1

Integers and indices

1

How low can you go? Humans have explored much of the land areas of our planet, but have seen very little of the vast depths of Earth's oceans.

The highest point on the Earth's surface is the summit of Mt Everest, a height of 8848 m above sea level (+8848 m). The lowest point on Earth is the bottom of the Mariana Trench, in the Pacific Ocean. It is believed to reach at least 10 994 m below sea level (-10 994 m). In 1960, naval lieutenant Don Walsh and engineer Jacques Piccard made the first expedition to the bottom of the trench. They reached a depth of -10 916 m. When they resurfaced, the windows of their submersible were cracked from the pressure of the water. Because of the dangers of high pressures, much of the Earth's oceans have only been explored to a depth of -300 m. Only a few submersibles can go beyond -3000 m, and even military submarines usually travel no deeper than -500 m. Below these depths, an amazing underwater world is waiting to be explored.

Forum

Have you ever been in negative temperatures? If you have, where were you?

If you were able to pick up Mt Everest and place it in the Mariana Trench, it would be completely covered by water. How much water would there be between the top of Mt Everest and sea level?

Why learn this?

Positive numbers are only half the story! You need to be able to work with numbers that are less than zero—the negative side of the number line. Negative numbers are used to show actions that are opposite to positives, such as a move backwards instead of forwards, or a fall in price instead of a rise.

Indices (also called powers or exponents) provide a convenient way to write and work with very large numbers. Having rules for working with indices helps make calculations more efficient.

After completing this chapter you will be able to:

- use directed numbers (integers) in everyday situations
- do the four operations (+, −, ×, ÷) on directed numbers
- use a number line to help with directed number calculations
- apply the order of operations rules to directed number calculations
- understand and use index notation
- work efficiently with index notation by applying the appropriate rules.

Curriculum links

Australian CURRICULUM | VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Number and Algebra

Number and place value

ACMNA182/VCMNA272
Use index notation with numbers to establish the index laws with positive integral indices and the zero index

- evaluating numbers expressed as powers of positive integers

ACMNA183
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies

- using patterns to assist in finding rules for the multiplication and division of integers
- using the number line to develop strategies for adding and subtracting rational numbers

VCMNA273
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies and make estimates for these computations

- using patterns to assist in finding rules for the multiplication and division of integers
- using the number line to develop strategies for adding and subtracting rational numbers
- making an estimate for the total of a family weekly grocery bill with consideration of accuracy of the estimate, or for problems involving the circumference and area of a circle

General capabilities

- Literacy
- Numeracy
- Critical and creative thinking
- Personal and social capability

Big ideas

- Every number has both magnitude (size) and direction.
- The number line can be extended below zero into negative numbers.
- Negative numbers are helpful in capturing real-world scenarios such as debt and temperature drop.
- There are various laws that help us work with index numbers.

notes:

Recall 1

Prepare for this chapter by trying the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 To answer the following, it may help to draw a diagram.

(a) At 7 am on a winter morning, the temperature was 4 °C. At midday, the temperature was 13 °C. By midnight it had dropped to -2 °C.

(i) By how many degrees did the temperature increase, from 7 am to midday?

(ii) By how many degrees did the temperature decrease, from midday to midnight?

(iii) What was the difference between the temperature at 7 am and the temperature at midnight?

(b) A city building has 5 storeys above the ground floor and 2 basement levels below the ground floor.

(i) Jade parked her car in the 2nd basement level, got in the lift and moved up 5 floors. At which floor did she get out?

(ii) Erin got in the lift on the 4th floor. She went down 5 floors, then up 2 floors. At which floor did she get out?

2 Write the value of the following. **(a)** 2^3 **(b)** 3^4 **(c)** 5^2 **(d)** 10^6 **(e)** $\sqrt{36}$ **(f)** $\sqrt{49}$ **(g)** $\sqrt[3]{64}$ **(h)** $\sqrt[3]{8}$

3 Imagine that you are standing at -4 on a large number line. Which number would you land on if you walked:

(a) 7 places to the right **(b)** 5 places to the left?

