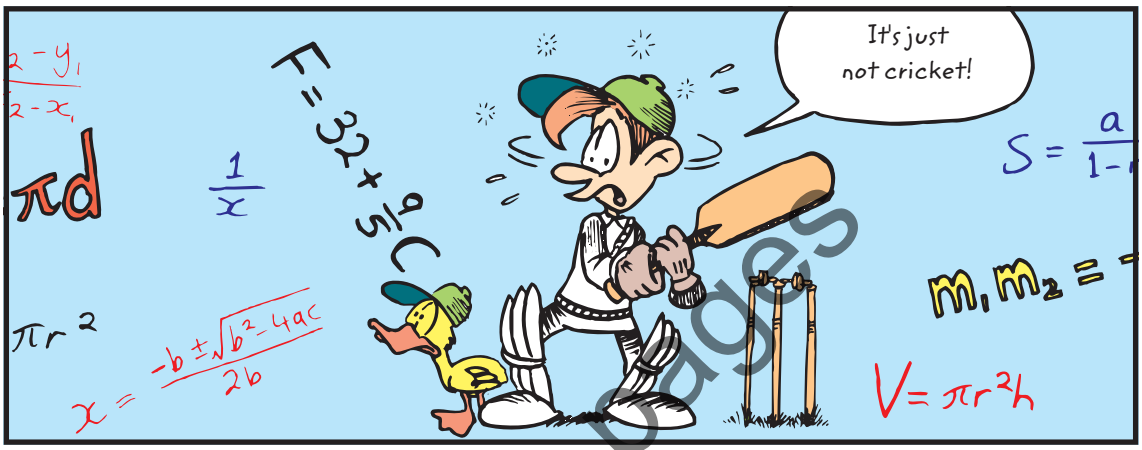
ALGEBRA

3



Contents

Syllabus references (See pages x–xv for details.)

Number and Algebra

Selections from ***Algebraic Techniques*** [Stages 5.2, 5.3§]

- Simplify algebraic expressions involving the four operations [Stage 4]
- Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)
- Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)
- Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)
- Add and subtract algebraic fractions with numerical denominators, including those with binomial numerators (NSW)
- Expand binomial products using a variety of strategies (ACMNA233)

Working Mathematically

• Problem Solving • Reasoning • Understanding • Fluency • Communicating

3:01 Simplifying algebraic expressions

PREP QUIZ 3:01

Simplify the following.

1 $7x + 2x$ **2** $9x - 8x$ **3** $3x \times 2y$ **4** $5x \times x$

5 $12x \div 4$ **6** $10ab \div 5a$ **7** $3a + 2b + 5a + 3b$ **8** $6x + 2y - x - y$

9 $3 \times (-2a) \times 4a$ **10** $3a \div (-9b)$

WORKED EXAMPLES

Remember that only like terms may be added or subtracted.

1 $5a + 2b - 3a + b = 5a - 3a + 2b + b$
$= 2a + 3b$

2 $5p^2 + 2p - 3p^2 = 5p^2 - 3p^2 + 2p$ (p^2 and p are not like terms.)
$= 2p^2 + 2p$

3 $6ab - 4ba = 6ab - 4ab$
$= 2ab$

4 $-7x \times -3xy^2 = 21x^2y^2$

5 $3pq \times 4qr = 12pqqr$
$= 12pq^2r$

6 $12ac \div 8ab = \frac{{}^{3}\cancel{12}\,\cancel{a}c}{{}_{2}\cancel{8}\,\cancel{a}b}$
$= \frac{3c}{2b}$

7 $-6x \div 18xy = \frac{-{}^{1}\cancel{6}\,\cancel{x}}{{}_{3}\cancel{18}\,\cancel{x}\,y}$
$= -\frac{1}{3y}$

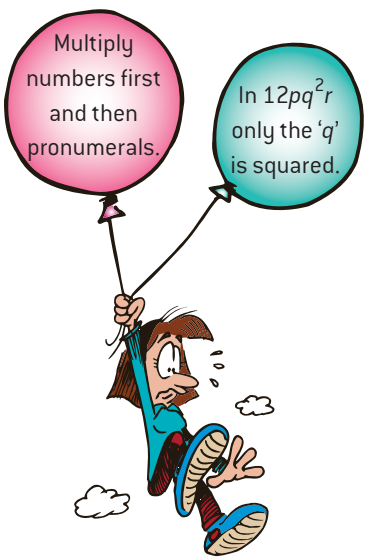
8 $10a - 3 \times 2a = 10a - 6a$
$= 4a$

9 $(5a + 7a) \times (3b - 2b) = 12a \times b$
$= 12ab$

10 $3m \times 2n \div mn = 6mn \div mn$
$= \frac{6\,\cancel{mn}^{1}}{\cancel{mn}_{1}}$
$= 6$

11 $\frac{7p + 8p - 3p}{2p \times 3q} = \frac{{}^{2}\cancel{12}\,\cancel{p}}{{}_{1}\cancel{6}\,\cancel{p}q}$
$= \frac{2}{q}$

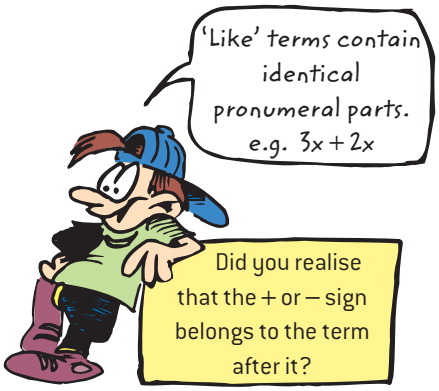
Remember the order in which operations should be done.
↓
Grouping symbols
↓
× ÷
↓
+ −



Exercise 3:01

1 Collect the like terms to simplify these expressions.

a $3x + 2x$	**b** $8a + 5a$
c $10p + 21p$	**d** $x + 7x$
e $7a - 4a$	**f** $9b - 3b$
g $11q - q$	**h** $12e + 9e$
i $3p + 5p - 6p$	**j** $4x + 2x + x$
k $10x - 9x + 3x$	**l** $x + 2x - 3x$
m $2a + p - a + 3p$	**n** $a + m - a + m$
o $8 + 2x - 5x - 7$	**p** $8y - 1 - 8y - 1$
q $x^2 + 2x + 2x^2 - x$	**r** $p^2 + 4p + 3p^2 + p$
s $3q^2 + 8q - 4q - q^2$	**t** $y^2 + y + y^2 - y$
u $7 - p^2 + p - 5$	**v** $2a + a^2 + 7 + a$
w $8x - 7 - 7x - 3x^2$	**x** $5ab - 7 + 3ba - 9$



2 Simplify these products.

a $8y \times 3$	**b** $4 \times 4a$	**c** $3x \times 2y$	**d** $8p \times 4q$
e $6a \times b$	**f** $5x \times x$	**g** $5a \times 3a$	**h** $ab \times ac$
i $3pq \times 2p$	**j** $5mn \times mp$	**k** $4mn \times \frac{1}{2}n$	**l** $9b \times a^2$
m $6a^2 \times (-7a)$	**n** $-5x \times -2x$	**o** $x \times 2y \times 3x$	**p** $14ab \times (-\frac{1}{2}ab)$
q $(-ab) \times (-bc)$	**r** $2k \times 3k \times 4k$	**s** $-2 \times 7x \times -5y$	**t** $\frac{1}{4}m \times 4n \times (-p)$

3 Simplify:

a $12x \div 4$	**b** $12x \div 4x$	**c** $9x^2 \div 3$	**d** $8x \div 8x$
e $15m \div 10n$	**f** $32a \div 12b$	**g** $5 \div 20a$	**h** $48ab \div 6b$
i $a \div 3a$	**j** $45ab \div 20ba$	**k** $-20p \div 4p$	**l** $-xy \div xz$
m $14a \div (-a)$	**n** $(-15x) \div (-5xy)$	**o** $-28mnp \div 7mp$	**p** $8a^2b \div 16ab^2$

4 Simplify:

a $mn \times np$	**b** $7 + m + 6 + 3m$	**c** $14 - 2a + 5$	**d** $5x^2 \times 0$
e $3xy \times 2yx$	**f** $8x^2 + 2x + 7x^2 + 3x$	**g** $3 \times 4y \times 5z$	**h** $-4x \times 7x$
i $15ab - 9ba + ab$	**j** $6m - 7m$	**k** $8b + 3b - 11b$	**l** $18ab \div 9bc$
m $x \div 3x$	**n** $2pq \times 9pq$	**o** $3a + b + 2a - c$	**p** $-3y \times (-5z)$
q $\frac{1}{2}y + \frac{1}{2}y$	**r** $m + n - m + n$	**s** $3a \times 2b \times c$	**t** $15at \div 10tx$

5 Write the simplest expression for:

a $(2a + 3a) \times 4$	**b** $(10x - 3x) \div 7$	**c** $(9b - 3b) \times 2$	**d** $(3m + 9m) \div 4$
e $12x \div (2x + x)$	**f** $5a \times (10a + 2a)$	**g** $3m \times (10m - 9m)$	**h** $15y \div (9y - 2y)$
i $5a \times 7 \div a$	**j** $8x \times 4y \div 2xy$	**k** $10a \div 5 \times 3a$	**l** $9xy \div 3x \times 2y$
m $2x + 3x \times 4$	**n** $5x \times 3x + 10x^2$	**o** $20y - 5 \times 2y$	**p** $18m - 12m \div 6$
q $3 \times 2n + 5n \times 4$	**r** $7x + 3 \times 2x - 10x$	**s** $8x \div 4 - x$	**t** $11m + 18m \div 2$
u $\dfrac{6 \times 3x}{2x \times 5}$	**v** $\dfrac{3p + 2p - 1p}{2 \times 2p}$	**w** $\dfrac{11y - y}{6y + 4y}$	**x** $\dfrac{5a \times 4b \times 2c}{10c \times b \times 8c}$

3:02 Algebraic fractions

3:02A Addition and subtraction

PREP QUIZ 3:02A

Answer the following:

1 $\frac{3}{5}+\frac{1}{5}$ 2 $\frac{7}{10}-\frac{3}{10}$ 3 $\frac{1}{4}+\frac{1}{3}$

4 $\frac{1}{2}+\frac{3}{8}$ 5 $\frac{2}{5}-\frac{1}{4}$ 6 $\frac{7}{12}-\frac{1}{3}$

Simplify the expressions:

7 $7x+4x$ 8 $3ab+ab$ 9 $6x-5x$ 10 $9a-a$

Rewrite each fraction as two equivalent fractions with a common denominator, then add or subtract the numerators.

