sea level.

4 A 283 g volleyball is hit into the opposition court with a velocity of 9.50 m s^{-1}. Calculate the kinetic energy of the volleyball as it leaves the player's hand.

5 Calculate the gravitational potential energy that a 3.00 kg watermelon has when it has travelled 45.0 m up into the air after having been fired from a slingshot.

continued over page

3.5 Review *continued*

Analysis

6 The 11 t Hubble telescope is in a circular orbit at an altitude of approximately 600 km above the surface of the Earth. A geosynchronous weather satellite of the same mass is in an orbit at an altitude of approximately 3600 km. Select the statements that are correct. More than one correct answer is possible.

A The gravitational potential energy of the geosynchronous satellite is six times that of the Hubble telescope, relative to the surface of the Earth.

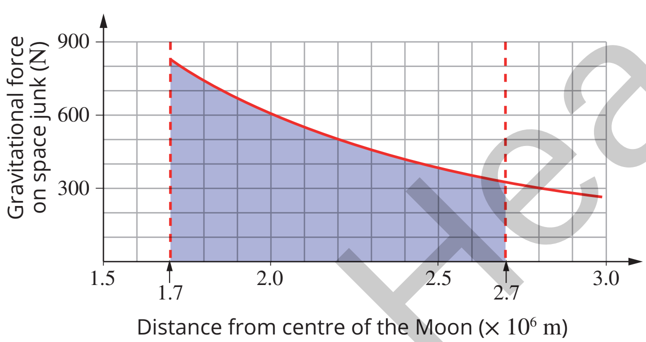
B The Hubble telescope's orbital speed is greater than that of the weather satellite.

C The kinetic energy of the weather satellite is greater than that of the Hubble telescope.

D The weather satellite has more gravitational potential energy than the Hubble telescope, relative to the surface of the Earth.

7 A 500 kg lump of space junk is plummeting towards the Moon. Its speed when it is 2.7×10^6 m from the centre of the Moon is 250 m s^{-1}. The Moon has a radius of 1.7×10^6 m.

The gravitational force–distance graph for the space junk is shown below.

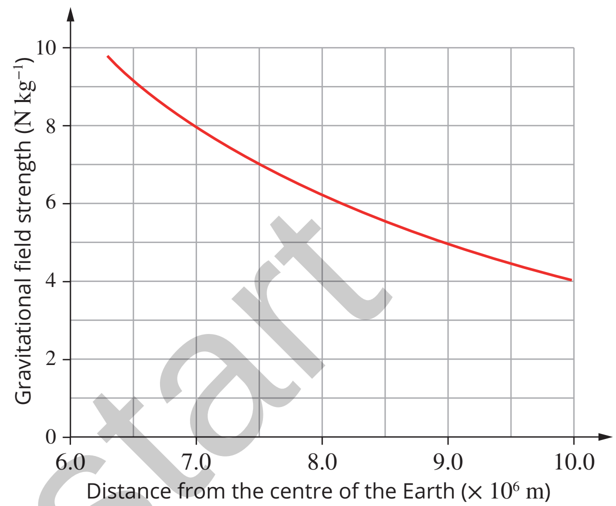


a Calculate the kinetic energy of the junk when it is travelling at 250 m s^{-1}.

b Calculate the increase in kinetic energy of the junk as it falls from 2.7×10^6 m from the Moon's centre to 1.7×10^6 m from the Moon's centre.

c Calculate the speed of the junk as it crashes into the Moon.

8 A 20 t piece of space junk is in orbit at an altitude of 600 km above the surface of the Earth. In order to remove it from the path of an oncoming satellite, it is shifted into an orbit of 2600 km above the surface of the Earth. Calculate the work done in moving the space junk into the higher orbit. The surface of the Earth is 6.4×10^6 m from the centre of the Earth.



3.6 Conservation of energy

We can classify everything we know about the universe as either matter or energy. In his famous equation $E = mc^2$, Einstein showed that matter is actually a store of energy, so everything in the universe is really just energy. This section explores the law of conservation of energy, a fundamental principle that can be applied to all interactions between objects.

THE LAW OF CONSERVATION OF ENERGY

Energy comes in many forms, such as heat, light, sound, chemical and electrical. It is a scalar quantity and is measured in joules (J). Energy is also associated with the motion and position of an object. Collectively this energy is called the **mechanical energy** of the object. In the motion problems explored in this chapter, moving objects are described as having kinetic energy. An object can also have access to stored or gravitational potential energy because of its position in a gravitational field. For instance, a building crane lifting a steel beam several stories is doing work against the gravitational field, giving the beam access to the gravitational potential energy stored in the gravitational field. If the lifting chain were to break, the field will then do work on the beam and increase its kinetic energy as it accelerates under the influence of gravity.

