Answers

Exercise 3:04

3 **a** 2 **b** 7 **c** 9, 15 **d** 15
e $6 = 3 + 3, 8 = 5 + 3, 10 = 5 + 5, 12 = 5 + 7, 14 = 7 + 7, 16 = 5 + 11, 18 = 7 + 11$

4 **a** $3 \times 2 \times 2$ **b** $2 \times 2 \times 5$
c $2 \times 3 \times 3$ **d** $2 \times 2 \times 2 \times 2$

5 **a** $2 \times 3 \times 5$ **b** $2 \times 2 \times 3 \times 3$
c $2 \times 2 \times 5 \times 5$ **d** $2 \times 2 \times 2 \times 2 \times 3 \times 3$
e $2 \times 2 \times 2 \times 2 \times 2 \times 3$
f $3 \times 3 \times 5 \times 5$

6 **a** $2^3 \times 3^3$ **b** $3^3 \times 5^2$
c $3 \times 5 \times 7^2$ **d** $2^3 \times 3^2 \times 7$
e $3^2 \times 5^3$ **f** $2^4 \times 7 \times 11$
g $3^3 \times 7^2$ **h** $3^3 \times 5^3$

Extension

Goldbach's Conjecture

A conjecture is a mathematical opinion that is thought to be true but has not yet been proven. Question 3e introduces students to Goldbach's Conjecture, which states:

'*Every even integer greater than 2 can be expressed as the sum of two primes.*'

As an extension activity, students can test this theory for numbers larger than 24. Students may also consider why this does not apply for all odd integers. The reason is that prime numbers, except for the number 2, are odd. The sum of two odd numbers is even.

Which odd numbers can be expressed as the sum of two primes?

Is there a quick way of finding the odd numbers that can be expressed as the sum of two primes? Explain.

[Communicating, Reasoning]

Perfect numbers

Question 6 states that 6 is known as a perfect number because its factors, except for itself, add up to 6.

e.g. The factors of 6 are 1, 2, 3 and 6.

$1 + 2 + 3 = 6$

What is the next perfect number?

Answer: 28

$1 + 2 + 4 + 7 + 14 = 28$

[Problem Solving]

Digital resources

eBook
- Foundation worksheet 3:05 Divisibility tests

3 **a** Write all of the even prime numbers.
b How many odd prime numbers are less than 20?
c List all the odd composite numbers that are less than 20.
d How many composite numbers are between 20 and 40?
e Write each of the even numbers from 6 to 24 as the sum of two prime numbers, e.g. $12 = 5 + 7$.
(The mathematician Christian Goldbach suggested that every even number greater than 4 is the sum of two odd prime numbers. This is thought to be true, but it has never been proved.)

4 Complete these factor trees.

a 12 → 3, 4
12 = … × … × …

b 20 → 4
20 = … × … × …

c 18
18 = … × … × …

d 16 → 4
16 = … × … × … × …

5 Use factor trees to write these numbers as products of their prime factors.
a 30 **b** 36 **c** 100 **d** 144 **e** 96 **f** 225

6 Another way to write a number as a product of its prime factors is to continue to divide by prime factors (starting from the smaller ones) until you get 1 as a result. Use this method (as shown at right) to write these numbers as products of prime factors:
a 216 **b** 675 **c** 735 **d** 504
e 1125 **f** 1232 **g** 1323 **h** 3375

Example: 360

2)360
2)180
2) 90
3) 45
3) 15
5) 5
1

$360 = 2^3 \times 3^2 \times 5$

6 is called a *perfect number* because the sum of all of its factors (other than itself) is equal to 6, i.e. $1 + 2 + 3 = 6$.