WORKED EXAMPLES

1 $\frac{3x}{5}+\frac{2x}{5}=\frac{3x+2x}{5}$

$=\frac{^{1}\cancel{5}x}{_{1}\cancel{5}}$

$=x$

2 $\frac{5}{a}-\frac{3}{a}=\frac{5-3}{a}$

$=\frac{2}{a}$

3 $\frac{x}{3}+\frac{x}{2}=\frac{x\times 2}{3\times 2}+\frac{x\times 3}{2\times 3}$

$=\frac{2x}{6}+\frac{3x}{6}$

$=\frac{5x}{6}$

4 $\frac{4a}{5}-\frac{a}{3}=\frac{4a\times 3}{5\times 3}-\frac{a\times 5}{3\times 5}$

$=\frac{12a}{15}-\frac{5a}{15}$

$=\frac{7a}{15}$

5 $\frac{5m}{8}+\frac{m}{2}=\frac{5m}{8}+\frac{m\times 4}{2\times 4}$

$=\frac{5m}{8}+\frac{4m}{8}$

$=\frac{9m}{8}$

6 $\frac{3x}{4}-\frac{2y}{3}=\frac{9x}{12}-\frac{8y}{12}$

$=\frac{9x-8y}{12}$

7 $\frac{9}{x}+\frac{2}{3x}=\frac{27}{3x}+\frac{2}{3x}$

$=\frac{29}{3x}$

8 $\frac{5a}{2x}-\frac{2a}{3x}=\frac{15a}{6x}-\frac{4a}{6x}$

$=\frac{11a}{6x}$

Exercise 3:02A

Foundation worksheet 3:02A
Simplifying algebraic fractions

1 Simplify the following.

a $\frac{3a}{2}+\frac{a}{2}$ b $\frac{3x}{5}-\frac{2x}{5}$ c $\frac{a}{3}+\frac{4a}{3}$ d $\frac{9m}{10}-\frac{3m}{10}$

e $\frac{x}{4}+\frac{y}{4}$ f $\frac{5a}{3}-\frac{2b}{3}$ g $\frac{2}{a}+\frac{3}{a}$ h $\frac{7}{x}+\frac{1}{x}$

i $\frac{3}{y}-\frac{2}{y}$ j $\frac{9}{m}-\frac{1}{m}$ k $\frac{5a}{x}+\frac{2a}{x}$ l $\frac{2x}{y}-\frac{3x}{y}$

m $\frac{5}{3n}+\frac{7}{3n}$ n $\frac{3}{2x}-\frac{1}{2x}$ o $\frac{8a}{5b}+\frac{2a}{5b}$ p $\frac{7m}{4x}-\frac{3m}{4x}$

2 Reduce each of these expressions to its simplest form.

a $\frac{x}{3}+\frac{x}{5}$ b $\frac{a}{2}+\frac{a}{5}$ c $\frac{y}{3}-\frac{y}{4}$ d $\frac{m}{2}-\frac{m}{4}$

e $\frac{2a}{3}+\frac{a}{2}$ f $\frac{5x}{3}+\frac{2x}{4}$ g $\frac{3n}{8}-\frac{n}{4}$ h $\frac{4p}{5}-\frac{3p}{10}$

i $\frac{x}{4}+\frac{y}{3}$ j $\frac{2a}{3}-\frac{3b}{2}$ k $\frac{3m}{5}+\frac{n}{2}$ l $\frac{k}{6}-\frac{21}{4}$

m $\frac{2}{x}+\frac{4}{3x}$ n $\frac{1}{3a}+\frac{2}{4a}$ o $\frac{7}{2m}-\frac{2}{5m}$ p $\frac{5}{8x}-\frac{1}{2x}$

q $\frac{2a}{3x}+\frac{3a}{2x}$ r $\frac{x}{3m}-\frac{2x}{m}$ s $\frac{5m}{2n}+\frac{3m}{4n}$ t $\frac{2x}{3a}+\frac{y}{4a}$

FUN SPOT 3:02 — TRY THIS MATHS-WORD PUZZLE

Hidden in the maze of letters there are many words used in mathematics. Make a list of the words you find and, at the same time, put a line through the letters you use. Words may be written in any direction: up, down, backwards, even diagonally. Also, a letter may be used more than once, but you cannot change direction in order to form a word (i.e. the letters must be in a straight line).

When you have found all the words there should be four letters that have not been used. These four letters can be arranged to form another 'mystery' maths word.

R	E	T	E	M	A	I	D	C
L	E	L	C	R	I	C	G	U
E	T	C	X	R	Y	O	H	B
L	E	I	T	R	A	N	T	E
L	S	Q	U	A	R	E	G	L
A	L	P	L	A	N	E	N	A
R	O	L	A	I	I	G	E	U
A	P	U	L	C	N	A	L	Q
P	E	S	M	M	E	T	R	E

3:02B Multiplication and division

PREP QUIZ 3:02B

Answer the following:

1 $\frac{1}{2} \times \frac{3}{4}$ **2** $\frac{2}{5} \times \frac{3}{4}$ **3** $\frac{4}{9} \times \frac{3}{8}$

4 $\frac{1}{2} \div \frac{3}{4}$ **5** $\frac{3}{5} \div \frac{3}{10}$ **6** $\frac{2}{3} \div \frac{5}{4}$

Simplify these expressions.

7 $5 \times 6x$ **8** $3a \times 2a$ **9** $15a \div 5$ **10** $12ab \div 6b$

When *multiplying*:
- cancel any common factors, then
- multiply the numerators together and multiply the denominators together.

When *dividing*:
- turn the second fraction upside down, then
- multiply as above (invert and multiply).

WORKED EXAMPLES

1 $\frac{2}{a} \times \frac{5}{b} = \frac{2 \times 5}{a \times b}$

$= \frac{10}{ab}$

2 $\frac{5}{x} \times \frac{x}{10} = \frac{\cancel{5}^{1}}{\cancel{x}_{1}} \times \frac{\cancel{x}^{1}}{\cancel{10}_{2}}$

$= \frac{1 \times 1}{1 \times 2}$

$= \frac{1}{2}$

3 $\frac{3b}{2} \times \frac{4}{5b} = \frac{3\cancel{b}^{1}}{{}_{1}\cancel{2}} \times \frac{\cancel{4}^{2}}{5\cancel{b}_{1}}$

$= \frac{3 \times 2}{1 \times 5}$

$= \frac{6}{5}$ or $1\frac{1}{5}$

4 $\frac{ab}{2} \div \frac{b}{5} = \frac{a\cancel{b}^{1}}{2} \times \frac{5}{\cancel{b}_{1}}$

$= \frac{a \times 5}{2 \times 1}$

$= \frac{5a}{2}$

5 $\frac{8a}{3b} \div \frac{2a}{9b} = \frac{{}^{4}\cancel{8}\,\cancel{a}}{{}_{1}\cancel{3}\,\cancel{b}} \times \frac{{}^{3}\cancel{9}\,\cancel{b}}{{}_{1}\cancel{2}\,\cancel{a}}$

$= \frac{4 \times 3}{1 \times 1}$

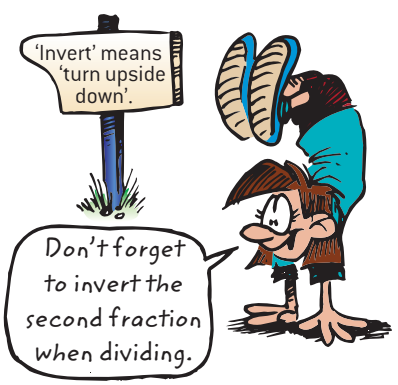
$= 12$

6 $\frac{2a}{3b} \times \frac{b}{4c} \div \frac{10a}{9c} = \frac{4a}{3b} \times \frac{b}{2c} \times \frac{9c}{10a}$

$= \frac{{}^{1}\cancel{2}\,\cancel{4}\,\cancel{a}}{{}_{1}\cancel{3}\,\cancel{b}} \times \frac{{}^{1}\cancel{b}}{{}_{1}\cancel{2}\,\cancel{c}} \times \frac{{}^{3}\cancel{9}\,\cancel{c}}{{}_{5}\cancel{10}\,\cancel{a}}$

$= \frac{1 \times 1 \times 3}{1 \times 1 \times 5}$

$= \frac{3}{5}$



Exercise 3:02B

Foundation worksheet 3:02B
Simplifying algebraic fractions

1 Simplify these products.

a $\frac{x}{2} \times \frac{y}{3}$ **b** $\frac{a}{4} \times \frac{b}{3}$ **c** $\frac{m}{2} \times \frac{m}{5}$ **d** $\frac{a}{4} \times \frac{a}{10}$

e $\frac{3}{a} \times \frac{4}{m}$ **f** $\frac{2}{x} \times \frac{1}{y}$ **g** $\frac{1}{p} \times \frac{4}{p}$ **h** $\frac{1}{n} \times \frac{1}{3n}$

i $\frac{p}{q} \times \frac{x}{y}$ **j** $\frac{2}{a} \times \frac{a}{4}$ **k** $\frac{m}{5} \times \frac{10}{n}$ **l** $\frac{3x}{5} \times \frac{2}{9x}$

m $\frac{ab}{3} \times \frac{2}{b}$ **n** $\frac{x}{y} \times \frac{y}{x}$ **o** $\frac{6m}{5a} \times \frac{15a}{2m}$ **p** $\frac{8x}{5p} \times \frac{2a}{3x}$