The transformation of gravitational potential energy to kinetic energy is an illustration of the **law of conservation of energy**, a fundamental principle of nature. This law states that energy is neither created nor destroyed. However, it can change from one form to another, or in other words**, transform**. As the gravitational potential energy available to a falling object decreases, its kinetic energy increases. The total amount of mechanical energy remains constant, that is, it is conserved.

While energy is never destroyed, it can be transformed into other energies that are not easily recoverable. For instance, the kinetic energy of a vehicle is reduced as it encounters friction, with the energy transformed into heat in the tyres. It could also be transformed into heat in the brakes as the vehicle stops. The mechanical energy before and after an event is only the same under ideal conditions, but in many cases, this equality is a useful approximation.

Problems combining gravitational potential and kinetic energy

Energy is a scalar quantity and hence easier to work with than a vector quantity. Therefore it is worth analysing a problem to see if calculations involving energy are possible without resorting to techniques involving forces and other vectors.

The sum of the potential and kinetic energy of an object is its mechanical energy, and this is constant unless work is done by an external force:

$$E_m = E_k + E_g = \frac{1}{2}mv^2 + mg\Delta h$$

Energy is frequently transformed from potential energy to kinetic energy and vice versa. But in any transformation, total mechanical energy is conserved. To illustrate this, consider a 60 g tennis ball dropped from a height of 1.0 m (Figure 3.6.1 on p. 148). Before it is released, its kinetic energy is 0 J and its gravitational potential energy (assuming that $g = 9.8\,\mathrm{m\,s^{-2}}$) is:

$$E_g = mg\Delta h = (0.060)(9.8)(1.0) = 0.59\,\mathrm{J}$$

Thus the ball's mechanical energy is $0 + 0.59 = 0.59$ J.

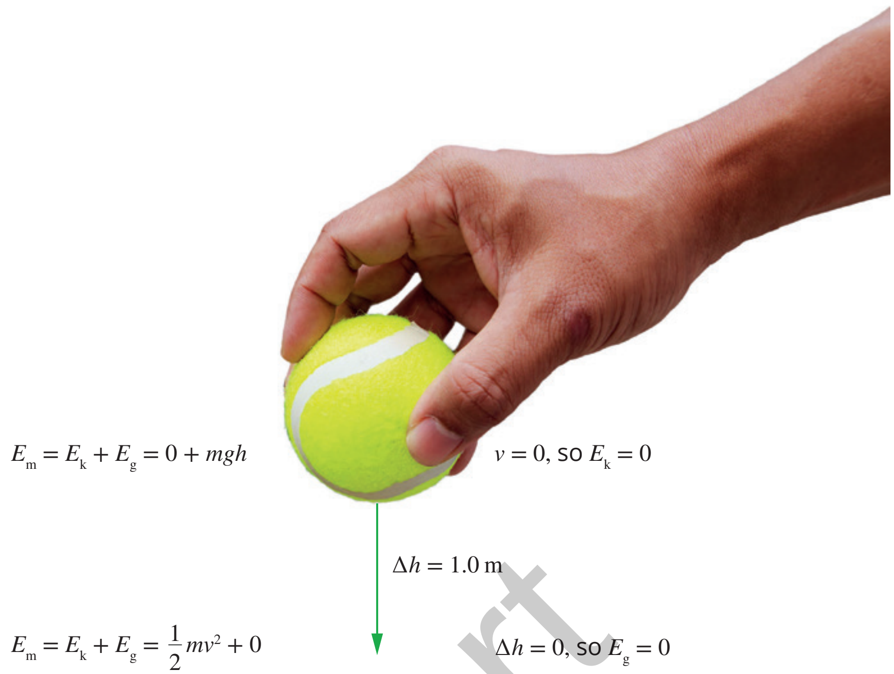


FIGURE 3.6.1 A falling tennis ball provides an illustration of the conservation of mechanical energy.

At the instant the ball hits the ground, the total mechanical energy consists of the gravitational potential energy available to it, which would be 0 J, and the kinetic energy it has just prior to hitting the ground. To calculate its kinetic energy, its velocity just before it hits the ground needs to be calculated using an appropriate equation of motion. Knowing that $s = -1.0\,\text{m}$, $a = -9.8\,\text{m s}^{-2}$ and $u = 0\,\text{m s}^{-1}$, the final velocity can be calculated as:

$$v^2 = u^2 + 2as$$
$$= (0)^2 + 2(-9.8)(-1.0)$$
$$v = \sqrt{19.6} = 4.43\,\text{m s}^{-1}$$

Therefore the kinetic energy of the ball just before it hits the ground is:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(0.0600)(4.43)^2 = 0.59\,\text{J}$$

Notice that the total mechanical energy prior to the ball's release is the same as its total mechanical energy as it hits the ground. Before release:

$$E_m = E_k + E_g = 0 + 0.59 = 0.59\,\text{J}$$

On hitting the ground:

$$E_m = E_k + E_g = 0.59 + 0 = 0.59\,\text{J}$$

In fact, mechanical energy is constant throughout the drop. To see this, consider the tennis ball when it has fallen halfway to the ground. At this point, $h = 0.50\,\text{m}$, $v = 3.13\,\text{m s}^{-1}$ and its mechanical energy is:

$$E_m = E_k + E_g$$
$$= \frac{1}{2}(0.060)(3.13)^2 + (0.060)(9.8)(0.50)$$
$$= 0.294 + 0.294$$
$$= 0.59\,\text{J}$$

Note that at the halfway point, the mechanical energy is evenly split between kinetic energy (0.294 J) and gravitational potential energy (also 0.294 J).

