FURTHER EQUATIONS



Contents

- 11:01 Linear equations involving algebraic fractions
- 11:02 Quadratic equations: Solution using factors
- 11:03 Solution by completing the square
- 11:04 The quadratic formula
- Investigation 11:04 How many solutions? 11:05 Choosing the best method
- Fun spot 11:05 What is an Italian referee? 11:06 Problems involving quadratic equations Challenge 11:06 Equations reducible to quadratics
- Investigation 11:06 Simple cubic equations $ax^3 = k$
- Syllabus references (See pages x-xiii for details.)

Number and Algebra

- Selections from *Equations* [Stage 5.3[§]]
- Solve complex linear equations involving algebraic fractions (NSW)
- Solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)
- Solve simple cubic equations (NSW)
- Rearrange literal equations (NSW)
- Solve simultaneous equations, where one equation is non-linear, using algebraic and graphical techniques, including the use of digital technologies (NSW)

Working Mathematically

 Communicating Problem Solving • Understanding Fluency Reasoning

Key ideas

- Linear equations can be solved by applying inverse operations.
- Linear equations with algebraic fractions can be solved using inverse operations and common denominators.
- The Null Factor Law can be used to solve quadratic equations. The Null Factor Law states that if $a \times b = 0$ then at least one of a and b must be zero. Quadratics must be factorised in order for the Null Factor Law to be applied.
- Quadratics can be factorised using the cross method or by recognising features of special quadratics such as perfect squares and difference of two squares.

• When guadratics cannot be factorised then solutions can be found by completing the square.

11.07 Simultaneous equations involving a non-linear

Fun spot 11:09 Did you know that 2 = 1?

Maths terms, Diagnostic test, Assignments

equation

11:08 Solving literal equations (1)

11:09 Solving literal equations (2)

- The discriminant $\Delta = b^2 = 4ac$ determines the number of solutions to a quadratic equation.
- A linear and non-linear equation may have a point or many points of intersection. These possible points of intersection can be found by solving the two equations simultaneously using the substitution method.
- A literal equation has two or more pronumerals. Literal equations or formulas are rearranged to make a given pronumeral the subject using inverse operations and factorisation.

CHAPTER FOCUS

Students will solve linear, quadratic, simultaneous and literal equations in this chapter. They will simplify and solve linear equations involving algebraic fractions, and will substitute values to determine an unknown or to check an answer. Students will factorise and use inverse operations to solve linear equations and literal equations, including those derived from formulas and worded questions. They will also solve a variety of guadratic expressions using different techniques, including completing the square and the quadratic formula. Simultaneous equations involving linear and non-linear equations will be explored and solved using algebraic and graphical techniques and technology.

Outcomes

Equations [Stages 5.2, 5.3[§]]

- MA5.2-1WM selects appropriate notations and conventions to communicate mathematical ideas and solutions
- MA5.2-2WM interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems
- MA5.2-3WM constructs arguments to prove and justify results
- MA5.2-8NA solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques
 - MA5.3-1WM uses and interprets formal definitions and generalisations when explaining solutions and/ or conjectures
- MA5.3-2WM generalises mathematical ideas and techniques to analyse and solve problems efficiently
- MA5.3-3WM uses deductive reasoning in presenting arguments and formal proofs
- MA5.3-7NA solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations

11:01 Content statements

Solve linear equations involving simple algebraic fractions (ACMNA240) [Stage 5.2]

• solve linear equations involving one or more simple algebraic fractions,

$$\frac{x-2}{3} + 5 = 10, \frac{2x+5}{3} = 10, \frac{2x}{3} + 5 = 10,$$
$$\frac{x}{3} + \frac{x}{2} = 5, \frac{2x+5}{3} = \frac{x-1}{4}$$

Solve complex linear equations involving algebraic fractions (NSW) [Stage $5.3^{\$}$]

• solve a range of linear equations, including equations that involve two or more

fractions, eg $\frac{2x-5}{3} - \frac{x+7}{5} = 2$, $\frac{y-1}{4} - \frac{2x+3}{3} = \frac{1}{2}$

Answers

PREP QUIZ 11:01

- **1** x = 12 **2** m = 16 **3** a = 10 **4** z = 7**5** x = 2 **6** n = 1 **7** t = 2 **8** m = -1
- **5** x = 2 **8** n = 1**9** x = 20 **10** y = 15
- **9** x = 20 **10** y = 13

Lesson starter

Mind reader Part 1

Ask students to pick a number between 1 and 10 and complete the following calculations:

- add 6
- multiply by 2
- subtract 8
- divide by 2
- subtract the original number

Students should all have the same answer of 2.



Why has this happened? How can this trick be written algebraically? Can you come up with a similar trick?

P Digital resources

eBook

• Foundation worksheet 11:01 Equations with fractions



11:01 Linear equations involving algebraic fractions

Equations involving fractions were studied in Year 9. These are reviewed and extended in this section. The Prep quiz should remind you of some basic types of algebraic equations.

PREP QUIZ 11:01 Solve each equation. All solutions are integers. 1 $\frac{x}{3} = 4$ 2 $\frac{m}{2} - 3 = 5$ 3 $\frac{3a}{5} = 6$ 4 $\frac{z+3}{2} = 5$ 5 $\frac{10-x}{4} = 2$ 6 $\frac{2n+3}{5} = 1$ 7 $\frac{t}{2} + 3 = 2t$ 8 $\frac{m-3}{4} = m$ 9 $\frac{x-5}{3} = \frac{x}{4}$ 10 $\frac{y}{3} + \frac{y}{5} = 8$

Remember these rules when solving equations with fractions.

- If there is only one denominator, multiply both sides of the equation by this denominator.
- If there is more than one denominator, multiply both sides of the equation by the lowest common denominator (LCD).



298 Australian Signpost Mathematics New South Wales 10 Stages 5.1–5.2

Language

algebraic fractions coefficient completing the square decimal approximation denominator equations factor factorising formula

highest common factor integer inverse operations literal equations non-linear equation null factor law numerator parentheses pronumeral quadratic quadratic formula rearrange simultaneous equations solution solve substitution surd trinomial