2 Simplify these divisions.

a $\frac{m}{2} \div \frac{m}{4}$ **b** $\frac{n}{3} \div \frac{n}{5}$ **c** $\frac{5n}{3} \div \frac{2n}{9}$ **d** $\frac{x}{5} \div \frac{3x}{10}$

e $\frac{5}{a} \div \frac{2}{a}$ **f** $\frac{3}{2m} \div \frac{1}{3m}$ **g** $\frac{a}{b} \div \frac{2a}{b}$ **h** $\frac{3x}{5y} \div \frac{x}{10y}$

i $\frac{a}{b} \div \frac{x}{y}$ **j** $\frac{2p}{3q} \div \frac{8p}{9q}$ **k** $\frac{10k}{3n} \div \frac{2k}{9n}$ **l** $\frac{a}{2} \div \frac{a}{3}$

m $\frac{xy}{2} \div \frac{y}{4}$ **n** $\frac{b}{2} \div \frac{ab}{6}$ **o** $\frac{xy}{c} \div \frac{y}{cx}$ **p** $\frac{9a}{b} \div \frac{4a}{3b}$

3 Simplify these expressions.

a $\frac{a}{3} \div \frac{12}{5a}$ **b** $\frac{2}{p} \times \frac{p}{3}$ **c** $\frac{15}{x} \div 5$ **d** $3b \div \frac{6}{b}$

e $\frac{xy}{z} \times \frac{2z}{x}$ **f** $\frac{ab}{c} \div \frac{a}{c}$ **g** $\frac{9m}{2} \times \frac{4m}{3}$ **h** $\frac{2x}{y} \div \frac{x}{2y}$

i $\frac{4}{pq} \times \frac{p}{q}$ **j** $\frac{3}{a} \times \frac{2}{b}$ **k** $\frac{4ab}{x} \times \frac{xy}{2ac}$ **l** $\frac{9bc}{2a} \div \frac{6b}{4a}$

m $\frac{2}{x} \times \frac{x}{3} \times \frac{9}{4}$ **n** $\frac{b}{c} \times \frac{c}{a} \times \frac{a}{b}$ **o** $\frac{8bc}{3a} \times \frac{9a}{b} \times \frac{1}{4c}$ **p** $\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3}$

q $\frac{2m}{3n} \times \frac{5n}{6p} \div \frac{8m}{9p}$ **r** $\frac{6a}{15} \div \frac{3a}{10b} \times \frac{3}{4b}$ **s** $\frac{xy}{yz} \times \frac{xz}{ty} \times \frac{tz}{tx}$ **t** $\frac{2a}{3b} \div \frac{3a}{2b} \div \frac{4a}{9b}$

Algebra is important in the design and construction of buildings.

3:03 Simplifying expressions with grouping symbols

PREP QUIZ 3:03

Simplify:

1 $7x + 3x$ **2** $4a^2 - a^2$ **3** $4x + 3 + 2x + 5$

4 $2x + 7 - x - 5$ **5** $3y^2 + 5y + 2y^2 - y$ **6** $7 - 3a + 6 + 5a$

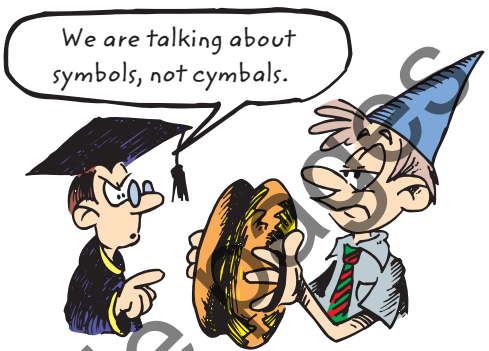
Expand:

7 $3(x - 7)$ **8** $9(2 - 5y)$ **9** $2a(a + 3)$ **10** $-5(x + 7)$

The two most commonly used grouping symbols are:

parentheses	( )
brackets	[ ]

$a(b \pm c) = ab \pm ac$



To expand an expression, such as $a(b + c)$, each term inside the grouping symbols is multiplied by the term outside the grouping symbols.

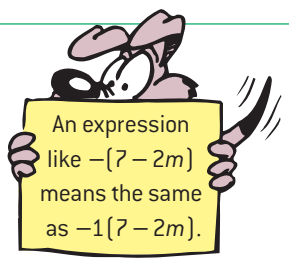
WORKED EXAMPLES

1 $p(p + 3) = p \times p + p \times 3$
$= p^2 + 3p$

2 $3a(5 - 2a) = 3a \times 5 - 3a \times 2a$
$= 15a - 6a^2$

3 $-5(3x + 4) = (-5) \times 3x + (-5) \times 4$
$= -15x - 20$

4 $-(7 - 2m) = (-1) \times 7 - (-1) \times 2m$
$= -7 + 2m$



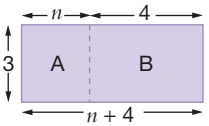
5 $x(x - 1) - x^2 + 5 = x^2 - x - x^2 + 5$
$= -x + 5$

6 $2a(a + b) - a(3a - 4b) = 2a^2 + 2ab - 3a^2 + 4ab$
$= 6ab - a^2$

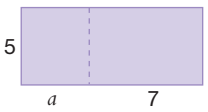
Exercise 3:03

P Foundation worksheet 3:03
Grouping symbols
Challenge worksheet 3:03
Fractions and grouping symbols

1 The area of rectangle A = $3 \times n = 3n$
The area of rectangle B = $3 \times 4 = 12$
The area of the combined rectangle = $3(n + 4)$
$\therefore\ 3(n + 4) = 3n + 12$



Following the example above, write the area of each of the following rectangles in two ways.

a


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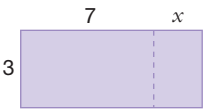

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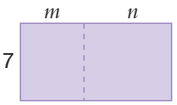

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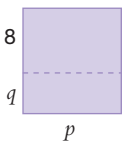

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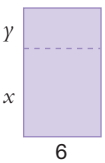

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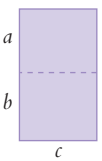


2 Expand:

a $2(x + 3)$	**b** $3(a + 5)$	**c** $5(x - 1)$	**d** $7(m - 3)$
e $2(3a + 2b)$	**f** $5(x + y)$	**g** $7(3x - 5y)$	**h** $6(7m - 8n)$
i $x(x + 7)$	**j** $a(a - 1)$	**k** $m(m + 10)$	**l** $n(n - 3)$
m $3a(2a - 1)$	**n** $9x(2x + 7)$	**o** $8p(2 - 5p)$	**p** $7q(3 + 2q)$
q $x(a + b)$	**r** $y(2y + x)$	**s** $2m(m + n)$	**t** $5a(2a + 3b)$

3 Expand by removing the parentheses.

a $-2(x + 3)$	**b** $-3(a + 5)$	**c** $-2(y - 1)$	**d** $-5(p - 3)$
e $-7(3a + 2)$	**f** $-5(2x - 1)$	**g** $-7(3 + 2m)$	**h** $-4(7 - x)$
i $-(a + 1)$	**j** $-(3x + 7)$	**k** $-(8 - 2p)$	**l** $-(3a + 2b)$
m $-x(x + 10)$	**n** $-y(5 - y)$	**o** $-3x(2x + 7)$	**p** $-10n(8m - n)$

4 Simplify:

a $2(a + 3) + 5a + 2$	**b** $3(x + 5) + 7x - 8$	**c** $5(y - 2) + 3y + 7$
d $4(a - 1) + 6a - 5$	**e** $3(p + 2) - 2p + 4$	**f** $10(m + 3) - 11m - 15$
g $5a + 6 + 2(a + 7)$	**h** $2x + 7 + 5(x - 1)$	**i** $7n - 4 + 3(n - 1)$
j $4h - 1 + 7(h + 2)$	**k** $6x + 2(x + 1) + 5$	**l** $4y + 6(y + 2) - 10$
m $3a + 10 - 2(a + 1)$	**n** $10m + 4 - 5(m + 4)$	**o** $6 - 2(y - 4) + 4y$
p $20 - 4(x - 2) + 5x$	**q** $5x + 7 + 2(2x + 7)$	**r** $4(3a + 1) - 10a + 2$
s $10m + 6 - 3(2m - 1)$	**t** $8x - 3(1 - 2x) + 10$	

5 Simplify each expression by expanding the grouping symbols and then collecting like terms.

a $3(x + 2) + 2(x + 1)$	**b** $5(y + 2) + 3(y + 4)$	**c** $2(a + 1) + 5(a - 1)$
d $8(m - 3) + 5(m + 2)$	**e** $4(3x + 2) + 5(x - 4)$	**f** $6(x + 7) + 2(2x - 1)$
g $5(x + 7) - 3(x + 4)$	**h** $6(m + 1) - 3(m + 2)$	**i** $9(a + 5) - 7(a - 3)$
j $5(n - 5) - 3(n + 7)$	**k** $x(x + 3) + 3(x + 1)$	**l** $a(a + 3) + 7(a - 3)$
m $m(m + 3) - 4(m + 3)$	**n** $t(t - 5) - 4(t - 5)$	**o** $a(a + 2b) + a(2a + b)$
p $x(x + y) + y(x + y)$		

FUN SPOT 3:03 — WHAT IS TAKEN OFF LAST BEFORE YOU GET INTO BED?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Write the expression that is:

H 2 more than x
L twice x
K half of x
N 2 less than x
R the square of x

Find the value of $u + 10t$ if:

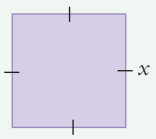
R $u = 12, t = 4$
R $u = -10, t = 2$

Simplify:

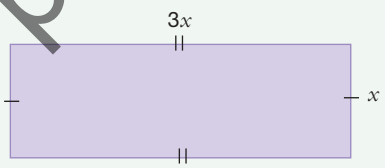
T $x \div x$
T $11x - x$
T $x + x + x$
F $x \times x \times 2$
F $x^2 - x - x$
F $7x + 3y - x + 3y$
F $3x \times 2y$
A $-2x \times -3x$
A $12x \div 2$
Y $12m \div 4m$
O $14 - 3x + 6$
E $\frac{x}{2} + \frac{x}{2}$
O $\frac{2x}{5} - \frac{x}{10}$
O $\frac{5}{x} \times \frac{x}{10}$
O $x - \frac{x}{10}$
U $x \div \frac{1}{8}$
E $\frac{x}{3} \div \frac{x}{6}$
E $\frac{x}{2} \div 2$

Write an expression for the perimeter of:

E



E



3	$\frac{9x}{10}$	$8x$	x^2	$6xy$	$\frac{x}{4}$	$4x$	1	$6x^2$	52	x	$3x$	$6x$	$\frac{x}{2}$	$8x$	$x - 2$

$\frac{3x}{10}$	$6x + 6y$	$2x^2$	$10x$	$x + 2$	2	$x^2 - 2x$	$2x$	$\frac{1}{2}$	$20 - 3x$	10

3:04 Further algebraic fractions

PREP QUIZ 3:04

Expand:

1 $3(x+5)$ **2** $5(m-7)$ **3** $2(6y+1)$ **4** $9(2p-3q)$

Find the lowest common multiple (LCM) of:

5 4 and 5 **6** 4 and 6 **7** 4 and 8

Simplify:

8 $\frac{x}{4}+\frac{x}{5}$ **9** $\frac{a}{4}-\frac{a}{6}$ **10** $\frac{m}{8}+\frac{n}{4}$

In Section 3:02A all of the fractions had a single term in the numerator. If there is more than one term in the numerator we use the skills met in the last section for expanding grouping symbols.

WORKED EXAMPLES

Simplify the following:

1 $\frac{x+1}{2}+\frac{x+4}{3}=\frac{(x+1)\times 3}{2\times 3}+\frac{(x+4)\times 2}{3\times 2}$

$=\frac{3(x+1)}{6}+\frac{2(x+4)}{6}$

$=\frac{3x+3}{6}+\frac{2x+8}{6}$

$=\frac{5x+11}{6}$

2 $\frac{2a+5}{3}-\frac{3a}{5}=\frac{(2a+5)\times 5}{3\times 5}-\frac{3a\times 3}{5\times 3}$

$=\frac{5(2a+5)}{15}-\frac{9a}{15}$

$=\frac{10a+25}{15}-\frac{9a}{15}$

$=\frac{a+25}{15}$

LCM of 3 and 5 is 15

3 $\frac{2x+5}{6}+\frac{x-4}{3}=\frac{2x+5}{6}+\frac{2(x-4)}{6}$

$=\frac{2x+5}{6}+\frac{2x-8}{6}$

$=\frac{4x-3}{6}$

4 $\frac{m}{3}-\frac{m+1}{4}=\frac{4m}{12}-\frac{3(m+1)}{12}$

$=\frac{4m-3(m+1)}{12}$

$=\frac{4m-3m-3}{12}$

$=\frac{m-3}{12}$

Note the minus sign!

Exercise 3:04

1 Simplify the following.

a $\frac{x+4}{2}+\frac{x}{3}$ b $\frac{a}{5}+\frac{a+3}{2}$ c $\frac{n-1}{3}+\frac{2n}{7}$

d $\frac{x+4}{5}+\frac{x+1}{3}$ e $\frac{m+4}{5}+\frac{m-3}{7}$ f $\frac{p+1}{3}-\frac{p}{4}$

2 Simplify:

a $\frac{a+4}{4}+\frac{a}{2}$ b $\frac{w}{5}+\frac{w+2}{10}$ c $\frac{d-1}{3}+\frac{5d}{6}$

d $\frac{x+3}{4}+\frac{x+2}{6}$ e $\frac{m+2}{8}+\frac{m-1}{6}$ f $\frac{q+1}{6}-\frac{q}{9}$

3 Simplify these expressions.

a $\frac{2x+3}{2}+\frac{x+4}{3}$ b $\frac{3a+5}{5}+\frac{2a+1}{4}$ c $\frac{5n-1}{3}+\frac{4n+3}{4}$

d $\frac{4t-4}{5}+\frac{3t-1}{3}$ e $\frac{7u-2}{8}+\frac{2u-3}{3}$ f $\frac{5q+2}{10}+\frac{2q-1}{9}$

g $\frac{2a+3}{6}+\frac{a+1}{3}$ h $\frac{3n+5}{8}+\frac{n-1}{4}$ i $\frac{5g-1}{3}+\frac{4g+3}{9}$

j $\frac{4y-1}{6}+\frac{3y-1}{8}$ k $\frac{5-2x}{6}+\frac{2x-3}{10}$ l $\frac{1-2a}{8}+\frac{2-a}{10}$

4 Simplify these fractions.

a $\frac{x}{2}-\frac{x+1}{3}$ b $\frac{3y}{5}-\frac{y+3}{2}$ c $\frac{n+1}{3}-\frac{n+2}{5}$

d $\frac{x+4}{5}-\frac{x-1}{6}$ e $\frac{m-1}{5}-\frac{m-3}{7}$ f $\frac{2p-1}{3}-\frac{p-4}{4}$

g $\frac{2q+3}{6}-\frac{q+1}{3}$ h $\frac{3t+2}{8}-\frac{t-1}{4}$ i $\frac{5w-2}{3}-\frac{4w-6}{9}$

j $\frac{4+y}{6}-\frac{2y-1}{8}$ k $\frac{1-2x}{6}-\frac{2-3x}{10}$ l $\frac{3-2z}{8}-\frac{2-3z}{10}$

5 Simplify these expressions.

a $\frac{x+3}{2}+\frac{x+4}{3}+\frac{x+1}{5}$ b $\frac{3a+5}{5}+\frac{2a+1}{4}+\frac{a-2}{3}$ c $\frac{2m+3}{3}+\frac{m+4}{6}+\frac{3m+1}{2}$

d $\frac{3y+1}{5}+\frac{2y-1}{4}-\frac{y-3}{2}$ e $\frac{5x-1}{4}-\frac{x+2}{3}+\frac{2x+1}{6}$ f $\frac{3z+4}{5}-\frac{2z-1}{4}-\frac{4-z}{10}$

CHALLENGE 3:04 THESE ARE A FRACTION TRICKIER!

Consider how you simplified expressions in Exercise 3:02A that involved pronumerals in the denominator, and attempt to simplify the following expressions.

1 $\frac{x+4}{2x}+\frac{x+1}{3}$ 2 $\frac{x+4}{2x}+\frac{x+1}{3x}$ 3 $\frac{a+5}{5x}+\frac{a-2}{2x}$ 4 $\frac{x+5}{5x}+\frac{a+1}{2a}$

5 $\frac{x-4}{3x}-\frac{x-6}{5x}$ 6 $\frac{3n+4}{4n}-\frac{5-n}{6n}$ 7 $\frac{a+b}{a}+\frac{a-b}{b}$ 8 $\frac{3x-5y}{2x}-\frac{2y+x}{3y}$

3:05 Factorising using common factors

PREP QUIZ 3:05

Expand:

1 $2(x + 5)$ **2** $x(4x - 1)$

3 $3a(2a + 7)$ **4** $2m(m + 3n)$

Write the factors of:

5 12 **6** 20 **7** 30

Write the highest common factor (HCF) of:

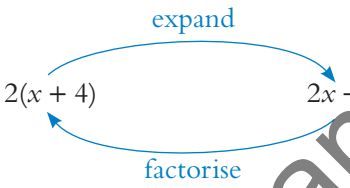
8 12 and 20 **9** 12 and 30 **10** 20 and 30

A factor of a given number is a number that divides into the given number with no remainder.
e.g. {1, 2, 3, 6, 9, 18} is the set of factors of 18.

To factorise an algebraic expression we must determine the highest common factor (HCF) of the terms and insert grouping symbols, usually parentheses.

If we expand the expression $2(x + 4)$, we obtain $2x + 8$. To factorise $2x + 8$ we simply reverse this procedure. The highest common factor of $2x$ and 8 is 2, so 2 is written outside the parentheses. The remainder is written inside the parentheses.