In reality, as a ball drops through the air, a small amount of its energy is transformed into heat and sound, and the ball slows down slightly. This means that mechanical energy is not entirely conserved. However, this small effect can be considered negligible for many falling objects.

Using conservation of mechanical energy to calculate velocity

The speed of a falling object does not depend on its mass. This can be demonstrated by applying the law of conservation of energy to mechanical energy.

Consider an object of mass m dropped from a height of h. At the moment it is dropped, its initial kinetic energy is zero. At the moment before it hits the ground, its final gravitational potential energy is zero. From the conservation of mechanical energy it follows that:

$$E_{\text{m initial}} = E_{\text{m final}}$$

$$E_{\text{k initial}} + E_{\text{g initial}} = E_{\text{k final}} + E_{\text{g final}}$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

This equation can be used to find the final velocity of a falling object. Note that the equation does not mention the mass of the falling object. Thus if air resistance is negligible, any object will have the same final velocity when it is dropped from the same height, whatever its mass.

Conservation of mechanical energy in complex situations

Knowing that mechanical energy is conserved allows us to determine outcomes in non-linear situations where equations of motion cannot be used. For example, consider a pendulum with a bob displaced from its mean position such that its height has increased by 20 cm (Figure 3.6.2).

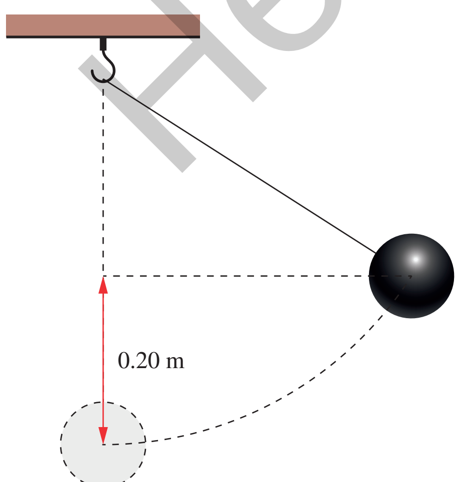


FIGURE 3.6.2 A falling pendulum is an example of conservation of mechanical energy.

Since a pendulum involves transforming gravitational potential energy into kinetic energy, the conservation of mechanical energy applies to the situation. Therefore the formula developed earlier for the velocity of a falling object can be used to find the velocity of the pendulum bob at its lowest point:

$$v = \sqrt{2gh} = \sqrt{2(9.8)(0.20)} = 2.0\,\mathrm{m\,s^{-1}}$$

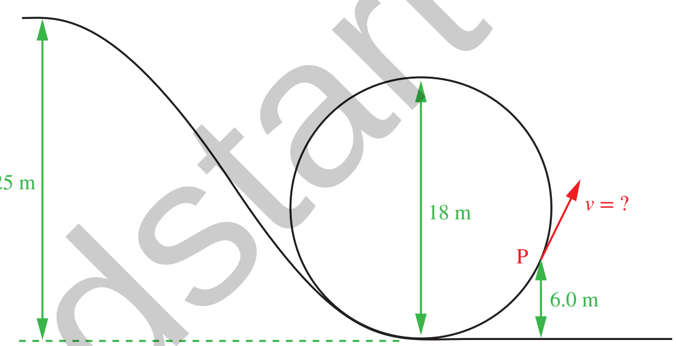
The speed of the pendulum bob will be approximately $2.0\,\mathrm{m\,s^{-1}}$. However, unlike a falling object, in this case the direction of the bob's motion will be horizontal instead of vertical at its lowest point.

The equations of motion relate to linear motion and cannot be applied to the motion of the pendulum.

Worked example 3.6.1

APPLYING THE LAW OF CONSERVATION OF ENERGY

Consider a rollercoaster with a lift hill of height 25 m and a loop height of 18 m. At the top of the lift hill, a rollercoaster car has zero velocity just before it begins to roll down the hill. Calculate the speed of the car at point P on the loop when the car is 6.0 m above the ground. Assume that friction is negligible.