3:05 Divisibility tests

PREP QUIZ 3:05

1 List all the factors of 40.
2 Write the next number after 60 that is divisible by 10.
3 Write the next three multiples of 6 after 12.
4 What is the largest even number that is less than 1000?
5 What is the next odd number after 999?
6 In this number sentence at right, which number is the quotient? $255 \div 5 = 51$
7 Which is the divisor? 8 Which is the dividend?
9 318 813 ÷ 11 10 102 102 102 ÷ 9

3:05 Content statements

Investigate index notation and represent whole numbers as products of powers of prime numbers (**ACMNA149**)

- determine and apply tests of divisibility for 2, 3, 4, 5, 6 and 10

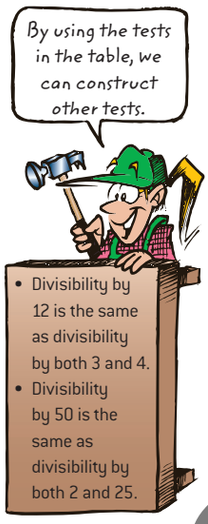
Answers

PREP QUIZ 3:05

1 1, 2, 4, 5, 8, 10, 20, 40
2 70 **3** 18, 24, 30 **4** 998
5 1001 **6** 51 **7** 5
8 255 **9** 28 983
10 11 344 678

If 4 divides a number and leaves no remainder, then the number is divisible by 4.
Here are some simple divisibility tests you should know.

Divisor	Divisibility test	Example
2	The number must be even, i.e. it must end in 0, 2, 4, 6 or 8.	4136 is divisible by 2 because it is even.
3	The sum of the digits is divisible by 3.	30 012 is divisible by 3 because (3 + 0 + 0 + 1 + 2) is divisible by 3.
4	The number formed by the last two digits must be divisible by 4.	76 112 is divisible by 4 because 12 is divisible by 4.
5	The last digit must be 5 or 0.	11 225 is divisible by 5 because it ends with a 5.
6	The number must be divisible by both 2 and 3.	40 002 is even and its digit sum is divisible by 3.
8	The number formed by the last three digits is divisible by 8.	963 216 is divisible by 8 because 216 is divisible by 8.
9	The sum of the digits is divisible by 9.	142 128 is divisible by 9 because (1 + 4 + 2 + 1 + 2 + 8) is divisible by 9.
10	The last digit must be 0.	814 710 is divisible by 10 because it ends with 0.
11	The sum of the digits in odd-numbered places is equal to the sum of the digits in even-numbered places, or will differ by a multiple of 11.	7 081 426 is divisible by 11 because (7 + 8 + 4 + 6) − (0 + 1 + 2) is 22, which is divisible by 11.
25	The last two digits must be 00, 25, 50 or 75.	80 925 is divisible by 25 because it ends in 25.
100	The last two digits must be 00.	81 700 is divisible by 100 because it ends in 00.



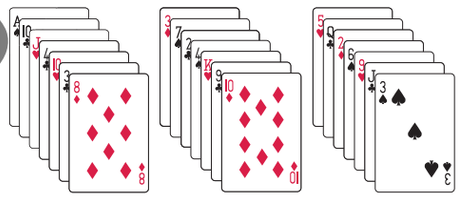
Lesson starter

Card trick

You will need: 21 playing cards

Start the lesson with a card trick that uses prime numbers.

1. Ask one student to select a card from a pack and, without showing you, place it back in the pack.
2. Shuffle the cards and lay them out in 3 groups of 7. Ask the student which pile his or her card is in.
3. Pick up the cards, making sure that the pile containing the student's card is the middle pile you pick up. Do not shuffle again.
4. Now, place the cards in 3 groups of 7 again, laying them down 1, 1, 1, 2, 2, 2, … Ask the student which pile his or her card is in. Again, pick this pile up so that it is the middle pile in your hand.



5. Flip the cards over one at a time, counting 1, 2, 3, 4, etc. as you flip them. The student's card will be the 11th card turned over.

It is a good idea to bring a number of packs of playing cards to the class as students will want to try the trick for themselves. Have students explore how this trick works.

Teaching strategies

Teach divisibility tests one at a time

The divisibility tests are often easier to understand if they are approached one at a time. Start with the prior knowledge of divisibility test for 2. Students will be aware of even and odd numbers. Explicitly teach that an even number is a number that can divide into 2 without a remainder.

