## FURTHER EQUATIONS



Contents
11:01 Linear equations involving algebraic fractions 11:02 Quadratic equations: Solution using factors
11:03 Solution by completing the square
11:04 The quadratic formula
Investigation 11:04 How many solutions?
11:05 Choosing the best method
Fun spot 11:05 What is an Italian referee?
11:06 Problems involving quadratic equations
Challenge 11:06 Equations reducible to quadratics Investigation 11:06 Simple cubic equations $a x^{3}=k$

Syllabus references (See pages $x$-xiii for details.)
Number and Algebra
Selections from Equations [Stage 5.3 ${ }^{\S}$ ]

- Solve complex linear equations involving algebraic fractions (NSW)
- Solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)
- Solve simple cubic equations (NSW)
- Rearrange literal equations (NSW)
- Solve simultaneous equations, where one equation is non-linear, using algebraic and graphical techniques, including the use of digital technologies (NSW)


## Working Mathematically

- Communicating • Problem Solving - Reasoning - Understanding • Fluency


## Key ideas

- Linear equations can be solved by applying inverse operations.
- Linear equations with algebraic fractions can be solved using inverse operations and common denominators.
- The Null Factor Law can be used to solve quadratic equations. The Null Factor Law states that if $a \times b=0$ then at least one of $a$ and $b$ must be zero. Quadratics must be factorised in order for the Null Factor Law to be applied.
- Quadratics can be factorised using the cross method or by recognising features of special quadratics such as perfect squares and difference of two squares.
- When quadratics cannot be factorised then solutions can be found by completing the square.
- The discriminant $\Delta=b^{2}=4 a c$ determines the number of solutions to a quadratic equation.
- A linear and non-linear equation may have a point or many points of intersection. These possible points of intersection can be found by solving the two equations simultaneously using the substitution method.
- A literal equation has two or more pronumerals. Literal equations or formulas are rearranged to make a given pronumeral the subject using inverse operations and factorisation.


## CHAPTER FOCUS

Students will solve linear, quadratic, simultaneous and literal equations in this chapter. They will simplify and solve linear equations involving algebraic fractions, and will substitute values to determine an unknown or to check an answer. Students will factorise and use inverse operations to solve linear equations and literal equations, including those derived from formulas and worded questions. They will also solve a variety of quadratic expressions using different techniques, including completing the square and the quadratic formula. Simultaneous equations involving linear and non-linear equations will be explored and solved using algebraic and graphical techniques and technology.

## Outcomes

Equations [Stages 5.2, 5.3 ${ }^{\S}$ ]
MA5.2-1WM selects appropriate notations and conventions to communicate mathematical ideas and solutions
MA5.2-2WM interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems
MA5.2-3WM constructs arguments to prove and justify results
MA5.2-8NA solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques
MA5.3-1 WM uses and interprets formal definitions and generalisations when explaining solutions and/ or conjectures
MA5.3-2WM géneralises mathematical ideas and techniques to analyse and solve problems efficiently
MA5.3-3WM uses deductive reasoning in presenting arguments and formal proofs
MA5.3-7NA solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations

## 11:01 Content statements

Solve linear equations involving simple algebraic fractions (ACMNA240) [Stage 5.2]

- solve linear equations involving one or more simple algebraic fractions,
$\frac{x-2}{3}+5=10, \frac{2 x+5}{3}=10, \frac{2 x}{3}+5=10$,
$\frac{x}{3}+\frac{x}{2}=5, \frac{2 x+5}{3}=\frac{x-1}{4}$
Solve complex linear equations involving algebraic fractions (NSW) [Stage $5.3^{\S}$ ]
- solve a range of linear equations, including equations that involve two or more
fractions, eg $\frac{2 x-5}{3}-\frac{x+7}{5}=2$, $\frac{y-1}{4}-\frac{2 x+3}{3}=\frac{1}{2}$


## Answers

## PREP QUIZ 11:01

$$
\begin{array}{llrlllll}
1 & x=12 & 2 & m=16 & 3 & a=10 & 4 & z=7 \\
5 & x=2 & 6 & n=1 & 7 & t=2 & 8 & m=-1 \\
9 & x=20 & 10 & y=15 & & & &
\end{array}
$$

## Lesson starter

## Mind reader Part 1

Ask students to pick a number between 1 and 10 and complete the following calculations:

- add 6
- multiply by 2
- subtract 8
- divide by 2
- subtract the original number

Students should all have the same
 answer of 2 .

Discuss:
Why has this happened? How can this trick be written algebraically? Can you come up with a similar trick?

## P Digital resources

## eBook

- Foundation worksheet 11:01 Equations with fractions


## 11:01 Linear equations involving algebraic fractions

Equations involving fractions were studied in Year 9. These are reviewed and extended in this section. The Prep quiz should remind you of some basic types of algebraic equations.

## PREP QUIZ 11:01

Solve each equation. All solutions are integers
$1 \frac{x}{3}=4$
$2 \frac{m}{2}-3=5$
$3 \frac{3 a}{5}=6$
$4 \frac{z+3}{2}=5$
$5 \frac{10-x}{4}=2$
$6 \frac{2 n+3}{5}=1$
$7 \frac{t}{2}+3=2 t$
$8 \frac{m-3}{4}=m$
9 $\frac{x-5}{3}=\frac{x}{4}$
$10 \frac{y}{3}+\frac{y}{5}=8$

Remember these rules when solving equations with fractions.

- If there is only one denominator, multiply both sides of the equation by this denominator.
- If there is more than one denominator, multiply both sides of the equation by the lowest common denominator (LCD).


## WORKED EXAMPLE 1

Examine the solution of each equation.

$$
\begin{aligned}
& \text { a } \begin{array}{rlr}
\frac{3 x+8}{4} & =5 & \begin{array}{l}
\text { Multiply both sides } \\
\text { by the denominator. }
\end{array} \\
& \times 4 & \text { b }
\end{array} \\
& \text { b } \\
& \text { b } \begin{aligned}
\quad \frac{a-5}{3} & =a-4 \\
\times 3 & \times 3 \\
\frac{\beta(a-5)}{\not 2} & =3(a-4) \\
a-5 & =3 a-12 \\
7 & =2 a \\
\therefore a & =3 \frac{1}{2}
\end{aligned} \\
& \text { d } \\
& \text { d } \begin{aligned}
\frac{3 m-1}{4} & =\frac{2 m+3}{6} \\
\times 12 & \times 12 \\
\frac{{ }^{3} 12(3 m-1)}{A_{1}} & =\frac{{ }^{2} 12(2 m+3)}{\not 6_{1}} \\
3(3 m-1) & =2(2 m+3) \\
9 m-3 & =4 m+6 \\
5 m & =9 \\
m & =\frac{9}{5} \text { or } 1 \frac{4}{5}
\end{aligned}
\end{aligned}
$$

298
Australian Signpost Mathematics New South Wales 10 Stages 5.1-5.2

| L_anguage |  |
| :--- | :--- |
| algebraic fractions highest common factor <br> coefficient integer <br> completing the square inverse operations <br> decimal approximation literal equations <br> denominator non-linear equation <br> equations null factor law <br> factor numerator <br> factorising parentheses <br> formula pronumeral |  |

## Language

algebraic fractions coefficient
completing the square decimal approximation denominator equations factorising formula
highest common factor integer inverse operations literal equations non-linear equation null factor law numerator pronumeral
quadratic quadratic formula rearrange
simultaneous equations solution solve substitution
surd trinomial

