3.1 Variables and expressions

The language of algebra

- An **expression** is made up of one or more **terms**. The expression $2x + 7$ is made up of two terms, $2x$ and 7.
- An **equation** links two expressions with an equal sign

  $2x + 7 = 25$ ← equation

  expressions

- The **coefficient** in an algebraic term is the number at the front.

  $5x$ means 5 lots of x

  $2ab^2$ means 2 lots of ab^2

If there is no number written in an algebraic term, we say the coefficient is 1, so x means 1 lot of x.

- **Variables** are unknown amounts (or amounts that can change, or vary) represented by symbols, usually letters.

  $5xy^2 + ax$: The variables are x, y and a.

- A **constant** term has no variables, so its value cannot change.

  $2a - b - 4$: The constant term is -4.

Algebraic conventions

- **Multiplication**

  $a \times 7 \times b = 7ab$

- **Powers**

  $5 \times a \times a \times a = 5a^3$

- **Division**

  $(2x + 9) \div 3y = \frac{2x+9}{3y}$

Words to algebra

In changing from words to algebraic expressions:

- define your variables (e.g. 'Let x be the number of jellybeans that Bill has')
- look for key words to decide which operation to use.

Words	Operation	Algebraic expression
Sara has three times as many jellybeans as Bill.	$\times$	Sara's number of jellybeans: $x \times 3 = 3x$
Mary has twice as many (or double the number of) jellybeans as Bill.	$\times$	Mary's number of jellybeans: $x \times 2 = 2x$
The total number of jellybeans for Bill and Sarah or how many they have altogether.	+	Total jellybeans for Bill and Sarah: $x + 3x$
Rick has five more jellybeans than Bill.	+	Rick's number of jellybeans: $x + 5$
Harry has 8 fewer than Bill	–	Harry's number of jellybeans: $x - 8$
Bill shared his jellybeans between 6 people	÷	Each person's share: $\frac{x}{6}$

1 State whether each statement is true (**T**) or false (**F**) by circling the correct answer.

a $2x + 3y = 8$ is an expression. **T or F**

b The number of terms in $5x^2 - 3x + 4$ is 3. **T or F**

c The coefficient of x^2 in $3x^2 + 5x - 1$ is 5 **T or F**

d The constant term in $8x - 9$ is -9. **T or F**

2 The variables in $6a^2b^2 + 5ab + 3$ are:

A a^2b^2 and ab

B a^2b^2, ab and 3

C a^2, b^2 and ab

D a and b

3 **a** Write down the coefficient of each variable term: $2x^2 + xy - 5x + 3$

> **Tip** Where a term is subtracted, the coefficient is negative.

x^2: ☐ xy: ☐ x: ☐

b The constant term in $2x^2 + xy - 5x + 3$ is:

☐

4 Write the number of terms in each expression.

> **Tip** Terms are separated by plus or minus symbols.

a $2 + b$ ☐

b $4x^2 - xy + a$ ☐

c $3x^2yz^5$ ☐

5 Rewrite each expression using algebraic conventions.

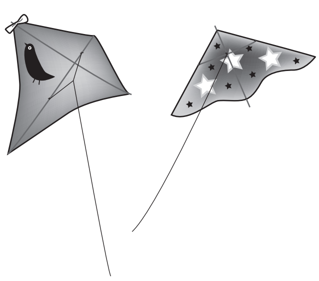
a $4 \div (b \times a) =$ ☐ **b** $x \times 5 \times x =$ ☐

c $(3y + x) \div 5 =$ ☐

6 Write an algebraic expression for each of the following given that k is the number of kites flying in the park on Saturday.

a On Sunday there were twice as many kites as Saturday.

☐



b On Friday there were six fewer kites than Saturday.

☐

c On Monday there were half as many kites as Saturday.

☐

d The following Saturday there were five more than double the kites as the first Saturday.

☐

7 If m is the mass of a muffin in grams, write a meaning for each expression:

a $6m$

☐

b $m - 30$

☐

c $\frac{m}{2}$

☐

NAPLAN-ready

Shade the box beneath the correct answer.

If a number is represented by x, an expression for 5 fewer than double the number is:

$2x - 5$ ☐	$x - 10$ ☐
$\frac{x-5}{2}$ ☐	$2(x - 5)$ ☐

> **Tip** Double means multiply by 2. 5 fewer means subtract 5.

3.2 Substitution for variables

Replacing variables with numbers

When we 'substitute' a variable, we replace it with an actual number value in an expression.

If $a = -3$ and $b = 4$: $a + b = -3 + 4$

Evaluating: $-3 + 4 = -1$

- Replacing the multiplication symbol

  For terms such as $2x$ or xy, when we replace the variables with numbers we must insert the multiplication symbols.

