

Student name: _____

Class: _____

Date: _____

4:01 Number patterns

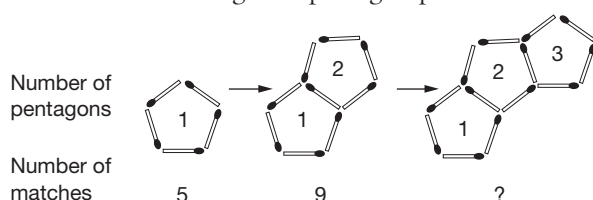
Some number patterns can be formed by adding or subtracting the same number, or multiplying by the same number at each stage to go from one term to the next term.

In the sequence 4, 7, 10, 13, the rule can be written as 'Start with 4, and then add 3 each time'.

The most interesting patterns arise from real-life situations, and the patterns can be used to make predictions.

Example:

These matches are arranged in pentagon patterns.



The rule is 'Start with five matches, and then add four each time'.

1 Using the first number, and the rule given, write down the next *three* numbers in each pattern.

- a 6; add 2 _____
- b 18; subtract 5 _____
- c 2; multiply by 3 _____
- d 72; divide by 2 _____
- e 4, multiply by 2 and subtract 3 _____
- f 73; add 1 and divide by 2 _____

2 For each number pattern, write down the next number, and the rule you used to get that number.

- a 5, 11, 17, 23, _____
- b 100, 87, 74, 61, _____
- c 3, 6, 12, 24, _____
- d 2000, 400, 80, _____

3 Write down the next six terms in this sequence:

1, 4, 9, 16, 25, _____

4 Dots can be arranged in the shape of a hexagon. The first two shapes in the pattern are shown.



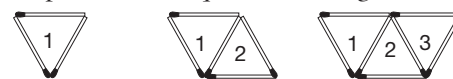
a Draw the next shape in the pattern.

b Write down the number of dots in each of the first three of these 'hexagon' patterns.

c Describe the pattern in your own words.

d Write down the number of dots in each of the first 12 of these shapes.

5 This match pattern uses equilateral triangles.



a Draw the next shape in the pattern.

b Complete this table to show how many matches are used at each stage.

Shape number	1	2	3	4
Number of matches	3			

c How many matches would you expect to use to make shape number 8 in this pattern? _____

d Describe how you can work out the number of matches when you know which shape number you want.

e Write down the rule that links the number of matches to the shape number.

Number of matches = _____

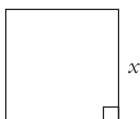
Result /25

4:02 Variables

Variables are used to represent numbers. We usually use letters of the alphabet (e.g. x) for variables. A variable can stand for a different number in different rules or questions.

Example:

We can write a rule that shows how to calculate the perimeter of a square when we know the length of one side.



The formula for finding the perimeter of a square is $P = 4x$.

Both P and x are variables. P stands for the perimeter and x stands for the side length.

$$\text{If } x = 5 \rightarrow P = 4 \times 5$$

$$P = 20$$

$$\text{If } P = 36 \rightarrow 36 = 4 \times x$$

So x would have to be 9.

- 1 Find the number represented by each variable:

a $x + 7 = 13$ _____

b $x - 4 = 20$ _____

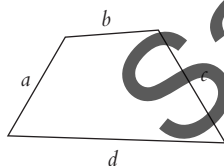
c $5 \times y = 35$ _____

d $p + p = 18$ _____

e $d \div 3 = 5$ _____

f $r \times r = 64$ _____

- 2 The rule $P = a + b + c + d$ gives the perimeter of a quadrilateral.



- a Find the perimeter if $a = 7$, $b = 4$, $c = 8$ and $d = 10$.

- b What is the value of a if $P = 101$, $b = 24$, $c = 31$ and $d = 17$?

- 3 Write expressions for these amounts of dollars using algebra.

- a The cost of buying five CDs at $\$x$ each.

- b The change when paying with a $\$20$ note to buy x toffee apples at $\$3$ each.

- c The price of a DVD if four of them cost a total of $\$x$.

- d The total cost of buying x books at $\$16$ each and posting them in one packet to a friend for $\$7$.

- 4 A 55 cm length of licorice has four lengths of y cm cut from it. What is the length of the remaining piece?

