3 PROPERTIES OF GEOMETRICAL FIGURES 1

Student name: ....

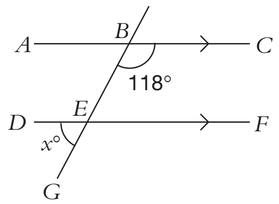
Class: .... Date: ....

3:01A Deductive reasoning in numerical exercises: Exercises using parallel lines

This exercise on angles on lines, at points, in triangles and in quadrilaterals involves two or more steps of reasoning.

Example:

Find the value of x in the diagram. Give reasons.

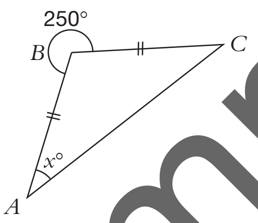


Answer:

$\angle ABE = 62°$ (adj. supp. ∠'s)
$\angle DEG = 62°$ (corresp. ∠'s, || lines)
$\therefore\ x = 62$

Example:

Find the value of x in the diagram. Give reasons.

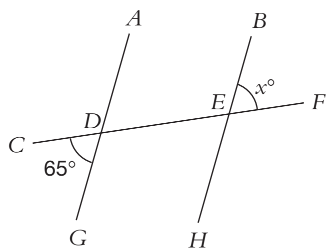


Answer:

$\angle ABC = 110°$ (∠'s at a point)
$\angle BAC = \angle ACB = 35°$ (base ∠'s, isos. Δ)
$\therefore\ x = 35$

1 Find the value of x in each diagram. Give reasons for each step of your reasoning.

a



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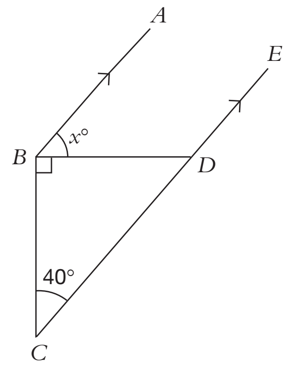

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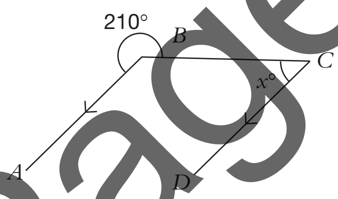

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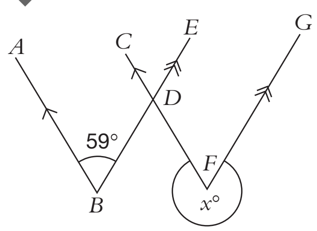

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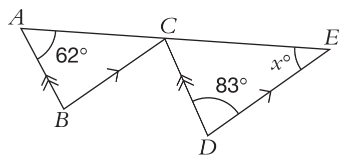

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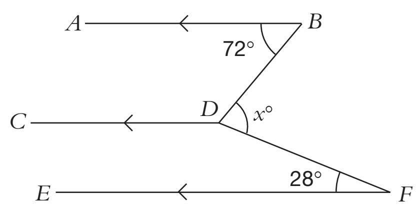
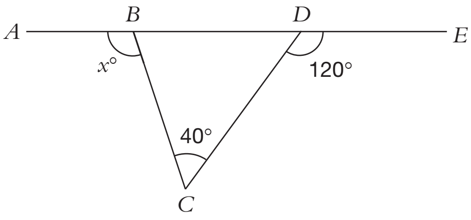


3:01B Exercises using triangles

2 Find the value of x in each diagram. Give reasons for each step of your reasoning.

a



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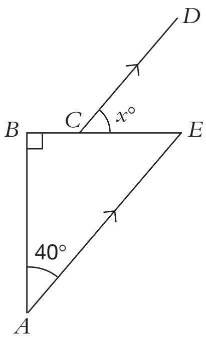

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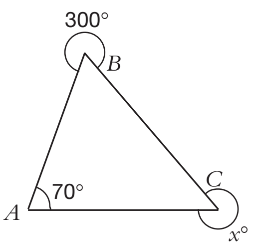

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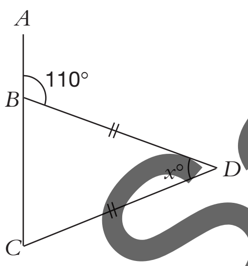