  If we substitute $x = 3$ into the expression $2x$:

  $2x = 2 \times 3$
  $= 6$

If we substitute $x = 4$ and $y = -5$ into the expression xy:

$xy = 4 \times -5$
$= -20$

- Powers of negative numbers

  When a negative number replaces a variable in a power, brackets are needed so that the power applies to the entire value.

  When $x = -3$, $x^2 = (-3)^2$
  $= -3 \times -3$
  $= 9$

Substitution using a rule

For $y = 2x - 6$, substitute each value of x in the table into the expression $2x - 6$, and then evaluate. The result is the corresponding value for y.

x	-4	0	3
y	-14	-6	0

-4	0	3
$2x - 6$	$2x - 6$	$2x - 6$
$= 2 \times -4 - 6$	$= 2 \times 0 - 6$	$= 2 \times 3 - 6$
$= -8 - 6$	$= 0 - 6$	$= 6 - 6$
$= -14$	$= -6$	$= 0$

Word Bank

Substitute
➜ To substitute means to replace a variable with an actual value.

Evaluate
➜ To evaluate means to find the answer, i.e. a single value for the expression.

1 What is the value of the following expressions if $x = 10$?

a $x - 25$
$= \_\_\_\_ - 25$
$= \square$

b $3x$
$= 3 \times \_\_\_\_$
$= \square$

2 Evaluate each expression when $a = 12$, $b = -2$, $c = 3$, $d = 1$.

a $a + d$
$= \_\_\_\_ + \_\_\_\_$
$= \square$

b $3a$
$= 3 \times \_\_\_\_$
$= \square$

c bc
$= \_\_\_\_ \times \_\_\_\_$
$= \square$

d $\frac{a}{4}$
$= \frac{\_\_}{4}$
$= \square$

e $\frac{bc}{6} + \frac{6d}{c}$
$= \frac{\_\_ \times \_\_}{6} + \frac{6 \times \_\_}{\_\_}$
$= \_\_\_\_ + \_\_\_\_$
$= \square$

f $\frac{8}{b} - \frac{a}{2d}$
$= \frac{8}{\_\_} - \frac{\_\_}{2 \times \_\_}$
$= \_\_\_\_ - \_\_\_\_$
$= \square$

3 Evaluate each expression when $x = 2, y = -3$.

Tip When substituting a negative number into a power, place brackets around the number including the negative sign.

a x^2
$= 2^2$
$= 2 \times 2$
$=$ ☐

b y^2
$= (-3)^2$
$= -3 \times -3$
$=$ ☐

c x^2y
$= (\_\_\_\_)^2 \times \_\_\_\_$
$= \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$
$=$ ☐

d $\frac{x^3}{4}$
$= \frac{\_\_\_^3}{4}$
$= \frac{\_\_ \times \_\_ \times \_\_}{4}$
$=$ ☐

e $2x^3 + y^2$
$= 2 \times \_\_\_\_^3 + (\_\_\_\_)^2$
$= 2 \times \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ + \_\_\_\_ \times \_\_\_\_$
$= \_\_\_\_ + \_\_\_\_$
$=$ ☐

4 One of the following shows correct substitution. For each incorrect substitution, write the correct expression. For the correct one, write 'correct'.

a If $x = 3$ and $y = -5$:
$xy = 3 - 5$
☐

b If $a = 9$:
$2a^3 = 29^3$
☐

c If $p = 6$:
$3p + \frac{p}{2} = 3 \times 6 + \frac{6}{2}$
☐

d If $t = -7$:
$t^2 = -7^2$
☐

5 Complete the tables of values for each rule.

Tip Substitute each value in the x-row into the expression and evaluate. Write the answer in the y-row below the matching x-value.

a $y = x + 7$

x	-4	0	3
y	3		

$y = x + 7$	$y = x + 7$	$y = x + 7$
$= \_\_ + 7$	$= \_\_ + 7$	$= \_\_\_\_$
$= 3$	$= \_\_$	$= \_\_$

b $y = -2x$

x	-2	0	5
y			

☐ ☐ ☐

c $y = 3x - 5$

x	0	2	4
y			

☐ ☐ ☐

NAPLAN-ready

Shade the box beneath the correct answer.

$L = \frac{a + b}{a - b}$

Which values of a and b give $L = 5$?

$a = 12, b = 8$ ☐

$a = 22, b = 18$ ☐

$a = 17, b = 3$ ☐

$a = 37, b = 23$ ☐

Tip Find the values for the numerator and denominator first.

3.3 Using formulas

A formula is an equation that is used in a practical situation.

The **formula** for converting temperature in degrees Celsius, c (°C), to degrees Fahrenheit, f (°F), is:

$$f = \frac{9c}{5} + 32$$

To find a temperature of 25°C in °F:

- Write the given information in symbols.