4 Calculate: **(a)** 2×8 **(b)** 7×9 **(c)** 38×6 **(d)** 15×19 **(e)** $18 \div 3$ **(f)** $54 \div 9$ **(g)** $324 \div 3$ **(h)** $616 \div 4$

5 Calculate: **(a)** $(8 \div 4) + (35 \div 5)$ **(b)** $6 \times 8 + 2 \times 3$ **(c)** $50 - 5 \times 2 + 3$

6 Use a factor tree to find the prime factors of the following, then write each number as a product of its prime factors in index form.

(a) 24 **(b)** 36 **(c)** 120

7 Find the value of the following. **(a)** $4^2 + 3^2$ **(b)** $2^2 + 5^2$ **(c)** $8^2 - 6^2$ **(d)** $9^2 - 3^2$ **(e)** $5^2 \times 2^4$ **(f)** $3^3 \times 5 \times 7$ **(g)** $9^2 \div 3^2$ **(h)** $2^3 \times 7 \div 2^2$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Comparing powers

In this activity, you will explore what can make numbers raised to powers greater or smaller.

Integers review

1.1

A **positive** (+) or a **negative** (-) sign is used to show the direction of a number. Some words associated with negative numbers are: down, loss, below, decrease, lose, withdrawal. Some words associated with positive numbers are: up, profit, above, increase, gain, deposit.

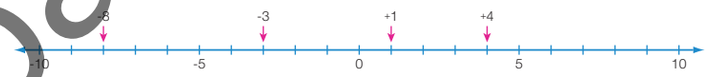
For example, a deposit of $50 into your bank account could be written as +50, and a withdrawal of $180 could be written as -180.

Date	Transaction	Balance
		$500
17 July	ATM Withdrawal -$180	$320
20 July	Birthday money from Mum +$50	$370

What is an integer?

The **integers** are all of the positive whole numbers 1, 2, 3, … , the negative whole numbers -1, -2, -3, … , and zero. (The use of '…' in mathematics shows that the sequence of numbers continues forever.) Positive and negative integers are also called *directed numbers*.

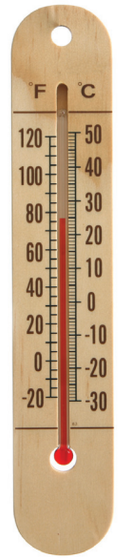
Integers can be represented on a number line:



The numbers on a number line get larger in value as you move from left to right.

If a number has no sign in front of it, assume it is positive. 4 = +4

From the number line above, you can write the statement +4 > -8. You could put this in words as 'positive 4 is greater than negative 8'. You could also write the statement -3 < +1, which is the same as saying 'negative three is less than positive one'. Number lines can also be written vertically, with the positive numbers above the negative ones. A thermometer is an example of a vertical number line.



Adding and subtracting integers

Adding and subtracting positive integers is the straightforward addition you have been doing for years.

To *add a positive integer*, imagine moving that many places to the *right* on the number line (or upwards on a vertical number line), in the positive direction.

To *subtract a positive integer*, imagine moving that many places to the *left* on the number line (or downwards on a vertical number line), in the negative direction.

Adding a negative integer is the opposite of adding a positive one. It means you move in the negative direction, to the *left* on the number line (or downwards on a vertical number line). You can see that this is the same movement as subtracting a positive integer.

Subtracting a negative integer is the opposite of subtracting a positive one. It means you move in the positive direction, to the *right* on the number line (or upwards on a vertical number line).

Resources

Recall Worksheets

- R1.1 Interpreting changes in temperature
- R1.2 Indices, roots and cube roots
- R1.3 Walking on a number line
- R1.4 Multiplying and dividing
- R1.5 Order of operations
- R1.6 Factor trees
- R1.7 Calculations involving indices