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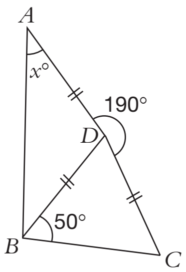

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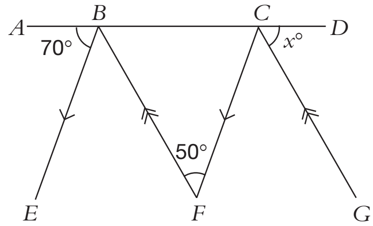

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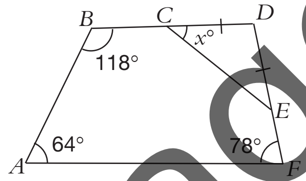
3:01C Exercises using quadrilaterals

3 Find the value of x, and y (if present), in each diagram. Give reasons for each step of your working.

a



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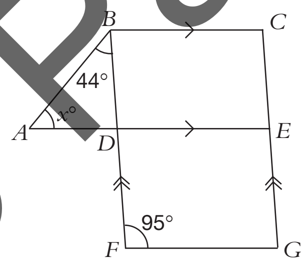

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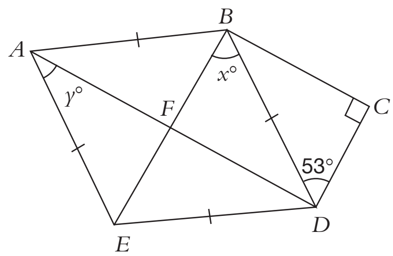
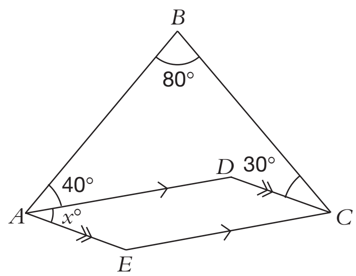


(*BCDF* is a rectangle.)

d



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Result /32

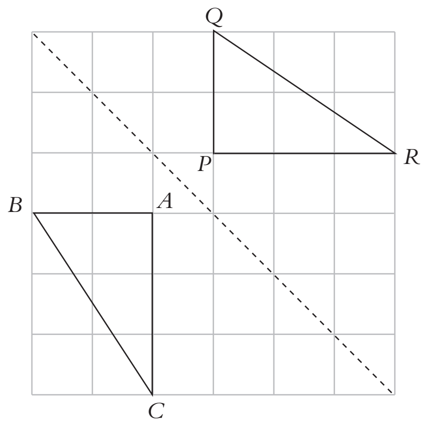
Student name: ....

Class: .... Date: ....

3:02 Congruent triangle proofs

Congruent shapes have the same shape and size.

In transformations such as reflection, rotation and translation the image is always congruent to the original figure.



Triangles *ABC* and *PQR* are congruent. Each one is a reflection of the other in the dashed mirror line.

In a pair of congruent triangles, matching sides and angles are equal.

$$\angle B = \angle Q$$
$$AC = PR$$

When checking whether a pair of triangles are congruent you have to decide whether the given information implies the triangles must be congruent. Some pairs of triangles can have equal sides or angles but do not have to be congruent.

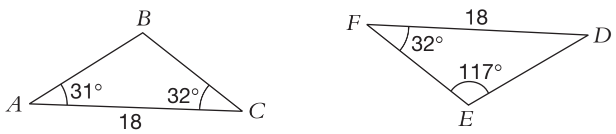
We can check whether triangles are congruent by comparing their measurements and applying one of four tests.

- **SSS** side, side, side (matching sides equal)
- **SAS** side, angle, side
- **ASA** angle, side, angle
- **RHS** right-angle, hypotenuse, side

Note that these 'three-letter' tests apply to matching sides and angles; so, in ASA for example, the two given angles must be on either side of the given side.

Example:

Explain why triangles *ABC* and *DEF* are congruent.



Answer:

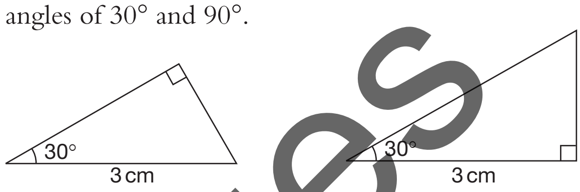
$\angle ABC = 117°$, $\angle DEF = 117°$

$AC = DF$

$\angle ACB = 32°$, $\angle DFE = 32°$

This fits the test ASA, so $\Delta ABC \equiv \Delta DEF$.