The divisibility for 3 is taught next, and then 6 (as this requires divisibility by 2 and 3).

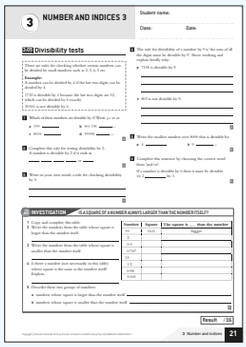
Teacher's notes

Answers

Exercise 3:05

1 3842, 5816, 9000, 8774, 8166, 7008
2 3006, 7110, 21 441, 8145, 211 002, 78
3 1004, 6124, 8156, 61 852
4 30 024, 81 810, 41 238, 765, 936
5 6633, 10 406, 92 180, 61 809
6 **a** 4, 5, 8, 11 **b** 11 **c** 4, 11
d 3, 5, 9, 11 **e** 3, 11
7 **a** 1078, 7600, 13 476
b 13 476, 33 885 **c** 7600, 13 476 **d** 13 476
e 33 885 **f** 7600 **g** 1078
h 7600 **i** 7600 **j** 7600
8 Test for divisibility of both 4 and 6.
9 **a** 3 and 5 **b** 2 and 9 **c** 3 and 4
10 **a** 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720
b 1, 2, 173, 346
c 1, 2, 3, 5, 6, 9, 10, 15, 18, 27, 30, 45, 54, 90, 135, 270
d 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 28, 30, 35, 36, 42, 45, 60, 63, 70, 84, 90, 105, 126, 140, 180, 210, 252, 315, 420, 630, 1260
e 1, 3, 7, 9, 21, 27, 63, 81, 189, 567
f 1, 3, 5, 7, 9, 15, 21, 25, 35, 45, 63, 75, 105, 175, 225, 315, 525, 1575
g 1, 3, 9, 27, 81, 243, 729
h 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 15, 16, 20, 22, 24, 30, 32, 33, 40, 44, 48, 55, 60, 66, 80, 88, 96, 110, 120, 132, 160, 165, 176, 220, 240, 264, 330, 352, 440, 480, 528, 660, 880, 1056, 1320, 1760, 2640, 5280
11 **a** 1002 **b** 1008 **c** 1008
d 1001 **e** 1025 **f** 1100
g 1005 **h** 1008
12 **a** 990 **b** 162 **c** 900
d 1, 3, 5, 7, 15, 21, 25, 35, 75, 105, 175, 525
e 1, 3, 9, 11, 27, 33, 99, 297

Homework 3:05


3 NUMBER AND INDICES 3

Student name:

Class: Date:

Divisibility tests

INVESTIGATION 4 A SQUARE OF A NUMBER ALWAYS LARGER THAN THE NUMBER ITSELF?

Result /25

3 Number and indices 21

Exercise 3:05

P Foundation worksheet 3:05 Divisibility tests

1 Which of these numbers are divisible by 2?
571 3842 5816 2221 887 9000 374 555 8774 8166 7008

2 Which of these numbers are divisible by 3?
7114 830 3006 7110 21 441 30 031 8145 60 001 211 002 78

3 Which of these numbers are divisible by 4?
1004 67 814 7118 2222 6124 8156 98 61 852 934

4 Which of these numbers are divisible by 9?
9994 31 024 30 024 81 810 41 238 3333 727 411 765 936

5 Which of these numbers are divisible by 11?
4115 8003 6633 7228 860 74 186 10 406 92 180 999 61 809

6 Find which of the numbers 3, 4, 5, 6, 8, 9 and 11 will exactly divide:
a 440 **b** 3883 **c** 3916 **d** 1485 **e** 3993

7 Which of the numbers 1078, 7600, 13 476 and 33 885 are divisible by the following?
a 2 **b** 3 **c** 4 **d** 6 **e** 9
f 10 **g** 11 **h** 25 **i** 100 **j** 50

8 What divisibility test could we use for 24?

9 The test for divisibility by 6 is that the number must be divisible by both 2 and 3.
a What divisibility test could we use for 15?
b What divisibility test could we use for 18?
c What divisibility test could we use for 12?

