CHAPTER

05 Gravity

Gravity is the force that drives the universe. It was gravity that first caused atoms to congregate together to form the first nebulas, stars and planets. An understanding of gravity is fundamental to understanding the universe.

This chapter centres on Newton's law of universal gravitation. This will be used to predict the size of the gravitational force experienced by an object at various locations on Earth and other planets. It will also be used to develop the idea of a gravitational field. And because the field concept is also used to describe other basic forces, such as electromagnetism, this will provide an important foundation for further study in physics.

Syllabus subject matter

Topic 1 • Gravity and motion

GRAVITATIONAL FORCE AND FIELDS

- recall Newton's Law of Universal Gravitation
- solve problems involving the magnitude of the gravitational force between two masses
- define the term *gravitational fields*
- solve problems involving the gravitational field strength at a distance from an object.

SCIENCE AS A HUMAN ENDEAVOUR

- Developing understanding of planetary motion

5.1 Newton's law of universal gravitation

BY THE END OF THIS MODULE, YOU SHOULD BE ABLE TO:

- recall Newton's law of universal gravitation
- solve problems involving the magnitude of the gravitational force between two masses
- calculate the strength of the gravitational force between two masses
- calculate acceleration due to gravitational force.

Gravity is one of the four fundamental forces in the universe. Ordered from strongest to weakest forces, these four forces are:

- strong nuclear force
- electromagnetic (EM) force
- weak nuclear force
- gravitational force.

Electromagnetic forces, which include light and magnetism, are covered in Unit 2 (Chapter 11) and Unit 3 (Chapters 8 and 9). Strong and weak nuclear forces act within the nucleus of atoms, as covered in Unit 1 (Chapter 3) and Unit 4 (Chapter 13).

The **gravitational force** (gravity) is by far the weakest force of the four. However, because it acts at great distances (unlike the two nuclear forces), and because it is always attractive (unlike the EM force), gravity is the force that drives the universe we live in.

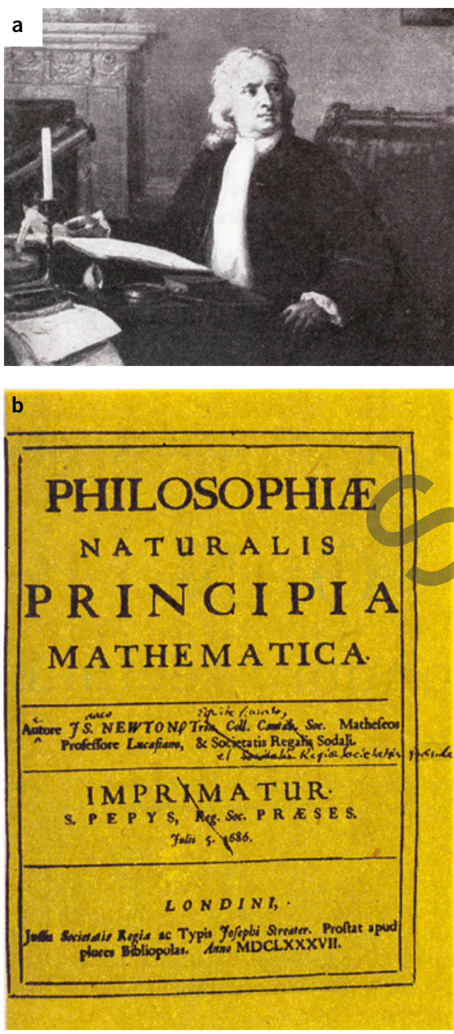


FIGURE 5.1.1 (a) Sir Isaac Newton. (b) The *Principia* is one of the most influential books in the history of science.

UNIVERSAL GRAVITATION

Sir Isaac Newton's law of universal gravitation was introduced in his book *Philosophiæ Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) in 1687 (Figure 5.1.1).

Newton's law of universal gravitation says that any two bodies attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Mathematically, Newton's law of universal gravitation is expressed as:

$$F_g = G\frac{m_1 m_2}{r^2}$$

where

F_g is the gravitational force (N)

m_1 is the mass of object 1 (kg)

m_2 is the mass of object 2 (kg)

r is the distance between the centres of m_1 and m_2 (m)

G is the gravitational constant, 6.67×10^{-11} (N m^2 kg^{-2})

The fact that distance r appears in the denominator of the equation indicates an inverse relationship. The greater the distance between the two objects, the smaller the force. As r is squared, this relationship is known as an **inverse square law**. The result is that as r increases, F_g decreases dramatically. Inverse square laws will reappear again later in the chapter, when gravitational fields are examined in detail, and also in Chapter 7.

Worked example 5.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL AND LARGE OBJECTS

In *Principia*, Isaac Newton used this deceptively simple proof to show that the same force that acted on the orbiting Moon (orbital period about Earth = 27.3 days, average radius of the Moon's orbit about Earth = 3.84×10^8 m) also acted on a falling object (let's say, an apple) here on Earth (radius = 6.37×10^6 m).

Newton also showed that the force acting from the Earth on both the Moon and the apple was inversely proportional to the distance. Here, we will do the same.

Compare the force on a falling apple to the force of the Moon from the Earth's gravity.

Thinking	Working
Newton used the orbit of the Moon to confirm his law of inverse squares. Use the equation for the centripetal acceleration of an object travelling in a circle (from Chapter 4). Here, *d* is used instead of *r* for the radius of the Moon's orbit.	$a_c = \frac{v^2}{d}$ $= \frac{\left(\frac{2\pi d}{T}\right)^2}{d}$ $= \frac{4\pi^2 d}{T^2}$
Substitute the known values into the equation. Use: *d* (radius of the Moon's orbit) = 3.84×10^8 m *T* (period of the Moon's orbit) = 27.3 days = 2.36×10^6 s	$a_{c(Moon)} = \frac{4\pi^2 d}{T^2} = \frac{4 \times \pi^2 \times 3.84 \times 10^8}{(2.36 \times 10^6)^2}$ $a_{c(Moon)} = 2.72 \times 10^{-3}\ m\,s^{-2}$
Compare the acceleration of the Moon with that of a falling apple on Earth by calculating the ratio $\frac{g_{Earth}}{a_{c\ Moon}}$. Note the measured acceleration of a falling object (e.g. an apple) on Earth is: $g_{Earth} = 9.8\ m\,s^{-2}$ In Newton's time this value was well known. Galileo first measured this acceleration in 1604.	$\frac{g_{Earth}}{a_{c\ (Moon)}} = \frac{9.8}{2.72 \times 10^{-3}}$ ≈ 3600 The acceleration of the Moon in orbit is approximately 3600 times smaller than the acceleration of the falling apple.
Compare the two distances that the forces are acting over by calculating the ratio $\frac{d_{Earth-Moon}}{r_{Earth}}$	Calculate $\frac{d_{Earth-Moon}}{r_{Earth}}$ where: $d_{Earth-Moon} = 3.84 \times 10^8$ m $r_{Earth} = 6.37 \times 10^6$ m $\frac{d_{Earth-Moon}}{r_{Earth}} = \frac{3.84 \times 10^8}{6.37 \times 10^6}$ ≈ 60 The distance from Earth to the Moon is approximately 60 times greater than the distance from the centre of Earth to its surface.
Square the ratio of the distances to compare with the ratio of the accelerations.	Newton noted that the acceleration of a falling object on Earth was 3600 times greater than the acceleration of the Moon in orbit. He also noted that the distance to the Moon was 60 times greater than the radius of the Earth. And, of course, 60 squared is 3600. That is, said Newton, the same force from Earth acts on both the orbiting Moon and a falling object at Earth's surface. And the strength of that force is inversely proportional to the square of the distance.

➤ Try yourself 5.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL AND LARGE OBJECTS

Apply Isaac Newton's calculations as if you were a Martian scientist comparing the acceleration of Mars's small, close-orbiting moon Phobos with the acceleration of a falling Martian apple.

Use 9.38×10^6 m as the radius of the orbit of Phobos, and 7 hours 40 minutes as the period of the moon's orbit. Use $3.71\,\text{m s}^{-2}$ as the acceleration of a falling apple on the Martian surface and 3.39×10^6 m as the radius of Mars.

Even the great Isaac Newton could not quantify the **gravitational constant** G when he developed his law of universal gravitation, because the mass of Earth was not then accurately known. All Newton could say was that the force was 'proportional' to the distance between two objects, with the component of proportionality being the product $G \times M_{\text{Earth}}$.

The value of G was first determined a century later by the British scientist Henry Cavendish, who used a sensitive torsion balance (a twisting wire) to find the gravitational attraction between two known masses held a small distance apart. By finding the constant G, Cavendish's experiment also enabled the first accurate measurement of the mass of the Earth.