1 These two triangles each have a side measuring 3 cm and angles of 30° and 90°.



a Are these two triangles congruent? ____________

b Do the two triangles have three pairs of equal angles? Explain. ____________

____

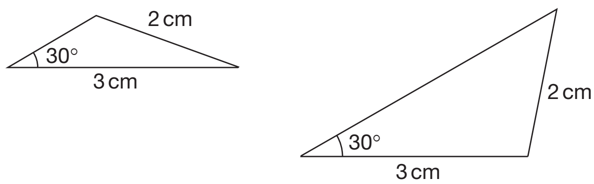
____

c If the three angles in a pair of triangles are the same, do the two triangles have to be congruent? ____________

____

4

2 These two triangles each have sides of 2 cm and 3 cm, and an angle of 30°. They have been drawn to scale.



a Use a ruler to measure the third side on each triangle, and a protractor to measure the two angles that are not 30° in each triangle. Write the measurements in this table.

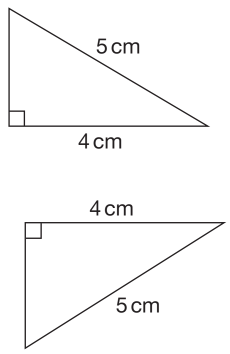
	Angles	Sides
Triangle 1	30°, ______ ______	3 cm, 2 cm, ______
Triangle 2	30°, ______ ______	3 cm, 2 cm, ______

b Are the two triangles congruent? ____________

7

3 The following pairs of triangles are congruent. Write the three letter test that establishes the congruence.

a



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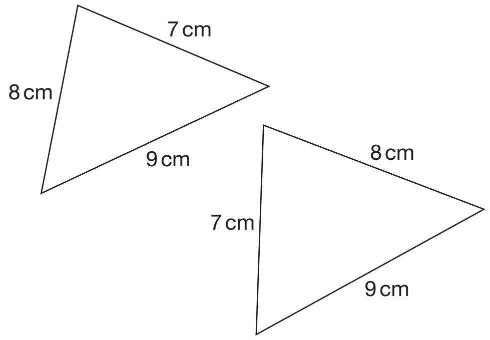

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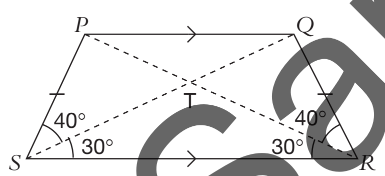


a Explain why ΔPQS is congruent to ΔQPR.

____

____

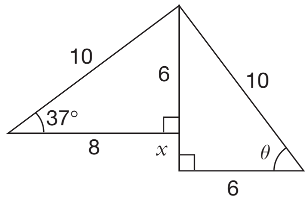
____

____

b Which triangle is congruent to ΔQTR? ______________

3

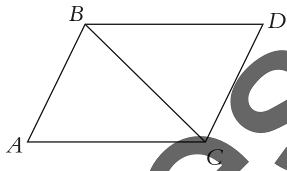
5 There are two congruent triangles in this diagram. Write the values of x and θ.



$x =$ ______________ $\theta =$ ______________

2

6 $ABCD$ is a parallelogram. Prove that $\Delta ABC \equiv \Delta DCB$.



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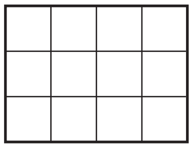
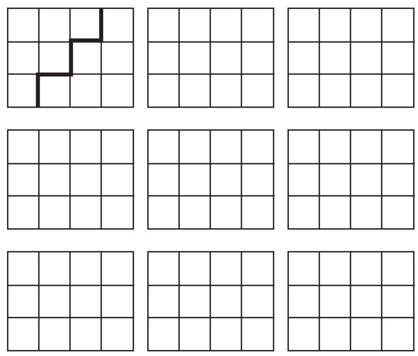
FUN SPOT PATHFINDER

There are nine different ways that this shape can be cut in half so that both pieces are congruent and each one is made up of six small squares.



One pathway is shown. Draw in different pathways on each of the eight shapes below. Use a pencil because you might need to try this several times before getting it right!



2

Result /23

3 PROPERTIES OF GEOMETRICAL FIGURES 3

Student name: ....