$2x + 8 = 2(x + 4)$



WORKED EXAMPLES

1 $2x + 6 = 2 \times x + 2 \times 3$ (HCF is 2)
$= 2(x + 3)$

2 $10x + 15y = 5 \times 2x + 5 \times 3y$ (HCF is 5)
$= 5(2x + 3y)$

3 $2mn + m = m \times 2n + m \times 1$ (HCF is m)
$= m(2n + 1)$

4 $9ax + 6a = 3a \times 3x + 3a \times 2$ (HCF is $3a$)
$= 3a(3x + 2)$

5 $5x^2 - 10xy = 5x \times x - 5x \times 2y$ (HCF is $5x$)
$= 5x(x - 2y)$

6 $-4a - 8 = -4 \times a + -4 \times 2$ (HCF is -4)
$= -4(a + 2)$

7 $-a^2 + 3a = -a \times a - (-a) \times 3$ (HCF is $-a$)
$= -a(a - 3)$

8 $4xy + 6x^2 - 2xz$ (HCF is $2x$)
$= 2x \times 2y + 2x \times 3x - 2x \times z$
$= 2x(2y + 3x - z)$

$ab + ac = a(b + c)$ and $ab - ac = a(b - c)$

Exercise 3:05

Foundation worksheet 3:05
Common factors

1 Complete the following.

a $4x + 8 = 4(\quad)$ **b** $6a + 18 = 6(\quad)$ **c** $8a - 12 = 4(\quad)$
d $5x + 10y = 5(\quad)$ **e** $21x - 14y = 7(\quad)$ **f** $12pq - 15x = 3(\quad)$
g $mn - np = n(\quad)$ **h** $ab - ac = a(\quad)$ **i** $2ax + 4ay = 2a(\quad)$
j $y^2 - 5y = y(\quad)$ **k** $5at - 3a^2t = at(\quad)$ **l** $7mn - mn^2 = mn(\quad)$

2 Factorise the following by taking out the HCF.

a $2x + 10$ **b** $6a + 4$ **c** $7y + 21$ **d** $28 + 4x$
e $27 - 3y$ **f** $24x + 6$ **g** $9x - 45$ **h** $16 - 12a$
i $9x + 3y$ **j** $5a + 10b$ **k** $15m - 20n$ **l** $4b - 6a$
m $mp + mn$ **n** $ax + ay$ **o** $x^2 + xy$ **p** $p^2 - pq$
q $ap + 3a$ **r** $5x + ax$ **s** $4m - mn$ **t** $xt - t$

3 Factorise completely.

a $3ax + 6ay$ **b** $5mn - 10mp$ **c** $4ab - 6bc$ **d** $9pq - 6qr$
e $5x^2 - 10xy$ **f** $3ab + 6a^2$ **g** $10m^2 - 4mn$ **h** $12x^2 + 4xy$
i $abc + bcd$ **j** $apq - bpq$ **k** $xyz + xy$ **l** $mn - mnp$
m $x^2a - xay$ **n** $5ax - 10xy$ **o** $a^2p - 5ap$ **p** $xy^2 + xyz$
q $10ab - 15bc$ **r** $5x^2y - 3xy^2$ **s** $ap^2 - a^2p$ **t** $5ab - a^2b^2$

4 Factorise the following by taking out the negative common factor.

a $-2a - 6$ **b** $-5x - 15$ **c** $-8m - 12$ **d** $-10x - 5$
e $-8x + 4$ **f** $-3n + 9$ **g** $-7y + 35$ **h** $-6a + 4$
i $-x^2 - 3x$ **j** $-m^2 - m$ **k** $-3x^2 + 2x$ **l** $-5y^2 + 10y$
m $-4p - p^2$ **n** $-3x - 2x^2$ **o** $-m + 7m^2$ **p** $-4a + 18a^2$

5 Factorise each of the following.

a $ab + ac + ad$ **b** $3x + xy + xz$ **c** $m^2 - 3m + mn$ **d** $7a - ab + a^2$
e $p^2 + pq - 5p$ **f** $2x + 4y - 6z$ **g** $10a - 5b + 15c$ **h** $9x^2 + 6x - 12$
i $8 - 4x + 6x^2$ **j** $25 + 15y - 20y^2$ **k** $x^2y - 3xy + x$ **l** $2ab - 4ac + 10a$
m $3x^2 + 6xy - 9x$ **n** $x^2y + xy + xy^2$ **o** $a^2b^2 + 3a^2b + 2ab^2$ **p** $mn + 4m^2n - 8mn^2$

6 Examine this example.

If $3(x + 2) - a(x + 2)$ is to be factorised then $(x + 2)$ is common to both terms, so it can be taken out as a common factor:

$3(x + 2) - a(x + 2) = (x + 2)(3 - a)$

Similarly:

$x(a + 1) + (a + 1) = x(a + 1) + 1(a + 1)$
$= (a + 1)(x + 1)$

Note:
$[a+1][x+1] = [x+1][a+1]$

Now factorise these similar types.

a $a(a + 2) + 3(a + 2)$ **b** $m(m + 2) + 4(m + 2)$ **c** $x(x - 1) + 5(x - 1)$
d $b(b + 1) - 5(b + 1)$ **e** $7(y - 2) - y(y - 2)$ **f** $t(t - 7) - 9(t - 7)$
g $4(2m - 3) + 3m(2m - 3)$ **h** $2x(7x + 1) - 5(7x + 1)$ **i** $x(a + 3) - (a + 3)$
j $y(2y - 1) - (2y - 1)$ **k** $p(p - 3) - 3(p - 3)$ **l** $(5x + 3) - x(5x + 3)$

3:06 Binomial products

PREP QUIZ 3:06

Simplify:

1 $5x + 7x$ **2** $2a - a$ **3** $x^2 + 3x - 5x + 3$

Expand:

4 $2(x + 5)$ **5** $x(x - 2)$ **6** $-3(a + 1)$ **7** $-y(5 - y)$

Expand and simplify:

8 $x(x + 1) + 3(x + 1)$ **9** $5(a + 5) - a(a + 5)$ **10** $2x(3x - 2) - 5(3x + 2)$

A binomial expression is one that contains two terms, such as $2x - 7$ or $a + b$.
A binomial product is the product of two such expressions, such as $(2x + 7)(a + 5)$.

Long multiplication is like a binomial product.

$26 \times 19 = (20 + 6) \times (10 + 9)$
$= 20(10 + 9) + 6(10 + 9)$
$= [20 \times 10] + [20 \times 9] + [6 \times 10] + [6 \times 9]$
$= 200 + 180 + 60 + 54$
$= 494$

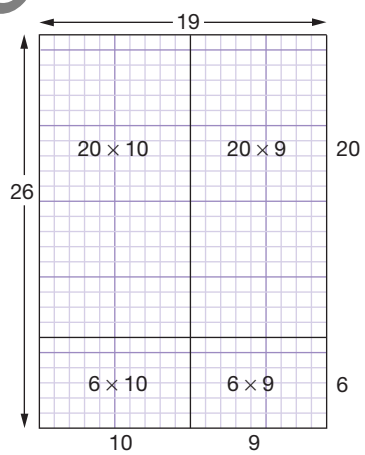
Each part of one number must multiply each part of the other.

$(20 + 6)\quad (10 + 9)$

As you can see, the products form a 'face'.

$$\begin{array}{r} 19 \\ \times\ 26 \\ \hline 54 \\ 60 \\ 180 \\ 200 \\ \hline 494 \\ \hline \end{array}$$

26×19 by areas

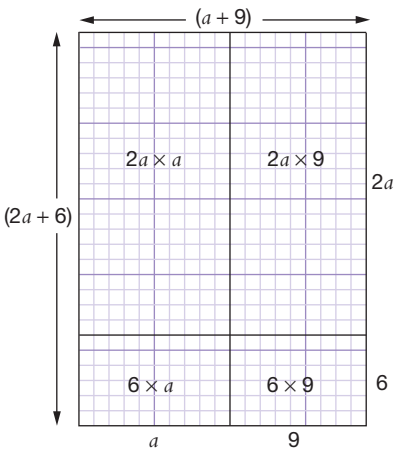


Multiplying binomial expressions

The expansion of binomial products may also be demonstrated by considering the area of a rectangle. This rectangle has dimensions $(2a + 6)$ and $(a + 9)$.

- The area of the whole rectangle must be equal to the sum of the four smaller areas.
- Area $= (2a + 6)(a + 9)$
  $= 2a(a + 9) + 6(a + 9)$
  $= 2a^2 + 18a + 6a + 54$
  $= 2a^2 + 24a + 54$
- We can see that the product of two binomials yields four terms. Often two of these may be added together to simplify the answer.

$(2a + 6)(a + 9)$ by areas



WORKED EXAMPLES

1 $(a+2)(b+4) = a(b+4) + 2(b+4)$
$= ab + 4a + 2b + 8$

2 $(a-2)(a+7) = a(a+7) - 2(a+7)$
$= a^2 + 7a - 2a - 14$
$= a^2 + 5a - 14$

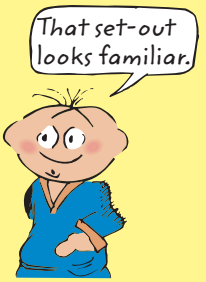
3 $(x+2y)(2x+y) = x(2x+y) + 2y(2x+y)$
$= 2x^2 + xy + 4xy + 2y^2$
$= 2x^2 + 5xy + 2y^2$

4 $(1-x)(x-3) = 1(x-3) - x(x-3)$
$= x - 3 - x^2 + 3x$
$= 4x - x^2 - 3$

You should notice that each term in the first binomial is multiplied by each term in the second; i.e.