Worked example 5.1.2

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Mark has a mass of 90.0 kg and his dance partner Bec has a mass of 75 kg. The distance between their centres is 85 cm. Calculate the magnitude of the force of gravitational attraction between the dancers.

Thinking	Working
Recall the formula for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Identify the information required, and convert values into appropriate units where necessary.	$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$ $m_1 = 90\,\text{kg}$ $m_2 = 75\,\text{kg}$ $r = 85\,\text{cm} = 0.85\,\text{m}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{90 \times 75}{0.85^2}$
Solve the equation.	$F_g = 6.2 \times 10^{-7}\,\text{N}$ This force is of the order of a millionth of a newton. To give an idea of scale, that's about ten-thousandth the weight of a feather.

➤ Try yourself 5.1.2

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

In Henry Cavendish's 1798 experiment, a large lead mass was suspended beside a smaller mass so that the centres of the two balls were 230 mm apart. Ball 1 had a mass of 160 kg and ball 2 had a mass of 0.73 kg. Calculate the magnitude of the force of gravitational attraction between them.

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Gravitational forces between everyday objects are so small (as seen in Worked example 5.1.2) that forces are hard to detect without specialised equipment. Such forces are often considered to be negligible. For the gravitational force to become significant, at least one of the objects must have a very large mass—for example, a planet (Figure 5.1.2).

FIGURE 5.1.2 Gravitational forces become significant when at least one of the objects has a large mass, for example forces between Earth and the Moon. Note that distances are not shown to scale.

Worked example 5.1.3

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Calculate the magnitude of the force of gravitational attraction between the Moon and Earth given the following data:

$m_{Moon} = 7.35 \times 10^{22}$ kg

$m_{Earth} = 5.97 \times 10^{24}$ kg

$r_{Moon–Earth} = 3.84 \times 10^{8}$ m

Thinking	Working
Recall the formula for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Identify the information required.	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $m_1 = 7.35 \times 10^{22}$ kg $m_2 = 5.97 \times 10^{24}$ kg $r = 3.84 \times 10^{8}$ m
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{7.35 \times 10^{22} \times 5.97 \times 10^{24}}{(3.84 \times 10^{8})^2}$
Solve the equation.	$F_g = 1.98 \times 10^{20}$ N This force is close to 200 million trillion newtons.

➤ Try yourself 5.1.3

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Calculate the magnitude of the force of gravitational attraction between Earth and the Sun given the following data:

$m_{Earth} = 5.97 \times 10^{24}$ kg

$m_{Sun} = 1.99 \times 10^{30}$ kg

$r_{Sun–Earth} = 1.50 \times 10^{11}$ m

The forces in Worked example 5.1.3 are trillions of trillions times greater than those in Worked example 5.1.2, illustrating the difference in the gravitational force when at least one of the objects has a very large mass.

Explaining the structure of the universe

The greatest achievement of Newton's law of universal gravitation was to explain the observed movement of planetary bodies. The three laws of planetary motion laid down by Johannes Kepler in 1609 (considered in detail in Chapter 6) had accurately predicted the movement of the planets, but before Newton no-one knew *why* the planets followed these orbits.

Newton's simple law of universal gravitation mathematically explained the movement of planets in ellipses, and all other aspects of Kepler's laws. It explained the orbit of the Moon about Earth, Earth about the Sun, the moons around Jupiter, and all other observable planetary motion.

There is one exception. Isaac Newton's laws did not properly explain very subtle quirks in the orbit of Mercury. For that, we needed Albert Einstein (you will learn more about this in Module 5.2).

EFFECT OF GRAVITY

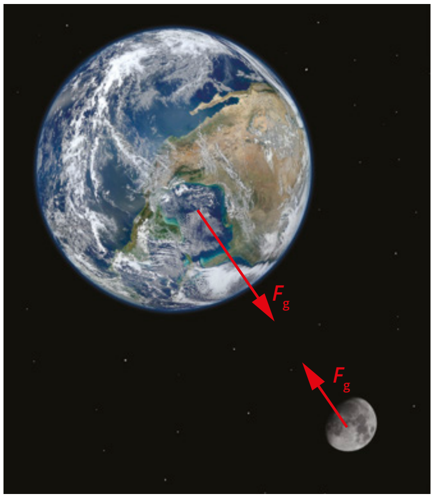


FIGURE 5.1.3 Earth and the Moon exert gravitational forces on each other.

Recall that according to Newton's third law of motion, all forces occur in equal action–reaction pairs.

An example of such a pair is shown in Figure 5.1.3. Earth exerts a gravitational force on the Moon and, conversely, the Moon exerts an equal and opposite force on the Earth.

Using Newton's *second* law of motion ($F = ma$), you can determine that the equal gravitational force between the two bodies results in a much smaller acceleration of the Earth than of the Moon.

Worked example 5.1.4

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

As you saw in Worked example 5.1.3, the force of gravitational attraction between the Moon and Earth is 1.98×10^{20} N. Calculate the acceleration of Earth and the Moon caused by this force. Compare these accelerations by calculating the ratio $\frac{a_{Moon}}{a_{Earth}}$.

Use the following data:

$m_{Moon} = 7.35 \times 10^{22}$ kg

$m_{Earth} = 5.97 \times 10^{24}$ kg

Thinking	Working
Recall the formula for Newton's second law of motion.	$F = ma$
Transpose the equation to make *a* the subject.	$a = \frac{F}{m}$
Substitute values into this equation to find the accelerations of the Moon and of Earth.	$a_{Moon} = \frac{1.98 \times 10^{20}}{7.35 \times 10^{22}} = 2.70 \times 10^{-3}\,\text{m s}^{-2}$ and the direction is towards Earth. $a_{Earth} = \frac{1.98 \times 10^{20}}{5.97 \times 10^{24}} = 3.32 \times 10^{-5}\,\text{m s}^{-2}$ and the direction is towards the Moon.
Compare the two accelerations by calculating the ratio $\frac{a_{Moon}}{a_{Earth}}$	$\frac{a_{Moon}}{a_{Earth}} = \frac{2.70 \times 10^{-3}}{3.32 \times 10^{-5}} = 81.2$ That is, the acceleration of the Moon due to the Earth is approximately 80 times greater than the acceleration of the Earth due to the Moon.

➤ Try yourself 5.1.4

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between Earth and a rugby ball kicked into the air is 4.90 N. Calculate the acceleration of the ball, and of Earth, caused by this force. Compare these accelerations by calculating the ratio $\frac{a_{ball}}{a_{Earth}}$.

Use the following data:

$m_{ball} = 500.0$ g

$m_{Earth} = 5.97 \times 10^{24}$ kg

Gravity in the solar system

Accelerations caused by gravitational forces between astronomical bodies, as calculated in Worked example 5.1.3, created the motion of the solar system.

In the Earth–Moon system, the acceleration of the Moon is many times greater than that of Earth, which is why the Moon orbits Earth. Although the Moon's gravitational force causes a much smaller acceleration of Earth, it does have other significant effects, such as the tides.

Similarly, the Earth and other planets orbit the Sun because their masses are much smaller than the Sun's mass. However, the combined gravitational effect of the planets of the solar system (and Jupiter in particular) causes the Sun to wobble slightly as the planets orbit it.

WEIGHT AND GRAVITATIONAL FORCE

You will have seen previously (Unit 2 Chapter 7 and Unit 3 Chapter 3) that the weight of an object is calculated using the formula $F_g = mg$.

'Weight' is simply another name for the gravitational force acting on an object near Earth's surface.

Worked example 5.1.5 below shows that the formula for weight $F_g = mg$ and Newton's law of universal gravitation $F_g = G\frac{m_1 m_2}{r^2}$ give the same answer for the gravitational force acting on objects on Earth's surface. It is important to note that the distance used in these calculations is the distance between the centres of the two objects, which is effectively the radius of Earth.

Weight is the force due to gravity.

Mass is the amount of material contained in a body.

In distant space, far from any large body, your weight would be zero, but your mass would be the same as here on Earth (and so, for example, your inertia would be the same, if you were trying to change direction in space).

Worked example 5.1.5

GRAVITATIONAL FORCE AND WEIGHT

Compare the weight of a 30.0 kg child, calculated using $F_g = mg$, with the gravitational force on the child due to Earth, calculated using $F_g = G\frac{m_1 m_2}{r^2}$.

Use the following dimensions of Earth in your calculations:

$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$

$g = 9.8\ \mathrm{m\,s^{-2}}$

$M_{Earth} = 5.97 \times 10^{24}\ \mathrm{kg}$

$r_{Earth} = 6.37 \times 10^{6}\ \mathrm{m}$

Thinking	Working
Apply the weight equation.	$F_g = mg$ $= 30.0 \times 9.8$ $= 290\ \mathrm{N}$
Apply Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24} \times 30.0}{(6.37 \times 10^{6})^2}$ $= 294\ \mathrm{N}$
Compare the two values.	Both equations give the same result (to within 2 or 3 significant figures).