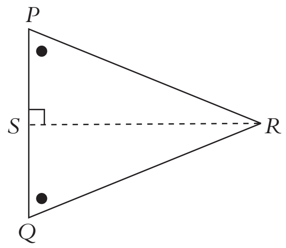
Class: .... Date: ....

3:03 Deducing properties of the special triangles and quadrilaterals

Geometric definitions and properties of triangles can be used to prove various geometric facts.

Example:

An isosceles triangle can be defined as one that has a pair of equal angles. Use this definition to prove it must have a pair of equal sides.



Answer:

Data: ΔPQR is isosceles with $\angle PQR = \angle QPR$.

Aim: To prove that $PR = QR$

Construction: Draw RS perpendicular to PQ, meeting PQ at S.

Proof:

In Δs PSR and QSR:

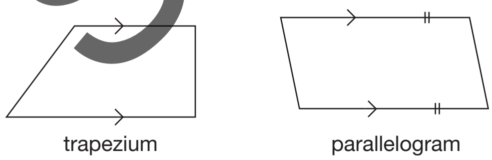
- $RS = RS$ (common to both)
- $\angle PSR = \angle QSR$ ($RS \perp PQ$)
- $\angle SPR = \angle SQR$ (data)

$\therefore$ $\Delta PSR \equiv \Delta QSR$ (ASA)
$\therefore$ $PR = QR$ (matching sides of congruent Δs)

Definitions of quadrilaterals are minimum definitions.

Example:

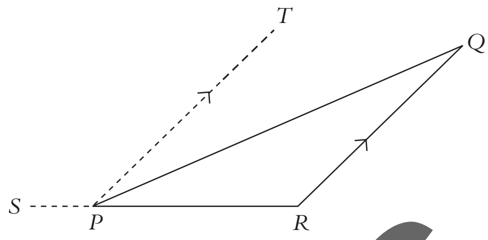
A parallelogram is a trapezium with the parallel sides equal in length.



Definitions are not unique. Here are two other ways of defining a parallelogram:

- A parallelogram is a quadrilateral with rotational symmetry.
- A parallelogram is a quadrilateral with both pairs of opposite sides equal.

1 Prove that the sum of the interior angles of a triangle is 180°. Complete the proof started below.



Data: ΔPQR is any triangle with angles $\angle P$, $\angle Q$ and ______.

Aim: To prove ______ + ______ + ______ = ______

Construction: Extend RP to S.

Draw PT ____________ to ______.

Proof: $\angle QPT = \angle Q$, (________________________________)

$\angle SPT =$ ______ (________________________________)

$\angle QPR =$ ______ (data)

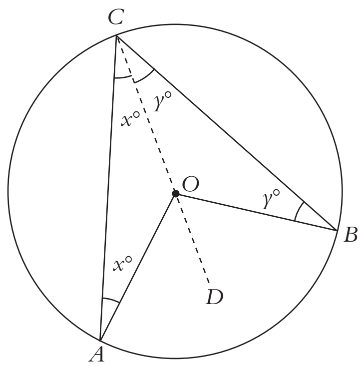
$\angle QPT + \angle SPT + \angle QPR =$ ______ (________________________)

$\therefore$ ______ + ______ + ______ = ______

$\therefore$ The angle sum of a triangle is ______

9

2



The diagram shows an arc AB and angles drawn in from the arc to the centre $\angle AOB$ and to the circumference $\angle ACB$.

a Explain why the two angles marked x are equal.

____

____

b Explain why the two angles marked y are equal.

____

____

c $\angle AOD$ is the exterior angle of ΔAOC. Write $\angle AOD$ in terms of x.

____

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d Explain why $\angle DOB = 2y$.

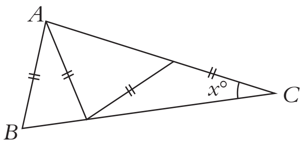
e Write $\angle AOB$ in terms of x and y.

f Explain why $\angle AOB$ is twice the size of $\angle ACB$.

g Describe in your own words the relationship between an angle at the centre of a circle and an angle at the circumference when both are drawn from the same arc.