Exploration Task

- Comparing powers

Lightbook Starter

- Before you begin 1

Answers

Recall 1

1 (a) (i) $13 - 4 = 9\,°C$

(ii) $13 - -2 = 15\,°C$

(iii) $4 - -2 = 6\,°C$

(b) (i) $-2 + 5 = 3$, 3rd floor

(ii) $4 - 5 + 2 = 1$, 1st floor

2 (a) 8 (b) 81

(c) 25 (d) 1 000 000

(e) 6 (f) 7

(g) 4 (h) 2

3 (a) +3 (b) -9

4 (a) 16 (b) 63

(c) 228 (d) 285

(e) 6 (f) 6

(g) 108 (h) 154

5 (a) $2 + 7 = 9$

(b) $48 + 6 = 54$

(c) $50 - 10 + 3 = 43$

6 (a) $24 = 2^3 \times 3$

(b) $36 = 2^2 \times 3^2$

(c) $120 = 2^3 \times 3 \times 5$

7 (a) $16 + 9 = 25$

(b) $4 + 25 = 29$

(c) $64 - 36 = 28$

(d) $81 - 9 = 72$

(e) $25 \times 16 = 400$

(f) $27 \times 5 \times 7 = 945$

(g) $81 \div 9 = 9$

(h) $8 \times 7 \div 4 = 6 \div 4 = 14$

1.1

Resources

eWorked examples

- Simplifying integer expressions (positive integers)
- Simplifying integer expressions (negative integers)

Lessons

- Integers

Lightbook Starter

- Check-in 1.1

Recap

Question	Answer
1 Calculate: $\sqrt{25}$	5
2 Calculate: $\sqrt[3]{27}$	3
3 Write in expanded form: 7^3	$7 \times 7 \times 7$
4 Calculate: $2^3 + 2^2$	$8 + 4 = 12$
5 Calculate: $3^2 \times 3^2$	$9 \times 9 = 81$

Suggested examples

1 *Simplify the following and calculate the answer.*

(a) $-3 + (+12)$

(b) $-6 + (+2)$

(c) $+1 - (+7)$

(d) $-4 - (+20)$

(e) $+8 + (-5)$

(f) $-15 + (-5)$

(g) $+10 - (-20)$

(h) $-7 - (-7)$

Answers:

(a) $-3 + 12 = 9$

(b) $-6 + 2 = -4$

(c) $1 - 7 = -6$

(d) $-4 - 20 = -24$

(e) $8 - 5 = 3$

(f) $-15 - 5 = -20$

(g) $10 + 20 = 30$

(h) $-7 + 7 = 0$

1.1

Simplifying addition

When adding and subtracting integers, brackets are often placed around the second number and its sign, to separate it from the addition or subtraction symbol; for example, +5 + (+9) or +7 – (+3). However, as you can write a positive integer without the + sign in front, you can drop the positive signs in front of the numbers, remove the brackets, and simply write 5 + 9 or 7 – 3.

You can see from the previous number lines that subtracting a negative integer is the same as adding the positive integer. This means that you could write 8 – (-2) as 8 + 2.

You can also see that adding a negative integer is the same as subtracting the positive integer. This means you could write 4 + (-10) as 4 – 10.

+ (+) and – (-) can be replaced with + – (+) and + (-) can be replaced with –

You could also say:

When the two signs are the *same*, add. When the two signs are *different*, subtract.

Worked example 1 W.E. 1

Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) -4 + (+11) **(b)** -9 + (+3) **(c)** +2 – (+8) **(d)** -5 – (+11)

Thinking		Working
(a) 1	Simplify the addition by writing positive integers without their signs, and removing the brackets.	*(a)* -4 + (+11) = -4 + 11
2	Imagine (or draw) a number line. Start at the first number (-4), then move the number of places indicated by the second number (11) in the positive direction (to the right). Write your answer.	= 7
(b) 1	Simplify the addition by writing positive integers without their signs, and removing the brackets.	*(b)* -9 + (+3) = -9 + 3
2	Imagine (or draw) a number line. Start at the first number (-9), then move the number of places indicated by the second number (3) in the positive direction (to the right). Write your answer.	= -6
(c) 1	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	*(c)* +2 – (+8) = 2 – 8
2	Imagine (or draw) a number line. Start at the first number (2), then move the number of places indicated by the second number (8) in the negative direction (to the left). Write your answer.	= -6
(d) 1	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	*(d)* -5 – (+11) = -5 – 11
2	Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (11) in the negative direction (to the left). Write your answer.	= -16

Worked example 2 W.E. 2

Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) +9 + (-7) **(b)** -12 + (-1) **(c)** +4 – (-6) **(d)** -5 – (-7)

Thinking		Working
(a) 1	Remove the brackets and write a single subtraction symbol between the numbers. Write positive integers without their signs.	*(a)* +9 + (-7) = 9 – 7
2	Complete this straightforward subtraction. Write your answer.	= 2
(b) 1	Remove the brackets and write a single subtraction symbol between the numbers. Write positive integers without their signs.	*(b)* -12 + (-1) = -12 – 1
2	Imagine (or draw) a number line. Start at the first number (-12), then move the number of places indicated by the second number (1) in the negative direction (to the left). Write your answer.	= -13
(c) 1	Remove the brackets and write a single addition symbol between the numbers. Write positive integers without their signs.	*(c)* +4 – (-6) = 4 + 6
2	Complete this straightforward addition.	= 10

Teaching strategies

Directed numbers

Integers are often called directed numbers as they have both magnitude (size) and direction.