- 5 A cinema sells drinks for $\$2$ each, cartons of popcorn for $\$5$ and movie tickets for $\$18$ each. The formula for buying a combination of these three items is $C = 2d + 5p + 18t$, where d , p and t are the numbers of each item, and C is the total cost in dollars. Complete this table to show the total cost for three different groups who go to the cinema.

	Group 1	Group 2	Group 3
d	2	7	3
p	3	5	0
t	4	8	3
c			

FUN SPOT

APPLES, ORANGES AND PEACHES

Six peaches weigh the same as five apples.
Seven oranges weigh the same as four peaches.

- 1 Write the three types of fruit in order from lightest to heaviest.

- 2 Write a sentence using whole numbers only that compares the weights of apples and oranges

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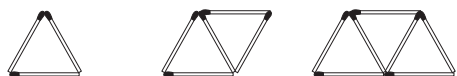
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4.03 Using variables

We use rules to explain how to calculate some quantity.

To write a rule in a simple form we use symbols to represent the numbers in the pattern. These symbols are called pronumerals and are usually letters of the alphabet.

Example:



In this pattern, the rule linking the number of matches to the number of triangles is:

Number of matches = 1 plus number of triangles \times 2

If we used m to represent the number of matches and t to represent the number of triangles, then the rule would be:

$$m = 1 + 2 \times t$$

The table of values would be:

t	1	2	3	4
m	3	5	7	9

- 1** Complete these tables of values using the rule given at the top of each table.

a $y = x + 4$

x	1	2	3	4
y				

b $y = 4 \times x + 1$

x	1	2	3	4
y				

- 2** Use the rules below to find the value of y if x is given the value 8.

a $y = x + 1$ _____ **b** $y = 30 - x$ _____

c $y = 5 \times x$ _____ **d** $y = 24 \div x$ _____

e $y = 3 \times x - 11$ _____

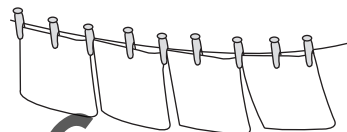
- 3** Rewrite these rules using suitable pronumerals. See the underlined letters to decide what pronumeral to use.

a daughter's age = father's age $- 29$

- b** number of tyres at a car yard = five times the number of cars

c speed = distance travelled divided by time

- 4** Nappies are hung out to dry on a clothesline. Each nappy takes three clothes pegs. The two pegs at each end can be shared with the nappy next to them.



- a** Use the diagram to work out how many pegs are needed for four nappies. _____
- b** A formula that gives the number of pegs for n nappies is:
 $p = 2 \times n + 1$
Use this formula to work out the number of pegs needed for:

i 1 nappy _____

ii 10 nappies _____

- c** Paula looks at the diagram and says: 'The formula $p = (n - 1) \times (n - 1)$ also works!' Explain whether she is right.

- 5** For the table of values shown below, the rule would be $y = x + 6$.

x	1	2	3	4
y	7	8	9	10

Write down the rule, using the pronumerals given, for each table of values below.

a

x	1	2	3	4
y	7	14	21	28

b

x	1	2	3	4
y	8	7	6	5

4:04 Algebraic abbreviations

Algebra uses pronumerals to express statements. It is a type of maths short hand.

A number with 6 added on $x + 6$

The result of taking 2 away from a number or '2 less than a number' $x - 2$

7 lots of x or 7 times x $7x$

x divided by 15 $\frac{x}{15}$

Notice how we don't need to use a \times or \div symbol in algebra.

Examples:

Show what these expressions mean by using multiplication and/or division signs.

a $5x + 2$ **b** $3(y + 4)$ **c** $\frac{x+1}{4}$

Answers:

a $5x + 2 = 5 \times x + 2$
b $3(y + 4) = 3 \times (y + 4)$
c $\frac{x+1}{4} = (x + 1) \div 4$