7

3 Calculate the size of angle x. The large triangle ABC is isosceles, with $AC = BC$.



2

4 Write the names of the quadrilaterals with these definitions.

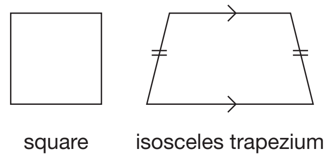
a a rhombus with one angle a right-angle

b a quadrilateral with one diagonal as an axis of symmetry

c a parallelogram with adjacent angles equal

3

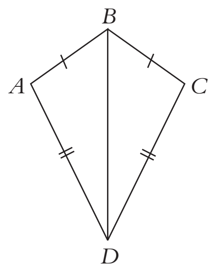
5 Complete this table for a square and an isosceles trapezium. Write 'yes', 'no' or 'one pair' in each space.



Property	Square	Isosceles trapezium
Opposite angles equal		
Opposite sides parallel		
Opposite sides same length		
Diagonals bisect each other		
Diagonals same length		
Diagonals perpendicular		

12

6 A kite is defined to be a quadrilateral with two pairs of equal adjacent sides. Use congruent triangles to prove that a kite has a pair of equal opposite angles.



3

7 True or false? 'The axis of symmetry of a kite is always the longer of the two diagonals.'

If your answer is false, draw a kite where the shorter of the two diagonals is the axis of symmetry.

2

Result /38

Student name: ....

Class: .... Date: ....

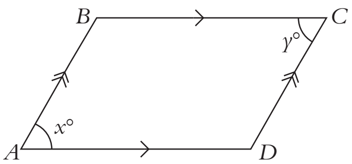
3:04 Proving general results

In some geometrical problems you use pronumerals to represent the size of angles. You can then use geometric reasons and algebraic skills to 'prove' relationships between angles.

Example:

ABCD is a parallelogram. Prove that $x = y$.

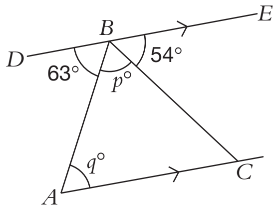
Note: This gives proof that the opposite angles of a parallelogram are equal.



Answer:

$x + \angle ABC = 180°$ (coint. ∠s, || lines)
$x = 180° - \angle ABC$
$y + \angle ABC = 180°$ (coint. ∠s, || lines)
$y = 180° - \angle ABC$
∴ $x = y$ (both equal to $180° - \angle ABC$)

1



The diagram shows two parallel lines *DE* and *AC*.
$\angle ABD = 63°$, and $\angle CBE = 54°$.
Prove that ΔABC is isosceles.
Complete the steps in this proof:

$q = 63°$ (reason: ______________)

$p =$ __________° (reason: ∠'s on line)

∴ $p = q$

∴ $\angle ABC = \angle BCA$

∴ $BC =$ ____

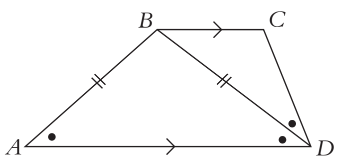
∴ ΔABC is isosceles.

3

2

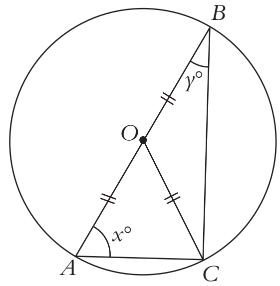
- $AB = BD$
- $BC \parallel AD$
- *BD* bisects $\angle ADC$.

Prove that $\angle ABD = \angle BCD$.



3

3 *AB* is a diameter.
Prove that
$\angle OAC + \angle OBC = 90°$.

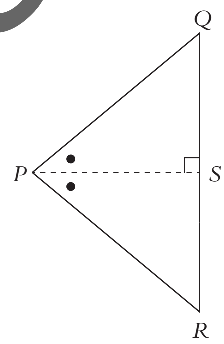


3

4 ΔPQR has the following properties:

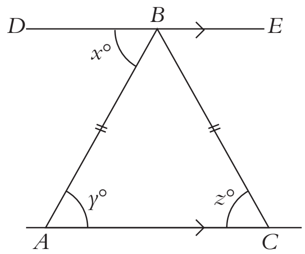
- *PS* bisects $\angle QPR$
- $PS \perp QR$

Use congruent triangles to prove that $PQ = PR$.



3

5 ΔABC is isosceles, with $AB = BC$, and $AC \parallel DE$.
Prove that the angles marked x and z are equal. Refer to angle y in your proof.