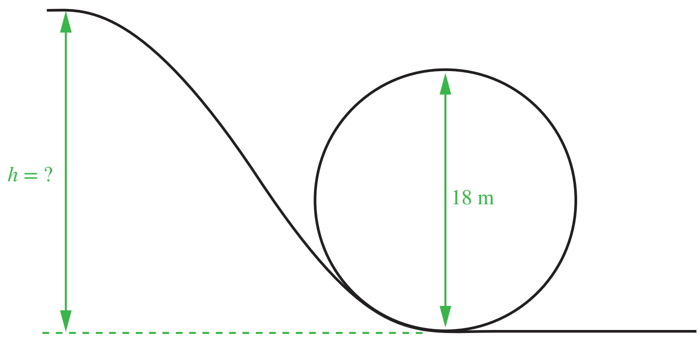


Thinking	Working
Because of the law of conservation of mechanical energy, the total mechanical energy, E_m, of the car before rolling down the hill can be equated with the total mechanical energy at point P.	$E_{\mathrm{m\ before}} = E_{\mathrm{m\ at\ P}}$
Expand the equation and cancel m from both sides.	$\frac{1}{2}mu^2 + mg\Delta h = \frac{1}{2}mv^2 + mg\Delta h$ $\frac{1}{2}u^2 + g\Delta h = \frac{1}{2}v^2 + g\Delta h$
Substitute the given values into the equation.	$\frac{1}{2}(0)^2 + (9.8)(25.0) = \frac{1}{2}v^2 + (9.8)(6.0)$
Rearrange the equation and solve for v.	$(0) + (245.0) = \frac{1}{2}v^2 + (58.8)$ $v = \sqrt{2(245.0 - 58.8)}$ $v = \sqrt{372.4}$
Present your answer with the correct number of significant figures and the correct units.	$v = 19\,\mathrm{m\,s^{-1}}$

Worked example: Try yourself 3.6.1

APPLYING THE LAW OF CONSERVATION OF ENERGY

Use the law of conservation of energy to determine the height of the lift hill required to ensure that the speed of a rollercoaster car at the top of the 18 m loop is 25 m s^{-1}. Assume that the velocity of the car at the top of the hill is zero just before it begins to roll down the hill, and that friction is negligible.



Worked Example 3.6.2

USING THE CONSERVATION OF ENERGY TO ANALYSE PROJECTILE MOTION

A cricket ball of mass 142 g is thrown upwards at a speed of 15 m s^{-1}. Calculate the speed of the ball when it has reached a height of 8.0 m above the ground. Assume that the ball is thrown from a height of 1.5 m above the ground and that $g = 9.8$ m s^{-2}.

Thinking	Working
Equate the total mechanical energy, E_m, of the cricket ball as it is released with the total mechanical energy at a height of 8.00 m.	$E_{\text{m before}} = E_{\text{m at 8.00 m}}$
Expand the equation and then cancel *m* from both sides.	$\frac{1}{2}mu^2 + mg\Delta h = \frac{1}{2}mv^2 + mg\Delta h$ $\frac{1}{2}u^2 + g\Delta h = \frac{1}{2}v^2 + g\Delta h$
Substitute the given values into the equation.	$\frac{1}{2}(15)^2 + (9.8)(1.5) = \frac{1}{2}v^2 + (9.8)(8.0)$
Rearrange the equation and solve for *v*.	$(112.5) + (14.7) = \frac{1}{2}v^2 + (78.4)$ $v = \sqrt{2(112.5 + 14.7 - 78.4)}$ $v = \sqrt{97.6}$
Present your answer with the correct number of significant figures and the correct units.	$v = 9.9$ m s^{-1}

Worked Example: Try yourself 3.6.2

USING THE CONSERVATION OF ENERGY TO ANALYSE PROJECTILE MOTION

An arrow of mass of 35 g is fired into the air at 80 m s^{-1} from a height of 1.4 m above the ground. Calculate the speed of the arrow when it is 30 m above the ground. Assume that $g = 9.8$ m s^{-2}.

Loss of mechanical energy

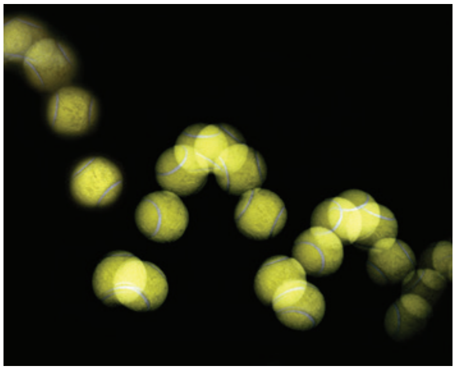


FIGURE 3.6.3 Mechanical energy is lost with each bounce of a tennis ball.

Mechanical energy is not conserved in every situation. This can be seen in the fact that when a tennis ball bounces a number of times, each bounce is lower than the bounce before it (Figure 3.6.3).

While mechanical energy is largely conserved as the ball moves through the air, a significant amount of kinetic energy is transformed into heat and sound when the ball compresses and decompress each time it bounces. This means that the ball does not have as much kinetic energy when it leaves the ground as it did when it landed. Therefore the gravitational potential energy that can be stored on the second bounce will be less than the gravitational potential energy that was stored initially, and so the second bounce is lower.