➤ Try yourself 5.1.5

GRAVITATIONAL FORCE AND WEIGHT

Compare the weight of a 5.0 kg mass on Earth's surface calculated using the formulas $F_g = mg$ and $F_g = G\frac{m_1 m_2}{r^2}$. Use the following dimensions of Earth, where necessary:

$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$

$g = 9.8\,\text{m s}^{-2}$

$m_{\text{Earth}} = 5.97 \times 10^{24}\,\text{kg}$

$r_{\text{Earth}} = 6.37 \times 10^{6}\,\text{m}$

Be aware that Newton's law of universal gravitation, $F_g = G\frac{m_1 m_2}{r^2}$, can also be expressed as $F_g = \frac{GMm}{r^2}$. This form will be introduced and used in Chapter 6.

Worked example 5.1.5 shows that the standard acceleration due to gravity at Earth's surface, *g*, can be derived directly from the dimensions of Earth. An object with mass *m* sitting on the surface of Earth is a distance of 6.37×10^6 m from its centre.

Given that Earth has a mass of 5.97×10^{24} kg, then:

$$\text{Weight} = Fg$$

$$mg = G\frac{m_{\text{Earth}}m}{(r_{\text{Earth}})^2}$$

$$g = G\frac{m_{\text{Earth}}}{(r_{\text{Earth}})^2}$$

$$6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{(6.37 \times 10^6)^2}$$

$$= 9.81\,\text{ms}^{-2}$$

This calculation shows that the acceleration of objects near the surface of Earth, *g*, is a result of Earth's mass and radius. A planet with a different mass or different radius will therefore have a different value for *g*.

The measured value for *g* is not constant everywhere on Earth. It is higher at the poles than at the equator, because of the slightly 'squashed' shape of the Earth (i.e. the distance from pole to centre is shorter than the distance from equator to centre). Also *g* varies according to nearby valleys, mountains and more dense or less dense rock. Globally, the average value for *g* is $9.8\,\text{m s}^{-2}$. You will learn more about variations in gravitational field strength in Module 5.2.

Likewise, if an object is above Earth's surface, the value of *r* will be greater and therefore the acceleration due to gravity will be smaller, following the inverse square law. This is why the strength of Earth's gravity reduces as you travel away from Earth.

Multi-body systems

So far, only gravitational systems involving two objects have been considered, such as the Moon and Earth.

In reality, objects experience a gravitational force from every other object around them. Usually, most of these forces are negligible and only the gravitational effect of the largest object nearby (i.e. in everyday life, Earth) needs to be considered.

When there is more than one significant gravitational force acting on a body, the gravitational forces must be added together as vectors to determine the net gravitational force (Figure 5.1.4).

For example, the gravitational forces acting on *you* right now include the pull of Earth, the pull of the Sun (about 1500 times smaller), the pull of the Moon (almost 200 times smaller again) and the pull of Jupiter (100 times smaller than that).

The direction and relative magnitude of the net gravitational force in a multi-body system depends on the masses and positions of all of the attracting objects (i.e. m_1, m_2 and m_3 in Figure 5.1.4).

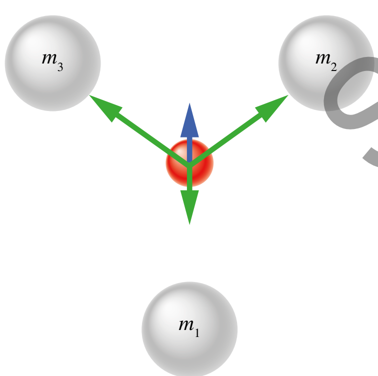


FIGURE 5.1.4 For three objects of equal mass ($m_1 = m_2 = m_3$) with the relative positions shown, the gravitational forces acting on the central red ball are indicated by the green arrows. The vector sum of the green arrows is indicated by the blue arrow. This will be the direction of the net (or resultant) gravitational force on the red ball due to the other three masses.

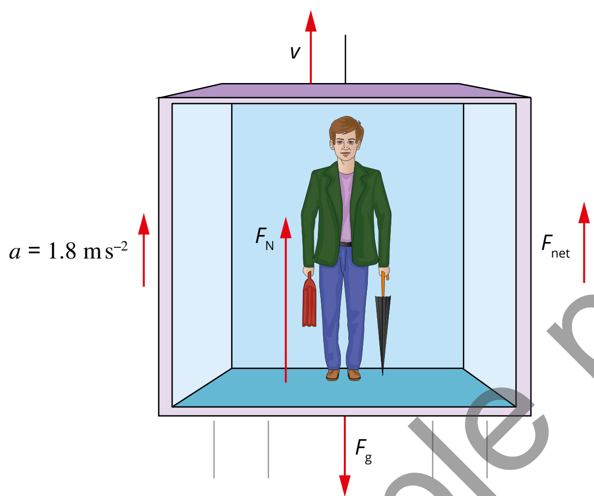
APPARENT WEIGHT

Scientists use the term ‘weight’ simply to mean ‘the force due to gravity’. It is also correct to interpret weight as the contact force (or normal reaction force) between an object and Earth’s surface. In most situations these two definitions are effectively the same. However, there are some cases—for example, when a person is accelerating up or down in a lift—when they give different results. In these situations, the normal force (F_N) is referred to as the **apparent weight**, as this is the force that the person will feel through their feet.

Worked example 5.1.6

APPARENT WEIGHT

A 50.0 kg person is standing in a lift that is accelerating upwards at 1.8 m s^{-2}.
Calculate the weight and apparent weight of the person.
Use $g = 9.8$ m s^{-2}.



Thinking	Working
Calculate the weight of the person using $F_g = mg$.	$F_g = mg = 50.0 \times 9.8 = 490$ N downwards
Calculate the force required to accelerate the person upwards at 1.8 m s^{-2}.	$F_{net} = ma = 50.0 \times 1.8 = 90$ N
The net force that causes the acceleration results from the normal reaction force (upwards) and the weight force (downwards). Since the lift is accelerating upwards, $F_N > F_g$. Recall that the normal reaction force gives the apparent weight.	$F_{net} = 90$ $F_N - F_g = 90$ $F_N - 490 = 90$ $F_N = 490 + 90$ F_N = apparent weight = 580 N upwards, since it in the same direction as the normal reaction force.

➤ Try yourself 5.1.6

APPARENT WEIGHT

Calculate the apparent weight of a 100.0 kg person in a lift that is accelerating downwards at 0.20 m s^{-2}. Use $g = 9.8$ m s^{-2}.

SCIENCE AS A HUMAN ENDEAVOUR

Hunting exoplanets

In recent years, scientists have been interested in discovering whether other stars have planets like those in our own solar system. One of the ways in which these 'extrasolar planets' (or 'exoplanets') can be detected is by the gravitational effect they have on their host star.

When any planet orbits a star, it causes the star to 'wobble' enough for this wobble to be detected on Earth. At the time of writing, thousands of exoplanets have been discovered using this technique. These exoplanets range in size from many times the mass of Jupiter to Earth-sized.

Consider Newton's law of universal gravitation: $F_g = G\frac{m_1 m_2}{r^2}$. You will see that the larger the mass of the planet, the greater the gravitational force between that planet and its host star, and therefore the more it will cause its host star to wobble. The inverse square relationship to distance means that the closer the planet's orbit is to the star, the greater the wobble too. So it won't be surprising to hear that the easiest exoplanets to spot are those known as 'hot Jupiters'—planets the mass of Jupiter or larger, and orbiting their star closer than Mercury orbits the Sun. These are the planets that exert the greatest forces on their host star.

These hot Jupiters were the first exoplanets to be discovered using the gravitational wobble method. In fact, before hot Jupiters were detected, astronomers didn't believe that planets that large could ever form so close to a star. So in 1995 the very first exoplanet discovered orbiting a regular star, 51 Pegasi b, revolutionised our understanding of how solar systems form.

Since then astronomers' instruments have improved and can now detect the gravitational tug of much smaller planets.

In fact, in 2016 it was discovered that Proxima Centauri, the star closest to us, has at least one planet not much larger than Earth tugging at the star in time with the planet's 11-day orbit. With such a small planet, the effect on the star's orbit is tiny—a change in velocity of only $1.4\,\text{m}\,\text{s}^{-1}$. The instrumentation used by astronomers is precise enough to identify such small changes in velocity.

Instruments at the new observatory at Mt Kent in southern Queensland will be able to detect even smaller gravitational wobbles in distant stars, changes as small as $1\,\text{m}\,\text{s}^{-1}$, or about the same as a very slow walking pace.