$2x^2$ -15
$(x+5)(2x-3)$
$10x$
$-3x$

$= 2x^2 + 10x - 3x - 15$
$= 2x^2 + 7x - 15$



$(a+b)(c+d) = a(c+d) + b(c+d)$
$= ac + ad + bc + bd$

Exercise 3:06

1 Expand the following binomial products.

a $(a+2)(b+3)$	**b** $(x+1)(y+4)$	**c** $(m+7)(n+5)$	**d** $(a+3)(x+2)$
e $(p+5)(q+4)$	**f** $(2x+1)(y+3)$	**g** $(a+6)(3p+2)$	**h** $(4x+1)(2y+3)$
i $(3a+1)(2b-7)$	**j** $(7x+5)(2p+1)$	**k** $(5p+3)(x-4)$	**l** $(2x+y)(a+2b)$

2 Expand the following and collect the like terms.

a $(a+2)(a+3)$	**b** $(x+1)(x+5)$	**c** $(n+3)(n+4)$	**d** $(p+2)(p+5)$
e $(m+1)(m-3)$	**f** $(y+7)(y-2)$	**g** $(x+1)(x-6)$	**h** $(t+2)(t-4)$
i $(x-2)(x-4)$	**j** $(n-7)(n-1)$	**k** $(a-6)(a-3)$	**l** $(x-10)(x-9)$
m $(y-11)(y+7)$	**n** $(a-2)(a+1)$	**o** $(x-8)(x-8)$	**p** $(m-9)(m-2)$
q $(a-3)(a+3)$	**r** $(x-7)(x+3)$	**s** $(y+12)(y+5)$	**t** $(a-8)(a+8)$
u $(q+5)(q+5)$	**v** $(x-1)(x-9)$	**w** $(t+3)(t+10)$	**x** $(k-8)(k+11)$

3 Find these products and simplify.

a $(a+3)(2a+1)$	**b** $(2x+1)(x+2)$	**c** $(3m+2)(m+5)$
d $(y+3)(4y+1)$	**e** $(2x+1)(2x+3)$	**f** $(3n+2)(2n+1)$
g $(2x+3)(4x+3)$	**h** $(5t+2)(2t+3)$	**i** $(2x-2)(5x-1)$
j $(8p+1)(3p-2)$	**k** $(5m-2)(2m-5)$	**l** $(3q+1)(7q-2)$
m $(3x+2)(6x-2)$	**n** $(2n+3)(2n-3)$	**o** $(8y-1)(8y+1)$
p $(3k-2)(5k-3)$	**q** $(7p-1)(7p-1)$	**r** $(3x-1)(5x-3)$
s $(5x+4)(5x+4)$	**t** $(9y-4)(3y+2)$	**u** $(5p+2)(p-7)$
v $(10q-1)(q-10)$	**w** $(4a+3)(3a+4)$	**x** $(7p+5)(7p-5)$

4 Expand and simplify:

a $(3 + x)(4 + x)$ **b** $(5 - a)(2 - a)$ **c** $(7 + m)(1 - m)$
d $(3 - n)(3 + n)$ **e** $(4 + y)(y + 5)$ **f** $(x - 7)(5 - x)$
g $(9 + k)(k + 10)$ **h** $(2a + 1)(3 + a)$ **i** $(3n + 1)(7 - 2n)$
j $(x + y)(x + 2y)$ **k** $(2n + m)(n + 2m)$ **l** $(a - b)(2a + 3b)$
m $(2p - q)(2p + q)$ **n** $(3x + y)(2x - 5y)$ **o** $(3a + 2b)(2a + 3b)$
p $(9w - 5x)(9w - 5x)$

3:07 Special products

3:07A Perfect squares

PREP QUIZ 3:07A

Simplify:

1 4^2 **2** 7^2 **3** $(-2)^2$ **4** $(-10)^2$ **5** $(3x)^2$

Complete:

6 $(x + 7)(x + 7) = x^2 + 14x + \ldots$ **7** $(a - 3)(a - 3) = a^2 - 6a + \ldots$
8 $(2m - 1)(2m - 1) = \ldots m^2 - 4m + 1$ **9** $(n + 5)(n + 5) = n^2 + \ldots n + 25$
10 $(x - 3)(x - 3) = x^2 - \ldots x + 9$

When a binomial is multiplied by itself, we call this product a perfect square. If a perfect square is expanded, we get:

$$\begin{aligned}(x + y)^2 &= (x + y)(x + y)\\ &= x(x + y) + y(x + y)\\ &= x^2 + xy + yx + y^2\\ &= x^2 + 2xy + y^2\end{aligned}$$

Similarly: $(x - y)^2 = x^2 - 2xy + y^2$

The square of a binomial is equal to the square of the first term, plus twice the product of the two terms, plus the square of the second term.

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$

WORKED EXAMPLES

1 $(a+3)^2 = a^2 + 2[3a] + 3^2$
$= a^2 + 6a + 9$

first term squared — product of the two terms — second term squared

2 $(m-5)^2 = m^2 - 2[5m] + 5^2$
$= m^2 - 10m + 25$

first term squared — twice the product of the two terms — second term squared

3 $(3y-7)^2 = [3y]^2 - 2[21y] + [-7]^2$
$= 9y^2 - 42y + 49$

INVESTIGATION 3:07 THE SQUARE OF A BINOMIAL

The Prep quiz on the previous page suggests that there might be a pattern formed when a binomial is squared.

Copy and complete this table.

x	y	x^2	y^2	xy	$(x+y)^2$	$x^2 + 2xy + y^2$	$(x-y)^2$	$x^2 - 2xy + y^2$
5	3							
6	1							
10	4							

What are your findings?

Exercise 3:07A

1 Find the missing term in each example to make the statement true.

a $(x+2)^2 = x^2 + 4x + \ldots$
b $(a+6)^2 = a^2 + 12a + \ldots$
c $(y-3)^2 = y^2 - 6y + \ldots$
d $(m-10)^2 = m^2 - 20m + \ldots$
e $(x+1)^2 = x^2 + \ldots + 1$
f $(y+7)^2 = y^2 + \ldots + 49$
g $(n-2)^2 = n^2 - \ldots + 4$
h $(p-5)^2 = p^2 - \ldots + 25$
i $(q+8)^2 = \ldots + 16q + 64$
j $(x-4)^2 = \ldots - 8x + 16$
k $(x + \ldots)^2 = x^2 + 6x + 9$
l $(a + \ldots)^2 = a^2 + 18a + 81$
m $(y - \ldots)^2 = y^2 - 14x + 49$
n $(m - \ldots)^2 = m^2 - 22m + 121$
o $(2x+3)^2 = \ldots + 12x + 9$
p $(5n+1)^2 = \ldots + 10n + 1$
q $(3m+7)^2 = 9m^2 + \ldots + 49$
r $(4x+5)^2 = 16x^2 + \ldots + 25$
s $(2a-1)^2 = 4a^2 - \ldots + 1$
t $(9y-7)^2 = 81y^2 - \ldots + 49$

2 Expand these perfect squares and simplify.

a $(x+3)^2$	**b** $(x+5)^2$	**c** $(x+1)^2$
d $(x-6)^2$	**e** $(m-1)^2$	**f** $(n-5)^2$
g $(x+2)^2$	**h** $(n-8)^2$	**i** $(m+11)^2$
j $(a+12)^2$	**k** $(x+10)^2$	**l** $(p-9)^2$
m $(x+y)^2$	**n** $(a+m)^2$	**o** $(x+t)^2$
p $(a-b)^2$	**q** $(k-m)^2$	**r** $(p-q)^2$

3 Expand and simplify:

a $(2x+3)^2$	**b** $(2x+1)^2$	**c** $(3x+5)^2$
d $(4a+1)^2$	**e** $(3a+7)^2$	**f** $(7t+2)^2$
g $(2x-1)^2$	**h** $(3a-2)^2$	**i** $(5m-4)^2$
j $(4t-7)^2$	**k** $(6q-1)^2$	**l** $(9n+4)^2$
m $(2x+y)^2$	**n** $(a+3b)^2$	**o** $(3t-2x)^2$

3:07B Difference of two squares

PREP QUIZ 3:07B

Evaluate:

1 7^2-3^2	**2** $(7+3)(7-3)$
3 4^2-2^2	**4** $(4+2)(4-2)$
5 5^2-1^2	**6** $(5-1)(5+1)$
7 6^2-3^2	**8** $(6-3)(6+3)$
9 10^2-9^2	**10** $(10+9)(10-9)$

If the sum of two terms is multiplied by their difference, another special type of product is formed. If $(x+y)$ is multiplied by $(x-y)$ we get:

$$\begin{aligned}(x+y)(x-y) &= x(x-y)+y(x-y)\\ &= x^2-xy+yx-y^2\\ &= x^2-y^2\end{aligned}$$

The sum of two terms multiplied by their difference is equal to the square of the first term minus the square of the second term.

$$(x+y)(x-y)=x^2-y^2$$

WORKED EXAMPLES

1 $(x + 3)(x - 3) = x^2 - 3^2$

first term squared; second term squared

$= x^2 - 9$

2 $(2a - 3b)(2a + 3b) = (2a)^2 - (3b)^2$

first term squared; second term squared

$= 4a^2 - 9b^2$

3 $(p - 7)(p + 7) = p^2 - 7^2$

$= p^2 - 49$

4 $(5x + y)(5x - y) = (5x)^2 - y^2$

$= 25x^2 - y^2$

Exercise 3:07B

1 Expand these products and simplify.

a $(x + 4)(x - 4)$	**b** $(a + 1)(a - 1)$	**c** $(m + 2)(m - 2)$	**d** $(n + 7)(n - 7)$
e $(p - 5)(p + 5)$	**f** $(q - 6)(q + 6)$	**g** $(x - 3)(x + 3)$	**h** $(y - 9)(y + 9)$
i $(10 + x)(10 - x)$	**j** $(5 + a)(5 - a)$	**k** $(8 - x)(8 + x)$	**l** $(11 - m)(11 + m)$
m $(x + t)(x - t)$	**n** $(a - b)(a + b)$	**o** $(m + n)(m - n)$	**p** $(p - q)(p + q)$

2 Express as the difference of two squares.

a $(2a + 1)(2a - 1)$	**b** $(3x + 2)(3x - 2)$	**c** $(5m + 3)(5m - 3)$
d $(9q + 2)(9q - 2)$	**e** $(4t - 3)(4t + 3)$	**f** $(7x - 1)(7x + 1)$
g $(8n - 5)(8n + 5)$	**h** $(10x - 3)(10x + 3)$	**i** $(2x + y)(2x - y)$
j $(4a + 3b)(4a - 3b)$	**k** $(5p + 2q)(5p - 2q)$	**l** $(3m - n)(3m + n)$
m $(2m - 5n)(2m + 5n)$	**n** $(2p - 3q)(2p + 3q)$	**o** $(x - 5y)(x + 5y)$
p $(12x - 5y)(12x + 5y)$		

3:08 Miscellaneous examples

- It is important that you are able to expand and simplify algebraic expressions readily and accurately, if you are to use algebra in later problem-solving exercises.
- Work through the following miscellaneous questions after examining the following two examples.