CASE STUDY

Coefficient of restitution

The bounce of the ball is an important factor in many sports. Physicists describe the bounciness of balls using a concept known as the coefficient of restitution (e). This coefficient is the ratio of the speed of a ball directly after a bounce to its speed before that bounce:

$$e = \frac{v_2}{v_1}$$

where v_1 is the speed before the bounce and v_2 is the speed after the bounce.

Since the coefficient of restitution is defined in terms of speed, v, and kinetic energy is proportional to v^2, it follows that:

$$e = \frac{v_2}{v_1} = \sqrt{\frac{(E_k)_2}{(E_k)_1}}$$

If we consider a ball dropped from height H and rebounding to height h, then, according to conservation of mechanical energy, the kinetic energy as the ball hits the ground is the same as the gravitational potential energy at the top of the bounce. Therefore:

$$e = \sqrt{\frac{(E_k)_2}{(E_k)_1}} = \sqrt{\frac{(E_g)_2}{(E_g)_1}} = \sqrt{\frac{mgh}{mgH}} = \sqrt{\frac{h}{H}}$$

So the coefficient of restitution (CoR) can be calculated from the initial drop height and the height of the first bounce. This is how many sports bodies specify the acceptability of balls used in playing the sport.

For example, according to the rules of the International Table Tennis Federation, a table tennis ball must bounce between 24 and 26 cm when dropped from a height of 30.5 cm onto a steel block. This corresponds to a CoR between 0.89 and 0.92. Similarly, a basketball must have a CoR of between 0.81 and 0.85 before it can be used in competition. Likewise a tennis ball must have a CoR of between 0.73 and 0.76.

The CoR depends on both the ball and the surface it is bouncing on. A tennis ball bouncing on grass has a different CoR to one bouncing on clay. This is one reason why tennis players prefer to play on some surfaces rather than others.

3.6 Review

SUMMARY

- Energy is a scalar quantity and is measured in joules (J).
- Energy is not created or destroyed, but merely transformed. This is called the law of conservation of energy.
- When work is done on a body it gains mechanical energy.
- When the body does work, energy is dissipated to the environment—as, for example, heat, sound or deformation—and the body loses mechanical energy.
- The sum of the kinetic and potential energy (i.e. the total mechanical energy) of an isolated system is always conserved.
- Because it is simpler to work with scalars, it is often helpful to solve motion problems by considering the energy involved.

KEY QUESTIONS

Knowledge and understanding

1 Choose the best alternative to complete the following sentence. In physics, the law of conservation of energy entails that:

- **A** All energy must be converted from one form into only one other form.
- **B** When energy is converted from one form to another, any missing energy must have been destroyed.
- **C** When energy is converted from one form to another, any extra energy gained must have come from gravitational potential energy.
- **D** No energy is gained or lost when one form of energy is converted into another form.

2 A student drops two sticks—one brown; one green—from a bridge into the water below to see which one emerges first at the other side of the bridge. The brown stick is twice the mass of the green stick. In answering the following questions, ignore any air resistance and friction that might be involved.

- **a** Which stick would hit the water first if they were dropped at the same time?
- **b** Which stick would have access to the greatest amount of gravitational potential energy at the top of the bridge?
- **c** Which stick would hit the water with the greatest speed?
- **d** Which stick would have the greatest kinetic energy just before it hit the water?

3 A group of people decide to film themselves throwing various objects off tall places. In one video they drop a bowling ball from the top of a dam wall. The ball hits the ground with a speed of $45.5\,\mathrm{m\,s^{-1}}$. Calculate the height of the dam wall.

4 A high-diver steps off a 10.0 m high platform and plunges into the pool below. Calculate the speed at which the diver hits the water.

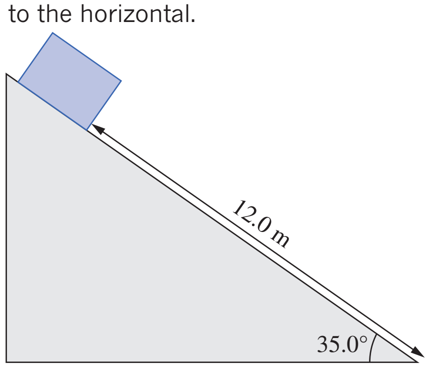
5 If a high-jumper with a mass of 63.0 kg just clears a height of 2.10 m, what was the high-jumper's speed as they left the ground?

6 A girl throws a 198.4 g softball directly up into the air. It leaves her hand at a speed of $21.7\,\mathrm{m\,s^{-1}}$.

- **a** Calculate the kinetic energy of the softball as it leaves the girl's hand.
- **b** If air resistance is ignored, what gain in gravitational potential energy occurs as the softball reaches the top of its flight?
- **c** If air resistance is ignored, calculate the height the softball reaches above the girl's hand.