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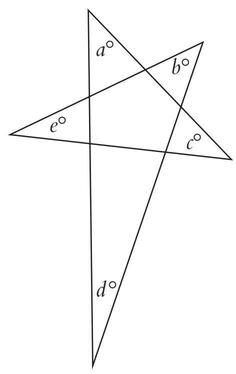
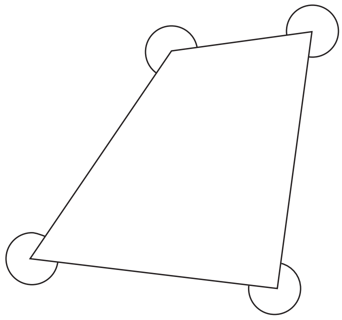
6 The diagram shows an irregular 5-pointed star. The angles at the five points are a, b, c, d and e.
Prove that $a + b + c + d + e = 180$.



3

INVESTIGATION BOUNDARY ANGLES OF POLYGONS

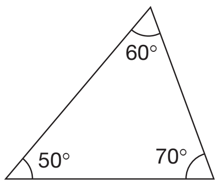
At any vertex of a polygon, there is an interior angle and a 'boundary' angle. The interior angle and the boundary angle form angles at a point, and their sum is 360°.



Don't confuse an exterior angle of a polygon with the 'boundary' angles referred to here!

This investigation is about finding the rule for the sum of the boundary angles in any polygon. The rule should be written in terms of n, the number of sides of the polygon.

1 Write the three boundary angles for this triangle, and their sum.



____

____

____

2 What is the size of each boundary angle of a square? What is the sum of the four boundary angles of a square?

____

____

____

3 True or false? Boundary angles of a quadrilateral are always reflex angles.
If true, explain why. If false, draw a quadrilateral that has a reflex boundary angle.

____

____

4 Copy and complete this table for regular polygons.

Regular polygon	**Number of sides**	**Each interior angle**	**Each boundary angle**	**Sum of boundary angles**
Equilateral triangle	3	60°		
Square	4			
Pentagon	5			1260°
Hexagon	6			
Octagon	8			
Decagon	10			
n-gon	n	$\frac{(n-2)\times 180°}{n}$		

5 Is the sum of the boundary angles for a regular polygon with n sides the same as the sum of the boundary angles of a polygon with n sides which is not regular?

____

____

6 Show how the boundary angle formula can be derived from the formula for the interior angles of a polygon with n sides.

6

Result /24

Student name: ....

Class: .... Date: ....

3:05 Definitions and proofs in deductive geometry

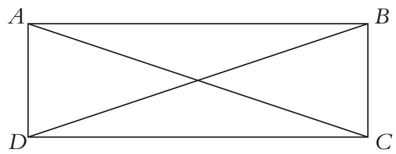
We can use deductive geometry to prove various properties of triangles, quadrilateral and other polygons.

Example:

A rectangle can be defined as a quadrilateral that has two pairs of opposite sides equal and with all angles being right angles.

Prove that the diagonals of a rectangle are equal in length.

Answer:



Consider ΔABC and ΔDCB

$AB = DC$ (opp sides of rectangle are equal)
$BC = CB$ (same length)
$\angle ABC = \angle DCB$ (both = 90°)

Therefore ΔABC congruent to ΔDCB

Therefore $AC = BD$

Here is a check list for properties that some of the special quadrilaterals have and others do not:

Sides:
- Is there a pair of adjacent sides equal?
- Are the opposite sides equal?
- Are the opposite sides parallel?

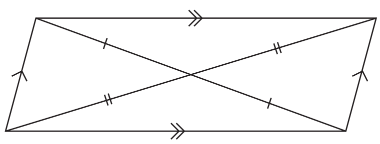
Angles:
- Are adjacent angles equal?
- Are opposite angles equal?

Diagonals:
- Are the diagonals equal in length?
- Does each diagonal bisect the other?
- Are the diagonals perpendicular?
- Do the diagonals bisect the angles?

The only quadrilateral that has all of the above properties is the square.