When working with positive numbers the positive sign is not usually written—so write 5 rather than +5. When students are working with lots of positive and negative numbers, they may find it helpful to write the positive numbers with their sign.

Zero

Zero is neither positive nor negative.

Magnitude and size

Test that your students understand that -19 is less than -2, even though the magnitudes of 19 and 2 imply the opposite. You could plot the numbers on a number line and point out that the further right on a number line a number is, the larger it is. Alternatively, for an advanced class this could lead into a discussion about 'absolute value' (absolute value is the magnitude of an integer, and is enclosed in straight lines: ||). Thus, although $-19 < -2$, $|-19| > |-2|$.

Class activities

Vertical peg number line

To revise the concept of negative integers covered in Year 7, the vertical number line is extremely useful. Take a piece of thick string and place it vertically on the wall, then give one student a peg and ask him/her to put it on the string at zero. Ask other students to come up with different integers stuck on pegs and ask them to place them on the number line in the best position. This brings up the concept of equal spacing between integers. Take down the string and fold it where the pegs are to check that the spaces are equal.

Integer snap

Using a standard pack of playing cards, designate the black cards as positive integers and the red cards as negative integers. Play a game of snap in which you are able to snap when the value of the cards is zero. Picture cards are worth 10 each so that e.g. a red King and a black Queen could be snapped. Aces have a magnitude of one. Take out the Jokers if there are any in your pack. For example, the following sequence could be snapped: red King, black 5, black 7, red 2.

notes:

1.1

(d) 1 Remove the brackets and write a single addition symbol between the numbers. Write positive integers without their signs.

(d) -5 – (-7)
= -5 + 7

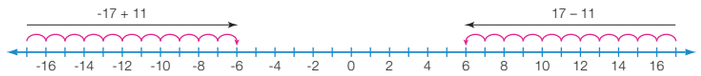
2 Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (7) in the positive direction (to the right). Write your answer.

= 2

Using number line symmetry

The number line is symmetrical about zero. A positive number is the same distance from zero as its negative opposite; for example, negative 6 is the same distance from zero as positive 6. This can be useful when adding integers, especially large ones.

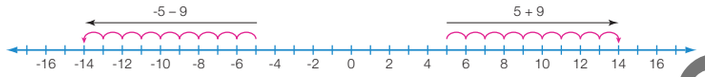
For example, if you show -17 + 11 on a number line, it is the mirror image of showing 17 – 11.



So, to calculate -97 + 64, you could simply do 97 – 64, then change the sign of the answer.

97 – 64 = 33 -97 + 64 = -33

Similarly, to calculate -5 – 9, you could consider the mirror image situation: 5 + 9.



So, to calculate -53 – 49, you could simply do 53 + 49, and change the sign of the answer.

53 + 49 = 102 -53 – 49 = -102

1.1 Integers review

Navigator

Answers p. 616

1 (columns 1–3), 2, 3 (columns 1–3), 4, 5, 6, 8, 9, 10, 13, 15, 17	1 (columns 3–4), 3 (columns 3–4), 4, 5 (row 2), 6, 7, 8, 10, 11, 14, 15, 16, 17	1 (columns 3–4), 3 (columns 3–4), 4, 5 (row 2), 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19

Fluency

W.E. 1

1 Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) +4 + (+9) **(b)** -7 + (+5) **(c)** +5 + (+3) **(d)** +2 – (+6)
(e) +10 – (+13) **(f)** -3 + (+8) **(g)** +12 + (+6) **(h)** -1 – (+9)
(i) -15 + (+8) **(j)** +8 – (+9) **(k)** -8 – (+13) **(l)** -19 + (+11)
(m) +16 – (+9) **(n)** -14 + (+7) **(o)** -4 – (+22) **(p)** -17 + (+23)