$c = 25$

- Substitute into the expression and evaluate.

$$\begin{aligned} f &= \frac{9c}{5} + 32 \\ &= \frac{9 \times 25}{5} + 32 \\ &= 45 + 32 \\ &= 77 \end{aligned}$$

- Write the answer in a sentence, including any units.

A temperature of 25°C is the same as 77°F.

1 If $a = 3$ and $b = 7$, what is the value of y in the following equations?

a $y = 2b - a$

$= 2 \times$ ____ $-$ ____

$=$ ________

$y =$ [ ]

b $y = ab$

$=$ ________________

$y =$ [ ]

2 Mia charges $12 to mow lawns in her neighbourhood. The money she earns could be described using the formula: $m = 12n$, where:

m = money earned in dollars
n = number of lawns

a Complete the table of values to show how much Mia could earn.

Tip Substitute each value of n into the formula $m = 12n$. e.g. When $n = 1$, $m = 12 \times 1$
$m = 12$

Number of lawns mowed (n)	1	2	3	4	5
Money earned in dollars (m)					

b One week Mia mowed 8 lawns. How much money did she make?

c How many lawns would she need to mow to earn over $100?

3 The formula for the distance travelled, d (km), for a car after a certain amount of time, t (hours), when travelling at an average speed of s (km/h) is $d = st$.

a Find the distance covered in 3 hours, if a car is travelling at an average speed of 80 km/h.

$d = st$

$=$ ____ $\times$ ____ (substitute in the values for s and t)

$=$ [ ] km

b Find the distance covered in 2 hours, if a car is travelling at an average speed of 50 km/h.

$d =$ ________________

$=$ ____ $\times$ ____

$=$ [ ] km

4 The cost of a taxi fare (F), in dollars, can be calculated using the formula $F = 1.2d + 3.30$, where d is the distance travelled in kilometres.

a What is the cost of a taxi fare if you travel 5 km?

b What is the cost of a taxi fare if you travel 20 km?

5 The formula used to calculate Annette's bonus (in dollars) at the end of the month is $b = 5(e - a)$, where e is the number of days she put in extra effort and a is the number of days she was absent. Find the amount of Annette's bonus if she put in extra effort on 10 days and was absent twice.

$e =$ ____, $a =$ ____

$b = 5(e - a)$

$=$ ________

$=$ ________

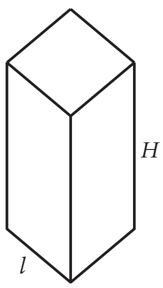
$=$ ____

Annette's bonus is [ ]

6 The formula for finding the total area (in cm^2) of the faces of a square-based prism is:

$$T = 2l^2 + 4lH$$

where l = length of the side of the square (cm) and H = height of the prism (cm).



Find T when $l = 15$ cm and $H = 5$ cm.

[ ]

7 The formula for finding the area (in cm^2) of a triangle, given the length of the sides is:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

where a, b and c are the lengths of the sides (cm) and s is the semi-perimeter (cm), $s = \dfrac{a+b+c}{2}$

Find the area of a triangle with side lengths of 5 cm, 6 cm and 7 cm, correct to one decimal place.

$a =$ ____, $b =$ ____, $c =$ ____

$s = \dfrac{a+b+c}{2}$

$s - a$	$s - b$	$s - c$
$=$ ___ $-$ ___	$=$ ___ $-$ ___	$=$ ___ $-$ ___
$=$ ____	$=$ ____	$=$ ____

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{\_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_}$

$A = \sqrt{\_\_\_\_}$

$\approx$ [ ] cm^2

NAPLAN-ready

Shade the box beneath the correct answer.

The formula for converting length, c in cm, to length i, length in inches, is $i = \dfrac{c}{2.54}$.

The length, in inches, of a 30 cm ruler is approximately:

11.8 inches ☐	12.2 inches ☐
75.8 inches ☐	76.2 inches ☐

Tip Substitute for c in the formula.

3.4 Simplifying expressions

Like terms

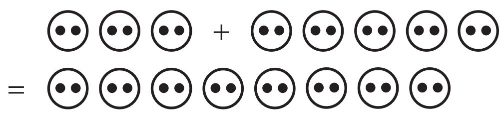
Like terms have algebraic factors that are *identical* in type and number. Only the coefficient can be different:

- **Like terms**
  $2x$ and $3x$
  $5ab$ and ab
  $8x^2$ and $-6x^2$
- **Not like terms**
  $3x$ and $3y$
  $2x$ and $5x^2$
  xy^2 and x^2

Collecting like terms

- **Addition**

The diagram shows that 3 groups of 2 collected with 5 groups of 2 gives 8 groups of 2.