Here is the check-list answered for a parallelogram:

Example:



Sides:
- Pairs of adjacent sides not equal
- Opposite sides equal and parallel

Angles:
- Pairs of adjacent angles not equal
- Opposite angles equal

Diagonals:
- Diagonals are not equal
- Diagonals bisect each other
- Diagonals are not perpendicular
- Diagonals do not bisect the angles

1 Complete the table by placing a tick in the box to represent a correct property:

Figure	Diagonals are perpendicular	At least one diagonal bisects the other	One or both pairs of opposite angles are equal
Rhombus			
Rectangle			
Isosceles trapezium			
Kite			

4

2 Here is a list of properties that some quadrilaterals have. For each property write all the quadrilaterals that have that property, using quadrilaterals from this group: {rectangle, rhombus, parallelogram, kite, isosceles trapezium}

a diagonals intersect at right-angles

____

b *one* pair only of parallel sides

____

c *one* pair only of opposite angles are equal

____

d both pairs of opposite angles are equal

____

e a diagonal bisects *one* pair only of opposite angles

____

f both pairs of opposite sides are equal

____

g diagonals are the same length

____

7

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3 My diagonals bisect each other at right angles, but are not equal in length. What is my name?

______________________________ 1

4 My diagonals bisect each other and are the same length, but are not perpendicular. What is my name?

______________________________ 1

5 My diagonals are the same length, but do not bisect each other and are not perpendicular. What is my name?

______________________________ 1

6 My diagonals bisect each other but are not the same length and are not perpendicular. What is my name?

______________________________ 1

7 A quadrilateral has all the properties of a rhombus and all the properties of a rectangle. What must it be?

______________________________ 1

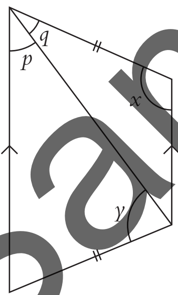
8 Which quadrilateral has both diagonals as axes of symmetry?

______________________________ 1

9 Which quadrilateral has two pairs of adjacent angles equal?

______________________________ 1

10 The diagram shows an isosceles trapezium.



a Write an expression for angle x in terms of angles p and q.

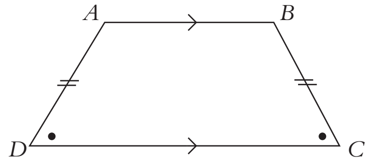
____

b Write an expression for angle y in terms of angles p and q.

______________________________ 2

11 An isosceles trapezium $ABCD$ has these properties:

- One pair of parallel sides: $AB \parallel CD$.
- The other pair of sides are the same length: $AD = BC$.
- The base angles are equal: $\angle ADC = \angle BCD$.

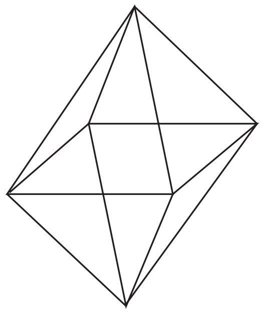


Use congruent triangles to prove the diagonals of an isosceles trapezium are equal in length.

3

FUN SPOT — OCTAHEDRONS

An octahedron is a three-dimensional solid with eight congruent faces.



1 Which geometric shape are the eight faces?

____

2 An octahedron is 'traversable', meaning it is possible to trace a path along all the edges without going over any edge twice. Show how, by labelling a starting point and numbering the edges, 1, 2, 3 etc. to show a possible path.

2

Result /25

Student name: ....

Class: .... Date: ....

3:06 Polygons

A polygon is a closed figure made up of straight sides.

Polygons are given special names when you know the number of sides.

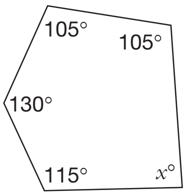
Number of sides	Name of polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
8	octagon
10	decagon

The angle sum of a polygon is given by the formula:

$$(n - 2) \times 180°$$

Example:

Calculate the size of the angle marked x.

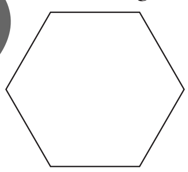


The shape is a pentagon (five sides).

Angle sum = $(5 - 2) \times 180° = 3 \times 180° = 540°$

$$x + (115 + 130 + 105 + 105) = 540$$
$$x + 455 = 540$$
$$x = 540 - 455$$
$$x = 85$$

1 This diagram shows a six-sided figure.

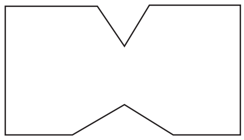


a Add lines to the diagram to show how to divide it into triangles. The triangles should share a common point and not overlap.

b How many triangles are there? ____________

2

2 a Name this polygon.