WORKED EXAMPLES

1 $(x + 3)^2 - (x - 1)(x + 2) = [x^2 + 6x + 9] - [x^2 + x - 2]$

$= x^2 + 6x + 9 - x^2 - x + 2$

$= 5x + 11$

2 $(3x + 5)(x - 1) + (x + 2)^2 - (2x + 1)(2x - 1) = [3x^2 + 2x - 5] + [x^2 + 4x + 4] - [4x^2 - 1]$

$= 3x^2 + 2x - 5 + x^2 + 4x + 4 - 4x^2 + 1$

$= 6x$

Exercise 3:08

Expand and simplify, where possible, each of the following expressions.

1

a $5x + 3(x - 7)$	**b** $(x + 2)(x - 1)$	**c** $(2x + 1)(x - 1)$
d $5(x + 2) - x(x + 1)$	**e** $(3x - 1)^2$	**f** $(x + 5)(x - 5)$
g $(2x - 7)(3x - 1)$	**h** $(5x - 1)(5x + 1)$	**i** $4x + 7 + x(x + 2)$
j $9x - (x + 5) + 5$	**k** $(x + 10)(x - 3)$	**l** $(9 - y)(9 + y)$
m $3x(x - 5) - 2x^2$	**n** $3(x + 2)(x + 1)$	**o** $(x + y)^2$
p $(x + 2y)(2x + y)$	**q** $5x - 2(x + y) + 2y$	**r** $(a + 2b)(a - 2b)$
s $a(x + 2) - x(a + 2)$	**t** $(3a + 7)(5a - 3)$	**u** $(2m - 5n)^2$
v $(1 - 5y)(1 + 5y)$	**w** $3x - 7(x - 3)$	**x** $(9x - 8y)(9x + 8y)$

2

a $(x + 1)^2 + 5(x + 2)$	**b** $(a - 3)^2 - 3(a + 1)$
c $(x + 2)(x + 3) - 7(x - 2)$	**d** $8(x + 2) + (x - 7)(x + 1)$
e $(x + 3)^2 + (x + 1)(x + 2)$	**f** $(a + 5)(a + 3) - (a + 4)^2$
g $(m + 6)^2 - (m - 1)(m + 1)$	**h** $(y + 7)(y - 7) - (y + 7)^2$
i $(x + 2)^2 + (x + 1)^2$	**j** $(a + 3)^2 - (a + 2)^2$
k $(x + 1)(x + 2) + (x + 2)(x + 3)$	**l** $(a + 1)(a - 2) + (a + 2)(a - 1)$
m $(x + 3)(x - 1) - (x + 2)(x - 5)$	**n** $(y + 7)(y - 2) - (y + 1)(y + 3)$
o $(2x + 1)^2 - 5(x + 3)$	**p** $2x(x + 5) + (x + 7)^2$
q $(5x + 1)(x - 3) + (2x + 1)^2$	**r** $(2x + 1)(3x + 1) - (2x - 1)(3x - 1)$
s $(p + 3)(p - 3) - (q + 3)(q - 3)$	**t** $(x + y)^2 - (x - y)(x + y)$
u $(a + b)(a + 2b) + (a + b)^2$	**v** $(m - n)^2 + (m + n)^2$
w $3(x + 1)^2 + 5(x + 1)$	**x** $2(x - 1)(x + 1) + 3(x + 1)^2$
y $(2x + 3y)^2 - (2x - 3y)(2x + 3y)$	**z** $(3a + 2b)(2a + 3b) - 6(a + b)^2$

3

a $(x + 1)^2 + (x + 2)^2 + (x + 3)^2$

b $(x + 1)(x + 2) + (x + 2)(x + 3) + (x + 3)(x + 4)$

c $(a - 1)(a + 1) + (a + 1)^2 + (a - 1)^2$

d $(x + 2)^2 + (x + 3)^2 - (x + 2)(x + 3)$

e $(3a + 2b)(2a + 3b) + (3a - 2b)(3a + 2b) + (2a + 3b)(2a - 3b)$

f $(4x + 1)(3x - 1) + (x + 2)^2 - (x - 3)(x + 3)$

g $5(m - 5)^2 - 8(m - 4)^2 + 3(m - 3)^2$

h $(3x + 2y)(3x - 2y) - (2x + y)(2x - y) - (x + 1)(x - 1)$

i $(x + 3y)^2 - (2x + 2y)^2 + (3x + y)^2$

j $2(x - y)(x + y) - (x + y)^2 - (x - y)^2$

CHALLENGE 3:08 PATTERNS IN PRODUCTS

The examples below involve the sum of a series of products. Can you see the patterns involved and, hence, find the simplest expression for each sum?

1 $(x + 1)^2 + (x + 2)^2 \ldots + (x + 9)^2 + (x + 10)^2$

2 $(x + 1)(x + 2) + (x + 2)(x + 3) + \ldots + (x + 9)(x + 10)$

3 $(a - 5)^2 + (a - 4)^2 + \ldots + a^2 + \ldots + (a + 4)^2 + (a + 5)^2$

4 $(5m - n)(5m + n) + (4m - 2n)(4m + 2n) + (3m - 3n)(3m + 3n)$
$+ (2m - 4n)\ (2m + 4n) + (m - 5n)(m + 5n)$

Perfect squares

Example

Using $(a \pm b)^2 = a^2 \pm 2ab + b^2$, evaluate $(103)^2$.

Solution

Writing 103 as $(100 + 3)$

Then $103^2 = (100 + 3)^2$

$= 100^2 + 2 \times 100 \times 3 + 3^2$

$= 10\,000 + 600 + 9$

$= 10\,609$

Similarly, the square of a number like 98 could be found by writing 98 as $(100 - 2)$.

Exercise

1 Following the example above, evaluate:

a 101^2 **b** 205^2 **c** 1004^2 **d** 72^2

e 98^2 **f** 199^2 **g** 995^2 **h** 67^2

Difference of two squares

Example

Using $(a - b)(a + b) = a^2 - b^2$, evaluate $100^2 - 97^2$.

Solution

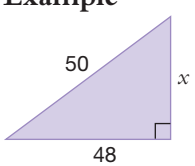
$100^2 - 97^2 = (100 - 97)(100 + 97)$

$= 3 \times 197$

$= 591$

This method can be useful when finding a shorter side of a right-angled triangle.

Example



Solution

$x^2 = 50^2 - 48^2$

$= (50 - 48)(50 + 48)$

$= 2 \times 98$

$= 196$

$\therefore x = \sqrt{196}$

$= 14$

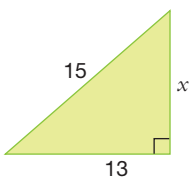
Exercise

1 Evaluate:

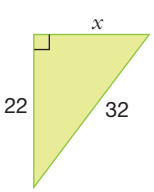
a $100^2 - 98^2$ **b** $73^2 - 67^2$ **c** $145^2 - 140^2$ **d** $651^2 - 641^2$

2 Use the method above to find the value of x for each triangle. (Leave your answer in surd form.)

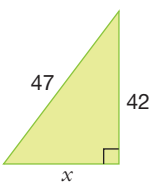
a



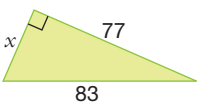
b



c



d



MATHS TERMS 3

algebra
- a branch of mathematics where numbers are represented by symbols

algebraic expression
- a group of terms and numbers that are joined by addition or subtraction signs

binomial
- an algebraic expression consisting of two terms,
  e.g. $2x + 4, 3x - 2y$

brackets
- the name given to these grouping symbols: [ ]

cancel
- to simplify a fraction by dividing the numerator and denominator by any common factor,
  e.g. $\frac{\overset{7}{\cancel{21} \div 3}}{\underset{10}{\cancel{30} \div 3}}$ so $\frac{21}{30} = \frac{7}{10}$

collect like terms
- to simplify an algebraic expression containing many terms by addition and/or subtraction,
  e.g. $5x + 3 + 7x - 4$
  $= 12x - 1$

difference of two squares
- the result of multiplying two binomials which are the sum and difference of the same terms,
  e.g. $(a + 3)(a - 3) = a^2 - 3^2$
  $= a^2 - 9$

denominator
- the bottom number of a fraction

expand
- to remove grouping symbols by multiplying each term inside the grouping symbols by the term outside

factorise
- to write an expression as a product
- the reverse of expanding

like terms
- terms that have identical pronumeral parts, e.g. $7x$ and $10x$, $5a^2b$ and $-3a^2b$

numerator
- the top number of a fraction

parentheses
- the name given to these grouping symbols: ( )

perfect square
- when a binomial is multiplied by itself, e.g. $(x + 5)^2$ or $(2a - 3b)^2$

pronumeral
- a symbol, usually a letter, used to represent a number

substitution
- the replacing of a pronumeral with a numeral in an expression,
  e.g. to substitute 3 for a in the expression $4a - 2$ would give:
  $4(3) - 2 = 10$

A machine counts coins by weight. What is the value of a pile of $M coins that weighs W grams if each coin weighs w grams?