10 Use divisibility tests where possible to list all of the factors of:
a 720 **b** 346 **c** 270 **d** 1260
e 567 **f** 1575 **g** 729 **h** 5280

11 Find the smallest number that is greater than 1000 and also:
a divisible by 3 **b** divisible by 8
c divisible by 9 **d** divisible by 11
e a multiple of 25 **f** a multiple of 100
g a multiple of 15 **h** a multiple of 12

12 **a** Find the lowest common multiple of 2, 5, 9, 10 and 11.
b Find the highest common factor of 810, 324 and 1944.
c Find the lowest common multiple of 2, 3, 4, 5, 9 and 100.
d List all factors of $3 \times 5^2 \times 7$. (There are twelve.)
e List all factors of $3^3 \times 11$.

Teacher's notes

3:06 HCF and LCM by prime factors

PREP QUIZ 3:06

1 List all factors of 30.
2 List all factors of 54.
3 What is the highest common factor of 30 and 54?
4 List all multiples of 4 that are less than 40.
5 List all multiples of 3 that are less than 40.
6 What is the lowest common multiple of 4 and 3?
7 Use a factor tree to write 48 as a product of its prime factors.

Write the following products of prime numbers using index notation.
8 $2 \times 2 \times 2 \times 2 \times 2$
9 $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
10 $3 \times 3 \times 5 \times 5 \times 5 \times 7$

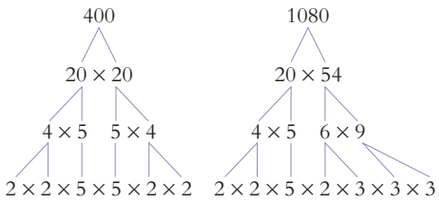
If we express numbers as products of their prime factors, it is sometimes easier to find their highest common factor (HCF) and their lowest common multiple (LCM).

WORKED EXAMPLE 1

Find the highest common factor of 400 and 1080.

Highest Common Factors (HCF) can also be called Greatest Common Divisors (GCD).

Solution



From the factor trees:

Number	Product of prime factors
400	$2 \times 2 \times 2 \times 2 \times 5 \times 5$ or $2^4 \times 5^2$
1080	$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$ or $2^3 \times 3^3 \times 5$

The two numbers have $2 \times 2 \times 2 \times 5$ in common. $2 \times 2 \times 2 \times 5 = 2^3 \times 5 = 40$, so the highest common factor of 400 and 1080 is 40.

Teacher's notes

3:06 Content statements

Investigate index notation and represent whole numbers as products of powers of prime numbers (**ACMNA149**)

- express a number as a product of its prime factors, using index notation where appropriate

Answers

PREP QUIZ 3:06

1 1, 2, 3, 5, 6, 10, 15, 30
2 1, 2, 3, 6, 9, 18, 27, 54
3 6
4 4, 8, 12, 16, 20, 24, 28, 32, 36
5 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39
6 12
7 $2 \times 2 \times 2 \times 2 \times 3$
8 2^5
9 $2^3 \times 3^4$
10 $3^2 \times 5^3 \times 7$

Lesson starter

What is special about the number 2520?

1 Use the divisibility tests from Exercise 3:05 and a calculator to find the first ten factors of 2520.

2520

Answer: 2520 can be divided evenly by all numbers from 1 to 10.

2 Multiply two of the factors of 2520. Is the result a factor of 2520?

Answer: Yes. For example, $6 \times 7 = 42$, so 42 is a factor of 2520.

3 State another number that has the same first ten factors as 2520.

Answer: 5040. This is double of 2520. Another number is 3 628 800 as this is the product of the numbers 1 to 10.

Digital resources

ProductLink
- Finding factors (Excel)