Queensland astronomer Jonti Horner (Figure 5.1.5) has discovered many exoplanets using gravitational wobble and other planet-hunting methods. But he is still impressed by the science behind these discoveries: 'Stars are trillions or quadrillions of kilometres away. Proxima Centauri, the nearest, is more than 40 trillion km distant. I find it astonishing we can measure something that is so distant and see a wobble that is comparable to the speed at which you or I would walk around the shops!'

FIGURE 5.1.5 Queensland astronomer Jonti Horner uses the gravitational pull on host stars to identify distant exoplanets.

Review

1 Explain the term 'hot Jupiters' and why they are reasonably easy for astronomers to detect.

2 Astronomers can discover planets orbiting distant stars by observing the effect of the exoplanet's gravitational pull on its host star. The huge exoplanet Hypatia has a mass of 1.68×10^{28} kg (almost 10 times the mass of Jupiter).

   a Calculate the magnitude of the gravitational force that the planet exerts on its host star, given that Hypatia orbits at an average distance of 195 000 000 km and the mass of its host star is 3.62×10^{30} kg.

   b Calculate the magnitude of the resulting acceleration of the planet's host star.

5.1 Review

SUMMARY

- All objects with mass attract one another with a gravitational force.
- The gravitational force acts equally on each of the masses.
- The magnitude of the gravitational force is given by Newton's law of universal gravitation: $F_g = G\frac{m_1 m_2}{r^2}$.
- Gravitational forces are usually negligible unless one of the objects is massive, e.g. a planet.
- The weight of an object on the Earth's surface is due to the gravitational attraction of Earth and, unless the object is accelerating, weight is equal to the normal reaction force: weight = $F_g = F_N$.
- The acceleration due to gravity of an object near the surface of Earth can be calculated using the dimensions of the Earth: $g = G\frac{m_{Earth}}{(r_{Earth})^2} = 9.8\,m\,s^{-2}$
- Objects can have an apparent weight that is greater or less than their normal weight. This occurs when they are accelerating vertically.

KEY QUESTIONS

Retrieval

1 Describe the relationships in Newton's law of universal gravitation. State what the force of attraction is proportional to. Determine what makes it greater and what makes it smaller.

2 Indicate what the symbol r represents in Newton's law of universal gravitation.

Comprehension

3 Describe what happens to the gravitational force acting between two masses m_1 and m_2 a distance r apart, in each case below.

a The mass of m_1 is doubled.

b The distance r is doubled.

c The distance r is quadrupled.

4 Explain why the acceleration of the Moon caused by the gravitational force of Earth is much larger than the acceleration of Earth due to the gravitational force of the Moon.

Analysis

5 Consider gravitational attraction between the Sun and Mars, given that the mass of the Sun is 1.99×10^{30} kg, the mass of Mars is 6.39×10^{23} kg and the average distance between the two is 2.28×10^{11} m.

a Calculate the gravitational attraction between the Sun and Mars.

b Calculate the acceleration of Mars.

6 On 14 April 2014, Mars came within 93.0 million km of Earth. At that point its gravitational effect on Earth was the strongest it had been for over 6 years. Use the following data to answer the questions below.

$$m_{Sun} = 1.99 \times 10^{30}\,kg$$
$$m_{Earth} = 5.97 \times 10^{24}\,kg$$
$$m_{Mars} = 6.39 \times 10^{23}\,kg$$

a Calculate the gravitational force between Earth and Mars on 14 April 2014.

b Calculate the force of the Sun on Earth, given that the distance between them is 150 million km.

c Compare your answers to parts **a** and **b** above by expressing the Mars–Earth force as a percentage of the Sun–Earth force.

d Calculate the distance r between Earth and Mars at the greatest distance between the two planets, where the gravitational force between Mars and the Earth is 5.03×10^{15} N.

7 Compare and explain the difference between the terms 'weight' and 'apparent weight', giving an example of a situation in which the magnitudes of these two forces would be different.

8 Two astronauts, Sandra and George, each of mass 150 kg (including their suits), float in outer space 1.00 m apart.

a Calculate the gravitational force between them.

b Calculate the resulting acceleration of each astronaut.

5.1 Review *continued*

9 Consider the force of gravity on Mercury, a small rocky planet.

- **a** State the equation you would use to calculate gravitational acceleration on the surface of Mercury.
- **b** Explain the effect the much smaller radius of Mercury has on gravitational acceleration. Mercury has a radius only a third of the radius of Earth.
- **c** Explain the effect the much smaller mass has on gravitational acceleration. The mass of Mercury is much smaller than that of Earth.
- **d** Calculate gravitational acceleration on the surface of Mercury, given that Mercury has a mass of 3.29×10^{23} kg and a radius of 2440 km.
- **e** Calculate the weight of Hermes, a 75.0 kg astronaut standing on the surface of Mercury.

10 Calculate the weight of Aphrodite, a 75.0 kg astronaut standing on the surface of Venus, given that the planet has a mass of 4.87×10^{24} kg and a radius of 6050 km.

11 Calculate the apparent weight of a 50.0 kg person in a lift under the following circumstances. Use $g = 9.8\,\text{m}\,\text{s}^{-2}$.

- **a** accelerating upwards at $1.24\,\text{m}\,\text{s}^{-2}$
- **b** moving upwards at a constant speed of $5.0\,\text{m}\,\text{s}^{-1}$

12 In 1846, Newton's laws of gravity allowed astronomers to predict the existence of Neptune, the eighth planet in the solar system, based on variations in the orbit of Uranus.

- **a** Calculate the maximum force between the two planets if the mass of Uranus is 8.68×10^{25} kg, the mass of Neptune is 1.02×10^{26} kg, and the shortest distance between the two planets is 1.63×10^{9} km.
- **b** Compare this with the gravitational force of the Sun on Uranus if the mass of the Sun is 1.99×10^{30} kg and the distance from the Sun to Uranus is 2.87×10^{9} km.

5.2 Gravitational fields

BY THE END OF THIS MODULE, YOU SHOULD BE ABLE TO:

- define 'gravitational fields'
- solve problems involving the gravitational field strength at distance from an object.

Newton's law of universal gravitation describes the force acting between two mutually attracting bodies. In reality, complex systems like the solar system involve a number of objects (e.g. the solar system has the Sun and planets (Figure 5.2.1)) that are all exerting attractive forces on each other at the same time.

In the 18th century, to simplify the process of calculating the effect of simultaneous gravitational forces, scientists developed a model known as the **gravitational field**. In the following centuries, the idea of a **field** was also applied to other forces and has become a very important concept in physics.

FIGURE 5.2.1 The solar system is a complex gravitational system.

GRAVITATIONAL FIELDS

A gravitational field is a region in which a gravitational force is exerted on all matter within that region. Every physical object has an accompanying gravitational field. For example, the space around your body contains a gravitational field because any other object that comes into this region will experience a (small) force of gravitational attraction to your body.

The gravitational field around a large object such as a planet is much more significant than that around a small object. Earth's gravitational field exerts a significant influence on objects on its surface and even up to thousands of kilometres into space.

Discovering Neptune

The planet Neptune was discovered through its gravitational effect on other planets and the application of Newton's law.

Two astronomers, Urbain Le Verrier of France and John Couch Adams of England, each independently identified that the observed orbit of Uranus varied significantly from predictions made based on the gravitational effects of the Sun and other known planets. Both astronomers suggested that this was due to the influence of a distant, undiscovered planet.

Le Verrier sent a prediction of the location of the new planet to Gottfried Galle at the Berlin Observatory and, on 23 September 1846, Neptune was discovered within 1 degree of Le Verrier's prediction (Figure 5.2.2).

a b

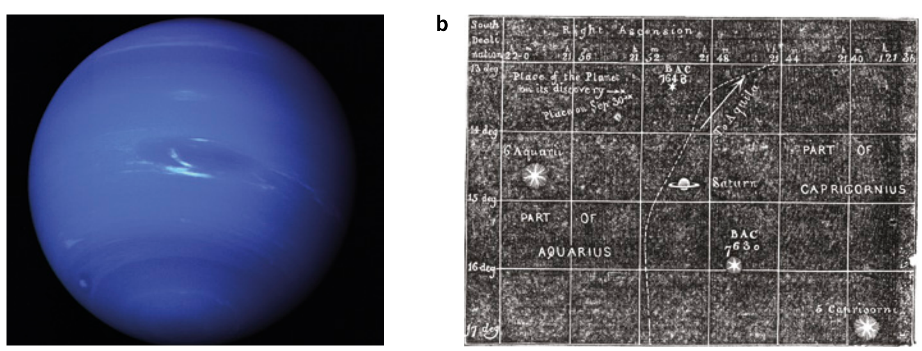


FIGURE 5.2.2 (a) Picture of Neptune taken by Voyager 2 in 1989. (b) This star chart published in 1846 shows the location of Neptune in the constellation Aquarius on its discovery, and its location one week later, on September 30.

Representing gravitational fields

Over time, scientists have developed a commonly understood method of representing fields using a series of arrows known as field lines (Figure 5.2.3). For gravitational fields, these are constructed as follows.

- The direction of the arrowhead indicates the direction of the gravitational force.
- The space between the arrows indicates the relative magnitude of the field:
  - closely spaced arrows indicate a strong field
  - widely spaced arrows indicate a weaker field.
- Parallel field lines indicate constant or uniform field strength.
- Gravitational field lines emanate from the source of the field.
- The lines never cross.

In theory, you could draw an infinite number of field lines, but in fact only a few are needed to represent the rest. The size of the gravitational force acting on a mass in the region of a gravitational field is determined by the strength of the field, and the force acts in the direction of the field.

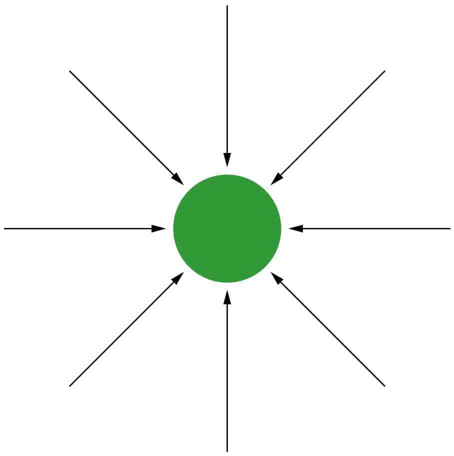
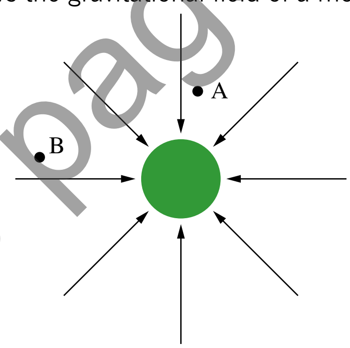


FIGURE 5.2.3 The arrows in this gravitational field diagram indicate that objects will be attracted towards the mass in the centre. The spacing of the lines shows that force will be strongest at the surface of the central mass and weaker further away from it.

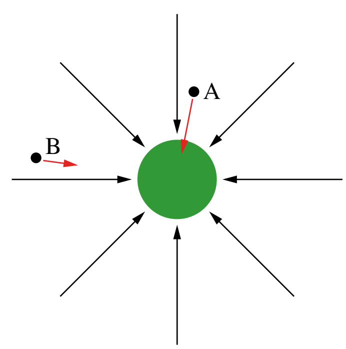
Worked example 5.2.1

INTERPRETING GRAVITATIONAL FIELD DIAGRAMS

The diagram below shows the gravitational field of a moon.



a Use arrows to indicate the direction of the gravitational force acting at points A and B.

Thinking	Working
The direction of the field arrows indicates the direction of the gravitational force, which is inwards towards the centre of the moon.	

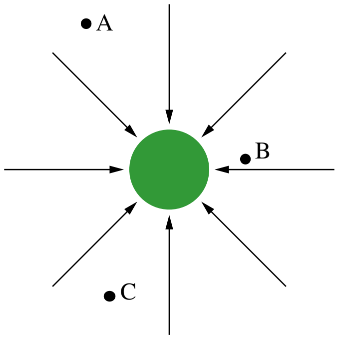
b Indicate the relative strength of the gravitational field at each point.

Thinking	Working
The closer the field lines, the stronger the force. The field lines are closer together at point A than they are at point B, as point A is closer to the moon.	The field is stronger at point A than at point B.

➤ Try yourself 5.2.1

INTERPRETING GRAVITATIONAL FIELD DIAGRAMS

The diagram below shows the gravitational field of a planet.



a Use arrows to indicate the direction of the gravitational force acting at points A, B and C.

b Indicate the relative strength of the gravitational field at each point.

GRAVITATIONAL FIELD STRENGTH

In theory, gravitational fields extend infinitely out into space. However, since the magnitude of the gravitational force decreases with the square of distance, eventually these fields become so weak as to become negligible.

In Module 5.1, you calculated the acceleration due to gravity of objects near the Earth's surface using the dimensions of the Earth: $g = G\frac{M_{Earth}}{(r_{Earth})^2} = 9.8\,m\,s^{-2}$.

This acceleration is known as the standard acceleration due to gravity, and is usually indicated by the letter g or g_{Earth}. This value is also used as a measure of the strength of the gravitational field, in which case it is written with the equivalent units of $N\,kg^{-1}$ rather than $m\,s^{-2}$. This means $g_{Earth} = 9.8\,N\,kg^{-1}$.

These units indicate that objects on the surface of the Earth experience 9.8 N of gravitational force for every kilogram of their mass.

Accordingly, the familiar equation $F_g = mg$ can be rearranged so that the **gravitational field strength**, g, can be calculated:

$$g = \frac{F_g}{m}$$

where

g is gravitational field strength ($N\,kg^{-1}$)

F_g is the force due to gravity (N)

m is the mass of an object in the field (kg)

Worked example 5.2.2

CALCULATING GRAVITATIONAL FIELD STRENGTH

When Maree hangs a 1.00 kg mass from a spring balance, the balance measures a downwards force of 9.8 N.

Calculate the gravitational field strength of the Earth at this location, according to this experiment.

Thinking	Working
Recall the equation for gravitational field strength.	$g = \frac{F_g}{m}$
Substitute in the appropriate values.	$g = \frac{9.8}{1.00}$
Solve the equation.	$g = 9.8\,N\,kg^{-1}$ downwards

> $g = G\frac{M}{r^2}$
>
> where
>
> g is the gravitational field strength ($N\,kg^{-1}$)
>
> G is the gravitational constant, $6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$
>
> M is the mass of the planet or moon (i.e. the central body; kg)
>
> r is the radius of the planet or moon (m)

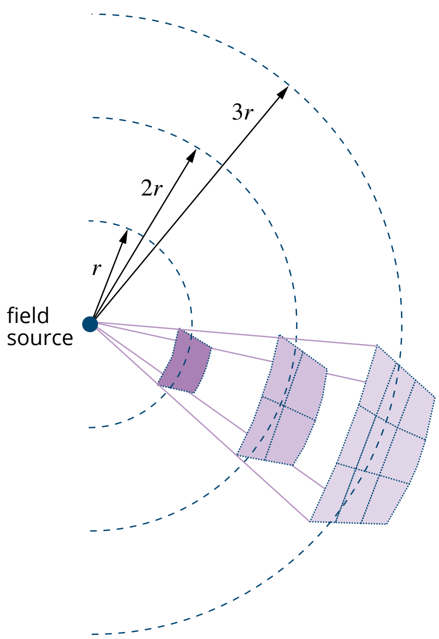


FIGURE 5.2.4 As the distance from the source of a field increases, the field is spread over an area that increases with the square of the distance from the source. This means that the strength of the field decreases by the same ratio.

➤ Try yourself 5.2.2

CALCULATING GRAVITATIONAL FIELD STRENGTH

> Dion uses a spring balance to measure the weight of a piece of wood as 2.53 N. If the piece of wood has a mass of 259 g, calculate the gravitational field strength indicated by this experiment.

As was shown earlier in this module and in Worked example 5.1.5, the formula for gravitational field strength, $g = \frac{F_g}{m}$, can be combined with Newton's law of universal gravitation, $F_g = G\frac{Mm}{r^2}$, to develop the formula for gravitational field strength:

$$g = \frac{F_g}{m} = \frac{G\frac{Mm}{r^2}}{m}$$

Therefore: $g = G\frac{M}{r^2}$

where

g is the gravitational field strength ($N\,kg^{-1}$)

G is the gravitational constant, $6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$

M is the mass of the planet or moon (i.e. the central body; kg)

r is the radius of the planet or moon (m).

The inverse square law

The concept of a field is a very powerful tool for understanding forces that act at a distance. It has also been applied to such forces as the electrostatic force between charged objects and the force between two magnets.

The study of gravitational fields introduces the concept of the inverse square law. From the point source of a field, whether it be gravitational or electric, the field will spread out radially in three dimensions. When the distance from the source is doubled, the field will be spread over four times the original area.

Figure 5.2.4 shows an increasing distance from the field source of r then $2r$ then $3r$. As you can see, a projection of one square at distance r increases to four squares (2^2) at distance $2r$ and increases to nine squares (3^2) at distance $3r$.

As the 'inverse' part of the inverse square law implies, at a distance $2r$ from the source the strength of the field will be reduced to a quarter of the field at distance r. The force the field would exert at that distance will also be reduced to a quarter. At a distance $3r$ from the source, the field will be reduced to one-ninth of the field at distance r, and so on.

> In terms of the gravitational field, the strength of the force varies inversely with the distance from the source of the field, squared:
>
> $F \propto \frac{1}{r^2}$
>
> where
>
> F is the force (N)
>
> r is the distance from the source of the gravitational field (m).
>
> This is referred to as an inverse square law. (Refer to the Skillbuilder on page 464 of *Pearson Physics 11 Queensland* for more information on proportional reasoning.)

One key difference between the gravitational force and other inverse square forces is that the gravitational force is always attractive, whereas like charges, or magnets with the same pole facing, repel one another.

Inverse square laws are an important concept in physics, not only in the study of fields but also in studying phenomena where energy is moving away from its source in three dimensions, such as in sound and other waves.

Variations in gravitational field strength of Earth

The gravitational field strength of Earth, g, is usually assigned a value of $9.8\,\mathrm{N\,kg^{-1}}$. However, the field strength experienced by objects on the surface of Earth can vary—it can be between $9.76\,\mathrm{N\,kg^{-1}}$ and $9.83\,\mathrm{N\,kg^{-1}}$, depending on the location.

Several factors influence the varying gravitational field strength. Earth is not a perfect sphere, but is 'flattened' at the poles (Figure 5.2.5). This means objects near the equator are slightly further from the centre than objects at the poles, so the gravitational field is slightly weaker at the equator than at the poles.

Geological formations can also create differences in gravitational field strength, depending on their composition. Geologists use a sensitive instrument known as a **gravimeter** (Figure 5.2.6) that detects small local variations in gravitational field strength to indicate underground features. Rocks with above-average density, such as those containing mineral ores, create slightly stronger gravitational fields, whereas less-dense sedimentary rocks produce weaker fields.

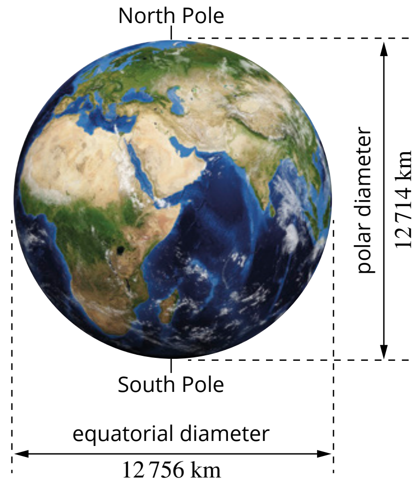


FIGURE 5.2.5 Earth is a flattened sphere, which means its gravitational field is slightly stronger at the poles than the equator.

FIGURE 5.2.6 A gravimeter can be used to measure the strength of the local gravitational field.

If Earth's surface is considered a flat surface as it appears in everyday life, then the gravitational field lines are approximately parallel, indicating a uniform field (Figure 5.2.7).

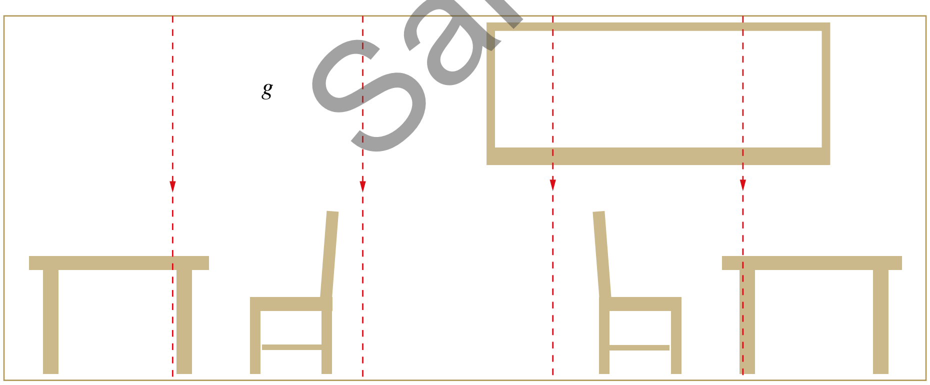


FIGURE 5.2.7 The uniform gravitational field, g, is represented by evenly spaced parallel lines in the direction of the force.

However, when Earth is viewed from a sufficiently large distance to see it as a sphere, it becomes clear that its gravitational field is not uniform at all (Figure 5.2.8). The increasing distance between the field lines indicates that the field becomes progressively weaker out into space.

This is because gravitational field strength, like gravitational force, is governed by the inverse square law:

$$g = G\frac{M_{\text{Earth}}}{(r_{\text{Earth}})^2}$$

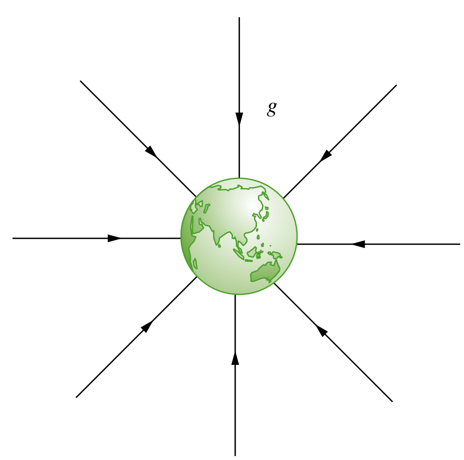


FIGURE 5.2.8 Earth's gravitational field becomes progressively weaker out into space.

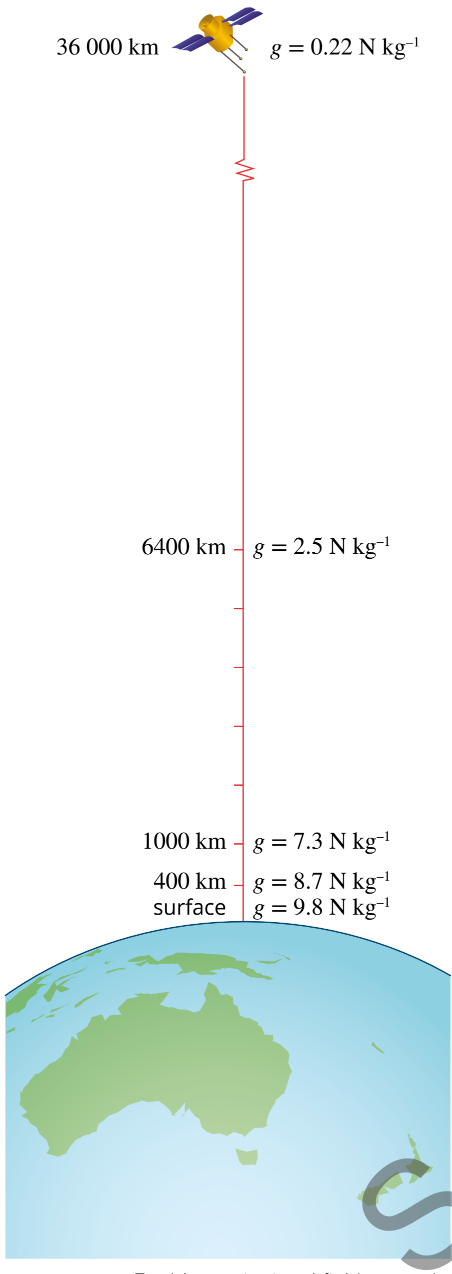


FIGURE 5.2.9 Earth's gravitational field strength is weaker at higher altitudes.

The gravitational field strength at different altitudes can be calculated by adding the **altitude** (height above the surface of Earth) to the radius of Earth to calculate the distance the object is from Earth's centre (Figures 5.2.9 and 5.2.10).