Analysis

7 A box slides down a frictionless plane that is inclined at 35.0° to the horizontal.



Use the law of conservation of energy to calculate the speed of the box after it has travelled 12.0 m down the plane.

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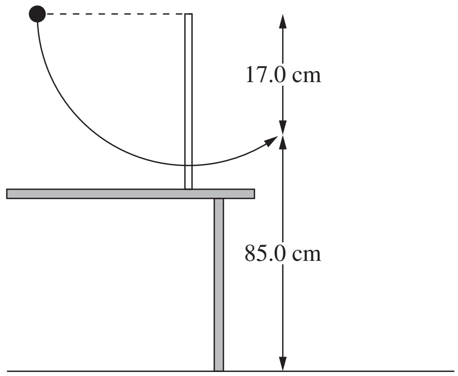
3.6 Review *continued*

8 A group of students sets up a pendulum with a thin 20.0 cm chain holding a heavy metal ball.



When the students tried to swing the pendulum for the first time, the chain broke just as the ball was 85.0 cm from the floor and 17.0 cm below the point at which it started to swing.

a Calculate the speed of the ball at the point at which the chain breaks.

b The students were not expecting the chain to break. Use the law of conservation of energy and the answer from part **a** to calculate the maximum height above the ground the ball would have achieved if the chain didn't break.

c Use the law of conservation of energy and the answer from part **b** to calculate the speed at which the ball strikes the ground.

d Use the law of conservation of energy and the starting point of the ball to calculate the speed at which the ball strikes the ground.

9 A 75.0 kg student swings out over a river on a rope attached to a tree on the riverbank. The student's final speed when they hit the water is 6.27 m s^{-1}.

a Calculate the kinetic energy of the student the moment before they hit the water.

b Determine the gravitational potential energy available to the student at the top of the riverbank before they began their swing.

c Using your answer to part **b**, calculate the height of the riverbank above the water level at the point where the student began their swing.

Chapter review

03

KEY TERMS

breaking point
conserved
deformation
elastic
elastic collision
elastic limit
elastic potential energy
gravitational potential energy
impulse
inelastic collision
isolated system
kinetic energy
law of conservation of energy
law of conservation of momentum
mechanical energy
momentum
spring constant
transform
work

REVIEW QUESTIONS

Knowledge and understanding

1 Arrange the following objects in order of decreasing momentum:
- **A** 10.0 kg dog running west at $5.00\,\mathrm{m\,s^{-1}}$
- **B** 42.0 kg child jogging south at $2.00\,\mathrm{m\,s^{-1}}$
- **C** 25.0 kg fish swimming east at $3\,\mathrm{m\,s^{-1}}$
- **D** 1250 kg car stationary at the traffic lights

2 Which alternative from the list below shows the unit for momentum that is equivalent to $\mathrm{kg\,m\,s^{-1}}$?
- **A** $\mathrm{J\,m}$
- **B** $\mathrm{N\,s^{-1}}$
- **C** $\mathrm{N\,s}$
- **D** $\mathrm{J\,s^{-1}}$

3 In an explosive collision, a combined mass separates into two masses. If one of the masses has momentum of $345\,\mathrm{kg\,m\,s^{-1}}$ south, what is the momentum of the other mass?

4 Which of the following statements correctly describes impulse? More than one correct answer is possible.
- **A** Impulse is the rate of change of momentum.
- **B** Impulse is the final momentum minus the initial momentum.
- **C** Impulse is a scalar.
- **D** Impulse can be calculated from the force and the time over which the force acts.

5 Use the concept of impulse to explain how airbags can help reduce injury during a car crash.

6 In the case of a person pushing against a solid brick wall, explain why no work is being done.

7 Contrast the meanings of the words ‘energy’ and ‘work’.

8 A group of students has conducted an investigation into the properties of an elastic band. They collected data by hanging different masses on the elastic band and measuring the extension from its original length. Unfortunately the students cannot agree on what the gradient of the graph represents and what the area under the graph represents. Explain how you could resolve their confusion using the equations for gradient and area, and the units for force and extension.

9 A squash ball that is repeatedly hit against a wall during a game becomes hot. Which of the following options explains this best?
- **A** The racquet gives the ball kinetic energy.
- **B** The impulse is positive.
- **C** The collisions are perfectly elastic.
- **D** Kinetic energy is not conserved in the collision.

10 A student carries a fully loaded backpack along a horizontal footpath for 450 m on their way home from school. What work was done by the student on the backpack during this journey if the student walked at a constant pace all the way?

11 An apple falls to the ground from a tree and strikes the ground with 45.5 J of kinetic energy. Ignoring any air resistance, how much gravitational potential energy did the apple have access to when it was on its branch.