1 Write down an algebra expression for each of these.

a a number with 1 subtracted from it

b a number multiplied by 9

c a number with 100 added to it

d a number divided by 22

2 Match each algebra expression with an English phrase from the box using **a**, **b**, **c** and **d**.

$x + 6$ •

$x - 6$ •

$\frac{x}{6}$ •

$6x$ •

• **a** 6 less than x

• **b** 6 times x

• **c** 6 more than x

• **d** x divided by 6

3 Write down these expressions without using multiplication or division signs.

a $2 \times x$ _____ **b** $x \div 3$ _____

c $50 \times y$ _____ **d** $6 \div x$ _____

4 Write these without using multiplication signs.

a $x \times 2$ _____ **b** $4 \times y$ _____

c $5 \times p + 12$ _____ **d** $8 \times (x + 2)$ _____

5 Write these divisions as fractions.

a $x \div 9$ _____ **b** $4y \div 3$ _____

c $(p + 2) \div 5$ _____ **d** $1 \div 2x$ _____

6 Explain in words what these expressions mean.

a $p - 4$ _____

b $\frac{p}{4}$ _____

7 True or false? $1x$ simplifies to x . _____

8 Explain what we use instead of a \div sign in algebra.

9 Simplify these expressions.

a $6 \times 2x$ _____ **b** $5 \times x$ _____

c $p \times q$ _____ **d** $4x \times y$ _____

e $1 \times p$ _____ **f** $5x \times 2y$ _____

g $c \times a \times b$ _____ **h** $2c \times 4e \times 5d$ _____

10 Write these expressions without multiplication signs.

a $3 \times x + 2$ _____ **b** $1 \times y - 6$ _____

c $p \times q + 1$ _____ **d** $a \times b - c$ _____

e $5 \times x + 6 \times y$ _____ **f** $10 \times p - 10 \times q$ _____

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4.05 Making sense of algebra

One way of looking at pronumerals is to let them represent an unknown number of sweets in a bag.

A bag contains x sweets. x is an unknown number: it might be 25, or 30 or 37—we don't know.

This table explains how we can use algebra to represent the total number of sweets.

Model	Number
	$x + 2$
	$2x + 2y$

- 1** Use pronumerals (x and y) to write the number of sweets in each of these.

a  _____

b  _____

c  _____

d  _____

e  _____

f  _____

g  _____

- 2** New shirts are sold in plastic packets and are fastened to a piece of cardboard by several pins. A shop uses a rule to decide how many pins it needs. The rule is:

$$\text{number of pins} = 8 \times \text{the number of shirts}$$

The shop writes down this rule as:

$$\text{number of pins} = 8x.$$

- a What does x stand for in this expression?

- b The shop is selling 20 shirts. How many pins will it need?

- c What is the value of $8x$ when x is 4?

- d The shop manager writes down $8 \times 9 = 72$. Explain in words what the manager is calculating.

- 3** An animal shelter gives cats two meals per day and dogs one meal per day. They use this rule for the total number of meals served each day:

$$\text{number of meals} = 2x + y$$

- a Explain what each of x and y stand for in this rule.

x _____ y _____

- b What is the value of $2x + y$ when $x = 10$ and $y = 8$?

- c Explain in words what you have worked out in part b.

- 4** Smartdrives Ltd charges for laying a cobblestone driveway using the formula $C = 2n + 35h + 560$. C is in dollars, n is the number of cobblestones required and h is the number of hours to do the job. The 560 represents a delivery charge.

- a What is the total cost if a job requires 1700 cobblestones and takes 9 hours?

- b Write down a formula for the total cost if Smartdrives Ltd gives a \$200 discount for prompt payment.

- c Cobbleco Ltd bases its charges only on the number of cobblestones laid. Write down a formula for their charges if each cobblestone laid costs \$3.

- d Which company is cheaper to use for a job that requires 1000 cobblestones and will take 6 hours? Show working and give the amount saved by choosing the cheaper company. Assume that the payment will be made promptly.

4:06 Substitution

A pronumeral (such as x) often represents a number.

The value of an expression such as $5x$ depends on the value of x .

Examples:

1 If $x = 6$, the value of $5x$ is 30.

This is called substituting.

When we evaluate algebraic expressions, we always do multiplication and division before addition and subtraction.

2 Substitute $y = 5$ into the following expressions.

$$\begin{array}{ll} \text{a } 10y + 3 = 10 \times 5 + 3 & \text{b } \frac{10}{y} - 1 = \frac{10}{5} - 1 \\ & = 50 + 3 \\ & = 53 \\ & = 2 - 1 \\ & = 1 \end{array}$$

1 The value of x is 12. Calculate the value of each of these expressions.

- a $x - 4$ _____
 b $x + 8$ _____
 c $2x$ _____
 d $\frac{x}{2}$ _____
 e $x \div 12$ _____
 f $30 - x$ _____

2 If $P = 4n - 5$, calculate the value of P when:

- a $n = 6$ _____ b $n = 20$ _____
 c $n = 1$ _____

3 Calculate the value of each of these expressions when $x = 8$.

- a $5x + 3$ _____ b $4x - 11$ _____
 c $\frac{x}{2} + 6$ _____ d $20 - 2x$ _____

4 $p = 8$, $q = 11$ and $r = 2$. Evaluate these.