2 Use your calculator to evaluate the following.

(a) -3 + -5 **(b)** 22 – 12 **(c)** 13 + -29
(d) 84 + 12 **(e)** 39 + +6 **(f)** -46 – -12
(g) -52 + -16 + 12 **(h)** 22 – 13 + 6 **(i)** 46 – +24 + -13 – -20

3 Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

W.E. 2

(a) +5 + (-4) **(b)** +7 + (-9) **(c)** +5 – (-11) **(d)** -7 – (-6)
(e) -4 – (-9) **(f)** -3 – (+8) **(g)** -14 – (+3) **(h)** +12 – (-8)
(i) +19 – (+13) **(j)** 0 – (-3) **(k)** -8 – (-16) **(l)** +15 + (-22)
(m) -11 – (-7) **(n)** -13 + (-9) **(o)** +25 + (-31) **(p)** -27 – (-16)

4 Write a negative or a positive integer to describe the following situations.

(a) 350 m above sea level
(b) a loss of $4800
(c) rewinding 6 seconds of an audio recording
(d) depositing $73 into your bank account
(e) 2 levels below the ground floor of a building
(f) 19 metres under water
(g) a company profit of $10 750
(h) skipping 34 minutes of a TV episode
(i) withdrawing $200 from your bank account
(j) a plane flying at an altitude of 8100 metres

5 Use the symbols greater than (>) or less than (<) between the following pairs of integers to show their relationship. A number line may be useful.

(a) +2 ___ +6 **(b)** -3 ___ +1 **(c)** +5 ___ -4 **(d)** -1 ___ -3
(e) +5 ___ -10 **(f)** -6 ___ -8 **(g)** +16 ___ -16 **(h)** -33 ___ -12

6 Write the following in descending order (largest to smallest).

(a) +4, 0, -7, +11, -2 **(b)** -23, 1, 0, -9, +7
(c) -3, 4, 0, 11, -15, 1 **(d)** -5, 8, 19, -43, -2, 6
(e) 14, -72, 5, 26, -1, -38 **(f)** 32, -19, 0, 17, -56, 4

7 Calculate:

(a) 2 + 7 – 5 **(b)** -3 + 10 – 5 **(c)** -6 + 4 – 8 **(d)** -15 + 9 + 8
(e) 11 + 14 – 23 **(f)** -7 – 8 – (-9) **(g)** 4 + 5 – (-5) **(h)** -6 + (-9) – (+9)

Answers

Exercise 1.1

1 **(a)** +13 **(b)** -2
(c) +8 **(d)** -4
(e) -3 **(f)** +5
(g) +18 **(h)** -10
(i) -7 **(j)** -1
(k) -21 **(l)** -8
(m) +7 **(n)** -7
(o) -26 **(p)** +6

2 **(a)** -8 **(b)** 10
(c) -16 **(d)** 96
(e) 45 **(f)** -34
(g) -56 **(h)** 15
(i) 29

3 **(a)** +1 **(b)** -2
(c) +16 **(d)** -1
(e) +5 **(f)** -11
(g) -17 **(h)** +20
(i) +6 **(j)** +3
(k) +8 **(l)** -7
(m) -4 **(n)** -22
(o) -6 **(p)** -11

4 **(a)** +350 **(b)** -4800
(c) -6 **(d)** +73
(e) -2 **(f)** -19
(g) +10 750 **(h)** +34
(i) -200 **(j)** +8100

5 **(a)** $+2 < +6$ **(b)** $-3 < +1$
(c) $+5 > -4$ **(d)** $-1 > -3$
(e) $+5 > -10$ **(f)** $-6 > -8$
(g) $+16 > -16$ **(h)** $-33 < -12$

6 **(a)** +11, +4, 0, -2, -7
(b) +7, 1, 0, -9, -23
(c) 11, 4, 1, 0, -3, -15
(d) 19, 8, 6, -2, -5, -43
(e) 26, 14, 5, -1, -38, -72
(f) 32, 17, 4, 0, -19, -56

7 **(a)** 4 **(b)** 2
(c) -10 **(d)** 2
(e) 2 **(f)** -6
(g) 14 **(h)** -24

notes: