

CHAPTER 1

Algebraic techniques

EXERCISE 1.1

- 1 $3x + 5 + 7x + 10$
 $= 3x + 7x + 5 + 10$
 $= 10x + 15$
- 3 $4a + b - a - 4b$
 $= 4a - a + b - 4b$
 $= 3a - 3b$
- 5 $3xy + 2xy - xy$
 $= 4xy$
- 7 $2x^2y + 3x^2y^2 - x^2y + 3x^2y^2$
 $= x^2y + 6x^2y^2$
- 9 $12mn + 3m - 6mn - m$
 $= 6mn + 2m$
- 11 $2x^2 + 5y^2 - 4x^2$
 $= 5y^2 - 2x^2$
- 13 $4(2x - y) - 6x$
 $= 8x - 4y - 6x$
 $= 2x - 4y$
- 15 $3(2x + 5y) + 4(x - y)$
 $= 6x + 15y + 4x - 4y$
 $= 10x + 11y$
- 17 $6(2a + 3b) + 3(a - b)$
 $= 12a + 18b + 3a - 3b$
 $= 15a + 15b$
- 19 $5x(x - 2y) + 3x(2x - y)$
 $= 5x^2 - 10xy + 6x^2 - 3xy$
 $= 11x^2 - 13xy$
- 21 $x + 5y - (3x + 2y)$
 $= x + 5y - 3x - 2y$
 $= -2x + 3y$
 $= 3y - 2x$
- 23 $15(x - 2) + 4(3x - 3)$
 $= 15x - 30 + 12x - 12$
 $= 27x - 42$
- 25 $3(x^2 + 5x - 1) - (2x^2 + x - 2)$
 $= 3x^2 + 15x - 3 - 2x^2 - x + 2$
 $= x^2 + 14x - 1$
- 27 **C**
 $a(a + 1) - 3(2a + 1)$
 $= a^2 + a - 6a - 3$
 $= a^2 - 5a - 3$

EXERCISE 1.2

- 1 $P = 2(20 + 12)$
 $= 2 \times 32$
 $= 64$
- 3 $F = 50 \times 0.2$
 $= 10$
- 5 $A = \pi(3.5^2)$
 $= \pi(12.25)$
 $= 38.5$
- 7 (a) $E = 10 \times 1.6^2$
 $= 25.6$
- (b) $c = \sqrt{\frac{E}{m}}$
 $= \sqrt{\frac{13.5}{1.5}}$
 $= 3$
- 9 (a) $s = 5 \times 2.4 + \frac{1}{2} \times 6 \times 2.4^2$
 $= 29.3$
- (b) $a = \frac{2(s - ut)}{t^2}$
 $= \frac{2(50 - 10 \times 2.5)}{2.5^2}$
 $= 8$
- 11 $s = \frac{1}{2}(2.6 + 3.2) \times 2.5$
 $= 7.3$
- 13 (a) $r = \sqrt{\frac{154}{\pi}}$
 $= 7.0$
- (b) $A = r^2 \times \pi$
 $= 1.75^2 \pi$
 $= 9.6$
- 15 (a) $t = 3.8 + (20 - 1) \times 0.2$
 $= 7.6$
- (b) $n = \frac{t - a}{d} + 1$
 $= \frac{25.6 - 5.6}{5} + 1$
 $= 5$
- 17 $t = 64 \times 0.5^5$
 $= 2$
- 19 $A = \pi(5.6^2 - 1.4^2)$
 $= \pi(29.4)$
 $= 92.4$
- 21 $V = \frac{1}{3}\pi \times 3^2 \times 3.5$
 $= 10.5\pi$
 $= 33.0$
- 23 $W = \frac{1}{3} \times 3 \times \pi \times \left(\frac{7}{11}\right)^2 \times \frac{11}{14}$
 $= 0.32\pi$
 $= 1.0$
- 25 $A = 1000\left(1 + \frac{10}{100}\right)^2$
 $= 1210$

EXERCISE 1.3

- 1 $(x + 5)(x + 1)$
 $= x^2 + 5x + x + 5$
 $= x^2 + 6x + 5$
- 3 $(a - 3)(a + 4)$
 $= a^2 - 3a + 4a - 12$
 $= a^2 + a - 12$
- 5 $(y + 7)^2$
 $= (y + 7)(y + 7)$
 $= y^2 + 7y + 7y + 49$
 $= y^2 + 14y + 49$
- 7 $(3x - 4)(x - 2)$
 $= 3x^2 - 6x - 4x + 8$
 $= 3x^2 - 10x + 8$
- 9 $(3x + 2)(3x + 2)$
 $= 9x^2 + 6x + 6x + 4$
 $= 9x^2 + 12x + 4$
- 11 $(3x + 2)(2x + 3)$
 $= 6x^2 + 4x + 9x + 6$
 $= 6x^2 + 13x + 6$

$$\begin{aligned}
 13 \quad & (3x+4)^2 \\
 & = (3x+4)(3x+4) \\
 & = 9x^2 + 12x + 12x + 16 \\
 & = 9x^2 + 24x + 16
 \end{aligned}$$

$$\begin{aligned}
 15 \quad & (3x^2 - 5x + 2)(2x - 4) \\
 & = 6x^3 - 12x^2 - 10x + 20x + 4x - 8 \\
 & = 6x^3 - 22x^2 + 24x - 8
 \end{aligned}$$

$$\begin{aligned}
 17 \quad & (x-1)(x-1)(x-2) \\
 & = (x-1)(x^2 - 3x + 2) \\
 & = x^3 - 3x^2 + 2x - x^2 + 3x - 2 \\
 & = x^3 - 4x^2 + 5x - 2
 \end{aligned}$$

$$\begin{aligned}
 19 \quad & (x^2 + 5)(x^2 - 2x - 3) \\
 & = x^4 - 2x^3 - 3x^2 + 5x^2 - 10x - 15 \\
 & = x^4 - 2x^3 + 2x^2 - 10x - 15
 \end{aligned}$$

$$\begin{aligned}
 21 \quad & (x^2 - y^2)^3 \\
 & = (x^2 - y^2)(x^2 - y^2)(x^2 - y^2) \\
 & = (x^2 - y^2)(x^4 - x^2y^2 - x^2y^2 + y^4) \\
 & = (x^2 - y^2)(x^4 - 2x^2y^2 + y^4) \\
 & = x^6 - 2x^4y^2 + x^2y^4 - x^4y^2 + 2x^2y^4 - y^6 \\
 & = x^6 - 3x^4y^2 + 3x^2y^4 - y^6
 \end{aligned}$$

$$\begin{aligned}
 23 \quad & \text{(a) incorrect} \\
 & (x+2y)^2 \\
 & = (x+2y)(x+2y) \\
 & = x^2 + 2xy + 2xy + 4y^2 \\
 & = x^2 + 4xy + 4y^2
 \end{aligned}$$

$$\begin{aligned}
 & \text{(b) correct} \\
 & (2y-x)^2 \\
 & = (2y-x)(2y-x) \\
 & = 4y^2 - 2xy - 2xy + x^2 \\
 & = 4y^2 - 4xy + x^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) incorrect} \\
 & (2x-y)^2 \\
 & = (2x-y)(2x-y) \\
 & = 4x^2 - 2xy - 2xy + y^2 \\
 & = 4x^2 - 4xy + y^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) correct} \\
 & (x-2y)^2 \\
 & = (x-2y)(x-2y) \\
 & = x^2 - 2xy - 2xy + 4y^2 \\
 & = x^2 - 4xy + 4y^2
 \end{aligned}$$

EXERCISE 1.4

$$\begin{aligned}
 1 \quad & a(x+2) + b(x+2) \\
 & = (a+b)(x+2)
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & p(a+b) + q(a+b) - r(a+b) \\
 & = (p+q-r)(a+b)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & ax + 4a + bx + 4b \\
 & = a(x+4) + b(x+4) \\
 & = (a+b)(x+4)
 \end{aligned}$$

$$\begin{aligned}
 7 \quad & 2xy + 2xz + y + z \\
 & = 2x(y+z) + 1(y+z) \\
 & = (2x+1)(y+z)
 \end{aligned}$$

$$\begin{aligned}
 9 \quad & 10y - 25y^2 + 4x - 10xy \\
 & = 5y(2-5y) + 2x(2-5y) \\
 & = (2x+5y)(2-5y)
 \end{aligned}$$

$$\begin{aligned}
 11 \quad & ac - 2bc - 2ad + 4bd \\
 & = c(a-2b) - 2d(a-2b) \\
 & = (c-2d)(a-2b)
 \end{aligned}$$

$$\begin{aligned}
 13 \quad & x^2 - 2xy - xz + 2yz \\
 & = x(x-2y) - z(x-2y) \\
 & = (x-z)(x-2y)
 \end{aligned}$$

$$\begin{aligned}
 15 \quad & 2mn + 2mp + pn^2 + p^2n \\
 & = 2m(n+p) + pn(n+p) \\
 & = (2m+pn)(n+p)
 \end{aligned}$$

$$\begin{aligned}
 17 \quad & p^2q - pq^2 + 5p - 5q \\
 & = pq(p-q) + 5(p-q) \\
 & = (pq+5)(p-q)
 \end{aligned}$$

$$\begin{aligned}
 19 \quad & x^2y + x^2 + y + 1 \\
 & = x^2(y+1) + (y+1) \\
 & = (x^2+1)(y+1)
 \end{aligned}$$

$$\begin{aligned}
 21 \quad & 2x - 6y - xy + 3y^2 \\
 & = 2(x-3y) - y(x-3y) \\
 & = (2-y)(x-3y)
 \end{aligned}$$

$$\begin{aligned}
 23 \quad & 2x^3 - 2x^2 - 2x + 2 \\
 & = 2x^2(x-1) - 2(x-1) \\
 & = (x-1)(2x^2-2) \\
 & = (x-1)2(x^2-1) \\
 & = 2(x-1)(x-1)(x+1) \\
 & = 2(x-1)^2(x+1)
 \end{aligned}$$

(a) incorrect (b) correct
(c) correct (d) incorrect

EXERCISE 1.5

$$\begin{aligned}
 1 \quad & m^2 - 1 \\
 & = m^2 - 1^2 \\
 & = (m-1)(m+1)
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & 64 - m^2 \\
 & = 8^2 - m^2 \\
 & = (8-m)(8+m)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & x^2 - 0.36 \\
 & = x^2 - 0.6^2 \\
 & = (x-0.6)(x+0.6)
 \end{aligned}$$

$$\begin{aligned}
 7 \quad & 9x^2 - 4y^2 \\
 & = (3x)^2 - (2y)^2 \\
 & = (3x-2y)(3x+2y)
 \end{aligned}$$

$$\begin{aligned}
 9 \quad & x^2 - y^2z^2 \\
 & = x^2 - (yz)^2 \\
 & = (x-yz)(x+yz)
 \end{aligned}$$

$$\begin{aligned}
 11 \quad & p^2 - \frac{1}{4} \\
 & = p^2 - \left(\frac{1}{2}\right)^2 \\
 & = \left(p - \frac{1}{2}\right)\left(p + \frac{1}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 13 \quad & (a+2)^2 - 4 \\
 & = (a+2)^2 - 2^2 \\
 & = (a+2-2)(a+2+2) \\
 & = a(a+4)
 \end{aligned}$$

$$\begin{aligned}
 15 \quad & 99^2 - 1 \\
 & = (99-1)(99+1) \\
 & = 98 \times 100 \\
 & = 9800
 \end{aligned}$$

$$\begin{aligned}
 17 \quad & a^3b - ab^3 \\
 & = ab(a^2 - b^2) \\
 & = ab(a-b)(a+b)
 \end{aligned}$$

$$\begin{aligned}
 19 \quad & 3x^2y - 27y \\
 & = 3y(x^2 - 9) \\
 & = 3y(x-3)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 21 \quad & a^2 - (a-b)^2 \\
 & = (a-(a-b))(a+(a-b)) \\
 & = b(2a-b)
 \end{aligned}$$

$$\begin{aligned}
 23 \quad & x^3 + 3x^2 - 4x - 12 \\
 & = x^2(x+3) - 4(x+3) \\
 & = (x+3)(x^2-4) \\
 & = (x-2)(x+2)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 25 \quad & a^2x - x \\
 & = x(a^2 - 1) \\
 & = x(a-1)(a+1)
 \end{aligned}$$

$$\begin{aligned}
 27 \quad & (1+h)^2 - 1 \\
 & = ((1+h)-1)((1+h)+1) \\
 & = h(2+h)
 \end{aligned}$$

- 29 D
 $(p+2)^2 - (p-2)^2$
 $= ((p+2) - (p-2))((p+2) + (p-2))$
 $= (p+2-p+2)(p+2+p-2)$
 $= 4 \times 2p$
 $= 8p$
- 31 $y^3 - 125$
 $= y^3 - 5^3$
 $= (y-5)(y^2 + 5y + 25)$
- 33 $8p^3 + 27$
 $= (2p)^3 + 3^3$
 $= (2p+3)(4p^2 - 6p + 9)$
- 35 $(x+5)^3 + (x-2)^3$
 $= (x+5+x-2)[(x+5)^2 - (x+5)(x-2) + (x-2)^2]$
 $= (2x+3)(x^2 + 10x + 25 - (x^2 + 3x - 10) + x^2 - 4x + 4)$
 $= (2x+3)(x^2 + 3x + 39)$
- 37 $b^6 - a^6$
 $= (b-a)(b+a)(b^4 + 2a^2b^2 + a^4 - a^2b^2)$
 $= (b-a)(b+a)((b^2 + a^2)^2 - a^2b^2)$
 $= (b-a)(b+a)(b^2 + a^2 + ab)(b^2 + a^2 - ab)$
 $= (b-a)(b+a)(b^2 + ba + a^2)(b^2 - ba + a^2)$
- 39 $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
 $= \frac{4}{3}\pi(R^3 - r^3)$
 $= \frac{4}{3}\pi(R-r)(R^2 + Rr + r^2)$
- 41 $x^6 + y^6$
 $= (x^2)^3 + (y^2)^3$
 $= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$
- 43 $a^3m^3 + a^3n^3 - b^3n^3 - b^3m^3$
 $= a^3(m^3 + n^3) - b^3(n^3 + m^3)$
 $= (m^3 + n^3)(a^3 - b^3)$
 $= (a-b)(m+n)(a^2 + ab + b^2)(m^2 - mn + n^2)$
- 45 $(x+h)^3 - x^3$
 $= (x+h-x)((x+h)^2 + x(x+h) + x^2)$
 $= h(x^2 + 2xh + h^2 + x^2 + xh + x^2)$
 $= h(3x^2 + 3xh + h^2)$
- 47 $(a+b)^3 - (a-b)^3$
 $= (a+b-(a-b))((a+b)^2 + (a+b)(a-b) + (a-b)^2)$
 $= 2b(a^2 + 2ab + b^2 + a^2 - b^2 + a^2 - 2ab + b^2)$
 $= 2b(3a^2 + b^2)$
- 49 $8 - (2-x)^3$
 $= 2^3 - (2-x)^3$
 $= (2 - (2-x))(2^2 + 2(2-x) + (2-x)^2)$
 $= x(4 + 4 - 2x + 4 - 4x + x^2)$
 $= x(12 - 6x + x^2)$
- 51 $2(x-y)^3 + 54$
 $= 2((x-y)^3 + 27)$
 $= 2((x-y)^3 + 3^3)$
 $= 2(x-y+3)((x-y)^2 - 3(x-y) + 3^2)$
 $= 2(x-y+3)(x^2 - 2xy + y^2 - 3x + 3y + 9)$
- 53 D
 $(2x+1)^3 + (2x-1)^3$
 $= (2x+1+2x-1)((2x+1)^2 - (2x+1)(2x-1) + (2x-1)^2)$
 $= 4x(4x^2 + 4x + 1 - (4x^2 - 1) + 4x^2 - 4x + 1)$
 $= 4x(4x^2 + 3)$

EXERCISE 1.6

- 1 $x^2 + 4x + 3$
 $m + n = 4$
 $mn = 3$
 $\therefore m = 3, n = 1$
 $(x+1)(x+3)$
- 3 $x^2 + 11x + 24$
 $m + n = 11$
 $mn = 24$
 $\therefore m = 8, n = 3$
 $(x+3)(x+8)$
- 5 $m^2 + 9m + 20$
 $m + n = 9$
 $mn = 20$
 $\therefore m = 5, n = 4$
 $(m+5)(m+4)$
- 7 $x^2 + 8x + 12$
 $m + n = 8$
 $mn = 12$
 $\therefore m = 6, n = 2$
 $(x+6)(x+2)$
- 9 $x^2 - 13x + 12$
 $m + n = -13$
 $mn = 12$
 $\therefore m = -12, n = -1$
 $(x-12)(x-1)$
- 11 $p^2 + 2p - 15$
 $m + n = 2$
 $mn = -15$
 $\therefore m = -3, n = 5$
 $(p+5)(p-3)$
- 13 $p^2 - 2p - 15$
 $m + n = -2$
 $mn = -15$
 $\therefore m = -5, n = 3$
 $(p-5)(p+3)$
- 15 $x^2 - 2x - 35$
 $m + n = -2$
 $mn = -35$
 $\therefore m = -7, n = 5$
 $(x-7)(x+5)$
- 17 $x^2 + 17x + 72$
 $m + n = 17$
 $mn = 72$
 $\therefore m = 9, n = 8$
 $(x+9)(x+8)$
- 19 $x^2 - 7x + 6$
 $m + n = -7$
 $mn = 6$
 $\therefore m = -6, n = -1$
 $(x-6)(x-1)$
- 21 $x^2 + 6x - 72$
 $m + n = 6$
 $mn = -72$
 $\therefore m = 12, n = -6$
 $(x+12)(x-6)$
- 23 $a^2 + 13a + 30$
 $m + n = 13$
 $mn = 30$
 $\therefore m = 3, n = 10$
 $(a+10)(a+3)$

$$25 \quad x^2 - 19x - 42$$

$$m + n = -19$$

$$mn = -42$$

$$\therefore m = -21, n = 2$$

$$(x - 21)(x + 2)$$

$$27 \quad D$$

$$x^2 - 11x - 42$$

$$m + n = -11$$

$$mn = -42$$

$$\therefore m = -14, n = 3$$

$$(x - 14)(x + 3)$$

EXERCISE 1.7

$$1 \quad \begin{array}{r} 2x \\ x \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} 1 \\ 1 \end{array}$$

$$(2x + 1)(x + 1)$$

$$3 \quad \begin{array}{r} 2x \\ x \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} 3 \quad 2 \quad 6 \quad 1 \\ 2 \quad 3 \quad 1 \quad 6 \end{array}$$

$$(2x + 3)(x + 2)$$

$$5 \quad \begin{array}{r} 3a \\ a \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} 1 \quad 2 \quad -1 \quad -2 \\ 2 \quad 1 \quad -2 \quad -1 \end{array}$$

$$(3a - 2)(a - 1)$$

$$7 \quad \begin{array}{r} 13c \\ c \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} -6 \quad 6 \quad -1 \quad 1 \quad -2 \quad 2 \quad -3 \quad 3 \\ 1 \quad -1 \quad 6 \quad -6 \quad 3 \quad -3 \quad 2 \quad -2 \end{array}$$

$$(13c + 6)(c - 1)$$

$$9 \quad \begin{array}{r} 3x \\ x \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} -10 \quad 10 \quad -1 \quad 1 \quad -2 \quad 2 \quad -5 \quad 5 \\ -1 \quad 1 \quad -10 \quad 10 \quad -5 \quad 5 \quad -2 \quad 2 \end{array}$$

$$(3x - 2)(x - 5)$$

$$11 \quad \begin{array}{r} 3x \\ x \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} -4 \quad 4 \quad -1 \quad 1 \quad -2 \quad 2 \\ 1 \quad -1 \quad 4 \quad -4 \quad 2 \quad -2 \end{array}$$

$$(3x + 1)(x - 4)$$

$$13 \quad \begin{array}{r} 2x \\ x \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} 2 \quad -2 \quad 1 \quad -1 \\ -1 \quad 1 \quad -2 \quad 2 \end{array}$$

$$(2x - 1)(x + 2)$$

$$15 \quad \begin{array}{r} 3x \\ 3x \\ 9x \\ x \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} -2 \quad 2 \quad -4 \quad 4 \quad -1 \quad 1 \\ -2 \quad 2 \quad -1 \quad 1 \quad -4 \quad 4 \\ -2 \quad 2 \quad -4 \quad 4 \quad -1 \quad 1 \\ -2 \quad 2 \quad -1 \quad 1 \quad -4 \quad 4 \end{array}$$

$$(3x - 2)(3x - 2)$$

$$= (3x - 2)^2$$

$$17 \quad \begin{array}{r} 3x \\ 2x \\ 6x \\ x \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} -14 \quad 14 \quad -1 \quad 1 \quad -7 \quad 7 \quad -2 \quad 2 \\ -1 \quad 1 \quad -14 \quad 14 \quad -2 \quad 2 \quad -7 \quad 7 \\ 14 \quad -14 \quad 1 \quad -1 \quad 7 \quad -7 \quad 2 \quad -2 \\ 1 \quad -1 \quad 14 \quad -14 \quad 2 \quad -2 \quad 7 \quad -7 \end{array}$$

$$(6x - 1)(x - 14)$$

$$19 \quad \begin{array}{r} 12y \\ y \\ 4y \\ 3y \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} 6 \quad -6 \quad 1 \quad -1 \quad 2 \quad -2 \quad 3 \quad -3 \\ -1 \quad 1 \quad -6 \quad 6 \quad -3 \quad 3 \quad -2 \quad 2 \\ 6 \quad -6 \quad 1 \quad -1 \quad 2 \quad -2 \quad 3 \quad -3 \\ -1 \quad 1 \quad -6 \quad 6 \quad -3 \quad 3 \quad -2 \quad 2 \end{array}$$

$$\begin{array}{r} 6y \\ 2y \end{array} \begin{array}{r} \times \\ \times \end{array} \begin{array}{r} 6 \quad -6 \quad 1 \quad -1 \quad 2 \quad -2 \quad 3 \quad -3 \\ -1 \quad 1 \quad -6 \quad 6 \quad -3 \quad 3 \quad -2 \quad 2 \end{array}$$

$$(6y - 2)(2y + 3)$$

$$= 2(3y - 1)(2y + 3)$$

$$21 \quad \begin{array}{r} 6x \\ x \\ 3x \\ 2x \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} 28 \quad -28 \quad 1 \quad -1 \quad 14 \quad -14 \quad 2 \quad -2 \quad 7 \quad -7 \quad 4 \quad -4 \\ 1 \quad -1 \quad 28 \quad -28 \quad 2 \quad -2 \quad 14 \quad -14 \quad 4 \quad -4 \quad 7 \quad -7 \\ 28 \quad -28 \quad 1 \quad -1 \quad 14 \quad -14 \quad 2 \quad -2 \quad 7 \quad -7 \quad 4 \quad -4 \\ 1 \quad -1 \quad 28 \quad -28 \quad 2 \quad -2 \quad 14 \quad -14 \quad 4 \quad -4 \quad 7 \quad -7 \end{array}$$

$$(3x - 4)(2x - 7)$$

$$23 \quad \begin{array}{r} 6x \\ x \\ 2x \\ 3x \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} 14 \quad -14 \quad 1 \quad -1 \quad 7 \quad -7 \quad 2 \quad -2 \\ 1 \quad -1 \quad 14 \quad -14 \quad 2 \quad -2 \quad 7 \quad -7 \\ 14 \quad -14 \quad 1 \quad -1 \quad 7 \quad -7 \quad 2 \quad -2 \\ 1 \quad -1 \quad 14 \quad -14 \quad 2 \quad -2 \quad 7 \quad -7 \end{array}$$

$$(6x - 14)(x - 1)$$

$$= 2(3x - 7)(x - 1)$$

$$25 \quad \begin{array}{r} 6p \\ p \\ 3p \\ 2p \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} 21 \quad 1 \quad 7 \quad 3 \\ 1 \quad 21 \quad 3 \quad 7 \\ 21 \quad 1 \quad 7 \quad 3 \\ 1 \quad 21 \quad 3 \quad 7 \end{array}$$

$$(p + 3)(6p + 7)$$

$$27 \quad \begin{array}{r} 12y \\ y \\ 6y \\ 2y \\ 3y \\ 4y \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} -5 \quad 5 \quad -1 \quad 1 \\ 1 \quad -1 \quad 5 \quad -5 \\ -5 \quad 5 \quad -1 \quad 1 \\ 1 \quad -1 \quad 5 \quad -5 \\ -5 \quad 5 \quad -1 \quad 1 \\ 1 \quad -1 \quad 5 \quad -5 \end{array}$$

$$(6y - 1)(2y + 5)$$

$$29 \quad \begin{array}{r} 15x \\ x \\ 3x \\ 5x \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} 6 \quad -6 \quad 1 \quad -1 \quad 3 \quad -3 \quad 2 \quad -2 \\ 1 \quad -1 \quad 6 \quad -6 \quad 2 \quad -2 \quad 3 \quad -3 \\ 6 \quad -6 \quad 1 \quad -1 \quad 3 \quad -3 \quad 2 \quad -2 \\ 1 \quad -1 \quad 6 \quad -6 \quad 2 \quad -2 \quad 3 \quad -3 \end{array}$$

$$(5x - 3)(3x - 2)$$

$$31 \quad \begin{array}{r} 9x \\ x \\ 3x \\ 3x \end{array} \begin{array}{r} \times \\ \times \\ \times \\ \times \end{array} \begin{array}{r} -10 \quad 10 \quad -1 \quad 1 \quad -2 \quad 2 \quad -5 \quad 5 \\ 1 \quad -1 \quad 10 \quad -10 \quad 5 \quad -5 \quad 2 \quad -2 \\ -10 \quad 10 \quad -1 \quad 1 \quad -2 \quad 2 \quad -5 \quad 5 \\ 1 \quad -1 \quad 10 \quad -10 \quad 5 \quad -5 \quad 2 \quad -2 \end{array}$$

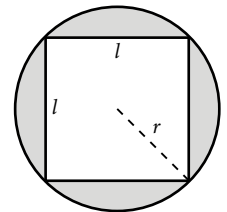
$$(3x + 5)(3x - 2)$$

33 C

$$\begin{array}{r}
 8x \quad \times \quad -3 \quad 3 \quad -9 \quad 9 \quad -1 \quad 1 \\
 x \quad \times \quad 3 \quad -3 \quad 1 \quad -1 \quad 9 \quad -9 \\
 \hline
 4x \quad \times \quad -3 \quad 3 \quad -9 \quad 9 \quad -1 \quad 1 \\
 2x \quad \times \quad 3 \quad -3 \quad 1 \quad -1 \quad 9 \quad -9 \\
 \hline
 (4x+3)(2x-3)
 \end{array}$$

EXERCISE 1.8

- 1 $x^2 - 3x$
 $= x(x - 3)$
- 3 $x^2 - 9$
 Difference of perfect squares
 $= (x - 3)(x + 3)$
- 5 $3x^2y - 12y^3$
 $= 3y(x^2 - 4y^2)$
 $= 3y(x - 2y)(x + 2y)$
- 7 $1 - (b + c)^2$
 Difference of perfect squares
 $= (1 - (b + c))(1 + (b + c))$
 $= (1 - b - c)(1 + b + c)$
- 9 $(a + b)^2 - b^2$
 $= ((a + b) - b)((a + b) + b)$
 $= a(a + 2b)$
- 11 $a^2 - a - 42$
 $m + n = -1$
 $mn = -42$
 $\therefore m = -7, n = 6$
 $(a - 7)(a + 6)$
- 13 $2x^3 + 14x^2 - 16x$
 $= 2x(x^2 + 7x - 8)$
 $m + n = 7$
 $mn = -8$
 $m = 8, n = -1$
 $2x(x + 8)(x - 1)$
- 15 $(x + 2y)^2 - 4$
 $= (x + 2y)^2 - 2^2$
 $= (x + 2y - 2)(x + 2y + 2)$
- 17 $x^2 - 36y^2$
 $= x^2 - (6y)^2$
 $= (x - 6y)(x + 6y)$
- 19 $4x^2 - 28x - 480$
 $= 4(x^2 - 7x - 120)$
 $m + n = -7$
 $mn = -120$
 $m = -15, n = 8$
 $4(x - 15)(x + 8)$
- 21 $6y^3 + 3y^2 - 3y$
 $= 3y(2y^2 + y - 1)$
 $2y \quad \times \quad -1 \quad 1$
 $y \quad \times \quad 1 \quad -1$
 $= 3y(2y - 1)(y + 1)$
- 23 $15a^2 - 60$
 $= 15(a^2 - 4)$
 $= 15(a^2 - 2^2)$
 $= 15(a - 2)(a + 2)$
- 25 $5a^2x - 125x$
 $= 5x(a^2 - 25)$
 $= 5x(a^2 - 5^2)$
 $= 5x(a - 5)(a + 5)$
- 27 $5t^3 + 5t^2 - 360t$
 $= 5t(t^2 + t - 72)$
 $= 5t(t + 9)(t - 8)$
- 29 $x^2(x + 3) - 4(x + 3)$
 $= (x + 3)(x^2 - 4)$
 $= (x + 3)(x^2 - 2^2)$
 $= (x - 2)(x + 2)(x + 3)$
- 31 D
 $4 - (x + 1)^2$
 $= 2^2 - (x + 1)^2$
 $= (2 - (x + 1))(2 + (x + 1))$
 $= (1 - x)(3 + x)$
- 33 $A = \pi r^2$ - area of square.
 Need to find side length (l)
 of square. Let h equal the
 diagonal of the square:
 $h = 2r$
 Use Pythagoras' theorem on the right-angled
 triangle formed by half of the square:
 $h^2 = l^2 + l^2$
 $h^2 = 2l^2$
 Substitute $h = 2r$ and solve for l :
 $(2r)^2 = 2l^2$
 $4r^2 = 2l^2$
 $2r^2 = l^2$
 $l = \sqrt{2}r$
 Area of square $= (\sqrt{2}r)^2$
 $= 2r^2$
 Therefore, area of shaded section:
 $= \pi r^2 - 2r^2$
 $= (\pi - 2)r^2$
- 35 Area of shaded section equals area of rectangle
 minus two half circles (i.e. one full circle).
 Length of rectangle $= 4r$, width $= 2r$:
 $A = 4r \times 2r - \pi r^2$
 $= 8r^2 - \pi r^2$
 $= (8 - \pi)r^2$



37 Area of shaded sections equals area of large circle minus four times area of a small circle:

$$A = \pi R^2 - 4\pi r^2$$

$$= \pi(R^2 - 4r^2)$$

Difference of perfect squares:

$$= \pi(R^2 - (2r)^2)$$

$$= \pi(R - 2r)(R + 2r)$$

39 Area = area of vertical rectangle plus area of horizontal rectangle:

$$A = a \times (b + a) + ca$$

$$= ab + a^2 + ac$$

$$= a(a + b + c)$$

EXERCISE 1.9

1 $\frac{8a-4b}{4}$

$$= \frac{4(2a-b)}{4}$$

$$= 2a - b$$

3 $\frac{14x-7y}{2x-y}$

$$= \frac{7(2x-y)}{2x-y}$$

$$= 7$$

5 $\frac{12ab-6b^2}{9ab}$

$$= \frac{6b(2a-b)}{9ab}$$

$$= \frac{2(2a-b)}{3a}$$

7 $\frac{3a-5b}{3a^2-5ab}$

$$= \frac{3a-5b}{a(3a-5b)}$$

$$= \frac{1}{a}$$

9 $\frac{mn-n^2}{n}$

$$= \frac{n(m-n)}{n}$$

$$= m - n$$

11 $\frac{x^2+xy}{2x}$

$$= \frac{x(x+y)}{2x}$$

$$= \frac{x+y}{2}$$

13 $\frac{x^2-y^2}{(x+y)^2}$

$$= \frac{(x-y)(x+y)}{(x+y)(x+y)}$$

$$= \frac{x-y}{x+y}$$

15 $\frac{x^2-9}{x^2+3x}$

$$= \frac{(x-3)(x+3)}{x(x+3)}$$

$$= \frac{x-3}{x}$$

17 $\frac{4x^2-4xy}{x^2-y^2}$

$$= \frac{4x(x-y)}{(x-y)(x+y)}$$

$$= \frac{4x}{x+y}$$

19 $\frac{a^2+ab}{ab+b^2}$

$$= \frac{a(a+b)}{b(a+b)}$$

$$= \frac{a}{b}$$

21 $\frac{x^2-1}{x^2-5x+4}$

$$= \frac{(x-1)(x+1)}{(x-4)(x-1)}$$

$$= \frac{x+1}{x-4}$$

23 $\frac{x^2+3x+2}{x^2-4}$

$$= \frac{(x+2)(x+1)}{(x-2)(x+2)}$$

$$= \frac{x+1}{x-2}$$

25 $\frac{x^2+4x+4}{x^2-3x-10}$

$$= \frac{(x+2)(x+2)}{(x-5)(x+2)}$$

$$= \frac{x+2}{x-5}$$

27 $\frac{12a+9}{15} \times \frac{5}{4a+3}$

$$= \frac{3(4a+3)}{5 \times 3} \times \frac{5}{4a+3}$$

$$= 1$$

29 C

$$\frac{m^2+m-2}{m^2-m}$$

$$= \frac{(m-1)(m+2)}{m(m-1)}$$

$$= \frac{m+2}{m}$$

31 $\frac{15x^2-5xy}{10xy} \div \frac{3x-y}{2y}$

$$= \frac{15x^2-5xy}{10xy} \times \frac{2y}{3x-y}$$

$$= \frac{5x(3x-y)}{10xy} \times \frac{2y}{3x-y}$$

$$= 1$$

33 $\frac{x^2-2x-3}{x^2-4x-5} \times \frac{x^2-25}{(x-3)(x+5)}$

$$= \frac{(x-3)(x+1)}{(x-5)(x+1)} \times \frac{(x-5)(x+5)}{(x-3)(x+5)}$$

$$= 1$$

35 (a) incorrect (b) incorrect
(c) incorrect (d) correct

$$\frac{m^2-9}{m^2-m-12} \div \frac{m^2-3m}{m^2-9m+20}$$

$$= \frac{m^2-9}{m^2-m-12} \times \frac{m^2-9m+20}{m^2-3m}$$

$$= \frac{(m-3)(m+3)}{(m-4)(m+3)} \times \frac{(m-5)(m-4)}{m(m-3)}$$

$$= \frac{m-5}{m}$$

37 $\frac{8x^2+4x+2}{8x^3-1}$

$$= \frac{2(4x^2+2x+1)}{(2x)^3-1^3}$$

$$= \frac{2(4x^2+2x+1)}{(2x-1)(4x^2+2x+1)}$$

$$= \frac{2}{2x-1}$$

39 $\frac{(x+h)^3-x^3}{h}$

$$= \frac{(x+h-x)[(x+h)^2+x(x+h)+x^2]}{h}$$

$$= \frac{h(x^2+2xh+h^2+x^2+xh+x^2)}{h}$$

$$= \frac{h(3x^2+3xh+h^2)}{h}$$

$$= 3x^2+3xh+h^2$$

EXERCISE 1.10

1 $\frac{x}{5} - \frac{x}{6}$

$$= \frac{6x}{30} - \frac{5x}{30}$$

$$= \frac{x}{30}$$

3 $\frac{a}{3} + \frac{4a}{5} - \frac{a}{6}$

$$= \frac{10a}{30} + \frac{24a}{30} - \frac{5a}{30}$$

$$= \frac{29a}{30}$$

5 $\frac{a+2}{5} - \frac{a-1}{3}$

$$= \frac{3(a+2)}{15} - \frac{5(a-1)}{15}$$

$$= \frac{3a+6-5a+5}{15}$$

$$= \frac{11-2a}{15}$$

$$\begin{aligned}
 7 \quad & \frac{3x+2}{6} - \frac{x+1}{4} \\
 &= \frac{2(3x+2)}{12} - \frac{3(x+1)}{12} \\
 &= \frac{6x+4-3x-3}{12} \\
 &= \frac{3x+1}{12}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad & \frac{x}{2} + \frac{y}{4} - \frac{x+y}{3} \\
 &= \frac{6x+3y-4(x+y)}{12} \\
 &= \frac{6x+3y-4x-4y}{12} \\
 &= \frac{2x-y}{12}
 \end{aligned}$$

$$\begin{aligned}
 11 \quad & \frac{3(a+b)}{4} - \frac{a-b}{6} \\
 &= \frac{3 \times 3(a+b) - 2(a-b)}{12} \\
 &= \frac{9a+9b-2a+2b}{12} \\
 &= \frac{7a+11b}{12}
 \end{aligned}$$

$$\begin{aligned}
 13 \quad & \frac{3}{a} + \frac{1}{a^2} \\
 &= \frac{3}{a} \times \frac{a}{a} + \frac{1}{a^2} \\
 &= \frac{3a}{a^2} + \frac{1}{a^2} \\
 &= \frac{3a+1}{a^2}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad & \frac{m}{n} - \frac{n}{m} \\
 &= \frac{m}{n} \times \frac{m}{m} - \frac{n}{m} \times \frac{n}{n} \\
 &= \frac{m^2}{mn} - \frac{n^2}{mn} \\
 &= \frac{m^2-n^2}{mn}
 \end{aligned}$$

$$\begin{aligned}
 17 \quad & \frac{5}{a^2b} - \frac{2}{ab^2} \\
 &= \frac{5}{a^2b} \times \frac{b}{b} - \frac{2}{ab^2} \times \frac{a}{a} \\
 &= \frac{5b}{a^2b^2} - \frac{2a}{a^2b^2} \\
 &= \frac{5b-2a}{a^2b^2}
 \end{aligned}$$

$$\begin{aligned}
 19 \quad & \frac{1}{x+1} + \frac{2}{3} \\
 &= \frac{1}{x+1} \times \frac{3}{3} + \frac{2}{3} \times \frac{x+1}{x+1} \\
 &= \frac{3}{3(x+1)} + \frac{2x+2}{3(x+1)} \\
 &= \frac{2x+5}{3(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 21 \quad & \text{B} \\
 & \frac{1}{ab} + \frac{a}{bc} \\
 &= \frac{1}{ab} \times \frac{c}{c} + \frac{a}{bc} \times \frac{a}{a} \\
 &= \frac{c}{abc} + \frac{a^2}{abc} \\
 &= \frac{a^2+c}{abc}
 \end{aligned}$$

$$23 \quad \text{LCM} = x(x-2)$$

$$\begin{aligned}
 25 \quad & x^2 - 4x \\
 &= x(x-4) \\
 & \text{LCM} = x(x-4)
 \end{aligned}$$

$$\begin{aligned}
 27 \quad & x^2 + 4x + 4 \\
 &= (x+2)(x+2) \\
 & \text{LCM} = (x+2)^2
 \end{aligned}$$

$$\begin{aligned}
 29 \quad & (x^2 - y^2) \\
 &= (x-y)(x+y) \\
 & x^2 + xy \\
 &= x(x+y) \\
 & xy - y^2 \\
 &= y(x-y) \\
 & \text{LCM} = xy(x-y)(x+y)
 \end{aligned}$$

$$\begin{aligned}
 31 \quad & \frac{1}{a-b} + \frac{1}{a+b} \\
 &= \frac{1}{a-b} \times \frac{a+b}{a+b} + \frac{1}{a+b} \times \frac{a-b}{a-b} \\
 &= \frac{a+b+a-b}{(a-b)(a+b)} \\
 &= \frac{2a}{(a-b)(a+b)}
 \end{aligned}$$

$$\begin{aligned}
 33 \quad & \frac{x}{x-y} + \frac{y}{x-y} \\
 &= \frac{x+y}{x-y}
 \end{aligned}$$

$$\begin{aligned}
 35 \quad & \frac{3a-b}{a^2-b^2} + \frac{1}{a-b} \\
 &= \frac{3a-b}{(a-b)(a+b)} + \frac{1}{a-b} \\
 &= \frac{3a-b}{(a-b)(a+b)} + \frac{1}{a-b} \times \frac{a+b}{a+b} \\
 &= \frac{3a-b+a+b}{(a-b)(a+b)} \\
 &= \frac{4a}{(a-b)(a+b)}
 \end{aligned}$$

$$\begin{aligned}
 37 \quad & \frac{x}{x^2-y^2} - \frac{y}{x^2-y^2} \\
 &= \frac{x-y}{x^2-y^2} \\
 &= \frac{x-y}{(x-y)(x+y)} \\
 &= \frac{1}{x+y}
 \end{aligned}$$

$$\begin{aligned}
 39 \quad & \frac{1}{x^2-4x+3} - \frac{1}{x^2-1} \\
 &= \frac{1}{(x-3)(x-1)} - \frac{1}{(x-1)(x+1)} \\
 &= \frac{1}{(x-3)(x-1)} \times \frac{x+1}{x+1} - \frac{1}{(x-1)(x+1)} \times \frac{x-3}{x-3} \\
 &= \frac{x+1}{(x-1)(x-3)(x+1)} - \frac{x-3}{(x-1)(x-3)(x+1)} \\
 &= \frac{x+1-x+3}{(x-1)(x-3)(x+1)} \\
 &= \frac{4}{(x-1)(x-3)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 41 \quad & \frac{1}{x+y} - \frac{1}{x-y} \\
 &= \frac{1}{x+y} \times \frac{x-y}{x-y} - \frac{1}{x-y} \times \frac{x+y}{x+y} \\
 &= \frac{x-y-(x+y)}{(x+y)(x-y)} \\
 &= \frac{-2y}{(x+y)(x-y)}
 \end{aligned}$$

$$\begin{aligned}
 43 \quad & \frac{1}{x-5} - \frac{1}{x+5} + \frac{x+10}{x^2-25} \\
 &= \frac{1}{x-5} - \frac{1}{x+5} + \frac{x+10}{(x-5)(x+5)} \\
 &= \frac{1}{x-5} \times \frac{x+5}{x+5} - \frac{1}{x+5} \times \frac{x-5}{x-5} + \frac{x+10}{(x-5)(x+5)} \\
 &= \frac{x+5-(x-5)+x+10}{(x-5)(x+5)} \\
 &= \frac{x+20}{(x-5)(x+5)}
 \end{aligned}$$

$$\begin{aligned}
 45 \quad & \frac{7a}{3a-4} - \frac{5a}{2a-3} \\
 &= \frac{7a}{3a-4} \times \frac{2a-3}{2a-3} - \frac{5a}{2a-3} \times \frac{3a-4}{3a-4} \\
 &= \frac{7a(2a-3) - 5a(3a-4)}{(3a-4)(2a-3)} \\
 &= \frac{14a^2 - 21a - 15a^2 + 20a}{(3a-4)(2a-3)} \\
 &= \frac{-a^2 - a}{(3a-4)(2a-3)} \\
 &= \frac{-a(a+1)}{(3a-4)(2a-3)}
 \end{aligned}$$

$$\begin{aligned}
 47 \quad & \frac{1}{x+4} + \frac{x}{x^2-16} \\
 &= \frac{1}{x+4} + \frac{x}{(x-4)(x+4)} \\
 &= \frac{1}{x+4} \times \frac{x-4}{x-4} + \frac{x}{(x-4)(x+4)} \\
 &= \frac{2x-4}{(x-4)(x+4)} \\
 &= \frac{2(x-2)}{(x-4)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 49 \quad & \frac{1}{a+3} + \frac{a+4}{a^2+5a+6} \\
 &= \frac{1}{a+3} + \frac{a+4}{(a+3)(a+2)} \\
 &= \frac{1}{a+3} \times \frac{a+2}{a+2} + \frac{a+4}{(a+3)(a+2)} \\
 &= \frac{a+2+a+4}{(a+3)(a+2)} \\
 &= \frac{2a+6}{(a+3)(a+2)} \\
 &= \frac{2(a+3)}{(a+3)(a+2)} \\
 &= \frac{2}{a+2}
 \end{aligned}$$

$$\begin{aligned}
 37 \quad & \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} - \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}} \\
 &= \frac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2}) - (\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} \\
 &= \frac{5 + 2\sqrt{10} + 2 - (5 - 2\sqrt{10} + 2)}{5 - 2} \\
 &= \frac{5 + 2\sqrt{10} + 2 - 5 + 2\sqrt{10} - 2}{3} \\
 &= \frac{4\sqrt{10}}{3}
 \end{aligned}$$

$$\begin{aligned}
 39 \quad & \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} - \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{5} + 2\sqrt{2}} \\
 &= \frac{\sqrt{5}(2\sqrt{5} - 3\sqrt{2}) + \sqrt{2}(2\sqrt{5} - 3\sqrt{2})}{\sqrt{5}(\sqrt{5} + 2\sqrt{2}) - \sqrt{2}(\sqrt{5} + 2\sqrt{2})} \\
 &= \frac{2\sqrt{25} - 3\sqrt{10} + 2\sqrt{10} - 3\sqrt{4}}{\sqrt{25} + 2\sqrt{10} - \sqrt{10} - 2\sqrt{4}} \\
 &= \frac{10 - \sqrt{10} - 3\sqrt{4}}{1 + \sqrt{10}} \\
 &= \frac{4 - \sqrt{10}}{1 + \sqrt{10}} \times \frac{1 - \sqrt{10}}{1 - \sqrt{10}} \\
 &= \frac{4 + 10 - 5\sqrt{10}}{1 - 10} \\
 &= \frac{14 - 5\sqrt{10}}{-9} \\
 &= \frac{5\sqrt{10} - 14}{9}
 \end{aligned}$$

$$\begin{aligned}
 41 \quad & \frac{2\sqrt{3}}{\sqrt{6} - \sqrt{3}} - \frac{\sqrt{3}}{2\sqrt{6} + 3\sqrt{3}} \\
 &= \frac{2\sqrt{3}}{\sqrt{6} - \sqrt{3}} \times \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} + \sqrt{3}} - \frac{\sqrt{3}}{2\sqrt{6} + 3\sqrt{3}} \times \frac{2\sqrt{6} - 3\sqrt{3}}{2\sqrt{6} - 3\sqrt{3}} \\
 &= \frac{2\sqrt{18} + 6}{6 - 3} - \frac{2\sqrt{18} - 9}{24 - 27} \\
 &= \frac{6\sqrt{2} + 6}{3} - \frac{6\sqrt{2} - 9}{-3} \\
 &= 2\sqrt{2} + 2 + 2\sqrt{2} - 3 \\
 &= 4\sqrt{2} - 1
 \end{aligned}$$

$$\begin{aligned}
 43 \quad & \frac{2\sqrt{5}}{\sqrt{10} - \sqrt{15}} - \frac{3\sqrt{7}}{\sqrt{35} - \sqrt{14}} \\
 &= \frac{2\sqrt{5}}{\sqrt{10} - \sqrt{15}} \times \frac{\sqrt{10} + \sqrt{15}}{\sqrt{10} + \sqrt{15}} - \frac{3\sqrt{7}}{\sqrt{35} - \sqrt{14}} \times \frac{\sqrt{35} + \sqrt{14}}{\sqrt{35} + \sqrt{14}} \\
 &= \frac{2\sqrt{5}(\sqrt{10} + \sqrt{15})}{10 - 15} - \frac{3\sqrt{7}(\sqrt{35} + \sqrt{14})}{35 - 14} \\
 &= \frac{2\sqrt{50} + 2\sqrt{75}}{-5} - \frac{3\sqrt{245} + 3\sqrt{98}}{21} \\
 &= \frac{-(10\sqrt{2} + 10\sqrt{3})}{5} - \frac{21\sqrt{5} + 21\sqrt{2}}{21} \\
 &= \frac{-21(10\sqrt{2} + 10\sqrt{3}) - 5(21\sqrt{5} + 21\sqrt{2})}{105} \\
 &= \frac{-210\sqrt{2} - 210\sqrt{3} - 105\sqrt{5} - 105\sqrt{2}}{105} \\
 &= \frac{-2\sqrt{2} - 2\sqrt{3} - \sqrt{5} - \sqrt{2}}{1} \\
 &= -3\sqrt{2} - 2\sqrt{3} - \sqrt{5} \\
 &= -(3\sqrt{2} + 2\sqrt{3} + \sqrt{5})
 \end{aligned}$$

CHAPTER REVIEW 1

$$\begin{aligned}
 1 \quad (a) \quad & (2x - 5)^2 \\
 &= (2x - 5)(2x - 5) \\
 &= 4x^2 - 10x - 10x + 25 \\
 &= 4x^2 - 20x + 25
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & (x + 3)(x - 7) \\
 &= x^2 + 3x - 7x - 21 \\
 &= x^2 - 4x - 21
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & (2y + 1)(3y + 4) \\
 &= 6y^2 + 3y + 8y + 4 \\
 &= 6y^2 + 11y + 4
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad & (5x - 4)(5x + 4) \\
 &= 25x^2 - 20x + 20x - 16 \\
 &= 25x^2 - 16
 \end{aligned}$$

$$\begin{aligned}
 (e) \quad & (2x - y)(x^2 - xy + y^2) \\
 &= 2x(x^2 - xy + y^2) - y(x^2 - xy + y^2) \\
 &= 2x^3 - 2x^2y + 2xy^2 - x^2y + xy^2 - y^3 \\
 &= 2x^3 - 3x^2y + 3xy^2 - y^3
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & \sqrt{(-3)^2 + (4)^2} \\
 &= \sqrt{9 + 16} \\
 &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 5 \quad (a) \quad & \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \\
 &= a^{\frac{1}{3}}\left(a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right) - b^{\frac{1}{3}}\left(a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \\
 &= a^{\frac{3}{3}} + a^{\frac{2}{3}}b^{\frac{1}{3}} + a^{\frac{1}{3}}b^{\frac{2}{3}} - a^{\frac{2}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} - b^{\frac{3}{3}} \\
 &= a - b
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & \left(a^{\frac{1}{2}} - b^{-\frac{1}{2}}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 - 2a^{\frac{1}{2}}b^{-\frac{1}{2}} + \left(b^{-\frac{1}{2}}\right)^2 \\
 &= a - 2a^{\frac{1}{2}}b^{-\frac{1}{2}} + \frac{1}{b} \\
 &= a - \frac{2\sqrt{a}}{\sqrt{b}} + \frac{1}{b}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad (a) \quad & E = \frac{13}{2}(17^2 - 15^2) \\
 &= 416
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & E = \frac{m}{2}(v^2 - u^2) \\
 & \frac{2E}{m} = v^2 - u^2 \\
 & v^2 = \frac{2E}{m} + u^2 \\
 & v = \sqrt{\frac{2E}{m} + u^2} \\
 & v = \sqrt{\frac{2(98)}{4} + 24^2} \\
 & = 25
 \end{aligned}$$

$$\begin{aligned}
 9 \text{ (a)} \quad & \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \\
 & = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} \\
 & = \frac{(\sqrt{5}-\sqrt{2})^2}{5-2} \\
 & = \frac{5-2\sqrt{10}+2}{3} \\
 & = \frac{7-2\sqrt{10}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{3^{\frac{5}{4}} \times 15^{\frac{3}{4}}}{5^{\frac{-3}{4}} \times 45^{\frac{1}{2}}} \\
 & = \frac{3^{\frac{5}{4}} \times (3 \times 5)^{\frac{3}{4}}}{5^{\frac{-3}{4}} \times (9 \times 5)^{\frac{1}{2}}} \\
 & = \frac{3^{\frac{5}{4}} \times 3^{\frac{3}{4}} \times 5^{\frac{3}{4}}}{5^{\frac{-3}{4}} \times 9^{\frac{1}{2}} \times 5^{\frac{1}{2}}} \\
 & = \frac{3^{\frac{8}{4}} \times 5^{\frac{3}{4}}}{5^{\frac{-3}{4}} \times 3 \times 5^{\frac{1}{2}}} \\
 & = \frac{3^2 \times 5^{\frac{3}{4}}}{5^{\frac{-3}{4}} \times 3 \times 5^{\frac{2}{4}}} \\
 & = 3^{(2-1)} \times 5^{\left(\frac{3}{4}-\frac{2}{4}-\frac{-3}{4}\right)} \\
 & = 3 \times 5 = 15
 \end{aligned}$$

11 D

$$\begin{aligned}
 & (\sqrt{5}+2\sqrt{2})(\sqrt{6}-\sqrt{5}) \\
 & = \sqrt{5}(\sqrt{6}-\sqrt{5})+2\sqrt{2}(\sqrt{6}-\sqrt{5}) \\
 & = \sqrt{30}-5+2\sqrt{12}-2\sqrt{10} \\
 & = \sqrt{30}-5+4\sqrt{3}-2\sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 13 \quad & (3\sqrt{2})^4 - 2(3\sqrt{2})^2 + 1 \\
 & = 81 \times 4 - 2 \times 18 + 1 \\
 & = 324 - 36 + 1 \\
 & = 289
 \end{aligned}$$

$$15 \text{ (a)} \quad 1 - \frac{4}{4+1} = \frac{1}{5}$$

$$\text{(b)} \quad \frac{64+36-16}{32 \times 3} = \frac{84}{8 \times 12} = \frac{7}{8}$$

$$\text{(c)} \quad H\pi\left(\frac{2R}{3}\right)^2\left(1-\frac{1}{R} \times \frac{2R}{3}\right) = \frac{4H\pi R^2}{27}$$

$$\text{(d)} \quad \frac{2x^2-2x-x^2-3}{(x-1)^2} = \frac{x^2-2x-3}{(x-1)^2}$$

$$\text{(e)} \quad \frac{1}{36} \times \frac{1225}{36^2-35^2} = \frac{1225}{36 \times 71} = \frac{1225}{2556}$$

$$\begin{aligned}
 \text{(f)} \quad & 1 + \frac{t^2}{t+1} - \frac{1}{t+1} \\
 & = \frac{t+1+t^2-1}{t+1} \\
 & = \frac{t+t^2}{t+1} \\
 & = \frac{t(t+1)}{t+1} \\
 & = t
 \end{aligned}$$

$$\text{(g)} \quad 2\left(\frac{8}{3}+2-\frac{1}{3}-1\right) = \frac{20}{3}$$

$$\begin{aligned}
 \text{(h)} \quad & (x+1)^2 + y^2 + (x-3)^2 + y^2 = 40 \\
 & x^2 + 2x + 1 + 2y^2 + x^2 - 6x + 9 = 40 \\
 & 2x^2 - 4x + 2y^2 + 10 = 40 \\
 & x^2 - 2x + y^2 = 15 \\
 & x^2 - 2x + 1 + y^2 = 16 \\
 & (x-1)^2 + y^2 = 4^2
 \end{aligned}$$

$$17 \text{ (a)} \quad 100000 \times 1.006^{120} - 780 \left(\frac{1.006^{120} - 1}{0.006} \right)$$

$$\text{(b)} \quad M = 2997.75$$

$$\text{(c)} \quad A_{20} = 269903.63$$

$$\begin{aligned}
 \text{(d)} \quad & \$500 \times \frac{1.003^{60} \left(\left(\frac{1.01}{1.003} \right)^{60} - 1 \right) \times 1.003}{1.01 - 1.003} \\
 & = \$500 \times \frac{1.003^{61} \left(\left(\frac{1.01}{1.003} \right)^{60} - 1 \right)}{0.007} \\
 & = \$44404.38 \approx \$44404
 \end{aligned}$$

$$19 \text{ For } n=1, \frac{1}{\sqrt{1+\sqrt{2}}} = \sqrt{2}-\sqrt{1};$$

$$\text{for } n=2, \frac{1}{\sqrt{2+\sqrt{3}}} = \sqrt{3}-\sqrt{2}; \text{ etc.}$$

$$\begin{aligned}
 & \frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \frac{1}{\sqrt{3+\sqrt{4}}} + \dots + \frac{1}{\sqrt{99+\sqrt{100}}} \\
 & = \sqrt{2}-\sqrt{1} + \sqrt{3}-\sqrt{2} + \sqrt{4}-\sqrt{3} + \dots + \sqrt{100}-\sqrt{99} \\
 & = \sqrt{100}-\sqrt{1} \\
 & = 10-1 \\
 & = 9
 \end{aligned}$$