DIAGNOSTIC TEST 3 ALGEBRA

Each part of this test has similar items that test a certain type of example.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Simplify: — 3:01

a $5a + 2b - 3a + b$ b $5p^2 + 2p - 3p^2$

c $6ab - 4ba$ d $6a - 2x + 5 + x - 2a - 7$

2 Simplify: — 3:01

a $8 \times 7m$ b $5a \times 6b$ c $10y \times y$ d $-4n \times 2y$

3 Simplify: — 3:01

a $6a \div 2$ b $15xy \div 3x$ c $12ac \div 8ab$ d $-6x \div 18xy$

4 Simplify: — 3:02A

a $\frac{3x}{5} + \frac{2x}{5}$ b $\frac{x}{3} - \frac{x}{2}$ c $\frac{4a}{5} - \frac{a}{3}$ d $\frac{5m}{8} + \frac{m}{2}$

5 Simplify: — 3:02B

a $\frac{3}{4} \times \frac{n}{3}$ b $\frac{2}{a} \times \frac{5}{b}$ c $\frac{5}{x} \times \frac{x}{10}$ d $\frac{3b}{2} \times \frac{4}{5b}$

6 Simplify completely: — 3:04B

a $\frac{3m}{2} \div \frac{1}{4}$ b $\frac{x}{3} \div \frac{x}{6}$ c $\frac{8a}{3b} \div \frac{2a}{9b}$ d $\frac{ab}{2} \div \frac{b}{5}$

7 Expand: — 3:03

a $9(x + 7)$ b $6(5a - 2)$ c $p(p + 3)$ d $3a(5 - 2a)$

8 Expand: — 3:03

a $-3(x + 2)$ b $-2(m - 8)$ c $-5(3x + 4)$ d $-(7 - 2m)$

9 Expand and simplify: — 3:03

a $x(x - 1) - x^2$ b $7n - 4 + 3(n - 1)$ c $2a(a + b) - a(3a - 4b)$

10 Simplify: — 3:04

a $\frac{x+4}{2} + \frac{x+3}{5}$ b $\frac{2a-5}{4} + \frac{3a+8}{6}$ c $\frac{3n+1}{4} - \frac{n-2}{3}$

11 Factorise completely: — 3:05

a $5m + 10$ b $x^2 - 3x$ c $6ab + 15a$ d $-8y - 12$

12 Expand and simplify: — 3:06

a $(x + 3)(x + 4)$ b $(a - 3)(2a - 1)$ c $(2 - y)(3 + y)$ d $(2x + y)(x - 3y)$

13 Expand and simplify: — 3:07A

a $(x + 2)^2$ b $(a - 7)^2$ c $(2y + 5)^2$ d $(m - n)^2$

14 Expand and simplify: — 3:07B

a $(x + 3)(x - 3)$ b $(y - 7)(y + 7)$ c $(2a + 5)(2a - 5)$ d $(x + y)(x - y)$

ASSIGNMENT 3A Chapter review

1 Simplify the following.

a $6a + a$
b $6x \times 3x$
c $a - 5a$
d $x^2 + x^2$
e $18x \div 3x$
f $12y \div 8$
g $2x + 3y$
h $3ab \times 2b$
i $12a^2b \div 6a$
j $5ab + 7ab$
k $6a^2 - a$
l $4x - 3y - 5x$
m $12 + 6x + 7 - x$
n $6x + 2x \times 3$
o $x^2 - 3x + 2x + 3x^2$
p $12x - 6x \div 3$

2 Simplify:

a $\frac{x}{2} + \frac{x}{3}$
b $\frac{2a}{5} - \frac{a}{10}$
c $\frac{3a}{2} \times \frac{5b}{6}$
d $\frac{10y}{3} \div 5y$
e $\frac{7x}{5} - \frac{x}{3}$
f $\frac{3m}{5} + \frac{m}{3} - \frac{m}{2}$
g $\frac{6n}{5} \times \frac{10}{7n} \div \frac{3}{2n}$
h $\frac{x+3}{2} + \frac{x+1}{3}$
i $\frac{2a-1}{5} + \frac{3a-2}{10}$
j $\frac{2n-1}{6} - \frac{2-n}{9}$

3 Factorise fully:

a $3a + 15$
b $6m + 9$
c $15 - 5y$
d $ax - 3x$
e $2x + 6xy$
f $4x^2 - 2x$
g $9ab - 6bc$
h $6x^2 - 9x + 3xy$

4 Expand and simplify:

a $x(x - 2)$
b $x - 2(x - 2)$
c $(x - 2)(x - 2)$
d $(x - 2)(x + 2)$
e $(x + 2)^2$
f $(2 - x)^2$

5 Expand and simplify where possible.

a $(x - 1)(x + 2)$
b $5x + 3(x - 1)$
c $2(x + 3) - 2x - 3$
d $(2x + 1)(x - 7)$
e $(x + 5)(x - 5)$
f $(3x + 2)^2$
g $x(x - 3) + 2(x + 1)$
h $(2 - x)(3 - x)$
i $(x + y)(y - x)$
j $(2x - y)^2$
k $5[x + 3(x + 1)]$
l $[3x - (x - 2)]^2$

If the width of one figure is x, what is the width of:

a 2 figures
b 4 figures
c half a figure?

ASSIGNMENT 3B Working mathematically

1 Refer to **ID Card 4** on page xxi to identify the mathematical terms numbered:

a 1 **b** 2 **c** 3
d 4 **e** 5 **f** 6
g 7 **h** 8 **i** 9
j 11

2 **a** What geometric shape has inspired the design of these coffee cups?

b What would you estimate the capacity of the cup to be?

3 Diane and Garry married and had three children. Each child married and had three children. Assuming that no one has died, how many people are now in this extended family altogether?

4 The numerals 1 to 10 are written on ten separate cards, one on each card.

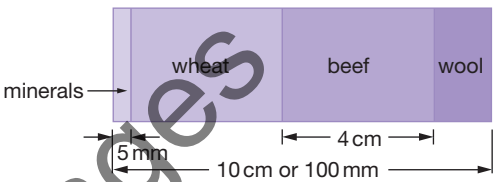
a How many pairs of cards have a sum of 10?

b How many groups of three cards are there that have a sum of 18?

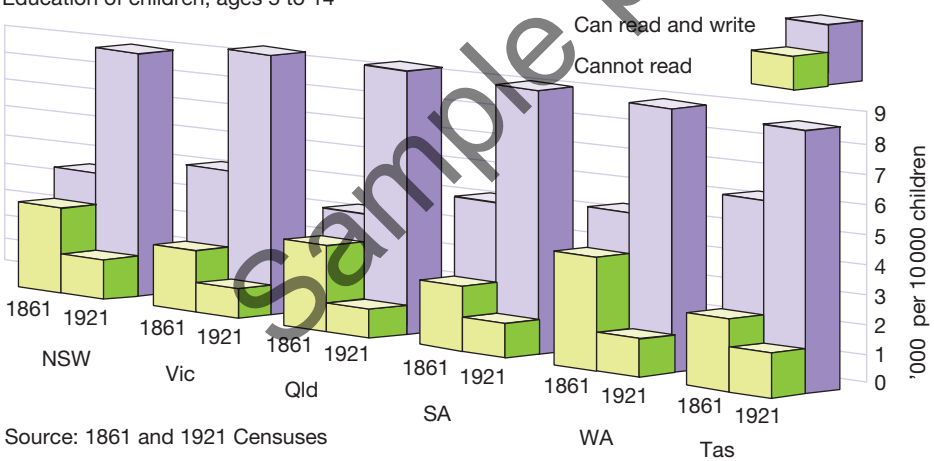
5 A particular country's exports are shown in the bar graph below (reduced in size). Find what percentage of the country's exports are taken up by:

a beef

b minerals.



6 Education of children, ages 5 to 14



Source: 1861 and 1921 Censuses

a In 1861, which state had the greatest number per 10 000 children who could read and write? What percentage was this?

b In 1921, which state had the greatest percentage of children who could read and write? What percentage was this?

c Which state had 4000 per 10 000 children who could read and write in 1861? About how many in that state could not read in 1861?

d Consider Western Australia in 1861. Approximately what percentage could read and write? Approximately what percentage could not read? (To determine this, measure the height of this column and measure this height on the scale.)

ASSIGNMENT 3C Cumulative revision

1 **a** Increase a wage of $900 by 6·5%. 1:01, 2:01

b In 1980 the population of a town was 42 000. By 2010 the population had decreased by 3·5%. What was the population of the town at that time?

c Which is larger?

i 15% or 20% **ii** $\frac{5}{12}$ or $\frac{2}{5}$ **iii** 0·65 or 0·0655

2 Change the following to decimals. 1:01

a $\frac{7}{8}$ **b** 6·8% **c** $\frac{5}{12}$

3 State the number of significant figures in the following measurements. 1:05

a 8·8 L **b** 123·45 m **c** 6 km **d** 4·0°C

4 In what range would each of the measurements in Question **3** lie? 1:10

5 Estimate the answers to the following calculations. 1:06, 1:07

a $\dfrac{9{\cdot}89 \times 3{\cdot}123}{5{\cdot}089}$ **b** $4{\cdot}95^2 + 2{\cdot}13 \times 5{\cdot}237$

c $\dfrac{212{\cdot}3}{\sqrt{104{\cdot}2}}$ **d** $\dfrac{48{\cdot}24 \times 0{\cdot}888}{(12{\cdot}5 - 7{\cdot}056)^2}$

6 Convert the following. 1:09

a 3·2 ML to L **b** 45 GW to W

c 3TB to MB **d** 15 MHz to kHz

7 Car A uses petrol at the rate of 4·8 L/100 km, whereas car B uses it at a rate of 10 L/100 km. What will the difference in petrol costs be in a year in which both cars travel 30 000 km, if the petrol costs on average $1.45/L? 1:04, 2:01

8 **a** A greengrocer buys 20 cases of oranges at a cost of $15 per case. Each case contains 10 kg of oranges. If he sells the oranges at $4/kg, how many kilograms must he sell before he makes a profit? If he sells all the oranges what will be his profit? 2:01

b Concrete is made by mixing volumes of cement, sand and gravel in the ratio 1 : 4 : 5. Jim calculates that he needs 2 m^3 of concrete to finish a job. How many cubic metres of sand and gravel does he need to order to make the concrete?

9 What is the last digit of the number 2^{2014}? 2:02

10 How many pairs of parallel edges are there in a rectangular prism? 2:02