12 A tennis ball is hit with the frame of a racquet and goes straight upwards. While it is travelling upwards it is slowing down until it reaches its maximum height, where its speed is zero. Where has all of the kinetic energy gone? In your answer ignore any air resistance.

13 A 70.0 kg rower steps out of a stationary boat with a velocity of $2.50\,\mathrm{m\,s^{-1}}$ onto a riverbank. The boat has a mass of 495 kg. With what velocity does the boat begin to move as the rower steps out?

14 A spacecraft of mass 1.00×10^4 kg that is initially at rest burns 5.00 kg of fuel to produce an equal mass of exhaust gases. The gases are ejected at a velocity of 6.00×10^3 m s^{-1}. Calculate the velocity of the spacecraft after this burn.

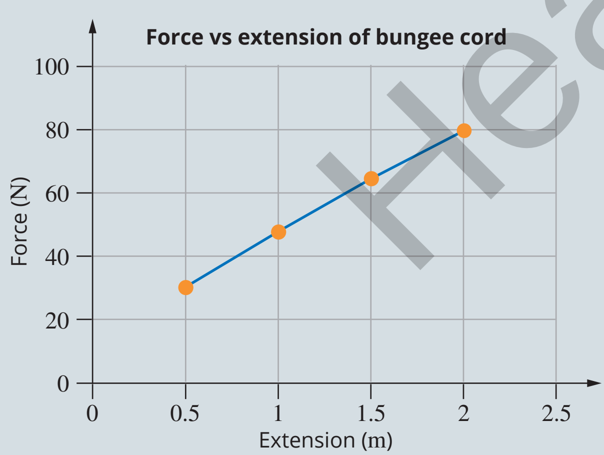
15 A batter blocks a 165 g cricket ball travelling towards him at 104 km h^{-1}. The ball leaves the bat at 20.0 km h^{-1}. Calculate the magnitude of the change in momentum of the ball.

16 Calculate the magnitude of the average force required to be applied by the brakes of a 15.0 kg bicycle carrying a 65.0 kg rider if the bike and rider are travelling at 12.0 m s^{-1} and come to rest in 2.00 s.

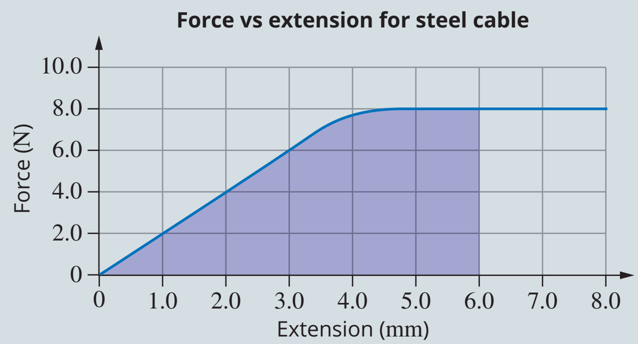
17 A polar research worker uses a tractor to drag a sled with supplies across a glacier. The harness is held at an angle of 60.0° to the horizontal and applies a force of 316 N on the sled, which is initially at rest. A constant frictional force of 105 N acts on the sled as it is dragged for a distance of 245 m.

- **a** For this distance, calculate the work done by the tractor on the 152 kg sled.
- **b** Find the speed of the sled after travelling 245 m.

18 A student wanting to increase their upper-body strength decides to stretch a piece of bungee cord 150 times each morning before school. From the force vs extension graph for the cord given below, estimate how much energy is expended in the workout if the student stretches the cord from 0.5 m to 1 m each time.



19 A steel cable 1.50 m long is stretched by fixing it at one end and applying a force to the other end. The graph of the force applied and the extension achieved is shown below.



Estimate the elastic potential energy stored in the cable when stretched by a distance of 6.0 mm.

20 The mass of a motorbike and its rider is 232 kg. If they are travelling at 80.0 km h^{-1}, calculate their combined kinetic energy.

21 A car of mass 1540 kg is travelling at 17.0 m s^{-1}. How much work would its engine need to do to accelerate the car to 28.0 m s^{-1}?

22 A 57.0 g tennis ball is thrown 8.20 m into the air.

- **a** Calculate the gravitational potential energy of the ball at the top of its flight.
- **b** Calculate the gravitational potential energy of the ball when it has fallen halfway back to the ground.

23 When climbing Mount Everest (h = 8848 m), a mountain climber stops to rest at North Base Camp (h = 5150 m). If the climber has a mass of 65.0 kg, how much gravitational potential energy will she gain in the her climb from North Base Camp to the summit? For simplicity, assume that g is 9.80 m s^{-2} for the whole climb.

Application and analysis

24 Two identical bowling balls, each of mass 4.00 kg, move towards each other across a frictionless horizontal surface with equal speeds of 3.00 m s^{-1}. During the collision 20.0 J of kinetic energy is transformed into heat and sound. After the collision the balls move in opposite directions.