- a $3q - 12$ _____

 b $qr + 5$ _____

 c $3pr$ _____

 d $4p + 5r$ _____

5 A firm of joiners makes doors. Each door needs three hinges. One rule the firm uses is: number of doors = $\frac{y}{3}$.

- a What does y stand for in this expression?

 b How many doors can be made if they have 60 hinges?

 c Explain whether the rule gives a sensible answer when $y = 68$.

d The firm also knows that each door needs two handles. Write down a rule that links the number of handles with the number of doors. Use x to represent the number of doors.

The number of handles = _____

6 This rule is used to work out the number of deliveries from a warehouse to a supermarket each day:

number of deliveries = number of trucks \times number of journeys

They write down this rule as: number of deliveries = tj

a Calculate the value of tj when $t = 4$ and $j = 5$.

b Explain in words what you have worked out in part a.

4.07 Simplifying algebraic expressions

Like terms include the same letter or combination of letters.

Like terms:

x and $5x$

$6p, 8p, 5p$

$ab, 10ab, 2ab$

Unlike terms:

$4x, 7$

$3x, 3y$

To simplify expressions, we add or subtract like terms.

$$p + p + p + p + p = 5p$$

$$8x - 7x = x$$

Like terms can be grouped together.

Examples:

- 1 Simplify the following.

$$8x + 5 + x + 2 = (8x + x) + (5 + 2) \\ = 9x + 7$$

Note: $9x$ and 7 are unlike terms so $9x + 7$ is the final answer. it can't be simplified further.

Expressions with like terms can have both $+$ and $-$ signs.

These signs go with the terms on their *right*.

- 2 Simplify the following.

$$2x + y - x + 8y = (2x - x) + (y + 8y) \\ = x + 9y$$

- 1 Write these expressions as simply as possible.

a $c + c + c + c + c + c$ _____

b $y + y + y$ _____

c $x + x$ _____

d $3h + 5h$ _____

e $4x - x$ _____

f $15y - 6y$ _____

g $10x + x + 2x$ _____

- 2 Simplify these expressions by adding and subtracting.
Do this by working from left to right.

a $20x + 4x - 6x$ _____

b $x + 5x - 3x$ _____

c $10x - 7x + 3x$ _____

d $5x - 2x - x$ _____

- 3 Write down the like terms from each group.

a $\{7x, 2y, x\}$ _____

b $\{2b, 4c, 4d, c\}$ _____

c $\{3ab, 5ac, 6ad, 5ab\}$ _____

- 4 For each of the following pairs of terms, say whether they are *like* or *unlike*.

a $6x, 10x$ _____

b $2p, 2q$ _____

c $5x, 4$ _____

d $2b, 3bc$ _____

- 5 Simplify these expressions by collecting like terms together and adding them.

a $2x + 7y + 3x + 5y$ _____

b $3p + 5q + q + q$ _____

c $6x + 8 + x + 1$ _____

d $x + x + x + y + y$ _____

e $6p + q + 2p$ _____

- 6 Simplify these expressions by collecting together like terms and adding or subtracting them.

a $5x + 2y - 3x + 8y$ _____

b $x + 9y + 7x - y$ _____

c $3x + 8 - 7 + x$ _____

d $3x + 5 - 2x - 4$ _____

e $5x + 4y - 4y - 4x$ _____

- 7 Mr and Mrs Smith each have a car. Mr Smith's car has needed six new tyres and two new batteries since it was purchased. Mrs Smith's car has needed nine tyres and one battery.

- a How many tyres and batteries have the Smiths needed altogether?

- b Use t to stand for a tyre, and b to stand for a battery. Write down an expression and then simplify it, to show how to get your answer in part a.

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4:08 The laws of arithmetic

The laws of arithmetic apply to variables as well as numbers. Using algebra allows us to show they apply to *any numbers*.