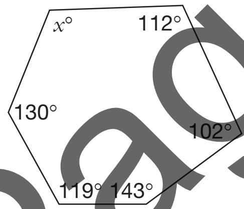
____

b Calculate the sum of the interior angles.

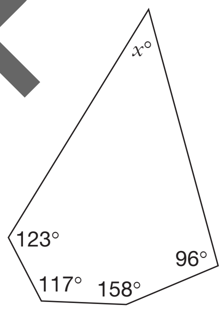
2

3 Work out the sizes of the marked angles.

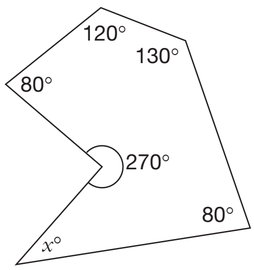
a



b

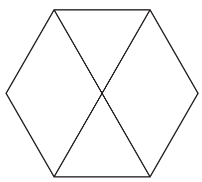


c

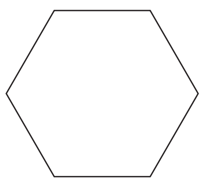


3

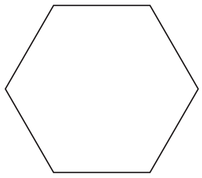
4 The diagram shows that a regular hexagon can be split up into two equilateral triangles and two rhombuses. Draw diagrams to show each of the following ways of splitting up a hexagon.



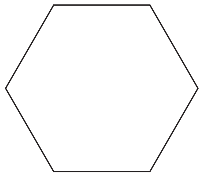
a two trapeziums



b a rectangle and two triangles



c a kite and two isosceles triangles



3

A polygon is **regular** if:
- each side is the same length
- each angle is the same size.

To calculate the size of each interior and exterior angle in a regular polygon, divide the sum by the number of sides.

Example:

Calculate the size of each exterior and interior angle in a regular octagon (eight sides).

Answer:

Exterior: $\frac{360}{8} = 45°$

Interior: $\frac{(8-2)\times 180}{8} = \frac{1080}{8} = 135°$

5 A rectangle has four equal angles. Explain why it is not regular.

____

1

6 Complete this table for exterior and interior angles of regular polygons.

Name	Number of sides	Each exterior angle	Each interior angle
Triangle			
	4		90°
Pentagon			
Hexagon			
12-gon	12		

5

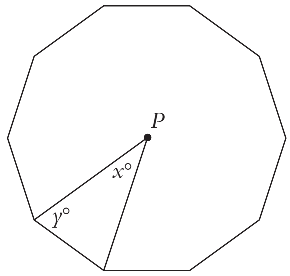
7 A regular polygon has 18 sides. Calculate the size of:

a each exterior angle

b each interior angle.

2

8 Calculate the sizes of angles x and y in this regular decagon (10 sides). P is the centre point.



2

9 The formula for the sum of the interior angles of a regular polygon (n sides) is $(n - 2) \times 180°$. Write a formula for the size of each interior angle.

Interior angle = _______________

1

10 Each exterior angle in a regular polygon is 15°.

a What is the size of each interior angle? _______________

b How many sides does the polygon have? _______________

2

11 Draw a pentagon with five equal sides which is not regular.

1

12 Explain whether it is possible for a regular polygon to have an interior angle of 165°.

____

____

1

INVESTIGATION THE SWISS CROSS

A tessellation is a tiling pattern in which identical shapes fit together perfectly with no gaps. Some polygons tessellate.

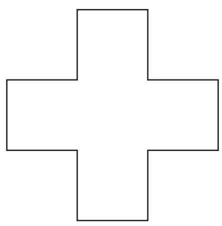
The shape shown is called a Swiss cross.

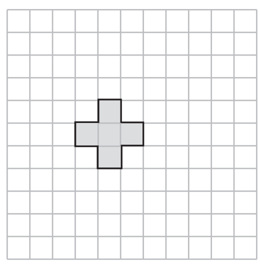
1 How many sides does it have? _______________

2 Is the Swiss cross a regular polygon? Explain.

____

3 Add several more Swiss crosses to this grid to show that it tessellates.





3

Result /28