We could write this as:

$2 \times 3 + 2 \times 5 = 2 \times (3 + 5)$
$= 2 \times 8$

Algebraically:

If we had any number of dots in each group, we could write it in algebra, where *x* stands for the number of dots in each group.

$3x + 5x = 8x$ ← (add the coefficients: $3 + 5 = 8$)

This is true for any variable or product of variables:

$3ab + 5ab = 8ab$

3 lots of *ab* plus 5 lots of *ab*, gives 8 lots of *ab*.

$3ab + 5ba = 8ab$ ($ab = ba = a \times b$)

$3x^2 + 5x^2 = 8x^2$

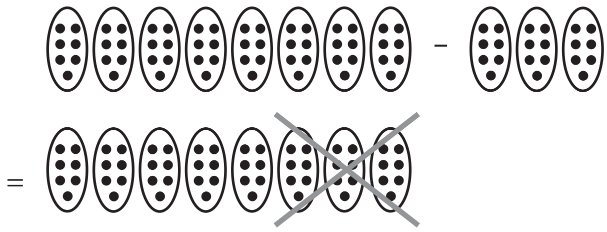
We can show it is true by substituting a value for *x*.

For example, if $x = 3$, then $x^2 = 9$

3 lots of 9 plus 5 lots of 9, gives 8 lots of 9

$3 \times 9 + 5 \times 9 = 8 \times 9$
$27 + 45 = 72$

- **Subtraction**



Subtraction is the reverse of addition.

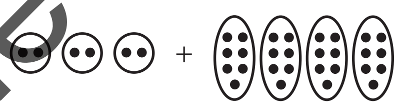
8 lots of any number minus 3 lots of the same number, gives 5 lots of the number.

Algebraically:

$8x - 3x = 5x$

This process of combining groups is called 'collecting like terms'.

Unlike terms



The number of dots in each group is different so we *cannot* group these any more simply.

To combine groups, we must have the same number of dots in each group.

Algebraically:

$3x + 4y$ cannot be simplified.

Neither can $2x + 6x^2$.

If $x = 7$, then $x^2 = 49$, we cannot combine 2 lots of 7 with 6 lots of 49.

1 Circle the like terms.

a $2x$ $2y$ $3x$ x

b $5ab$ $2bc$ $6ab$ $3a$

2 **a** Which of the following is a pair of like terms?

A $3x^2, 3y^2$ **B** $5xy, 2yx$
C $3ab, 9ab^2$ **D** $x^4, 4x$

b Which of the following is a pair of unlike terms?

A $3a, -a$ **B** $-3cd, 2cd$
C $4ab^2, ab^2$ **D** $2pn, 4pm$

3 Which of the following expressions can be simplified?

A $4r + r$
B $-3mn - nm$
C $5w^2 + w$
D $8de + de - 5de$

4 Simplify, if possible, by collecting like terms.

Tip Add (or subtract) the coefficients. Remember x means $1x$.

a $4e + 6e$

= ☐

b $-6f + 8f$

= ☐

c $15y - 12y$

= ☐

d $-2s - s$

= ☐

e $5t^2 + 9t^2$

= ☐

f $t^3 + 8t$

= ☐

g $9mn + mn$

= ☐

h $5a^2b - a^2b$

= ☐

5 Simplify by collecting like terms.

Tip Place like terms side by side first.

a $6e + 9xy + xy + 9e$

$= 6e + 9e + 9xy + xy$

$= 15e +$ ______

b $-8f + p + 2f + 8p$

$= -8f +$ ______ $+ p +$ ______

= $-6f +$ ______

c $11y - 5x + 2x - 10y$

= ______ $-$ ______ $- 5x +$ ______

= ☐

d $-3s - 5s^2t - s - 3s^2t$

= ______ $-$ ______ $-$ ______ $-$ ______

= ☐

e $4t^2 - 5t^2u - 5t^2u + 9t^2$

= ______ $+$ ______ $-$ ______ $-$ ______

= ☐

6 Check that collecting like terms has 'worked' by substituting $x = 3$ and $y = 5$ to find a value for each term, and then add the values in the last column.

a $3xy$

$= 3 \times 3 \times 5$

= ____

$4xy$

$= 4 \times 3 \times 5$

= ____

$7xy$

$= 7 \times 3 \times 5$

= ____

$3xy + 4xy = 7xy$

Check:

____ + ____ = ____

b x^2y

$= 3^2 \times 5$

=

=

$-6x^2y$

=

=

=

$-5x^2y$

=

=

=

$x^2y - 6x^2y = -5x^2y$

Check:

____ − ____ = ____

7 State whether each statement is true (T) or false (F) by circling the correct answer.

a $x^2y = xy^2$ **T** or **F**

b $5x - x = 5$ **T** or **F**

c $3mn = 3nm$ **T** or **F**

d $4a^2b^2 = 2ab^2$ **T** or **F**

NAPLAN-ready

Shade the box beneath the correct answer.