	Using variables	Number example
Adding zero	$x + 0 = x$	$7 + 0 = 7$
Multiplying by zero	$y \times 0 = 0$	$15 \times 0 = 0$
Multiplying by 1	$z \times 1 = z$	$3 \times 1 = 3$
Commutative properties (order reversal) for + and \times	$a + b = b + a$ $p \times q = q \times p$	$5 + 12 = 12 + 5$ $4 \times 9 = 9 \times 4$
Associative properties (grouping order) for + and \times	$c + (d + e) = (c + d) + e$ $(r \times s) \times t = r \times (s \times t)$	$2 + (3 + 5) = (2 + 3) + 5$ $(6 \times 2) \times 4 = 6 \times (2 \times 4)$
Distributive property	$x \times (y + z) = x \times y + x \times z$	$10 \times (5 + 3) = 10 \times 5 + 10 \times 3$

- 1 Find a numeral that would go into the box to make each number sentence true.

a $\square \times 5 = 5$

b $8 + \square = 8$

c $13 \times \square = 0$

d $7 + 8 = \square + 7$

e $(6 + 5) + 4 = 6 + (\square + 4)$

5

- 2 Which market garden has more pumpkins:

A has 6 rows with 93 pumpkins in each.

B has 93 rows with 6 pumpkins in each.

1

- 3 John lost 3 golf balls yesterday and 8 golf balls today.

Sunita lost 8 golf balls yesterday and 3 today. Which person lost more in total?

1

- 4 Write a number property to explain the answer to Question 3.

1

- 5 Use number properties to find the answers to these quickly.

a $999 + 673 + 1 = \underline{\hspace{2cm}}$

b $341 \times 5 \times 2 = \underline{\hspace{2cm}}$

c $1002 + 544 - 2 = \underline{\hspace{2cm}}$

d $7 + 614 + 93 = \underline{\hspace{2cm}}$

4

- 6 a Calculate each expression.

i $20 - (14 - 5) = \underline{\hspace{2cm}}$

ii $(20 - 14) - 5 = \underline{\hspace{2cm}}$

- b Explain what the answers to i and ii show about the associative property.

3

FUN SPOT

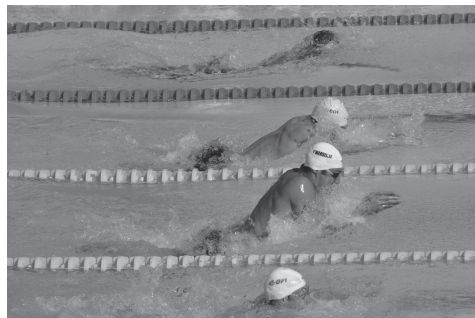
WHICH SWIMMER WAS THE WINNER?

Four swimmers in a race each made a prediction about the order in which they would finish.

- Peter said 'Sean will be last'.
- Quentin said 'Ralph will come third'.
- Ralph said 'Peter will be first'.
- Sean said 'Ralph's prediction is correct'.

The only correct prediction was made by the winner.

Which swimmer was the winner?



1

Result /15

4.09 Using grouping symbols

An expression like $7(3 + 10)$ can be worked out in two ways.

Usually, you would calculate the part inside the grouping symbols first.

$$\begin{aligned} 7(3 + 10) &= 7 \times 13 \\ &= 91 \end{aligned}$$

However, you get the same result if you multiply each number inside the grouping symbols by 7, and do the adding last.

$$\begin{aligned} 7(3 + 10) &= 7 \times 3 + 7 \times 10 \\ &= 21 + 70 \\ &= 91 \end{aligned}$$

This process is called expanding. It also works in algebra.

Examples:

$$1 \quad 8(x + y) = 8x + 8y \qquad 2 \quad 10(p - q) = 10p - 10q$$

Grouping symbols can also be removed when there are numbers inside.

$$\begin{aligned} 3 \quad 7(x + 8) &= 7 \times x + 7 \times 8 \\ &= 7x + 56 \end{aligned} \qquad \begin{aligned} 4 \quad 5(2x - 3) &= 5 \times 2x - 5 \times 3 \\ &= 10x - 15 \end{aligned}$$

Sometimes, there are pronumerals outside the grouping symbols, at the front.

$$\begin{aligned} 5 \quad p(x + y) &= p \times x + p \times y \\ &= px + py \end{aligned} \qquad \begin{aligned} 6 \quad x(3x - 4y) &= x \times 3x - x \times 4y \\ &= 3x^2 - 4xy \end{aligned}$$

$$\begin{aligned} 7 \quad 3x(x + 2) &= 3x \times x + 3x \times 2 \\ &= 3x^2 + 6x \end{aligned} \qquad \begin{aligned} 8 \quad 4x(2x - 5) &= 4x \times 2x - 4x \times 5 \\ &= 8x^2 - 20x \end{aligned}$$

Make sure that the expression is in its simplest form by collecting like terms.