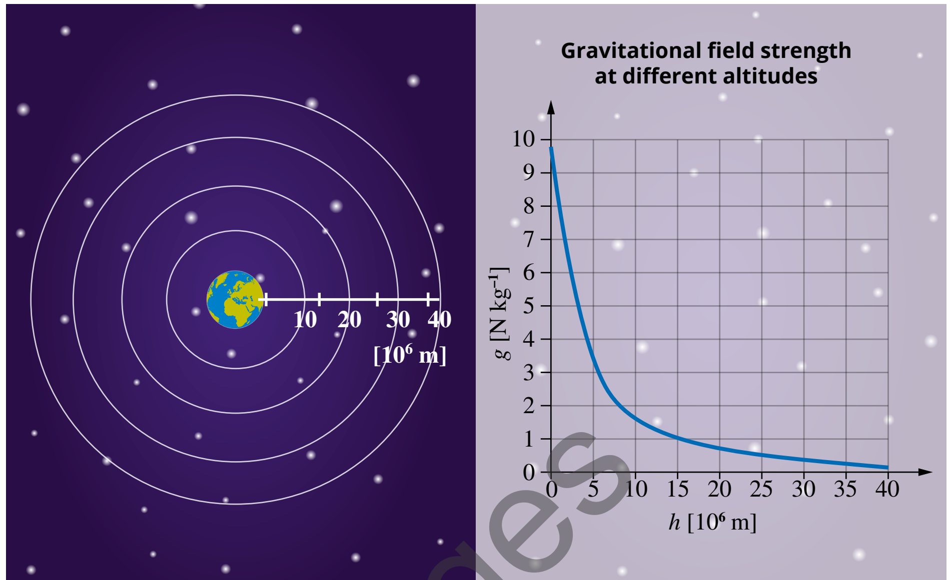


FIGURE 5.2.10 As the distance from the surface of Earth increases from 0 to 40 × 10^6 m, the value for g decreases rapidly from 9.8 N kg^{-1}, according to the inverse square law. The blue line on the graph gives the value of g at various altitudes (h).

Worked example 5.2.3

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Calculate the magnitude of Earth's gravitational field at the top of Mt Everest.

$r_{Earth} = 6.37 \times 10^6$ m

$M_{Earth} = 5.97 \times 10^{24}$ kg

height of Mt Everest = 8848 m

Compare the answer with average gravitational field strength at sea level, 9.8 N kg^{-1}.

Thinking	Working
Recall the formula for gravitational field strength.	$g = G\frac{M}{r^2}$
Add the height of Mt Everest to the radius of Earth.	$r = 6.37 \times 10^6 + 8848$ m $= 6.378 \times 10^6$ m
Substitute the values into the formula.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{(6.378 \times 10^6)^2}$ $= 9.79$ N kg^{-1}
Compare the field strength at the height of Mt Everest with the global average at sea level by calculating the ratio $\frac{g_{Everest}}{g_{average}}$.	$\frac{g_{Everest}}{g_{average}} = \frac{9.79}{9.8}$ $= 0.9986$ The gravitational field strength at the top of Mt Everest is 99.9% of the average strength at sea level.

$g = \frac{GM_{Earth}}{(r_{Earth} + \text{alititude})^2}$

➤ Try yourself 5.2.3

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

The International Space Station (ISS) orbits at an altitude of approximately 400 km. Calculate the magnitude of the gravitational field strength at this height.

$h_{ISS} = 400.0\,\text{km}$

$r_{Earth} = 6.37 \times 10^6\,\text{m}$

$M_{Earth} = 5.97 \times 10^{24}\,\text{kg}$

Compare the answer with the average gravitational field strength at sea level, $9.8\,\text{N kg}^{-1}$.

Consider whether astronauts are 'weightless' at this altitude.

Gravitational field strengths on another planet or moon

The gravitational field strength on the surface of the Moon is $1.62\,\text{N kg}^{-1}$, which is much less than on Earth. This is because the Moon's mass is so much less than the mass of Earth, more than making up for its smaller radius.

The formula $g = G\frac{M}{r^2}$ can be used to calculate the gravitational field strength on the surface of any astronomical object, such as Mars (Figure 5.2.11).

FIGURE 5.2.11 The gravitational field strength on the surface of Mars (shown here) is different from the gravitational field strength on the surface of Earth, which, in turn, is different from that on the Moon.

Worked example 5.2.4

GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON

Calculate the magnitude of the gravitational field on the surface of the Moon, given that the Moon's mass is $7.35 \times 10^{22}\,\text{kg}$ and its radius is 1740 km.

Compare the answer with Earth's average gravitational field strength $9.8\,\text{N kg}^{-1}$.

Thinking	Working
Recall the formula for gravitational field strength.	$g = G\frac{M}{r^2}$
Convert the Moon's radius to m.	$r = 1740\,\text{km}$ $= 1740 \times 1000\,\text{m}$ $= 1.74 \times 10^6\,\text{m}$
Substitute values into the formula.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{7.35 \times 10^{22}}{(1.74 \times 10^6)^2}$ $= 1.62\,\text{N kg}^{-1}$
Compare the field strength with Earth's average gravitational field strength by calculating the ratio $\frac{g_{Earth}}{g_{Moon}}$.	$\frac{g_{Earth}}{g_{Moon}} = \frac{9.8}{1.62}$ $= 6.1$ The gravitational field strength on the surface of Earth is six times stronger than on the Moon.

➤ Try yourself 5.2.4

GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON

Calculate the strength of the gravitational field on the surface of the distant exoplanet Kepler-10C, the largest known rocky planet, which is 17 times more massive than Earth.

$M_{Kepler\text{-}10C} = 1.0 \times 10^{26}\,\text{kg}$

$r_{Kepler\text{-}10C} = 15\,000\,\text{km}$

Compare the answer with Earth's average gravitational field strength $9.8\,\text{N kg}^{-1}$.

Introducing general relativity

Hold onto your hats. Here's a shock: Newtonian gravity is not the way the universe actually works! Newtonian gravity is a wonderfully useful approximation that works in all but the most extreme conditions. But in those extreme conditions, for example within black holes or very close to very massive objects such as stars, Newton's model of gravity stops working properly.

The system that really explains the way gravity functions was developed by Albert Einstein in 1915, and is known as *general relativity*.

When general relativity was first proposed, physicists struggled to accept that Newton's long-accepted laws were wrong.

However, the theory was accepted when Einstein's general relativity correctly explained the quirks in Mercury's orbit that Newtonian physics could not, and properly predicted the bending of light as it passed the Sun.

General relativity also predicted the existence of gravitational waves, which were detected by the LIGO facilities in late 2015, after a huge international scientific effort including many Australian researchers (see Science as a Human Endeavour).

There are two principles of general relativity that explain how mass and 'spacetime' (the fabric of the universe) affect each other:

1 Mass tells spacetime 'how to bend'.
2 The curvature of spacetime tells mass 'how to move'.

WS 3.1.7

You will learn more about special relativity in Chapter 10.

SCIENCE AS A HUMAN ENDEAVOUR

Gravitational waves and the Australian connection

The successful detection of gravitational waves in late 2015 was the culmination of a century of physics. When Albert Einstein predicted such waves in his 1915 theory of general relativity, he said they could never be detected, as the displacement would be too small to measure.

According to Einstein's theory of general relativity, objects with mass bend spacetime. Two massive objects, such as black holes, that had been orbiting each other, would cause ripples in spacetime if they collided. These ripples are gravitational waves.

Fast forward to 2015 and the instruments at LIGO (the Laser Interferometer Gravitational-Wave Observatory) are accurate enough to measure displacements as small as one-thousandth the width of a proton.

LIGO is made up of two large facilities: one in Washington state, in the northwest of the USA, and the other 3000 km southeast near New Orleans, Louisiana. Each facility comprises two 2 km long buildings at right angles, along which a laser is shone and reflected back. The two perpendicular beams should arrive simultaneously, with any tiny deviations indicating that the length of one building has increased or decreased.

And as a gravitational wave passes through the Earth, that is precisely what happens. Depending on the source of the wave, one building or the other becomes about one quadrillionth of a mm (10^{-18} m) longer than the other for a few milliseconds as the wave passes.

Australian physicists played a key part in the project, including working on the mirrors that position LIGO's lasers, the detectors that detect changes in path length, and software to interpret results. Australian gravitational waves research is coordinated by the ARC Centre of Excellence for Gravitational Waves Discovery (OzGrav).

The first gravitational waves were successfully detected in September 2015, and were calculated to be the result of a collision between two black holes: one of 36 solar masses (i.e. 36 times more massive than our Sun) and one of 29 solar masses.

Review

1 Explain how gravitational waves that pass through the Earth are detected by the LIGO observatory.

2 The first gravitational waves event to be measured, in September 2015, was found to be the result of a collision between two orbiting black holes: one 36 times more massive than our Sun (7.16×10^{31} kg) and another 29 times more massive than our Sun (5.77×10^{31} kg). Because black holes are so dense, at the moment they merged it is calculated their centres were only 350 km apart. Calculate the gravitational force between the two black holes at the moment they merged.

FIGURE 5.2.12 (a) Monash University (Australia) astrophysicist Chris Whittle in the optics vacuum chamber at LIGO, Washington USA, assisting with the replacement of an interferometer mirror. The cleanroom suit minimises dust, which can foul the equipment.
(b) LIGO's observatory in Washington state, USA, showing the two perpendicular arms of the facility.