Poppy has x 20-cent coins and $3x$ 50-cent coins.

The amount of money Poppy has in cents is:

$70x$	$170x$	$17x$	$300x$
☐	☐	☐	☐

Tip Find how much she would have with one 20-cent coin and three 50-cent coins first.

Multiplying and dividing algebraic terms

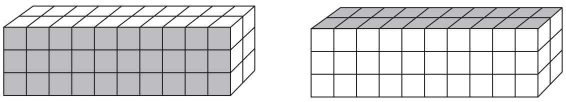
Multiplication with algebra

When multiplying, we can rearrange the calculation without changing the result.

$$2 \times 5 = 5 \times 2$$

We can also regroup factors. This may make the calculation easier to do, or help us to see common factors.

$$(3 \times 10) \times 2 = 3 \times (10 \times 2)$$



We can use this in algebra to simplify expressions. We multiply all coefficients together and then multiply all the variables.

$$5 \times 6a = (5 \times 6) \times a$$
$$= 30a$$

$$e \times 4f \times 2p = (4 \times 2) \times (e \times f \times p)$$
$$= 8efp$$

Don't forget to write variables in alphabetical order.

Division with algebra

Division is the reverse process of multiplication.

We use fraction rules to cancel out common factors of the numerator and denominator.

$$8ab \div 4bc = \frac{{}^{2}\cancel{8}ab}{{}_{1}\cancel{4}bc}$$ ← (writing the division as a fraction, dividing top and bottom by 4)

$$= \frac{2a\,{}^{1}\cancel{b}}{{}_{1}\cancel{b}c}$$ ← (dividing top and bottom by b)

$$= \frac{2a}{c}$$

We could have done the two divisions above in one step, by dividing by $4b$, which is the highest common factor (HCF) of the two expressions.

$$\frac{8ab}{4bc} = \frac{{}^{2}\cancel{8}a\cancel{b}}{{}_{1}\cancel{4}\cancel{b}c}$$

$$= \frac{2a}{c}$$

Word Bank

Highest common factor (HCF)
→ For two numbers, the HCF is largest number that divides into both. For two algebraic products, take the HCF of the numbers multiplied by as many shared algebraic factors as possible (e.g. the HCF of $6ef$ and $8et$ is $2e$).

1 Simplify each of the following.

Tip Rearrange and group the factors as needed.

a $5 \times 2a$

=

b $8b \times 4$

=

c $3 \times 7c$

=

d $6e \times 10a$

=

e $2t \times 11s$

=

f $9p \times 5m$

=

2 Simplify each of the following.

Tip Ignore the negative signs until the end. A single negative gives a negative answer. Two negatives multiplied together give a positive answer.

a $4 \times -9q$

=

b $-3m \times 5$

=

c $-5n \times -6d$

=

d $-6d \times -11h$

=

c $-7r \times 4g$

=

d $-y \times -z$

=

3 Simplify each of the following.

Tip Cancel **HCF** of the numbers and then any common algebraic factors.

a $\frac{20f}{5}$

$= \frac{{}^{4}\cancel{20}f}{{}_{1}\cancel{5}}$

= ______

b $\frac{14ch}{2}$

= ______

c $\frac{6bc}{c}$

= ______

d $\frac{8mx}{2m}$

$= \frac{\_\_\_}{m}$ (cancelling a common factor of 2)

= ______ (cancelling a common factor of m)

e $\frac{-6bj}{3j}$

$= \frac{\_\_\_}{\_\_\_}$ (cancelling a common factor of 3)

= ______ (cancelling a common factor of j)

f $\frac{-4jk}{-2j}$

$= \frac{\_\_\_}{\_\_\_}$

______

4 Simplify each of the following.

Tip Write as fractions first.

a $40by \div 8y$

$= \frac{{}^{5}\cancel{40}b\cancel{y}}{{}_{1}\cancel{8}\cancel{y}}$

= ______

b $-24pq \div -6p$

$= \frac{\_\_\_}{\_\_\_}$

= ______

c $90cw \div -3cw$

$= \frac{\_\_\_}{\_\_\_}$

= ______

5 Circle the letter next to the correct answer.

a $4ac \div 16c$ simplifies to:

A $\frac{4}{a}$ B $\frac{a}{4}$ C $4ac$ D $4a$

b $-5g \div -50gh$ simplifies to:

A $10h$ B $-10h$ C $-\frac{1}{10h}$ D $\frac{1}{10h}$

c $42kl \div -30dl$ simplifies to:

A $-\frac{7k}{5d}$ B $-\frac{7d}{5k}$ C $2k$ D $-35kd$

6 Simplify $(14pt \times -6kq \times br) \div 21pqr$.

Tip Write as a fraction. Cancel common factors

NAPLAN-ready

Shade the box beneath the correct answer.