- **a** Is momentum conserved in this collision?
- **b** Is this an elastic or inelastic collision? Explain your answer.
- **c** Calculate the speed of each ball after the collision.

25 An 80.0 kg student jumps from a bridge at the end of a bungee rope. When the student drops the full length of the 134 m bungee rope it then stretches by 10.0% as the student comes to a stop. Calculate the spring constant of the rope.

26 A student throws a basketball upwards with an initial speed of $u\,\mathrm{m\,s^{-1}}$ and notes that it reaches a maximum height above their hand of h m. If the student then throws the ball with an initial speed of $2u\,\mathrm{m\,s^{-1}}$, how high will the ball now go? Give your answer in terms of h.

27 Two children are standing on a bridge throwing stones into the river below. Susan throws a stone upwards, and Peter throws a stone downwards and at the same speed. Select the correct answer from the following options and justify your choice.

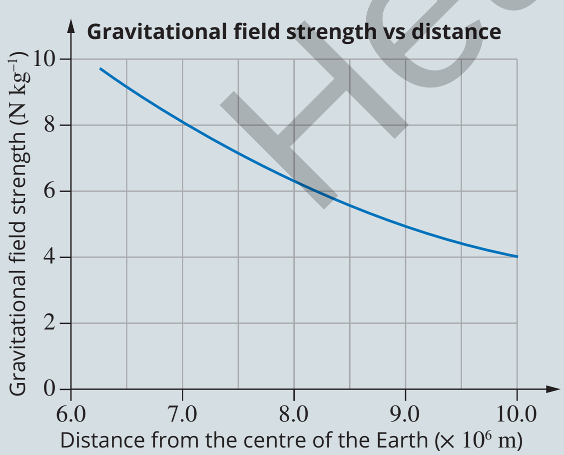
A Both stones will hit the water at the same speed.

B The stone that is thrown downwards by Peter will hit the water at a greater speed than Susan's stone which was thrown upwards.

C Susan's stone will hit the water at a greater speed than Peter's stone.

D More information is required to determine which stone hits the water at the greatest speed.

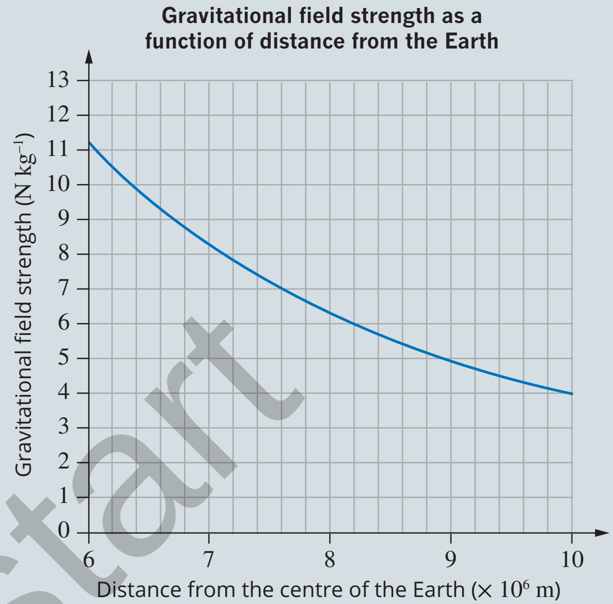
28 An 11.0 t satellite is in orbit at an altitude of 1130 km above the surface of the Earth. A booster rocket is fired putting the satellite into an orbit of altitude 2130 km. The graph below shows how the gravitational field changes as the distance from the centre of the Earth varies. The radius of the Earth is 6.37×10^6 m and the mass of the Earth is 5.98×10^{24} kg.



a Using the graph above, estimate the work done by the booster rocket in increasing the potential energy of the satellite.

b Calculate the kinetic energy of the satellite in its final orbit.

29 A new space telescope is 631 km above the surface of the Earth and in a circular orbit. Its mass is 1.10×10^7 kg. Use the graph below to estimate its gravitational potential energy relative to the surface of the Earth.



30 A 264 g toy truck with a springy bumper is travelling at $0.300\,\mathrm{m\,s^{-1}}$. It collides with a 112 g toy car travelling in the same direction at $0.200\,\mathrm{m\,s^{-1}}$. The car moves forwards at a speed of $0.300\,\mathrm{m\,s^{-1}}$.

a Calculate the speed of the truck after the collision.

b Calculate the total kinetic energy of the system before the collision.

c Calculate the total kinetic energy of the system after the collision.

d Complete the following statements by selecting the appropriate option from those in bold.

i The total kinetic energy before the collision is **more than**/**less than**/**equal to** the total kinetic energy after the collision.

ii The kinetic energy of the system of toys **is**/**is not** conserved.

iii The total energy of the system of toys **is**/**is not** conserved.

iv The total momentum of the system of toys **is**/**is not** conserved.

v The collision **is**/**is not** perfectly elastic because **kinetic energy**/**total energy**/**momentum** is not conserved.