5.2 Review

SUMMARY

- A gravitational field is a region in which a gravitational force is exerted on all matter within that region.
- A gravitational field can be represented by a gravitational field diagram.
  - The arrowheads indicate the direction of the gravitational force.
  - The spacing of the lines indicates the relative strength of the field. The closer the line spacing, the stronger the field.
- The strength of a gravitational field can be calculated using the following formulas: $g = \frac{F_g}{m}$ and $g = G\frac{M}{r^2}$.
- The gravitational field strength on Earth's surface is approximately $9.8\,\text{N kg}^{-1}$. This varies from location to location and with altitude.
- The gravitational field strength on the surface of any other planet depends on the mass and radius of the planet.

KEY QUESTIONS

Retrieval

1 State the units used to express acceleration due to gravity. State the average value for the acceleration due to gravity at the Earth's surface.

2 Name the person who proposed the new theory of gravity in 1915, which correctly explained even extreme gravitational fields.

Comprehension

3 Describe the direction in which gravitational field lines point within a classroom at the Earth's surface. Explain why we assume the field lines are parallel.

Analysis

4 Compare the magnitude of the gravitational field at a distance of 1200 km from the centre of a planet with that at a distance of 400 km.

5 Determine the gravitational field strength in the classroom of Imogen and Scott who use a spring balance to measure the weight of a 149 g set of slotted masses to be 1.45 N.

6 Calculate the magnitude of Earth's gravitational field for each different type of orbit listed in the table below.

$r_{\text{Earth}} = 6370\,\text{km}$
$M_{\text{Earth}} = 5.97 \times 10^{24}\,\text{kg}$

	Type of orbit	Altitude (km)
a	low-Earth orbit	2 000.0
b	medium-Earth orbit	10 000.0
c	semi-synchronous orbit	20 200
d	geosynchronous orbit	35 786

7 On 12 November 2014, after many attempts, the Rosetta spacecraft landed a probe on the comet 67P/Churyumov–Gerasimenko.

a Calculate the magnitude of the comet's gravitational field strength at its surface, assuming this comet is a roughly spherical object with a mass of 9.98×10^{12} kg and a diameter of 1.80 km.

b Compare this gravitational field strength with that at the Earth's surface, $9.8\,\text{m s}^{-2}$.

8 When a star dies, its atomic structure may collapse to form a very small, very dense body known as a neutron star. A typical neutron star may still have a mass larger than our Sun, but be smaller than a city.

a Compare the gravitational field strength on the surface of such a star with that on our own Sun.

b Calculate the gravitational field strength at the surface of a neutron star that has mass 3×10^{30} kg and radius 10 km.

c Compare this to the strength of the gravitational field at Earth's surface.
Use $g_{\text{Earth}} = 9.8\,\text{m s}^{-2}$.

d Calculate the distance from the neutron star where the gravitational field strength is the same as the gravitational field strength at the surface of Earth.
Use $g_{\text{Earth}} = 9.8\,\text{m s}^{-2}$.

9 A hypothetical planet in a distant solar system is distinctly non-spherical in shape. Its polar radius (5.0×10^3 km) is only half of its equatorial radius (1.00×10^4 km).

a Describe how different the gravitational field strength would be at the equator compared to the poles.

b Calculate the magnitude of the field at the equator, and confirm your answer from part **a**. The gravitational field strength at the poles is $8.10\,\text{N kg}^{-1}$.

10 Calculate the distance, in Earth radii, of an astronaut from the centre of Earth when the astronaut travels away from Earth to a region in space where the gravitational force due to Earth is only 1.0% of that at the Earth's surface.

Chapter review

05

KEY TERMS

altitude
apparent weight
field
gravimeter
gravitational constant
gravitational field
gravitational field strength
gravitational force
inverse square law
Newton's law of universal gravitation

KEY QUESTIONS

Retrieval

1 Newton's law of universal gravitation is used to calculate the gravitational force acting on a person standing on the surface of Earth. Assume the mass of Earth is 5.97×10^{24} kg and its radius is 6370 km.
- **a** State the equation you would use to calculate gravitational force.
- **b** Indicate what units your answer will be given in.

Comprehension

2 The planet Jupiter and the Sun exert gravitational forces on each other.
- **a** Determine, qualitatively, the force exerted on Jupiter by the Sun and compare it to the force exerted on the Sun by Jupiter.
- **b** Determine, qualitatively, the acceleration of Jupiter caused by the Sun and compare it to the acceleration of the Sun caused by Jupiter.

Analysis

3 A person standing on the surface of Earth experiences a gravitational force of 900 N. Determine which of the following gravitational forces this person will experience at a height of two Earth radii above Earth's surface.
- **A** 900 N
- **B** 450 N
- **C** zero
- **D** 100 N

4 During a tourist space mission, a wealthy astronaut of mass 100.0 kg initially accelerates at 30.0 $m\,s^{-2}$ upwards, then travels in a stable circular orbit at an altitude where the gravitational field strength is 8.20 $N\,kg^{-1}$.
- **a** Determine the apparent weight of the astronaut during lift-off.
  - **A** zero
  - **B** 980 N
  - **C** 2020 N
  - **D** 3980 N
- **b** Identify which of the following the astronaut will feel during the lift-off phase.
  - **A** lighter than usual
  - **B** heavier than usual
  - **C** the same as usual
- **c** Identify the mass of the astronaut during the lift-off phase.
  - **A** lower than usual
  - **B** greater than usual
  - **C** the same as usual
- **d** Determine the true weight of the astronaut during the orbit phase.
  - **A** zero
  - **B** 980 N
  - **C** 100.0 N
  - **D** 820 N

5 The gravitational force of attraction between Saturn and Dione, a moon of Saturn, is equal to 2.79×10^{20} N. Calculate the orbital radius of Dione. Use the following data:

mass of Dione = 1.05×10^{21} kg

mass of Saturn = 5.69×10^{26} kg

6 Of all the planets in the solar system, Jupiter exerts the largest force on the Sun: 4.16×10^{23} N. Calculate the acceleration of the Sun due to this force, using mass of the Sun = 1.99×10^{30} kg.

7 Calculate the acceleration due to gravity on the surface of Pluto if it has a mass of 1.31×10^{22} kg and a radius of 1190 km.

8 Calculate the apparent weight of a 50.0 kg person in a lift under the following circumstances. Use $g = 9.8\,m\,s^{-2}$.
- **a** accelerating downwards at 0.600 $m\,s^{-2}$
- **b** moving downwards at a constant speed of 2.00 $m\,s^{-1}$

9 A comet of mass 1100 kg is plummeting towards Jupiter. Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.15×10^{7} m. The comet is about to crash into Jupiter.

- **a** Calculate the magnitude of the gravitational force that Jupiter and the comet exert on each other.
- **b** Calculate the acceleration of the comet towards Jupiter.
- **c** Calculate the acceleration of Jupiter towards the comet.

10 Calculate the distance from the Sun (which has a mass of 1.99×10^{30} kg), at which an astronaut would experience the same gravitational field strength as they experience at the Earth's surface (namely 9.8 N kg^{-1}).

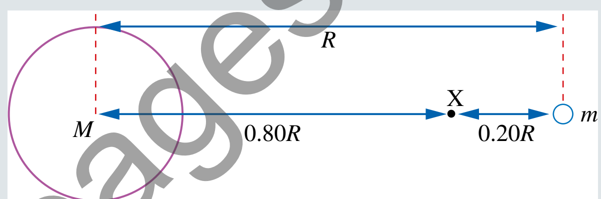
11 Determine what gravitational field strength has been assumed in the following setting. A set of bathroom scales is calibrated so that when the person standing on it has a weight of 6.00×10^{2} N, the scales read 61.5 kg.

12 Calculate the gravitational field strength at the surface of Neptune, which has a radius of 2.48×10^{7} m and a mass of 1.02×10^{26} kg.

13 Earth is a flattened sphere. Its radius at the poles is 6357 km, and at the equator the radius is 6378 km. Earth's mass is 5.97×10^{24} kg.

- **a** Calculate Earth's gravitational field strength at the equator.
- **b** Calculate how much stronger the gravitational field would be at the North Pole compared with the equator using the information in part **a**. Give your answer as a percentage of the strength at the equator to one significant figure.

14 Two stars of masses *M* and *m* are in orbit around each other. As shown in the following diagram, they are a distance *R* apart. A spacecraft located at point X experiences zero net gravitational force from these stars. Calculate the value of the ratio $\frac{M}{m}$.



Knowledge utilisation

15 The value for gravitational acceleration *g* decreases according to the inverse square law with increasing altitude above Earth, as shown in the diagram and graph. The blue line on the graph gives the value of gravitational acceleration (*g*) at various altitudes (*h*).

- **a** Determine the approximate altitude at which the gravitational field strength will be 4 N kg^{-1} using the graph.
- **b** Compare the strength of the gravitational field strength at ground level and at an altitude of 6500 km, using the graph. Predict what the difference will be.