The expression $(ab \times 7cd \times 5e) \div 10def$ simplifies to:

$\frac{14abc}{f}$	$7cd$	$\frac{7abc}{2def}$	$\frac{7abc}{2f}$
☐	☐	☐	☐

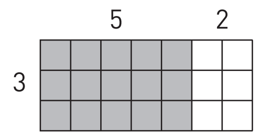
Tip Write as a fraction then cancel out common factors.

3.6 Expanding brackets A

The Distributive Law

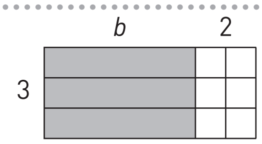
The 3×7 rectangle has the side of 7 split into $5 + 2$.

The total area can be written as
$3 \times 5 + 3 \times 2$ or $3 \times (5 + 2)$ or $3(5 + 2)$



This rectangle has a length of $b + 2$ and a width of 3.

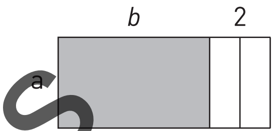
Its area can be written as $3 \times b + 3 \times 2$ or $3(b + 2)$.



This rectangle has a length of $b + 2$ and a width of a.

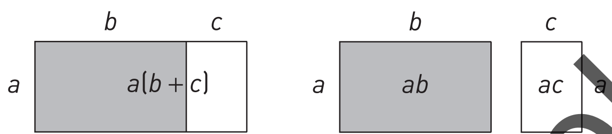
Its area can be written as $a \times b + a \times 2$ or $a(b + 2)$.

So $a(b + 2) = a \times b + a \times 2$
$= ab + 2a$



As a general rule: $a(b + c) = ab + ac$

This rule is known as the **distributive law**. We use it to multiply the terms inside a set of brackets by the term in front. This is called 'expanding the brackets'.



It is helpful to use the 'crab claws'.

$a(b + c) = ab + ac$

• **Mental maths**

We can use the distributive law to help multiply large numbers, using addition to break up the number.

$4 \times 102 = 4(100 + 2)$
$= 400 + 8$
$= 408$

1 Use the distributive law to calculate:

e.g. $5 \times 47 = 5(40 + 7)$
$= 200 + 35$
$= 235$

a 8×34

$8 \times 34 = 8($______ + ______$)$

= ______ + ______

= [   ]

b 7×603

$7 \times 603 = 7(600 +$ ______$)$

$= 4200 +$ ______

= [   ]

c 43×1001

$43 \times 1001 = 43(1000 +$ ______$)$

= __________ + ______

= [   ]

2 Complete the multiplication grid for 24×35.
Add each row and then calculate the total.

	20	4	Total
30	600		
5			

=

3 Use the distributive law to expand.

a $6(x+3) = 6 \times$ ______ $+ 6 \times$ ______

$=$ [ ]

b $3p(t+r) = 3p \times$ ______ $+ 3p \times$ ______

$=$ [ ]

4 Use the distributive law to expand the brackets.

> **Tip** Group the numbers and variables separately. Remember $x \times x = x^2$.

a $9b(2c+5)$

$= 9b \times$ ______ $+ 9b \times$ ______

$=$ [ ]

b $2b(8+b)$

$=$ ______ $\times$ ______ $+$ ______ $\times$ ______

$=$ [ ]

5 Simplify the following expressions.

> **Tip** Expand the brackets first then collect like terms.

a $3(x+5) - 2x$

$=$ ____ $+$ ____ $- 2x$

$=$ __ $- 2x +$ __

$=$ [ ]

b $5m(p+4) + m$

$=$ ____ $+$ ____ $+$ ____

$=$ [ ]

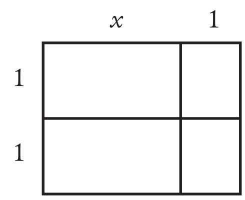
c $2a(6+a) + 3(a+4)$

$=$ ____ $+$ ____ $+$ ____

$=$ ____ $+ 12a + 3a +$ ____

$=$ [ ]

6 Demonstrate expanding brackets using the rectangle as an example.



The four individual areas are:

x, __, 1, __

Total area = __ + __

Length of rectangle: __ + 1

Width of rectangle: __

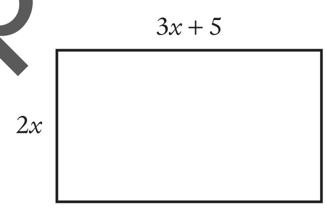
Area = length × width

= (__ + 1) × __

= __(__ + 1)

i.e. 2(__ + __) = __ + __

7 A rectangle is $(3x+5)$ cm long and $2x$ cm wide.



a Write an expression for the area of the rectangle in cm^2.