- 1** Write down what needs to go on each line to expand these expressions correctly.

a $5 \times (6 + 8) = \square \times 6 + 5 \times 8$

b $12 \times (3 + 2) = 12 \times 3 + \square \times 2$

c $7 \times (15 - 13) = 7 \times \square - 7 \times \square$

d $4(p + q) = \square \times p + 4 \times q$

e $6(x - 10) = 6 \times \square - 6 \times \square$

- 2** Expand these expressions.

a $2(x + y)$ _____

b $15(x + y)$ _____

c $9(p - q)$ _____

d $10(x + y + z)$ _____

e $8(c + d - e)$ _____

- 3** Expand these expressions.

a $2(x + 3)$ _____

b $5(x + 2)$ _____

c $4(x - 9)$ _____

d $10(x - 7)$ _____

e $12(x + 3)$ _____

f $6(3x - 4)$ _____

g $4(2x + 7)$ _____

h $10(12x - 5)$ _____

- 4** Expand these expressions.

a $x(y + z)$ _____

b $c(d - e)$ _____

c $x(p + 2)$ _____

d $x(x + 8)$ _____

e $x(2x - 3)$ _____

f $x(4x - 1)$ _____

g $2p(q + r)$ _____

h $2x(3x - 4)$ _____

i $6x(x + 2)$ _____

- 5** Expand and simplify these expressions.

a $5(x + y) + 2(x + y)$ _____

b $2(x + y) + 4(2x + y)$ _____

c $3(2x + y) + 6(x + 2y)$ _____

d $2(x + 3) + 4(x + 6)$ _____

e $6(x - 1) + 2(2x - 3)$ _____

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4:10 Simplifying expressions with index notation

We use index form to show repeated multiplication of the same number or pronumeral.

$$p^4 = p \times p \times p \times p$$

$$x \times x \times x \times y \times y = x^3 y^2$$

- Any numbers can be multiplied separately.
- Then letters can be combined, each one written once only, in index form.
- In the final, simplified answer, the number part is written first.

$$6p \times 3p = 6 \times 3 \times p \times p$$

$$= 18p^2$$

- 1** Shorten these expressions by writing them in index form.

a $c \times c \times c \times c \times c$ _____

b $x \times x$ _____

c $p \times p \times q \times q$ _____

d $r \times r \times s \times s \times r$ _____

e $a \times b \times b \times b \times c \times c$ _____

- 2** Write these expressions in full, using multiplication symbols.

a y^3 _____

b w^6 _____

c xy^3 _____

d x^3y _____

e p^4q^2 _____

5

- 3** The dots in this NSW place name stand for multiplication symbols. Write it in index form.

w.a.g.g.a.w.a.g.g.a _____

2

- 4** Write in full what this sheep is saying.



2

- 5** Simplify these by multiplying and writing in index form where possible.

a $4 \times x \times x$ _____

b $6 \times y \times 5 \times y$ _____

c $3x \times 5 \times 2x$ _____

d $2q \times 2q \times 2q$ _____

e $y \times 2y \times 3y$ _____

f $4 \times 2x \times x$ _____

g $3p \times 2q$ _____

h $5x \times y \times 3x \times 2y$ _____

8



FUN SPOT

TRAVEL ROUTES

You have been given the job of making the travel arrangements for the APEC Conference.

Here is the map of downtown. The triangles show the embassies. The circles show hotels, restaurants and airline terminals.

The Canadian delegation needs to go from the Canadian Embassy to the Carlton Hotel.

The US delegation needs to travel from the United Airlines terminal to the US Embassy.

The Russians have to drive from the Regent Hotel to the Russian Embassy.

The Australians have to drive from the Australian Embassy to the Autolodge.

The Japanese need to go from the Japanese Embassy to the Jade Garden Restaurant.

These journeys are all taking place at about the same time. The routes are not allowed to cross. They also must not travel through the same intersections.

Show how you would organise the travel routes. Use a pencil and have a rubber handy!