[ ]

b Expand your answer in **a**.

[ ]

c Write an expression for the perimeter of the rectangle in cm.

[ ]

d Expand and simplify your answer from **c**.

[ ]

3.6 Expanding brackets B

The distributive law and subtraction

If the bracket contains a subtraction, we apply the distributive law in the same way.

$a(b-c) = ab-ac$

$-a(b+c)$ has the opposite sign of $a(b+c)$

$-a(b-c)$ has the opposite sign of $a(b-c)$

So $-a(b+c) = -ab-ac$ ← (each positive term changes to negative)

and $-a(b-c) = -ab+ac$ ← (each term changes to the opposite sign)

- **Mental maths**

We can use the distributive law to help multiply large numbers, using subtraction to break down the number.

$4 \times 98 = 4(100-2)$
$= 400-8$
$= 392$

1 Use the distributive law to calculate the following.

Tip The distributive law can be used to perform multiplications, making it easier to do calculations in our head.
e.g. $3 \times 76 = 3 \times (80 - 4)$
$= 240 - 12$
$= 228$

a $4 \times 58 = 4(60-2)$
$=$ ____ $-$ ____
$=$ [ ]

b $7 \times 106 =$ ____ $(110-$ ____ $)$
$=$ ____ $-$ ____
$=$ [ ]

c $8 \times 199 = 8(200-$ ____ $)$
$= 1600-$ ____
$=$ [ ]

d $21 \times 97 = 21(100-$ ____ $)$
$=$ ____ $-$ ____
$=$ [ ]

2 Use the distributive law to expand the following.

a $3(a-2) = 3 \times$ ___ $- 3 \times$ ___
$=$ [ ]

b $4(2x-y) = 4 \times$ ___ $- 4 \times$ ___
$=$ [ ]

c $m(m-7) = m \times$ ____ $- m \times$ ____
$=$ [ ]

d $h(4a-h) = h \times$ ____ $- h \times$ ____
$=$ [ ]

3 Expand the brackets.

Tip Group the numbers and variables separately.
Remember $x \times x = x^2$.

a $7m(3n-r)$
$= 7m \times$ ____ $- 7m \times$ ____
$=$ [ ]

b $3f(4f-5c)$
$=$ ____ $\times$ ____ $-$ ____ $\times$ ____
$=$ [ ]

4 Expand the brackets and then collect like terms.

Tip Multiply each term in a bracket by the number in front of that bracket.

a $5(m+1)+4(3m-2)$

$= 5 \times$ ____ $+ 5 \times$ ____ $+ 4 \times$ ____ $- 4 \times$ ____

$=$ ____ $+$ ____ $+$ ____ $-$ ____

$=$ ____ $+$ ____ $+$ ____ $-$ ____ (collect like terms)

$=$

b $5s(3p-2)+7(4sp-6s)$

$= 5s \times$ ____ $- 5s \times$ ____ $+ 7 \times$ ____ $- 7 \times$ ____

$=$ ____ $-$ ____ $+$ ____ $-$ ____

$=$

c $8ab+3a(2b-7)$

$= 8ab + 3a \times$ ____ $- 3a \times$ ____

$=$ ____ $+$ ____ $-$ ____

$=$ ________ $-$ ________

d $2gh(4-3a)-8gh$

$=$ ____ $\times$ ____ $-$ ____ $\times$ ____ $- 8gh$

$=$ ______ $-$ ______ $- 8gh$

$=$

b $3(b-7)-2(b+8)$

c $2l(m-6)-3(2lm+3n)$

$= 2l \times$ ____ $- 2l \times$ ____ $-3 \times$ ____ $-3 \times$ ____

$=$ ____ $-$ ____ $-$ ____ $-$ ____

$=$ ____ $-$ ____ $-$ ____ $-$ ____

$=$

d $4hk-9k(6h-1)$

$= 4hk -$ ____ $\times$ ____ $+$ ____ $\times 1$

$=$ ________________________

$=$

5 Expand $6(p-2)+5(2p+4)$ and collect like terms.

6 Expand and collect like terms.

Tip When multiplying by a negative factor outside the brackets, the sign of each term in the bracket changes to the opposite sign.

a $2(a+4)-5(a-6)$

$= 2 \times$ ____ $+ 2 \times$ ____ $- 5 \times$ ____ $+ 5 \times$ ____

$=$ ____ $+$ ____ $-$ ____ $+$ ____

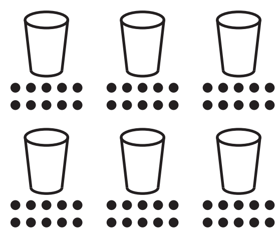
$=$ ____ $-$ ____ $+$ ____ $+$ ____ (collecting like terms)

$=$

NAPLAN-ready

Shade the box beneath the correct answer.

The illustration shows some cups, each of which contains the *same number* of counters, and some loose counters.



If x stands for the number of counters in each cup, the expression that does *not* match the diagram is:

$2(3x + 30)$ ☐	$6x + 10$ ☐
$6(x + 10)$ ☐	$6x + 60$ ☐

Tip Group the cups and counters to match each expression.

3.7 Factorising

Factorising algebraic expressions using the distributive law is the reverse of expanding.

When factorising an expression we need to:

- Identify the highest common factor (HCF) of the terms in the expression. The HCF could consist of numbers, variables or both.
- Write the HCF in front of a set of brackets and write the remaining factors inside the brackets.

$6x + 10xy$

$= 2 \times 3 \times x + 2 \times 5 \times x \times y$ ← (write each term as a product of individual factors)

$= \boxed{2} \times 3 \times \boxed{x} + \boxed{2} \times 5 \times \boxed{x} \times y$ ← (highlight the factors common to each term, HCF = $2x$)

$= 2 \times x(3 + 5 \times y)$ ← (place the common factors at the front of the brackets and the 'left over' factors inside)

$= 2x(3 + 5y)$ ← (remove multiplication symbols)

Always *check* factorising by expanding your answer.

$2x(3 + 5y) = 2x \times 3 + 2x \times 5y$
$= 6x + 10xy$

- **Powers**

When a term contains a power, write the power in expanded form so you can 'see' the individual factors.

$x^2 + 7xy$
$= \boxed{x} \times x + 7 \times \boxed{x} \times y$
$= x(x + 7y)$

- **Negative common factors**

When the first term is negative, write a negative sign to the factor in front of the brackets so that the first term in the brackets is positive. Each term will need to change to the opposite sign from its original sign.

$-6x + 5xy$
$= -6 \times \boxed{x} + 5 \times \boxed{x} \times y$
$= -x(6 - 5y)$ (plus in front of the second term becomes minus)

Word Bank

Highest common factor (HCF)
➜ For algebraic terms, find the HCF of the numbers multiplied by any common algebraic factors.

(e.g. $4ab + 6bc$, HCF = $2b$). When the HCF is removed from each term, no common factors remain.

1 Factorise:

Tip Write the numbers as products, if necessary, so you can 'see' the HCF.

a $9 + 3b$

$= 3 \times 3 + 3 \times b$

$= 3(\_\_\_\_ + \_\_\_\_)$

Check:

$3 \times 3 + 3 \times b$
$= 9 + 3b$

b $10m - 8n$

$= \_\_\_\_ \times 5 \times m - 2 \times \_\_\_\_ \times \_\_\_\_$

$= 2(\_\_\_\_ - \_\_\_\_)$

Check:

c $14d + 7$

$= \_\_\_\_ \times \_\_\_\_ \times d + 7 \times 1$

$= \_\_\_\_(\_\_\_\_ + 1)$

Check:

2 Choose the highest common factor (HCF) of each pair of terms.

a $24a$ and 20

A 20 **B** 10 **C** 4 **D** $4a$

b $6p$ and $18pq$

A 2 **B** **C** 6 **D** $6p$

3 Factorise:

Tip Remember to circle common factors for the numbers *and* the variables.

a $20g + 30gh$

$= 10 \times \_\_\_\_ \times \_\_\_\_ + \_\_\_\_ \times 3 \times \_\_\_\_ \times \_\_\_\_$

$= 10g(\_\_\_\_ + \_\_\_\_)$

Check:

b $15p^2 - 18pr$

$= 3 \times \_\_\_\_ \times \_\_\_\_ \times p - \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ \times r$

$= \_\_\_\_(\_\_\_\_ - \_\_\_\_)$

Check:

4 Factorise:

Tip If the first term is negative, take a negative factor outside the brackets. The terms inside will now have the opposites of their original signs.

$-16k + 12kn$

$= -\_\_\_\_ \times \_\_\_\_ \times k + \_\_\_\_ \times \_\_\_\_ \times k \times \_\_\_\_$

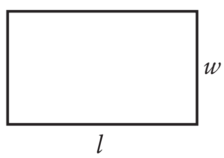
$= \_\_\_\_(\_\_\_\_ - \_\_\_\_)$

Check:

5 Use factorising to write the formula in another way.

Perimeter of a rectangle:

$P = 2l + 2w$



NAPLAN-ready

Shade the box beneath the correct answer.

Which of the expressions has highest common factor of 1:

$2xy + 15z^2$ ☐ $3x - 5ax$ ☐

$8ab + 50cd$ ☐ $xy - ay$ ☐

Tip Check the numbers and the variables in each pair of terms.